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International spot power trade: agreements between countries and the impact on the optimal design of a power system

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Resumen

Esta tesis analiza tres problemas vinculados con el comercio internacional de energía eléctrica. La primera parte trata el problema de negociación de precios en el comercio spot internacional entre dos países, en el contexto institucional de América del Sur, donde las transacciones tienen lugar entre operadores de dos sistemas eléctricos, y el principal problema económico es el reparto de los beneficios del comercio entre los países que comercian. Se analiza formalmente la ineficiencia de la negociación caso a caso ante cada oportunidad de comercio, y las condiciones de sostenibilidad de acuerdos de largo plazo para fijar precios. La segunda parte presenta algunas de las reglas de fijación de precios del comercio usadas en la región en las transacciones bilaterales que se han realizado hasta el presente y, dado que se requerirán acuerdos multilaterales en el futuro, estudia diferentes formas de definir transacciones y sus precios, cuando más de dos países comercian. Finalmente la tercera parte estudia el efecto de variaciones en los precios del comercio internacional de electricidad sobre el diseño óptimo de un sistema de generación, e investiga condiciones bajo las cuales las importaciones son complementarias o bien sustitutivas de la instalación de capacidad de generación eólica.

Abstract

This thesis analyzes three economic problems of international power trade. The first part addresses the problem of the negotiation of prices in international power spot trade, in the institutional context prevailing for such trade in South America, where the systems' operators of two countries decide the transactions and the main economic problem is the share of the benefits of trade between the countries. The analysis explains formally the inefficiencies of case by case bargaining, and finds conditions to be fulfilled by long run price setting rules to be stable. The second part reviews different price setting rules used in the past in the region for bilateral transactions between countries, and as efficient trade will require multilateral agreements, studies different methods to define transactions and energy prices, when more than two countries participate in trade. Finally the third part studies the effect of variations in the prices of international power trade on the optimal design of a country's generation system, and investigates conditions leading to imports to be a complement or a substitute of wind capacity.

Contents

1 GENERAL INTRODUCTION

In the last twenty years structural and regulatory reforms in many countries led to the emergence of competitive electricity markets of national scope: spot markets for short term and contract markets for long term supply. Transmission and distribution grids allow third party access. Apparently electricity has become another commodity. However this simplified view cannot be sustained when the technical singularities and the complexity of sectorial regulation to make those markets work are taken into account.

The electric system requires an instantaneous balance between demand and supply, which can only be achieved by a centralized coordination of generation and transmission operation. Therefore regulation must ensure this coordination and determines the way the load curve is supplied at every moment, and a system operator must perform the function of load dispatch.

The generation capacity to ensure short term reliability is a public good. The development of long term supply contracts is burdened with high transaction costs and uncertainty, and in many countries there is a reasonable concert that efficient markets for those contracts will not spontaneously emerge. These two market failures and the technical reasons discussed above justify that with no exception countries regulate power generation.

International power trade can only take place through interconnections, which require long construction periods and heavy investments. In most countries, interconnection capacity with neighboring countries is a small fraction of local generation capacity (Uruguay is one of the few exceptions), and every country relies mainly or exclusively on its own resources to supply its demand.

The need for regulation in national spot markets makes regulation in international spot power trade also a necessity.

In South America the main form of international power trade is currently spot trade, performed by system operators in the framework of bilateral agreements between countries. Spot trade consists in transactions decided day by day involving energy surpluses of the selling country, without a long term commitment or any obligation for the selling country to have generation capacity to supply the buyer permanently.

Therefore some of the problems of economic interest that arise in the study of international power trade are different depending on the institutional and regulatory context. This thesis analyzes three problems related to international power trade in the specific situation of the Mercosur region, and regarding the technical singularities of the Uruguayan power system.

1.1 The modeling of bargain in spot power trade

The first part of this thesis addresses the problem of the negotiation of prices between countries in international spot power trade.

In some regions of the world, as the European Union, regulations for international spot power trade aim at creating a single spot market unifying the national markets of the countries involved in trade. The main issue in this case is to ensure equal rights and non-discriminatory participation in trade in an open market for firms from every interconnected country. Among the theoretical economic problems that arise, perhaps the most important is the analysis of the effect on market power of a limited capacity interconnection between two markets

By contrast, in South America and specifically in the Mercosur countries, regulation of international spot power trade has given place to bilateral trade regimes, where countries explicitly determine prices for the energy traded between them. In this case it is essential to find long run agreements with rules to determine prices, to avoid a case by case bilateral bargain, each time a new trade opportunity appears. International spot trade in our region then raises the problem of the formal analysis of bargaining between countries to determine trade prices. The first part of this thesis is in this line of work.

Electricity production costs in each trading country and therefore the direction of the trade and its potential benefit are random, as a consequence of the randomness of hydraulic and wind generation, the availability of thermal plants, and in the long run of the unpredictable cycles of under and overinvestment in power generation capacity. Consequently the model of bargain we develop in the first part of this thesis is based on the existence of a sequence of trade opportunities with random duration and benefits.

The objectives of our analysis are: i) explaining formally the inefficiencies of case by case bargaining in spot power international trade when the direction of the trade flow and its potential benefits are random, and ii) finding conditions to be fulfilled by long run price setting rules, to be preferable to case by case bargaining.

1.2 The problem of design of multilateral agreements for international spot trade

In our region international spot power trade has been until now the subject of bilateral agreements.

Sometimes the multilateral nature of trade has hindered bilateral transactions: the requirement by Brazil of Argentinean energy in 2005 affected the Argentinean supply to Uruguay; the lack of Argentinean agreement to grid access prevented Paraguayan energy sales to Uruguay and Chile.

In 2015 a 500 kV new interconnection between Uruguay and Brazil will be completed, raising the total interconnection capacity to 570 MW. Argentine and Uruguay are linked by 500 kV lines with 2000 MW capacity. Argentine and Brazil have a 2000 MW link. These interconnection capacities can be compared with the 1200 MW average Uruguayan demand, to perceive the importance of power trade to Uruguay: the whole local demand could potentially be supplied by imports.

This triple strong interconnection requires a multilateral agreement for power trade. The mere superposition of independent bilateral agreements between countries could induce inefficiency or even worse leave indetermination in the transactions.

There is ample literature about energy integration and international power trade pursuing the ideal of a single energy market, where generators and consumers could trade without any country based discrimination. Besides, there are few analysis of spot trade in the South American institutional framework, where countries have to settle agreements to determine energy prices in trade, and the transactions are decided by power system operators.

The second part of this thesis aims at describing different methods to define transactions and energy prices for them, when more than two countries negotiate how to divide the benefits of energy trade, as in the case of the Mercosur countries. The transactions to be defined must implement the optimal power flows between countries, in the sense of producing a Pareto optimal allocation.

Using the fact that the graph formed by optimal flows through the interconnections has no cycles, different sets of economically meaningful and consistent transactions are defined, resulting in those energy flows, with different criteria to solve the problems of: i) power transits through third countries and the intermediation in energy and ii) the split of the benefits of trade between countries in these transactions.

The retribution for the use of grids is beyond the scope of this work. Although it is a subject of economic importance, it has a second order impact when compared with the prices of energy in trade.

1.3 The problem of generation portfolio design

The optimal design of a generation system is a problem of practical importance since in many countries, even with competitive markets for generation, the authorities conduct planning processes to shape the power system. The goal is to find the amount of capacity to be installed for every available kind of generation unit, assuming that capacity will be used optimally.

In Uruguay for both institutional and technical reasons, the expansion of the power generation system is strongly influenced by a process of centralized planning. In the last few years the national energy policy has set two main goals for generation planning: a massive expansion of wind generation capacity, and the construction of a GNL regasification plant.

The country´s energy policy has determined the expansion in wind capacity to reach 25% of energy supply from wind energy by 2017 (DNE, 2013). This goal has led to a series of competitive biddings performed by the state owned public utility UTE, to award long term purchase contracts to private generators, and to the development by the firm of its own wind generation projects. The construction of the GNL regasification plant required a competitive bidding, and the firm awarded with the construction of the plant received a long term contract, with UTE and the state owned oil company ANCAP purchasing the capacity. The size of the generation system is too small to allow multiple efficient scale thermal generation projects using natural gas to be developed in parallel, competing to supply the increasing needs of the demand, so either franchise biddings to grant power purchase agreements can be conducted, or UTE can develop its own projects. The latter is the present option of the government. The institutional and technical landscape we have described explains the importance of the problem of generation portfolio design in the country.

As Uruguay is a small country with strong interconnections with its neighbors, in this process of power generation planning, the conditions of international power trade play an important role.

The aim of the third part of this thesis is to develop an analytical model to study the effect of international energy trade on the optimal design of a country's power generation system.

As investments in power plants are irreversible, the problem is dynamic, since present investment decisions affect the future optimal short run performances of the system. In this thesis we will use a simplified static model, with two kinds of local generation resources (thermal and wind capacity), and with the possibility of international trade. The country can import energy form neighbor countries without restrictions, and export wind energy surpluses. In the model the energy imports are fully characterized by a single constant price.

The main subject under analysis is the effect of variations in the prices of international energy trade on the optimal amount of wind capacity to be installed, and particularly the determination of conditions leading to imports to be a complement or a substitute of wind capacity.

2 INTERNATIONAL POWER TRADE AS A SEQUENCE OF BARGAINING GAMES

2.1 Introduction

This chapter studies the negotiation between two risk neutral players to split the benefits in an infinite series of trade games developed over an infinite sequence of time periods. Each trade game has a random potential benefit per time unit, and a random duration, and the objective of the negotiation in each game is to determine the shares of these benefits between the players from the moment they reach an agreement to the end of that particular trade game. The potential benefits per time period of a trade game before the moment of the agreement are lost. The duration of a trade game results from Bernoulli trials at the end of each time period, to determine if the current game survives. After a trade game ends, another one starts, with a new potential benefit per period, a new probability of survival in Bernoulli trials and a new negotiation.

The sequence of games described above is motivated by the actual problem of the negotiation of prices between countries in international power spot trade, in the institutional context prevailing for such trade in South America. Interconnected countries face a permanent relationship in which both countries observe the repeated emergence of trade opportunities, of random duration. For each trade opportunity prices have to be determined, resulting in a partition of the benefits per period, as long as the trade opportunity survives.

The formal analysis developed here can be applied to the partition of any kind of benefits per period of time, and to the repetition of such games.

We understand by international power spot trade, the cross-border wholesale transactions of electricity between countries or firms on a spot basis, without any long run contractual obligation for the seller to supply. International spot power trade has two singularities when compared to the trade of almost any other good. First, electricity production costs in each trading country and therefore the direction of the trade and its potential benefit are random, as a consequence of the randomness of hydraulic and wind generation, the availability of thermal plants, and the cycles of under and overinvestment in power generation capacity. Consequently a model of international spot trade should be based on the existence of a sequence of trade opportunities with random duration and benefits. This randomness has been an essential feature of international electricity trade in the region in the past years. Second, for technical reasons power trade always requires some kind of regulation.

In some regions of the world, as the European Union, regulations for international power trade aim at creating a single spot market unifying the national markets of the countries involved in trade. A vast literature describes the institutional and economic problems of designing and implementing this single market for electricity in the EU, for instance Boucher and Smeers (2001), Glachant and Lévêque (2005), ERGEG (2006), and Meeus, Belmans and Glachant (2006). The main issue in this institutional setting is to ensure equal rights and non-discriminatory participation in trade in an open market for firms from every interconnected country. Among the theoretical economic problems that arise, perhaps the most important are the analysis of the effect on market power of a limited capacity interconnection between two markets (for instance Borenstein, Bushnell and Stoft (1999), Parisio and Bosco (2006)) and the assignation among competitors of the scarce transmission capacity (Joskow and Jean Tirole (2000)).

By contrast in South America, regulation has given place most frequently to bilateral trade regimes, where countries explicitly determine prices for the energy traded between them. A description of such regimes can be found in CIER (2004). In this case it is essential to find long run agreements with rules to determine prices, to avoid a case by case bilateral bargain, each time a new trade opportunity appears. To our knowledge, the formal analysis of bargaining between countries to determine prices in spot power trade, which is addressed in this chapter, has few references in the literature. An exception is an interesting paper by Moitre and Rudnick (2000) that studied prices for spot trade between Argentina and Chile, in the framework of the Nash bargaining solution.

The objectives of this chapter are: i) explaining formally the inefficiencies of case by case bargaining, when the players face an infinite sequence of trade games, and ii) finding conditions to be fulfilled by long run agreements with price setting rules, in order to be preferable for both players to case by case bargaining in each trade game.

In section 2 of this chapter, the formal model for each trade game (TG) is presented. The main result is that there is a correspondence between this TG, and the classic model of bargaining over a single amount or "pie" presented in Rubinstein (1982). Therefore the conclusions obtained by Rubinstein about the subgame perfect Nash equilibria (SPNE) can be extended to trade games.

In section 3, the correspondence between trade games and Rubinstein games is extended to the case described by Avery and Zemsky (1994), when in a Rubinstein game, a player can worsen the result of possible agreements each time his offer is rejected (an action of "money burning"), to improve his bargaining position. We will consider one form of money burning consisting of a player delaying the game when his offer is rejected. Avery and Zemsky show that such games have infinite SPNE with partitions belonging to a segment and with agreement in any period t between 0 and a higher bound depending on the parameters of the problem. Therefore there is theoretical support for the evidence that real bilateral negotiations will probably produce inefficient delayed outcomes. Using the correspondence between the extended games, we prove in this section that the existence of multiple inefficient SPNE found by Avery and Zemsky for Rubinstein games with money burning (RGM), can be extended to the trade games with money burning (TGM) we are interested in.

In section 4 we study a super game SG consisting of an infinite sequence of TGMs. In the nth trade game TGM_n there is an assignment of the roles of buyer and seller, a benefit per time unit b_n to split between the two players (equal to the difference between the avoided cost of the buyer and the incremental cost of the seller if trade takes place) and a probability p_n of the game surviving to the next period of time. The set of these parameters is a random variable denoted by v_n. When TGM_n ends, nature generates TGM_{n+1} with parameters resulting from a new random variable v_{n+1}. This sequence of trade games represents the repeated interaction of countries facing a practically infinite series of trade opportunities. We prove in section 4 that a SPNE of game SG consists in both players using in each TGM_n the strategies of an inefficient SNPE of this trade game. As this result of repeated inefficient delayed agreement seems undesirable for both players, we study the possibility of an efficient rule to play the TGMs, to be sustainable as a SNPE of SG. Such a rule should determine right away the split of the benefits (the energy price in international spot power trade) as a function of the observed parameters of the TGM. We find conditions for such a rule to be sustainable with certainty, and give an intuitive interpretation of the conditions in the context of our problem of international power trade.

2.2 Correspondence of a trade game with a Rubinstein game

In this section we will prove that each TG with risk neutral players having the same discount rate, can be put in correspondence with another game first defined by Rubinstein (1982) and widely studied by the literature: the bilateral bargaining for the partition of a fixed amount B, in which each player has to make in turn a proposal as to how it should be divided. After one player has made an offer the other must decide to accept it or reject it and continue the bargaining with the roles reversed. The sequence of offers and rejections can last infinitely if agreement is never reached. Both players have discounting rates so the delay to achieve an agreement is inefficient. Let us call this game Rubinstein game (RG).

Using the biunivocal correspondence between subgames, strategies and equilibria in both games (a TG and a certain RG), we will prove that the expected benefits for the players in a TG discounted to the beginning of any subgame when they play a pair of strategies, are respectively equal to the sure benefits in the corresponding RG when they play the corresponding strategies. Therefore the subgame perfect Nash equilibria (SPNE) of both games can also be put in a biunivocal correspondence. This allows us to employ the results found in the literature about SPNEs in RGs to the analysis of the TGs defined in this paper. Rubinstein obtained a very interesting result for RGs: there is a single SPNE in any RG, consisting of players reaching an immediate efficient agreement in the first period of time, with the partition close to one half for each player if discount rates are equal, with a small advantage to the player who makes the first proposal. The subsequent literature extended Rubinstein games to allow for the existence of inefficient delayed agreements, and is addressed in section 3 of the paper.

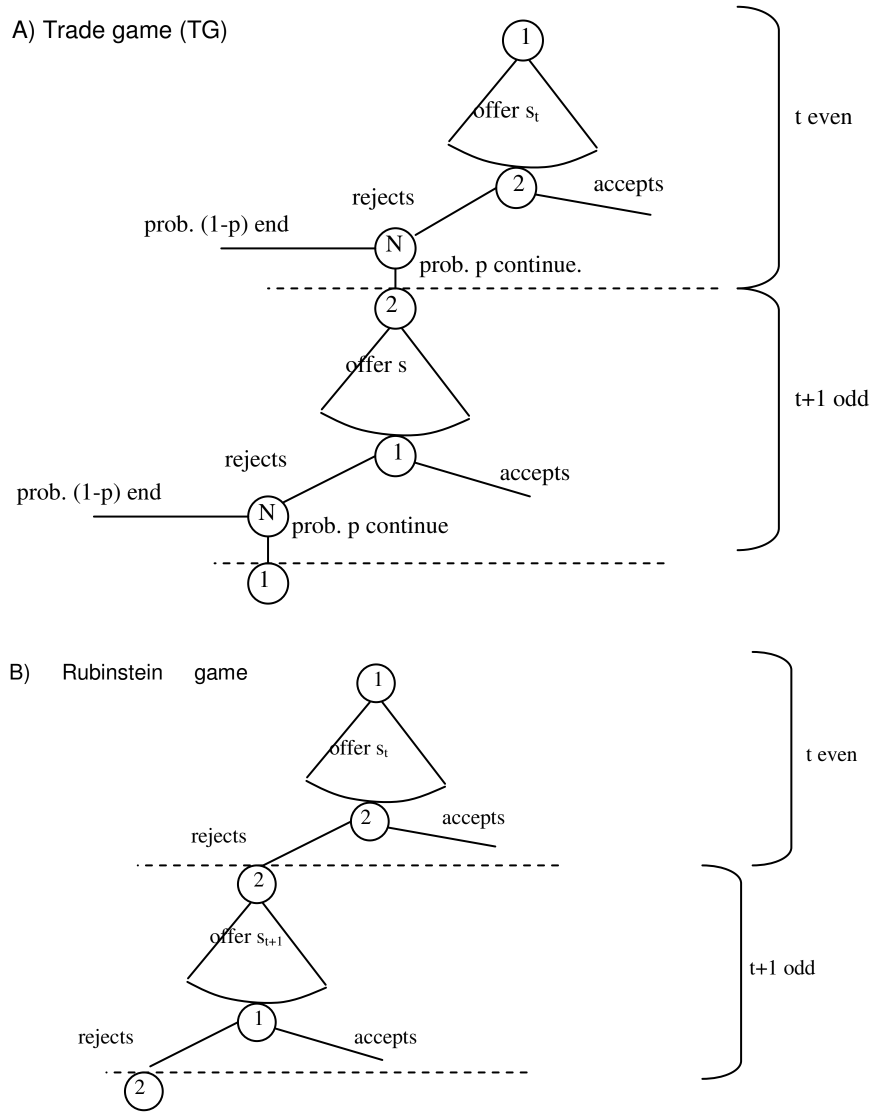
2.2.1 The trade game (TG)

Let us first describe more precisely the TG. Panel A) from Diagram 1 in the next page represents a trade game. The game develops through a potentially infinite sequence of equal duration periods of time, $= 0,1, \dots$. Let us call instant t the beginning of period t.

In a TG there is a benefit b per period to split between players 1 and 2. Both players are risk neutral, and have discount factors per time unit δ_1 and δ_2. The game starts at period $t = 0$ with an offer s made by player 1, about how to split the benefits. As long as no agreement is reached, at the beginning of each time period, one player i makes an offer s

to player j, that j can either accept or reject. If the offer is accepted, player 1 has a share s in the subsequent flow of benefits, and player 2 a share $(1-s)$. The benefit b of time period t is dated at instant t, the beginning the period. If the offer is rejected, in the following period $t+1$ the roles are reversed and j makes an offer to i.

Diagram 1



Therefore if the agreement is delayed for one period its benefit b is lost. When an offer s is accepted the players begin to receive benefits sb and $(1-s)b$ respectively per period in each of the following periods, as long as the game survives. The possibility of renegotiation is not considered. At the end of each period there is a probability p of the game surviving at least another period, and $(1-p)$ of its ending. This random result, which we model as a decision taken by a player nature, is the same whether an agreement has already been reached previously or not. Decisions by nature in different periods are independent random variables and are not affected by the players' strategies.

The probability of the TG born at period 0 surviving until period t is equal to p^t. The benefits for each player in the TG, discounted to the beginning of $t = 0$, given an agreement at $t = 0$, are random variables as the duration of the game after the agreement is random.

The expected values of the benefits for each player in a TG with discount factors δ_1 and δ_2, benefit per period b, and surviving probability p, discounted to the beginning of period 0, when agreement is reached at $t = 0$ with partition s, are:

$$B_1^{TG}(s, b, \delta_1, p) = sb \sum_{t=0}^{\infty} \delta_1^t p^t = \frac{sb}{1-p\delta_1} = sB^{\infty}(b, p\delta_1) \tag{1}$$

$$B_2^{TG}(s, b, \delta_2, p) = (1-s)b \sum_{t=0}^{\infty} \delta_2^t p^t = \frac{(1-s)b}{1-p\delta_2} = (1-s)B^{\infty}(b, p\delta_2)$$

Where $B^{\infty}(b, \rho) \coloneqq \frac{b}{1-\rho}$ is the value of an infinite flow of benefits b per period, beginning at $t = 0$, discounted to instant $t = 0$ with a discount factor ρ per period.

Let us introduce the restrictive assumption $\delta_1 = \delta_2 = \delta$,[1] resulting in:

$$B_1^{TG}(s, b, \delta, p) = \frac{sb}{1-p\delta} = sB^{\infty}(b, p\delta) \tag{2}$$

[1] This hypothesis is needed to find a correspondence of the trade game with a Rubinstein game in a straightforward way. It would be possible to extend the Rubinstein game to consider the negotiation about partition s of two "pies" of different size B_1 and B_2, one intended for each player, so that player 1 gets sB_1 and player 2 gets $(1-s)B_2$. With such and extension, the results of this paper could take into account different discount rates for both players. This extension is possible as no interpersonal comparison of benefits is needed to develop the results in Rubinstein games; each player only compares the outcomes for himself in different strategies, involving different delays to reach agreement.

$B_2^{TG}(s,b,\delta,p) = \frac{sb}{1-p\delta} = (1-s)B^{\infty}(b,p\delta)$

These values $B_1^{TG}(.)$ and $B_2^{TG}(.)$ are also the expected benefits of an agreement with partition s, obtained at any period t, discounted to the beginning of this period, given the event that the game has survived until period t.

We will denote by $TG(b,p,\delta)$ the trade game with benefit per period of time b, probability of survival of the game in each period p, and discount factor δ equal for both players.

Let us denote with $\mathbb{S}_{\mathbf{TG}}$ the sets of subgames of the trade game TG, and with Σ_1^{TG} and Σ_2^{TG} the sets of all possible strategies in trade game TG for each players.

Let us call $T^{TG}(H^{TG},\sigma_1^{TG},\sigma_2^{TG})$: $\mathbb{S}_{\mathbf{TG}} \times \Sigma_1^{TG} \times \Sigma_2^{TG} \rightarrow \{N \cup \infty\}$ the function determining the period of the agreement in $TG(b,p,\delta)$ (or infinite, meaning no agreement is ever reached) when strategies $\sigma_1^{TG},\sigma_2^{TG}$ (of the entire game) are played starting at the beginning of subgame H^{TG}, and nature plays always "continue". If subgame H^{TG} is dated at t, then $T^{TG}(H^{TG},\sigma_1^{TG},\sigma_2^{TG}) \geq t$.

Let us call $S^{TG}(H^{TG},\sigma_1^{TG},\sigma_2^{TG})$: $\mathbb{S}_{\mathbf{TG}} \times \Sigma_1^{TG} \times \Sigma_2^{TG} \rightarrow \{[0,1] \cup NA\}$ the function determining the partition reached in $TG(b,p,\delta)$, when strategies σ_1^{TG}, σ_2^{TG} are played starting at the beginning of subgame H^{TG}, and nature plays always "continue", with NA meaning no agreement.

The expected benefits for both players discounted to instant t, in trade game $TG(b,p,\delta)$, given the event of the game having reached the beginning of subgame H^{TG} at instant t, when strategies in the entire game are σ_1^{TG} and σ_2^{TG}, can be written as:

$$\Pi_1^{TG}\left(H^{TG},\sigma_1^{TG},\sigma_2^{TG}\right) = \delta^{T^{TG}(H^{TG},\sigma_1^{TG},\sigma_2^{TG})-t}\, p^{T^{TG}(H^{TG},\sigma_1^{TG},\sigma_2^{TG})-t}\, B_1^{TG}(S^{TG}\left(H^{TG},\sigma_1^{TG},\sigma_2^{TG}\right),b,\delta,p)$$

$$\Pi_2^{TG}\left(H^{TG},\sigma_1^{TG},\sigma_2^{TG}\right) = \delta^{T^{TG}(H^{TG},\sigma_1^{TG},\sigma_2^{TG})-t}\, p^{T^{TG}(H^{TG},\sigma_1^{TG},\sigma_2^{TG})-t}\, B_2^{TG}(S^{TG}\left(H^{TG},\sigma_1^{TG},\sigma_2^{TG}\right),b,\delta,p) \tag{3}$$

$\delta^{T^{TG}(.)-t}$ is the discount factor between instants t and $T^{TG}(.)$ for both players

$p^{T^{TG}(.)-t}$ is the probability of the trade game surviving until period $T^{TG}(.)$ when agreement happens, given the event of having reached period t.

Using (2) to substitute for $B_1^{TG}(.)$ and $B_2^{TG}(.)$ in (3), we have:

$$\Pi_1^{TG}\left(H^{TG},\sigma_1^{TG},\sigma_2^{TG}\right)=(p\delta)^{T^{TG}(H^{TG},\sigma_1^{TG},\sigma_2^{TG})-t}\,S^{TG}\left(H^{TG},\sigma_1^{TG},\sigma_2^{TG}\right)B^{\infty}(b,p\delta) \tag{4}$$

$$\Pi_2^{TG}\left(H^{TG},\sigma_1^{TG},\sigma_2^{TG}\right)=(p\delta)^{T^{TG}(H^{TG},\sigma_1^{TG},\sigma_2^{TG})-t}\left[1-S^{TG}\left(H^{TG},\sigma_1^{TG},\sigma_2^{TG}\right)\right]B^{\infty}(b,p\delta)$$

2.2.2 The Rubinstein game (RG)

Panel B) from Diagram 1 represents schematically the Rubinstein game RG. An RG is simpler than a TG in two features: first, the benefit to split comes from a single "pie", instead of a sequence of per period "pies"; second the RG ends at the moment of the agreement when the benefits are collected, while the TG survival, and the duration of the flow of benefits, depends on the random trials at the end of each period. Both games share the feature that both players alternate in the role of making proposals about how to divide the benefits.

In the RG there is an amount B to split between players 1 and 2, with discounting rates per time unit ρ_1 and ρ_2 respectively. As in a TG, bargaining develops through a sequence of equal duration periods of time, $=0,1,2\ldots.$. As long as no agreement is reached, at the beginning of each time period, one player i makes an offer s to player j, that j can either accept or reject. If the offer is accepted, player 1 has a single certain benefit sB and player 2 a single certain benefit $(1-s)B$, dated at instant t, the beginning of period t, and the game ends. If the offer is rejected, in the following period $t+1$ the roles are reversed and j makes an offer to i.

Let us introduce the restrictive assumption $\rho_1=\rho_2=\rho$. We will denote by $RG(B,\rho)$ the Rubinstein game with benefit B to divide, and discount factor ρ equal for both players.

Let us denote with $\mathbb{S}_{\mathbf{R}}$ the set of subgames of the Rubinstein game RG, and with Σ_1^R and Σ_2^R the sets of all possible strategies in the game RG for each player.

Let us call $T^R(H,\sigma_1,\sigma_2)$: $\mathbb{S}_{\mathbf{R}}\times\Sigma_1^R\times\Sigma_2^R\to\{N\cup\infty\}$ the function determining the period of the agreement (or infinite, meaning no agreement is ever reached), when strategies σ_1 and σ_2 are played, starting at the beginning of subgame $H\in\mathbb{S}_{\mathbf{R}}$. If subgame H is dated at t, $T^R(H,\sigma_1,\sigma_2)\geq t$.

Let us call $S^R(H, \sigma_1, \sigma_2)$: $\mathbb{S}_\mathbf{R} \times \Sigma_1^R \times \Sigma_2^R \rightarrow \{[0,1] \cup NA\}$ the function determining the partition reached, when strategies $\sigma_1$and σ_2 (of the entire game) are played, starting at the beginning of a subgame $H \in \mathbb{S}_\mathbf{R}$.

For any subgame H beginning at instant t the benefits for the players in the $RG(B, \rho)$, discounted to instant t, when players start playing strategies $\sigma_1$and σ_2 at the beginning of subgame H, are:

$$\Pi_1^R(H, \sigma_1, \sigma_2) = \rho^{T^R(H,\sigma_1,\sigma_2)-t} \, S^R(H, \sigma_1, \sigma_2)B \tag{5}$$

$$\Pi_2^R(H, \sigma_1, \sigma_2) = \rho^{T^R(H,\sigma_1,\sigma_2)-t} \, [\, 1 - S^R(H, \sigma_1, \sigma_2)]B$$

The following table compares the benefits in an RG with total benefit B and a TG with per period benefit b for an agreement reached at period t, with partition s.

Game	Player		Benefits in periods preceding the agreement 0, 1, …., t-1	Benefit in period t of the agreement	Benefits in periods following the agreement t+1, t+2,….
RG	1	Sure benefit	0	sB	--
	2	Sure benefit	0	(1-s)B	--
TG	1	Current benefit per time period as long as the game survives	0	sb	sb
		Expected benefit discounted to t		$sB^\infty(b, p\delta)$	
	2	Current benefit per time period as long as the game survives	0	(1-s)b	(1-s)b
		Expected benefit discounted to t		$(1-s)B^\infty(b,p\delta)$	

Rubinstein found that there is a single SPNE in any RG, consisting of players reaching an immediate efficient agreement in the first period of time, with the partition close to one half for each player if discount rates are equal, with a small advantage to the player who makes the first proposal. The conclusion that all bilateral negotiations end immediately reaching efficient outcomes in rather unrealistic. The subsequent literature extended Rubinstein games to allow for the existence of inefficient delayed agreements, and is addressed in section 3 of the paper.

2.2.3 Correspondence between a TG and a RG

We will show here there is a correspondence between any trade game $TG(b, p, \delta)$ and the Rubinstein game $RG(B^{\infty}(b, p\delta), p\delta)$.

First, there is a biunivocal correspondence $\mathfrak{B}_{\mathbf{H}}$ between the subgames of $TG(.)$ and the subgames of $RG(.)$ where corresponding subgames have at their beginning the same player choosing an action, have in their respective histories the same actions for each of the players, and in $TG(.)$ nature has played always "continue".

Second, for each player there is a biunivocal correspondence between his strategies in both games, where any strategy $\sigma_i \in \Sigma_i^R$ for player i in $RG(.)$ has a corresponding strategy in $TG(.)$ which we will denote by σ_i^{TG}: the one that chooses at the beginning of each subgame of $TG(.)$ where i plays the same action as σ_i at the beginning of the corresponding subgame in $RG(.)$ determined by $\mathfrak{B}_{\mathbf{H}}$. Those actions refer of course to the partition s to be offered and the rule to accept or reject offers.

Using these correspondences we will prove the following proposition.

<u>Proposition I</u>:

Given:

- A trade game $TG(b, p, \delta)$ with benefit per period of time b, probability of survival of the trade opportunity in each period p, and discount factor δ equal for both players.

- and a Rubinstein game $RG(B^{\infty}(b, p\delta), p\delta)$ splitting an amount $B^{\infty}(b, p\delta)$ where both players have discount factor $p\delta$ (with $B^{\infty}(b, p\delta)$ defined in (2)),

- then the expected benefits for both players discounted to the beginning of period t, in

$TG(b,p,\delta)$, given that the game is at the beginning of subgame $H^{TG} \in \mathbb{S}_{\mathbf{TG}}$ dated at t, when strategies σ_1^{TG}, σ_2^{TG} are played

- are respectively equal to the certain benefits discounted to the beginning of t in $RG(B^{\infty}(b,p\delta),p\delta)$ if this game is at the beginning of subgame H, the corresponding of H^{TG} in $\mathfrak{B}_{\mathbf{H}}$, and strategies $\sigma_1$and σ_2 the corresponding in $\mathfrak{B}_{\mathbf{\Sigma}}$ of σ_1^{TG} and σ_2^{TG} are played.
- This is valid in particular for the entire games $TG(.)$ and $RG(.)$, which are also subgames dated at $t=0$.

Proof.

As $\sigma_1$and σ_2 are the corresponding strategies in $\mathfrak{B}_{\mathbf{\Sigma}}$ of σ_1^{TG} and σ_2^{TG}, and H^{TG} and H are corresponding subgames related by $\mathfrak{B}_{\mathbf{H}}$, if nature plays "continue" until agreement is reached:

$$T^{TG}(H^{TG},\sigma_1^{TG},\sigma_2^{TG}) = T^R(H,\sigma_1,\sigma_2) \quad (6)$$

$$S^{TG}(H^{TG},\sigma_1^{TG},\sigma_2^{TG}) = S^R(H,\sigma_1,\sigma_2)$$

This is a result of strategies of both players in both games, prescribing the same rules to make offers and to accept or reject offers.

Using (5) applied to $RG(B^{\infty}(b,p\delta),p\delta)$ yields:

$$\Pi_1^R(H,\sigma_1,\sigma_2) = (p\delta)^{T^R(H,\sigma_1,\sigma_2)-t}\, S^R(H,\sigma_1,\sigma_2)B^{\infty}(b,p\delta)$$

$$\Pi_2^R(H,\sigma_1,\sigma_2) = (p\delta)^{T^R(H,\sigma_1,\sigma_2)-t}\,[\,1 - S^R(H,\sigma_1,\sigma_2)]B^{\infty}(b,p\delta) \quad (7)$$

Using (6) (which states that the resulting partition and time of agreement are equal in both games) and comparing (4) and (7), we have: $\Pi_k^{TG}(H^{TG},\sigma_1^{TG},\sigma_2^{TG}) = \Pi_k^R(H,\sigma_1,\sigma_2)$ for $k=1,2$; the expected benefits in $TG(b,p,\delta)$ are equal to the sure benefits in $RG(B^{\infty}(b,p\delta),p\delta)$ ■

As a consequence of this proposition, there is also a biunivocal correspondence between the SPNEs in $TG(b,p,\delta)$ and $RG(B^{\infty}(b,p\delta),p\delta)$ where sure benefits discounted to $t=0$ in the equilibria in $RG(.)$ are equal to the expected benefits discounted to $t=0$ in the corresponding equilibria in $TG()$. Moreover as there is only one SNPE in $RG(.)$ there is also a unique equilibrium in the corresponding $TG(.)$ with the same partition s and with immediate agreement.

Immediate agreement in real life bargaining is rarely obtained. It is therefore necessary to explain why inefficient outcomes with delayed agreement occur. In the next section we will consider modified Rubinstein games with inefficient outcomes studied in the literature, and settle correspondences between them and trade games modified accordingly.

The correspondences we have found between the subgame sets, strategy sets and SNPEs in the RG and TG games, will emerge again between the two new families of extended games.

2.3 Extension to trade games with money burning by delaying the offer

The result found by Rubinstein (a unique efficient SNPE) contradicts the intuitive perception that bargaining often leads to delayed agreements, or no agreement at all, so that outcomes are Pareto inefficient. Since then new models were developed with adequate changes in the hypothesis of the RG to allow the existence of multiple SPNE with delayed agreement.

Avery and Zemsky (1994) first synthesize the results of some of that literature: "Delayed agreements can result from outside options (Shaked, 1987), the ability to postpone bargaining (Kambe, 1992), …. We show that all of the cited examples conform to a general principle. Multiple equilibria arise because at least one player has the ability to take some action that reduces the value of the asset after her own offer is rejected. We refer to such an action as money burning"

The intuition is that when a player can worsen the result of possible agreements each time his offer is rejected (an action of "money burning"), there is an incentive to use this threat to improve his bargaining position. To our purposes we will consider one form of money burning presented by Avery and Zemsky, consisting of a player delaying the game when his offer is rejected.

Let us call RGM and TGM respectively the Rubinstein and trade games modified to allow money burning by delaying negotiation. In both games there is the possibility for player *i* to delay the game during k periods if his offer is rejected by player j.

We will show that there exists a correspondence between a TGM and a RGM defined adequately, therefore enabling us to use the results found in the literature for RGMs in the study of TGMs. Diagrams 2 and 3 try to describe graphically both games.

Diagram 2 - Rubinstein game with the option for 2 to delay the game k periods

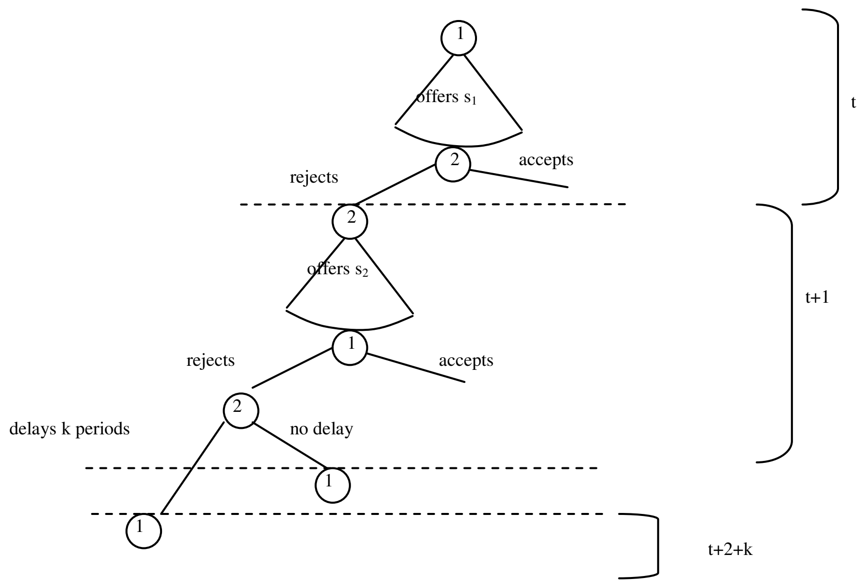
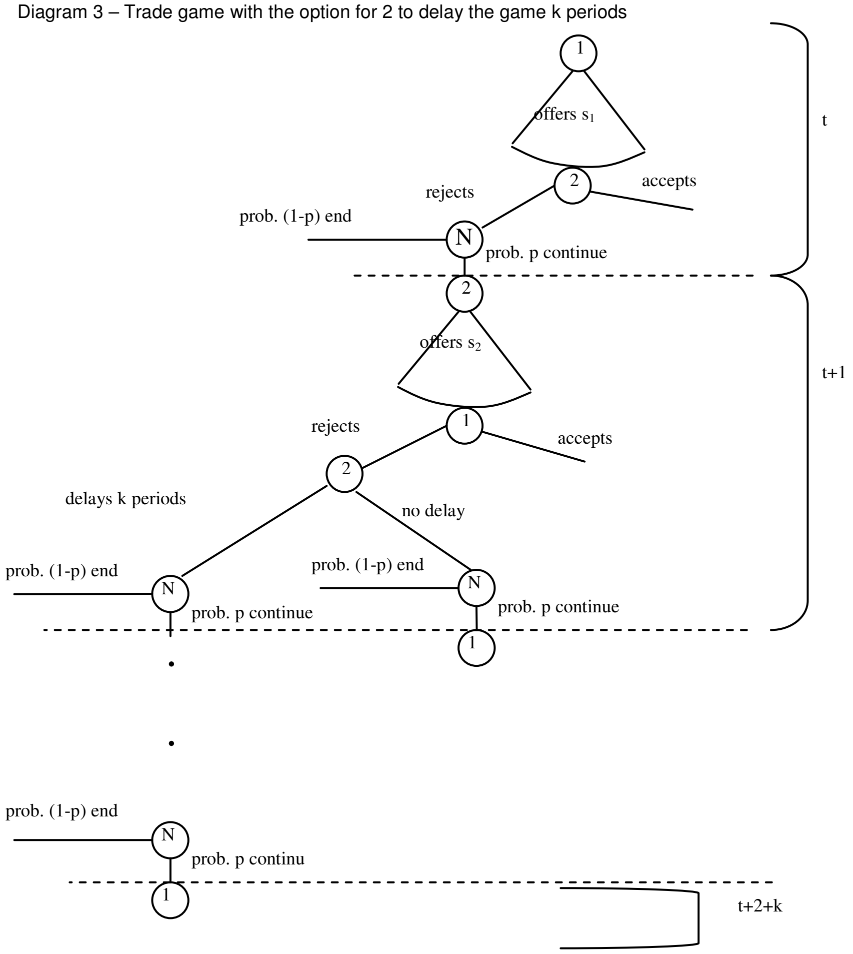


Diagram 3 – Trade game with the option for 2 to delay the game k periods



As in the original Rubinstein and trade games, there is a biunivocal correspondence between the subgames in the TGM and the RGM, where corresponding subgames have at their beginning the same player choosing an action, have in their respective histories the same actions for each of the players, and in TGM nature has played always "continue". We will call this correspondence $\mathfrak{B}_{\mathbf{HM}}$. Let us also call $\mathfrak{S}_{\mathbf{TGM}}$ and $\mathfrak{S}_{\mathbf{RM}}$ respectively the sets of all subgames in the TGM and in the RGM.

Let us call Σ_i^{TGM} and Σ_i^{RM} for $i = 1,2$, the respective sets of strategies for both players in both games.

For each player there is a biunivocal correspondence $\mathfrak{B}_{\mathbf{\Sigma M}}$ between the strategies for each player in both games, where any strategy $\sigma_i \in \Sigma_i^{RM}$ for player i in the RGM has a corresponding strategy σ_i^{TGM} in the TGM: the one that chooses at the beginning of each subgame of the TGM the same action as σ_i at the beginning of the corresponding subgame in RGM, determined by $\mathfrak{B}_{\mathbf{HM}}$. Those actions refer to the partition s to be offered, the rule to accept or reject offers, and now also the rules to delay the game. Then the following proposition (similar to Proposition I but extended to modified games) holds.

Proposition II

Given:

- A modified trade game $TGM(b, p, \delta)$ with the possibility of delaying the game, benefit per period of time b, probability of survival of the trade opportunity in each period p, and discount factor δ equal for both players.

- and a modified Rubinstein game $RGM(B^{\infty}(b, p\delta), p\delta)$ with the same possibility of delaying the game, splitting an amount $B^{\infty}(b, p\delta)$ where both players have discount factor $p\delta$ (with $B^{\infty}(b, p\delta)$ defined in (2)),

- then the expected benefits discounted to the beginning of period t, in $TGM(b, p, \delta)$, given that the game is at the beginning of subgame $H^{TGM} \in \mathfrak{S}_{\mathbf{TGM}}$ dated at t, when strategies σ_1^{TGM}, σ_2^{TGM} are played

- are respectively equal to the certain benefits discounted to the beginning of t in $RGM(B^{\infty}(b, p\delta), p\delta)$ when this game is at the beginning of subgame H, the corresponding of H^{TGM} in $\mathfrak{B}_{\mathbf{HM}}$, and strategies $\sigma_1$and σ_2 the corresponding in $\mathfrak{B}_{\mathbf{\Sigma M}}$ of σ_1^{TGM} and σ_2^{TGM} are played in $RGM(B^{\infty}(b, p\delta), p\delta)$. This is valid in particular for the entire games $TGM(.)$ and $RGM(.)$, which are also subgames dated at $t = 0$.

Proof.

Let

$T^{TGM}(H^{TGM}, \sigma_1^{TGM}, \sigma_2^{TGM})$: $\mathfrak{S}_{\mathbf{TGM}} \times \Sigma_1^{TGM} \times \Sigma_2^{TGM} \rightarrow \{N \cup \infty\}$

$S^{TGM}(H^{TGM}, \sigma_1^{TGM}, \sigma_2^{TGM})$: $\mathfrak{S}_{\mathbf{TGM}} \times \Sigma_1^{TGM} \times \Sigma_2^{TGM} \rightarrow \{[0,1] \cup NA\}$

be the functions determining the period of the agreement and the partition in $TGM(.)$, when starting at subgame $H^{TGM} \in \mathfrak{S}_{\mathbf{TGM}}$, the players use strategies σ_1^{TGM} and σ_2^{TGM}. Subgame H^{TGM} is dated at t so $T^{TGM}(.) \geq t$.

Let

$T^{RM}(H^{RM}, \sigma_1^{RM}, \sigma_2^{RM})$: $\mathfrak{S}_{\mathbf{RM}} \times \Sigma_1^{RM} \times \Sigma_2^{RM} \to \{N \cup \infty\}$

$S^{RM}(H^{RM}, \sigma_1^{RM}, \sigma_2^{RM})$: $\mathfrak{S}_{\mathbf{RM}} \times \Sigma_1^{RM} \times \Sigma_2^{RM} \to \{[0,1] \cup NA\}$

be the functions determining the period t of the agreement and the partition in $RGM(.)$, when starting at subgame $H^{RM} \in \mathfrak{S}_{\mathbf{RM}}$, and players use strategies σ_1^{RM} and σ_2^{RM}.

The delay for k periods in the game provokes a reduction by a factor $(p\delta)^k$ in the sure benefits for both players in $RGM(B^\infty(b, p\delta), p\delta)$, due to a discount factor $p\delta$. The same delay in $TGM(b, p, \delta)$ causes the same reduction in the expected benefits as the combined effect of the discount factor per period δ and a probability of the game ending after each period p. Then the analogues of (1) to (7) hold now, substituting indexes RM and TGM for R and TG respectively. ■

Therefore there is a biunivocal correspondence between the SPNEs of $TGM(b, p, \delta)$ and $RGM(B^\infty(b, p\delta), p\delta)$, where the sure benefits of equilibria in $RGM()$ are equal to the expected benefits in the corresponding equilibria in $TGM()$.

Avery and Zemsky show that in a $RGM()$ exist infinite SPNEs with partitions s belonging to a segment $[s_{min}, s_{max}]$ and with agreement in any period t between 0 and a higher bound depending on the parameters of the problem.

We will assume that the parameters of the problem are such that there exist SPNEs with inefficient equilibria in $RGM(B^\infty(b, p\delta), p\delta)$, the correspondent to our $TGM(b, p, \delta)$. As a consequence the inefficient equilibria in $TGM(b, p, \delta)$ also exist. The possibility of such inefficient outcomes of a case by case negotiation in trade games, is the motivation of the following section.

2.4 Sustainability of efficient rules of trade as an alternative to an infinite sequence of inefficient SNPEs

The following question of prime practical importance arises: would it be possible for both parts, knowing in advance they will face a practically infinite series of trade opportunities (represented by TGMs), to settle a permanent rule to split benefits without delay, avoiding the repetitive case by case bargain in every TGM, and fully exploit the potential benefits of trade? Such a rule should determine right away the energy price and partition of the following benefits as a function of the observed parameters of the TGM.

Therefore the motivation of the present section is to study an infinite sequence of TGMs with random parameters and random durations, to find conditions for the emergence of efficient rules to trade as alternatives to the inefficient SPNEs of each TGM.

We will consider a super game SG consisting of an infinite sequence of TGMs, and a rule determining strategies for both players to play each TGM. The rule does not induce a SPNE in each TGM. However, it is intuitive that the rule could persist in time if:

- Each player thinks his deviation from the rule would cause the game to fall to an infinite sequence of inefficient SPNEs, one for each TGM
- This sequence of SPNEs is undesirable for both players when compared with the survival of the rule.

The goal of this section is to find a condition for such a rule to be sustainable in this way, by ensuring that the pair of strategies consisting of the two players following the rule is a SPNE of total game SG.

2.4.1 Definition of SG

Let us denote again by $t = 0, 1, 2, ...$, an infinite sequence of periods of time of equal duration, and by $\{TGM_n\}, n = 1, ..$, an infinite sequence of TGMs between two players. Game TGM_{n+1} starts as soon as TGM_nends.

Let us call $v_1, v_2, \ldots..$, the infinite sequence of random variables, determining the parameters that describe the respective TGMs, all taking values in a set Ω, with the same distribution $\Phi(.)$, with $v_n = (r_n, b_n, p_n)$, where:

- r_n is the variable describing the roles of the players in TGM_n, who is the seller and who the buyer. This last fact can be relevant to determine which of the multiple equilibria is played, in other words who has the bargaining power.

- b_n is the benefit per period in TGM_n.

- p_n is the probability of trade opportunity in TGM_n surviving from one period to the next one.

Let us call $TGM(v)$ the trade game determined by a realization $v \in \Omega$ of the random variables; therefore $TGM_n = TGM(v_n)$.

<u>Definition I</u>

The game SG is defined by the following:

1) Initially $n = 1, t = 0$.

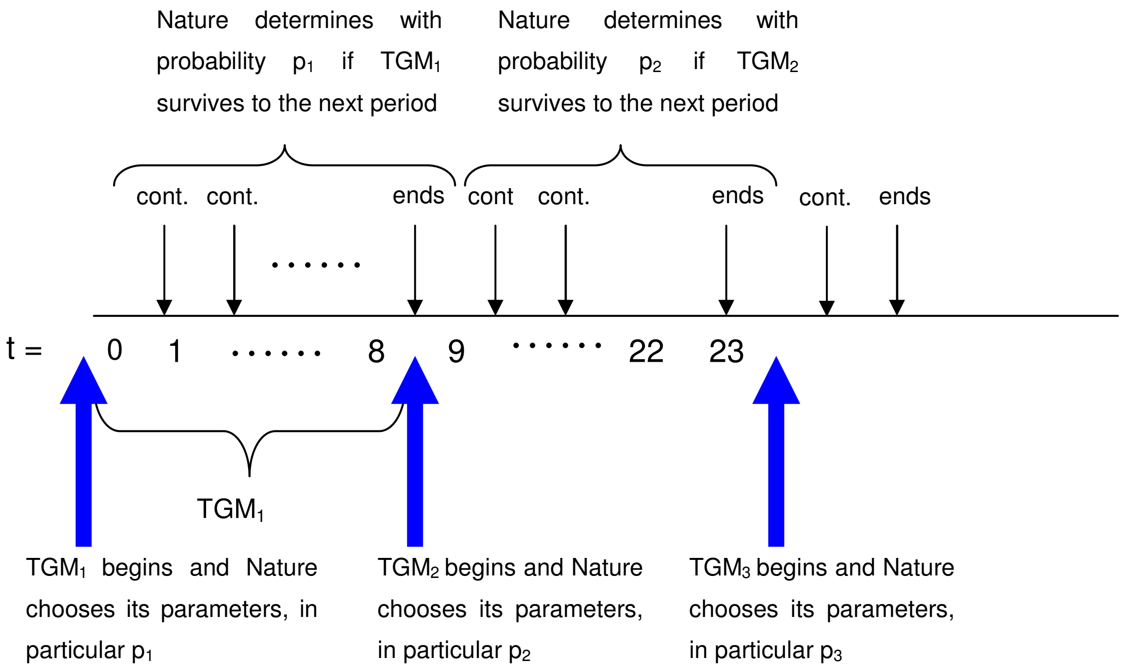
2) Nature chooses the parameters $v_n = (r_n, b_n, p_n)$, of the nth game, and a game $TGM_n = TGM(v_n)$ starts.

3) At period t the game TGM_n is played. At t an agreement could have already been achieved with partition s_n, leading both players 1 and 2 to receive benefits $s_n b_n$ and $(1 - s_n)b_n$ respectively. Or on the contrary, with no agreement, no benefits are collected and the sequence of offers and possibly delays is still going on.

4) At the end of period t Nature decides whether TGM_n continues in the following period. The choice "continue" has probability p_n.

- If TGM_n finishes by nature choosing "end", the game returns to step 2) to initiate a new trade game, n is increased by one, and period t is also increased by one.
- If TGM_n continues the game returns to step 3) with period t increased by one.

The following diagram describes the beginning of a realization of SG.



The period of time when TGM_n (the nth TGM) begins, denoted by t_n, is not predetermined but is a random variable, equal to the addition of $n-1$ waiting times to obtain the first failure in $n-1$ sequences of Bernoulli independent trials with success probabilities p_μ, with $\mu = 1, \ldots., n-1$.

At time period t, depending on the previous choices of Nature, the current TGM can be any TGM_n with n between 1 and $t+1$. In the former case Nature has always played "continue" and SG is still in TGM_1. In the latter Nature has played always "end" and SJ is in TGM_{t+1}.

A strategy for player i to play SG should determine an action at the beginning of each of the subgames of SG, in other words, for every t and every possible history of the game prior to t.

2.4.2 Inefficient SNPE in SG

Let us call $\Sigma_1^{TGM}(v)$ and $\Sigma_2^{TGM}(v)$ the strategy sets of both players in $TGM(v)$, and let us define the functions:

$$\sigma_1^*(v): \Omega \rightarrow \Sigma_1^{TGM}(v)$$

$$\sigma_2^*(v): \Omega \rightarrow \Sigma_2^{TGM}(v)$$

that for every $v \in \Omega$ determine a pair of strategies $(\sigma_1^*(v), \sigma_2^*(v))$ constituting a particular inefficient SPNE of game $TGM(v)$.

We have supposed there are infinite SPNEs in $TGM(v)$, with different partitions and delays until agreement. Let us also suppose now that (for any reason, maybe as a result of the previous experience playing SG) both parts expect a particular SPNE $(\sigma_1^*(v), \sigma_2^*(v))$, to be played in $TGM(v)$ when they are playing SG. Depending on the value of v the result of $(\sigma_1^*(v), \sigma_2^*(v))$ could be more or less favorable to one player, for instance benefits can be systematically greater for sellers as a result of the weak bargaining position of a country relying on imports to avoid energy rationing.

Definition II

The pair of strategies S_1^* and S_2^* to play SG consist in both players using in trade game $TGM_n = TGM(v_n)$, their respective strategy $\sigma_i^*(v_n)$, for $i = 1,2$, disregarding the outcomes of the previous TGMs.

Proposition III

The pair of strategies (S_1^*, S_2^*) is a SPNE in SG.

Proof.

Let us consider any subgame H of SG dated at period t. We must prove that (S_1^*, S_2^*) is a Nash equilibrium of H. A subgame dated at t is determined by the choices of both players and Nature up to period $t-1$ (if the subgame begins with an offer) or t (if the subgame begins with an acceptance or rejection of an offer).

Let us denote by TGM_n the TGM containing the beginning of subgame H.

Suppose that $(S_1^*,\ S_2^*)$ is not a Nash equilibrium in subgame H of SG. This means that one of the players, say player 1, has another strategy S_1' to play SG reporting him an expected discounted benefit strictly greater than his benefit from S_1^*, given the event of the game being at the beginning of subgame H, when player 2 plays S_2^*. The expected discounted benefit for 1 is the infinite addition of expected discounted benefits from the games: TGM_n (starting at H), TGM_{n+1}, TGM_{n+2}, …..

Let us call BS_m^* the contribution to player 1 expected discounted benefit corresponding to

TGM_m, when 1 plays S_1^* and 2 plays S_2^*, with $m = n, n+1, \ldots$. Similarly let us call BS'_m the contribution to player 1 expected discounted benefit from TGM_m, when 1 plays S'_1 and 2 plays S_2^*.

As the benefit for 1 in SG with strategy S'_1 is assumed strictly greater than with strategy S_1^*, there exists at least one $\mu \geq n$ so that in TGM_μ, BS'_μ is strictly greater than BS_μ^* .

- If $\mu = n$, this means that S'_1 is a better strategy than S_1^* in the subgame G of TGM_n starting with the beginning of H. But when they play S_1^* and S_2^* both players are using $\sigma_1^*(v_n)$ and $\sigma_2^*(v_n)$ which form a Nash equilibrium in G. Therefore there cannot exist any strategy S'_1 of SG reporting to player 1 in subgame G of TGM_n a greater benefit than $\sigma_1^*(v_n)$, when confronting with $\sigma_2^*(v_n)$.

- If $\mu > n$, this means that S'_1 is better for player 1 than S_1^* in TGM_μ to confront with S_2^*, against the hypothesis of $(\sigma_1^*(v_\mu), \sigma_2^*(v_\mu))$, being a SPNE of TGM_μ, for every realization of v_μ. ■

2.4.3 Sustainability of an efficient rule to play SG by means of a Nash reversion strategy

If benefits for both players in the SPNEs $(\sigma_1^*(v_n), \sigma_2^*(v_n))$ of the TGMs are poor enough, as agreement is badly delayed, other SNPEs in SG different from (S_1^*, S_2^*), with a cooperative nature, may exist. The following reasoning to explore the existence of other SNPEs in SG is of the "Nash reversion strategy" kind, often used to analyze repeated games: cooperative results are sustainable in the long run as an alternative to the fall to an infinite sequence of inconvenient non cooperative results (in this case the inefficient SPNE in each game).

Definition III

A rule or agreement $A = (A_1(v), A_2(v))$ to play $TGM(v)$, is a function $A: \Omega \to \Sigma_1^{TGM}(v) \times \Sigma_2^{TGM}(v)$, that for each possible value v of the parameters describing the TGM, determines the strategies $A_1(v)$ and $A_2(v)$, to be played by both players in the TGM. A rule A is Pareto optimal if for any $v \in \Omega$, both players agree an immediate partition $s(A, v)$ in the first period $t = 0$.

Let us call $N(t)$ the ordinal of the TGM which is being played at period t, starting with $N(0) = 1$. $N(t)$ is a random variable, with $N(t) \leq t + 1$.

Let us assume there is a Pareto optimal rule $= (A_1(v), A_2(v))$. Using (1) and (2), the expected benefits in $TGM(v)$, with $v = (r, v, p)$ discounted to the initial instant of this game, if the players use $A_1(v)$ and $A_2(v)$ are respectively equal to $s(A, v)\ B^{\infty}(b, p\delta)$ and $(1 - s(A, v))\ B^{\infty}(b, p\delta)$. Agreement is immediate and the discounted expected benefit to split is $B^{\infty}(b, p\delta)$.

Definition IV

Given a Pareto optimal rule $A = (A_1(v), A_2(v))$, let $G_i(A)$ be the following strategy for player i, to play SG.

- At $t = 0$, play $A_i(v_1)$, where v_1 is the set of parameters of TGM_1 determined by Nature.
- At $t > 0$:
    - Play $A_i(v_{N(t)})$, if in the entire previous history of the game SG, both players have played their respective $A_k(v_n)$, for every $n = 1, \ldots\ldots N(t)$, for $k = 1,2$
    - Play $\sigma_i^*(v_{N(t)})$ in any other case.

With this definition, when two players are using strategies $G_1(A)$ and $G_2(A)$ in SG, the result when a new TGM_n is born, in a subgame with a past of both having played $A_i(v_\mu)$ for every $\mu < n$, is:

- If both players abide to $A_i(v_n)$ in the new TGM, they reach immediate agreement with partition $s(A, v_n)$.
- If any of the players i deviates from his respective $A_i(v_n)$, both players turn in the next period to their respective strategies in the SPNE $\sigma_1^*(v_n), \sigma_2^*(v_n)$ and keep on playing $\sigma_1^*(v_m), \sigma_2^*(v_m)$ in the infinite sequence of the following TGM_m, for $m > n$.

Our problem is to find conditions for the pair of strategies $(G_1(A), G_2(A))$ to be a SPNE in SG with certainty, that is, for every possible realization of the random variables. We will now find conditions for $G_1(A)$ to be a better response to $G_2(A)$ in every subgame of SG for every possible realization of the random variables $v_n,\ n = 1, \dots \dots$

Let us call:

- $\overline{\Pi}_{1,n}(S_1'/v)$ the expected benefit for player 1 in TGM_n, discounted to the beginning of this trade game, if player 1 uses strategy S_1' from the beginning of this TGM, against strategy $G_2(A)$, given a certain observed value of the parameters v, which are revealed as soon as TGM_n begins, before players choose their actions. The expected value takes into account the randomness in the survival of TGM_n.
- $\Pi_1(s_1, s_2)$ the expected benefit for player 1 in any TGM_n, discounted to the beginning of this trade game, if player 1 uses strategy $s_1(v)$, against strategy $s_2(v)$. Strategy $s_i(v)$ provides a way to play each $TGM(v)$ as a function of the respective v. The expected value takes into account both the randomness of the values of the parameter v_n, which has a common distribution for all n, and the fact that game TGM_n duration is a random variable.
- $P(\mu, \tau/v)$, with $\mu > 0, \tau \geq \mu$ the conditional probability of game $TGM_{n+\mu}$ starting τ periods after period t, given the event that at period t trade game TGM_n begins and given a certain observed value of the parameters v of TGM_n.

The last two expressions defined above are independent of the ordinal n of the game, as s_1 and s_2 behavior depends only on the value of v, and the set Ω of possible values of v, and its distribution Φ are the same for all n.

Let us classify the subgames of SG that start with a move by player 1 into two sets:

- The set C_a of the subgames with a history of both players using always $G_1(A)$ and $G_2(A)$, that is, both players following rule A. As rule A is efficient they are subgames beginning at the first period of a TGM and where player 1 begins the TGM by making the first offer. (In the symmetric for player 2 the first offer in the TGM was made by player 1 following rule A and player 2 has to decide whether to accept it following rule A or to reject it).

- The set C_b of all the other subgames starting with a move by player 1, which have a history of deviation from rule A by at least one player.

Lemma I

The pair of strategies $(G_1(A), G_2(A))$, is a Nash equilibrium for every subgame in C_b.

Proof.

Let $H \in C_b$, be a subgame of SG starting at period t and belonging to the nth TGM. As the rule A has been broken, by Definition IV, the players use $\sigma_1^*(v_m)$ and $\sigma_2^*(v_m)$ in every TGM_m for every $m \geq n$, so in subgame H, $G_1(A)$ behaves as S_1^* and $G_2(A)$ behaves as S_2^*. Proposition III states that (S_1^*, S_2^*) are a SPNE in SG, so (S_1^*, S_2^*) is a Nash equilibrium in H subgame of SG. Therefore $(G_1(A), G_2(A))$ is also a Nash equilibrium in H. ■

Lemma II

The following **Condition I:**

$$\overline{\Pi}_{1,n}(S_1'/v) - s(A,v)B^{\infty}(b,p\delta)$$

$$+\sum_{\mu=1}^{\infty}\sum_{\tau=\mu}^{\infty}\delta^{\tau}P(\mu,\tau/v)[\Pi_1(\sigma_1^*,\sigma_2^*) - \Pi_1(A_1,A_2)] \leq 0$$

for all $v = (r, v, p) \in \Omega$.

and its symmetric for player 2) are necessary conditions for $(G_1(A), G_2(A))$ to be with certainty a Nash equilibrium in every subgame in C_a.

Proof.

Let us consider a subgame $H \in C_a$. H begins at t_n, the first period of TGM_n. Let S_1' be a strategy for player 1 to play SG different from $G_1(A)$. For $(G_1(A), G_2(A))$ to be with certainty a Nash equilibrium in H, the gain for player 1 from using S_1' instead of $G_1(A)$ must be non-positive for any S_1', for all $v \in \Omega$.

The gains Δ for player 1 if he deviates from $G_1(A)$ in TGM_n with parameters $v = (r, v, p)$ can be expressed as the addition $\Delta = \Delta_n + \Delta_{LR}$, of:

- An expected "opportunistic" gain Δ_n in TGM_n:
  $\Delta_n = \overline{\Pi}_{1,n}(S_1'/v) - s(A,v)B^{\infty}(b,p\delta)$ (8)
- An infinite sequence of long run effects Δ_{LR} resulting from the deviation, and player 2 response to it, taking place in games $TGM_{n+\mu}$, for $\mu = 1, \dots$.....

$$\Delta_{LR} = \sum_{\mu=1}^{\infty} \sum_{\tau=\mu}^{\infty} \delta^{\tau} P(\mu, \tau/v) \left[\Pi_{1,n+\mu}(S_1') - \Pi_1(A_1, A_2) \right] \tag{9}$$

Where $\Pi_{1,n+\mu}(S_1')$ is the expected benefit in $TGM_{n+\mu}$ if 1 plays S_1' and 2 plays $G_2(A)$ starting in subgame H. On the other side, if both players keep using rule A player 1 earns $\Pi_1(A_1, A_2)$ as the expected benefit in $TGM_{n+\mu}$, for $\mu = 1, \dots \dots \dots$.

The summation in μ covers all the TGM_μ following TGM_n. The summation in τ covers all the possible starting periods for game TGM_μ. Both summations take into account all the possible events concerning the duration of trade games.

As $\sigma_1^*(v_\mu), \sigma_2^*(v_\mu)$ is a Nash equilibrium in $TGM_{n+\mu}$ for all $\mu > 0$ and given that $G_2(A)$ plays $\sigma_2^*(v_\mu)$ in $TGM_{n+\mu}$ for all $\mu > 0$ as a result of the deviation from rule A then:

$$\Pi_{1,n+\mu}(S_1') \leq \Pi_1(\sigma_1^*, \sigma_2^*) \tag{10}$$

Therefore, the condition of non-positivity gains applied only to strategies playing $\sigma_1^*(v_{n+\mu})$ for $\mu > 0$,

$$\bar{\Pi}_{1,n}(S_1'/v) - s(A, v) B^{\infty}(b, p\delta) + \sum_{\mu=n+1}^{\infty} \sum_{\tau=\mu}^{\infty} \delta^{\tau} P(\mu, \tau/v) [\Pi_1(\sigma_1^*, \sigma_2^*) - \Pi_1(A_1, A_2)] \leq 0$$

implies the non positivity of gains for all other dominated strategies. ■

Condition I has an intuitive interpretation.

$\Pi_1(\sigma_1^*, \sigma_2^*)$ is the expected value of the benefit for player 1 in the inefficient SPNE $(\sigma_1^*(v), \sigma_2^*(v))$ of game $TGM(v)$.

$\Pi_1(A_1, A_2)$ is the expected benefit for player 1 if the pair of strategies of the Pareto optimal rule A, $(A_1(v), A_2(v))$ are played in $TGM(v)$.

As the rule A is the result of an agreement of both countries to overcome the drawback of inefficient SPNEs $(\sigma_1^*(v), \sigma_2^*(v))$ it natural to assume: $\Pi_1(\sigma_1^*, \sigma_2^*) - \Pi_1(A_1, A_2) < 0$

Therefore using (10) we have $\Delta_{LR} < 0$: the long run effect (in games $TGM_{n+\mu}$, for $\mu = 1, \dots \dots$.) of a deviation from rule A at the beginning of TGM_n, is negative.

Condition I says that for all possible $v = (r, v, p) \in \Omega$ the expected opportunistic gain in TGM_n, $\overline{\Pi}_{1,n}(S_1'/v) - s(A, v)B^{\infty}(b, p\delta)$ is not big enough to justify a deviation, when compared with the long run loses Δ_{LR}.

The non-fulfillment of Condition I means that there is a chance of player 1, after observing $v = (r, v, p)$ at the beginning of TGM_n, finding both a high value for s (as a result of a disadvantage for player 2 in the immediate negotiation determined by v) and a high probability p of the current TGM_n survival, so that deviation from rule A becomes profitable. A high value for p tends both to prolong the life of the convenient "present" TGM_n increasing the expected opportunistic gains, and to reduce the discounted value of $|\Delta_{LR}|$, and therefore the long run losses caused by opportunism.

In short, an efficient rule A will survive with certainty if there is no possible stroke of good luck for any player that compensates the future losses from abandoning the rule and falling to an infinite sequence of inconvenient SPNEs, with inefficient case to case negotiations, one in each game.

From Lemmas I and II results immediately the following proposition.

Proposition IV

Condition I and its symmetric for player 2) are necessary conditions for $(G_1(A), G_2(A))$ to be with certainty a SNPE in SG.

2.5 Conclusions

This chapter studies the negotiation between two risk neutral players to split the benefits in an infinite series of trade games developed over an infinite sequence of time periods. Each trade game has a random potential benefit per time unit, and a random duration, and the objective of the negotiation in each trade game is to determine the shares of these benefits between the players from the moment they reach an agreement to the end of that particular bargaining game, with no possibility of renegotiation in that trade game. After a trade game ends, another one starts, with a new potential benefit per period, a new probability of survival in Bernoulli trial and a new negotiation.

This sequence of trade games is motivated by the actual problem of the negotiation of prices between countries in international power spot trade, where interconnected countries face a permanent relationship in which both countries observe the repeated emergence of trade opportunities, of random duration. For each trade opportunity, prices have to be determined, resulting in a partition of the benefits as long as the trade opportunity survives.

We first proved that each trade game with players having the same discount rate, can be put in correspondence with another game first defined by Rubinstein (1982). Then we showed that if we extend trade games to allow money burning actions, as delaying the game when a player's offer is rejected, there is a similar correspondence between these extended trade games (TGMs) and Rubinstein games with the same possibility of money burning (RGMs).

This correspondence let us apply to TGMs the results found in the literature for RGMs with money burning by delaying the game: there are infinite inefficient SNPEs. The theory then explains a feature of real bilateral negotiations, including international spot power trade negotiations: it is likely to find inefficient delayed agreements, or to see a trade opportunity vanish without the players having found an agreement.

We then studied a super game SG, an infinite sequence of TGMs. We proved that a pair of strategies (S_1^*, S_2^*) for SG consisting in each player using in every TGM with parameters v its strategy $\sigma_i^*(v)$ from a particular SPNE $\left(\sigma_1^*(v), \sigma_2^*(v)\right)$ of this TGM, is a SNPE of SG.

We defined strategies $G_1(A)$ and $G_2(A)$ for the players, consisting in playing in each TGM according to a rule A, of efficient immediate agreement, as long as both players have maintained the rule in the previous TGMs, and to play $\sigma_i^*(v)$ otherwise. Such a rule should determine right away the partition of the following benefits as a function of the observed parameters v. We assumed that the expected benefits for both players in $TGM(v)$ if they use rule A and follow strategies $\left(A_1(v), A_2(v)\right)$ are Pareto superior to the results with $(\sigma_1^*(v), \sigma_2^*(v))$.

We found conditions for the pair of strategies $(G_1(A), G_2(A))$ to be a SNPE of SG with certainty. An intuitive interpretation of these conditions is the following. At the beginning of any trade game TGM_n the expected present opportunist gain obtained by a deviation from rule A (earned in TGM_n) must be smaller than the future expected losses from the fall to the infinite sequence of inefficient equilibria (experienced in $TGM_{n+1}, TGM_{n+2}, \dots$), for all possible random values of v_n. In other words there must not exist a possible stroke of good luck for any player that compensates the future losses from abandoning the rule.

The results can be applied to assess the feasibility of long run rules to determine prices for international energy spot trade:

- The more inefficient the results from case by case price negotiation (the longer the delays to achieve price agreement) the greatest incentive for countries to develop such long run rules.
- A situation of significant asymmetry of bargaining power between countries, as a result of one of them suffering a structural underinvestment crisis, with expected duration of at least a few years, (meaning a high value of the probability p) would probably generate incentives for the other country to avoid setting a rule or to abandon a previously existing one.
- A situation where both power systems have sufficient installed capacities, and trade is the consequence of random differences in renewable generation or thermal plant availability, seems more suitable for the emergence of trade rules. Wind power generation can change in a few hours. Hydroelectric generation conditions of abundance of drought can last a few months or a year. The shorter the expected duration of periods of good luck, the less probable a country is tempted to prefer opportunistic short run gains, to an efficient rule.

3 AGREEMENTS FOR INTERNATIONAL SPOT POWER TRADE IN SOUTH AMERICA

3.1 Introduction

In South America the main form of international power trade is currently spot trade, performed by system operators in the framework of bilateral agreements between countries. Spot trade consists in transactions decided day by day involving short term energy surpluses, without a long run commitment or any obligation for the selling country to have generation capacity to supply the buyer permanently.

In the recent past some power transactions in the region had a multilateral component: for instance Uruguay purchased energy from Brazil, through the Argentinean grid. Sometimes the multilateral nature of trade has hindered bilateral transactions: the requirement by Brazil of Argentinean energy in 2005 affected the Argentinean supply to Uruguay; the lack of Argentinean agreement to grid access prevented Paraguayan energy sales to Uruguay and Chile.

The present development of interconnections in South America will require the analysis of multilateral trade. In the Mercosur region a 500 kV new interconnection between Uruguay and Brazil will be completed in 2015, raising the total interconnection capacity to 570 MW, and completing an interconnection loop with the existing Argentine- Uruguay 500 kV lines (with 2000 MW capacity), and Argentine-Brazil 2000 MW link. In the Andean region the Ministers and senior officials from the energy sectors of Chile, Colombia, Ecuador, Peru, and Bolivia (as an observer) signed a commitment during the Meeting of the Council of Ministers for the Andean Electrical Interconnection System (SINEA) in September 2012, to move forward in an ambitious electrical interconnection project (INTAL, 2012).

Both cases of multiple strong interconnection between countries, require multilateral agreements for power trade. The mere superposition of independent bilateral agreements between countries could induce inefficiency or even worse leave indetermination in the transactions.

There is ample literature about energy integration and international power trade pursuing the ideal of a single energy market, where generators and consumers could trade without any country based discrimination. Perhaps the main example is the European Union (Jamasb y Pollitt, 2005; Nowak, 2010). Besides, there are few analysis of spot trade in the South American institutional framework, where countries have to settle agreements to determine energy prices in trade, and the transactions are decided by power system operators. An important reference is Moitre and Rudnik (2000).

The present paper aims at describing different methods to define energy transactions and their prices, when more than two countries negotiate how to divide the benefits of energy trade through a grid with limited capacities.

The retribution for the use of grids is beyond the scope of this work. Although it is a subject of economic importance, its impact is of second order when compared with the effect of energy prices in trade. The representation of the interconnection grid is then very simplified, retaining only the energy balance.

This chapter's contents can be described as follows. In section 2 some singularities of international power trade are outlined, leading to the necessity of regulation and agreements between countries. It is showed that there is a diversity of institutional frameworks to allow trade, and the situation in South America is described. Section 3 presents the formal definition of optimal flows in a simplified interconnection grid, in the sense of flows leading to Pareto optimal allocations for the countries. Section 4 describes the bilateral agreements applied in recent years in South America to determine prices for international spot trade. In Section 5 the problem of multilateral trade is addressed, and a family of methods to define economically meaningful and consistent transactions are presented, resulting in the optimal flows, with different criteria to solve the problem of power transits through third countries and to split the benefits of trade between countries. Section 6 contains the conclusions.

3.2 Institutional issues

3.2.1 The need for regulation in power generation and international power trade

Structural and regulatory reforms in different countries led to the emergence of multiple competitive electricity markets of national scope: spot markets for short term and contract markets for long term supply. Transmission and distribution grids allow third party access. Apparently electricity has become another commodity, even for international trade. However this simplified view cannot be sustained when the technical singularities and the complexity of sectorial regulation to make those markets work are taken into account.

The electric system requires an instantaneous balance between demand and supply, which can only be achieved by a centralized coordination of generation and transmission operation. Therefore regulation must ensure this coordination and determine the procedures to supply the load curve at every moment, and a system operator must perform the function of load dispatch. As a consequence, the marginal cost of production is known at every moment.

International power trade can only take place through interconnections, which require long construction periods and heavy investments. In most countries, interconnection capacity with neighboring countries is a small fraction of local generation capacity, and every country relies mainly or exclusively on its own resources to supply its demand.

The generation capacity to ensure short term reliability is a public good. Besides, the development of long term supply contracts is burdened with high transaction costs and uncertainty, and in many countries there is a reasonable concern that efficient markets for those contracts will not emerge spontaneously. These two market failures and the technical reasons discussed above justify that with no exception countries regulate power generation.

Regulation in national spot markets makes regulation in international spot power trade a necessity.

In South America this regulation involves the definition of spot prices. In a spot market with inelastic demands, the Pareto optimal prices, different in each node of the transmission system, are the so called nodal prices which are equal to the respective marginal costs of supply in each node (including the marginal cost of unserved energy).

Nodal prices generate implicit incomes for every transmission line, as a result of the difference of the values at nodal prices of outgoing and incoming power flows (Pérez-Arriaga y Meseguer (1997). When the flow through a line reaches its capacity this difference is usually economically significant and receives the name of congestion rent.

3.2.2 Diversity of institutional models

In a simplified vision, international spot power trade can follow two quite different institutional models, described in Ibarburu y García de Soria (2008).

In what we can call **single market model**, there is an economic integration process between the countries, and the main issue is to ensure equal treatment and no discrimination in a competitive market, to every generator and consumer in whatever country. International power trade is intended to be a particular case in a more general integration framework. The typical case is the European Union (EU) with its energy single market, although such an ideal is difficult to achieve (Comisión Europea, 2005; ETSO, 2006). Any generator can supply any consumer in another country. In theory the strategic interests of countries are subordinated to the logic of market integration. Spot prices result from the transactions in power exchanges, and system and market operators have the task to iteratively adjust power flows by means of balancing transactions, to achieve a technically feasible load dispatch, respecting grid restrictions.

The main problem to be solved is the allocation of international interconnection capacity among the firms demanding it, usually called congestion management problem.

In what we can call **country bargaining model**, international trade is performed without the existence of multilateral integration institutions and a relevant general framework for trade between the countries. The main issue is in this case the split of the gains from power trade between the countries and the agreements to set prices for the energy. Each country decides independently how to distribute its part of those gains between the national market's participants. That is the most frequent situation in South America. In each country a system operator determines both the load dispatch and the regulated spot prices, based on nodal prices.

In the single market model, restrictions to the exercise of market power by generators result from the competitive pressures in the extended multinational market, and the action of multilateral antitrust agencies. In the country bargaining model the restrictions for market power exerted by selling generators and countries, come from the rules to determine transaction prices. A situation in which no rules to set prices exist, and every trade opportunity begins a new bargain between the countries could be grossly inefficient, so in the following we will assume countries are determined to establish rules and to abide by them in every transaction.

3.2.3 Security of supply issues

Any agreement to trade energy between countries must take into account a series of problems related to security of supply. The risk of power outages and the way trade is performed in a situation of outage require carefully crafted provisions for such cases.

3.2.3.1 Restrictions to exports

A country's authorities may want to restrict energy exports, if they are deemed risky for the future local reliability of supply. That may be the case when the exported energy comes from hydraulic power plants and require the use of water reserves, but concerns may arise also for the wear and future reliability of thermal plants used to export. Trade agreements should specify explicitly any a priori restrictions a country may want to impose to its exports, based on security of supply issues.

3.2.3.2 Trade prices when the buyer is in a situation of energy outage

Rules to set the prices of international power trade frequently depend on the countries' marginal costs or avoided costs. In a situation of energy outage, if the cost of unserved energy of the buyer is taken into account, such rules can produce very high prices, many times greater than the usual variable costs and market prices in a normal situation, an outcome the buyer may consider inequitable and unacceptable. Agreements to set prices should take into account and avoid such situations, for instance by setting an upper bound to the buyers' relevant costs when applying the rules.

3.2.3.3 Priorities of supply in a situation of energy outage

Countries may have valuations of the cost of unserved energy differing much from each other, depending on the ability of the authorities to reduce consumption without affecting the country's production and welfare. In a situation where more than one country is rationing the demand, if trade is guided by the criterion of total cost minimization the country with the highest unserved energy cost would receive absolute priority in supply, which can be an unfair solution. An agreement for international trade must explicitly determine criteria to split energy surpluses from sellers when more than one potential buyer is in an outage situation.

3.3 Optimal flows through the interconnections

3.3.1 Definitions

The goal of an ideal multilateral trade system is to achieve power flows through the interconnections such that: i) a Pareto efficient allocation between the countries is the result of energy transactions, and ii) the split of the gains from trade results acceptable for all the countries.

The informational requirements to perform the calculation of those energy flows are reasonable in the institutional context of South America. In almost every country of the region the system operator determines a load dispatch minimizing the total cost of supply, including outage costs, based on audited generators costs (the exception is Colombia where the generators declare prices freely to the dispatch). As a result regulators and system operators already have the costs for every generation unit, and can calculate the total cost at every moment, as a function of total power generation in the country.

The Pareto optimal flows for a group of countries at a unit time interval, for instance an hour, are the ones minimizing the addition of the countries' total costs, including generation and rationing costs.

The following is a very simplified analytical formulation of the problem, assuming that:

- All dynamic problems due to generation unit operational constraints can be neglected
- The opportunity costs of water in the reservoirs of all hydraulic power plants have been determined by each country, in a way consistent with the expected international power trade flows. As a result, hydraulic energy has a unit opportunity cost fixed during the time period.
- Each country can be considered a single node in the electric grid, and has a single marginal cost at every one of its interconnections.

- If two countries have more than one interconnection, they can be simplified with no significant error to only one composite interconnection.

There is a set N of countries and each country $i \in N$ has a set of generation units C_i and a demand $D_i \in R$. Each generation unit c in country i has a maximum generation capacity P_c^{max}, and a non-decreasing marginal cost function $f_c: [0, P_c^{max}] \to R$. Energy rationing procedures are included as generation units with a marginal cost equal to the marginal social energy outage cost (EOC). The EOC can be many times greater than the variable cost of generation units, and is an increasing function of the magnitude of the outage.

As we are dealing with costs in a unit time period, for instance an hour, the numeric values of all energy and average power magnitudes during the period are respectively equal.

A **generation resource** r can be defined as a quartet (i_r, c_r, P_r, f_r), where i_r is a country, $c_r \in C_{i_r}$ is a generation unit in country i_r, P_r is a power less or equal to the maximum generation capacity $P_{c_r}^{max}$ of the unit and $f_r: [0, P_r] \to R$ is the non-decreasing function of the marginal cost of production of the resource. The different forms of rationing and their costs are also interpreted here as generation resources.

The **set of country i's generation resources**, G_i is the set

$$G_i = \{(i, c, P_c^{max}, f_c) \text{ for each } c \in C_i\}$$

consisting of all the generation units of the country at its maximum power.

Given any set Q of generation resources we can define:

- $P(Q)$**,** total power of the resources in Q, $P(Q) = \sum_{r \in Q} P_r$.
- $CT(Q, d)$**,** total cost to supply a power $d \le P(Q)$ during an hour, with generation resource set Q, $CT(Q, d): [0, \sum_{r \in Q} P_r] \to R$**,** resulting from the minimum of the problem:

$$Min_{\{\pi_r\}, r \in Q}\left[\sum_{r \in Q} \int_0^{\pi_r} f_r(x)dx\right] \qquad (1)$$

$$s.t: \quad \sum_{r \in Q} \pi_r \ge d\,; \qquad 0 \le \pi_r \le P_r \; for\ every\ r \in Q$$

- $CMg(Q,d)$**,** marginal cost of energy supplied by set Q of resources, the derivative respect to d of the function $CT(Q,d)$**.**

Let us call:

- $CT_i(g_i)$ the total generation cost in country i to supply energy g_i in an hour, using only its own generation units, or equivalently $CT_i(g_i) = CT(G_i, g_i)$
- $CMg_i(g_i)$ the marginal generation cost of country i; $CMg_i(g_i)$ is defined as the right derivative of total cost $CT_i(g_i)$.
- G_i^{SC} the set of resources used in country i without trade

  $G_i^{SC} = \{(i, c, P_c^{SC}, f_c): c \in C_i, P_c^{SC} > 0\}$

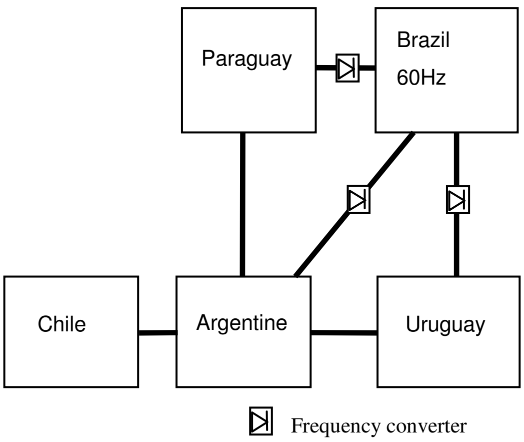
  where $\{P_c^{SC}\}$ are the solutions to problem (1) when $Q = G_i$ and $d = D_i$.
- $L \subseteq \{\{i,j\}, i \in N, j \in N\}$ the set of interconnections between countries.
- $\tau_{i,j}^i$, $\tau_{i,j}^j$, respectively the outgoing flow from i, in the direction $i \rightarrow j$ and the incoming flow to j in the direction $i \rightarrow j$. Only situations with $\tau_{i,j}^i . \tau_{j,i}^i = 0$ and $\tau_{i,j}^j . \tau_{j,i}^j = 0$ are economically and physically meaningful.

Interconnections are subject to:

- Physical losses that are assumed quadratic with the power transmitted, so that only a fraction of outgoing flow $\tau_{i,j}^i$ from i, arrives at j,
- Transmission capacity constraints, so that the outgoing flow $\tau_{i,j}^i$ from i, is limited by an upper bound $t_{i,j}^{i\,max}$.

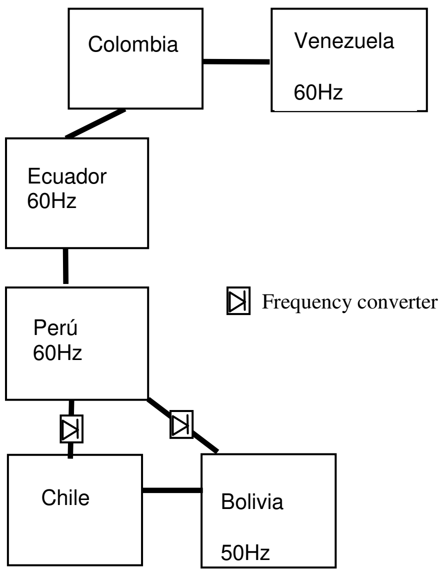
3.3.2 The case with controllable interconnection flows

Graphic 1



The particular case when there are no cycles in the graph of alternate current (AC) interconnection links having no frequency conversion, allows taking the flows through the interconnections as control variables of the problem. In the Mercosur region, the links between Brazil (with 60 Hz frequency) and Argentine, Paraguay and Uruguay on the other side (with 50 Hz) require frequency converters, power electronics devices that allow to control the power transmitted through them. Besides, Chile is interconnected only with Argentine. Graphic 1 shows this situation. In a case like this, the flows through the interconnections can be chosen freely, subject only to maximum capacity constraints.

Graphic 2



The same assumption can be made in the Andean region interconnection grid, shown schematically in Graphic 2, if we disregard the possibility of a direct strong interconnection between Peru and Colombia through the Amazonian region.

In this section we will explicitly formulate the problem of determining the optimal flows through interconnections, when they can be chosen as controllable variables.

Let us define the **optimal dispatch** for a set of countries N, with their sets of generation units C_i and demands D_i, for every country $i \in N$ and the set of interconnections L, as the set of powers generated by generation units and flows through every interconnection, that solve the following cost minimization problem, Problem (2) [2]:

$$Min \qquad \left[\sum_{i \in N} \sum_{c \in C_i} \int_0^{\pi_c} f_c(x)dx\right] \tag{2}$$

$\{\pi_c\}, for\ every\ c \in C_i, i \in N$

$\{\tau_{i,j}^i\}, \{\tau_{i,j}^j\}, for\ every\ \{i,j\} \in L$

$$s.t: \quad \sum_{c \in C_i} \pi_c + \sum_{\{i,j\} \in L_i} \tau_{j,i}^i - \sum_{\{i,k\} \in L_i} \tau_{i,k}^i - D_i \geq 0; \quad for\ every\ i \in N \tag{2a}$$

$$0 \leq \pi_c \leq P_c^{max}\ for\ every\ c \in C_i,\ for\ every\ i \in N \tag{2b}$$

$$\tau_{i,j}^j = \tau_{i,j}^i - r_{i,j}(\tau_{i,j}^i)^2 \qquad \tau_{j,i}^i = \tau_{i,j}^j - r_{j,i}(\tau_{i,j}^j)^2 \quad for\ every\ \{i,j\} \in L \tag{2c}$$

$$\tau_{i,j}^i \leq \tau_{i,j}^{max} \qquad \tau_{j,i}^j \leq t_{j,i}^{max} \qquad for\ every\ \{i,j\} \in L \tag{2d}$$

Where $r_{i,j}$ and $r_{j,i}$ are non-negative constants that determine the energy losses at interconnection $\{i,j\}$.

Let us call:

- $O^* = (\{\pi_c^*\}, \{t_{i,j}^{i\,*}\}, \{t_{i,j}^{j\,*}\})$ the optimal solution to problem (2); let us suppose the solution is unique.
- CMg_i^* the dual variable associated to constraint (2a) for node i with optimal international trade.
- G_i^* the set of resources with units from country i used in the optimal dispatch:

[2] Problem (2) can be reformulated to show that optimal flows can be obtained from marginal cost functions for each country; powers generated by each unit are no longer control variables:

$$Min \sum_{i \in N} \int_{D_i}^{\pi_i} CMg_i(x)dx$$

$\{\pi_i\}, i \in N$
$\{\tau_{i,j}^i\}, \{\tau_{i,j}^j\},\ for\ every\ \{i,j\} \in L$

$s.t: \quad \pi_i + \sum_{\{i,j\} \in L_i} \tau_{j,i}^i - \sum_{\{i,k\} \in L_i} \tau_{i,k}^i - D_i \geq 0;\ \ and\ to\ (2c)\ y\ (2c) \quad for\ every\ i \in N$

$G_i^* = \{(i, c, \pi_c^*, f_c^*): c \in C_i\,, P_c^* > 0\}$ where $f_c^*: [0, \pi_c^*] \to R$, so that $f_c^*(\pi) = f_c(\pi)$.

Countries with $\sum_{c\in C_i} \pi_c^* > D_i$ are defined as net exporters and we define the net exports for each one of them as $g_i^* = \sum_{c\in C_i} \pi_c^* - D_i$. The other countries are defined as net importers and we define the net imports of country i: $d_j^* = D_j - \sum_{c\in C_j} \pi_c^*$. Let us call N_X and N_M the sets of net exporters and net importers respectively.

Let us call graph $\mathscr{G}$ the directed graph having a node for each country in N, and one arc for each interconnection with nonzero flow in the optimal dispatch. Each arc has the same direction as the associated flow. In Annex I we present the proof of two propositions related to the directed graph of optimal flows $\mathscr{G}$:

Proposition I: For any arc $\{i, j\}$ with positive flow from i to j, $CMg_i^* \le CMg_j^*$,

and its immediate consequence,

Proposition II: If losses are nonzero $\mathscr{G}$ has no cycles.

3.3.3 Non controllable interconnection flows in meshed AC interconnection grids

In the case of multiple countries linked in alternating current by a non-radial, meshed grid, energy flows between countries cannot be chosen arbitrarily. Additionally to active power balance constraints in every node and grid losses, another set of constraints determine the power flow in the interconnected grid. As a result, if a country A supplies energy to country B, and there are multiple possible paths for the energy, through the grid of other countries, the flow in each path cannot be controlled arbitrarily. Then the assumption of controllable flows through all the interconnections is not valid, neither the simplified analysis presented above.

However Proposition I, and therefore Proposition II, also hold with AC grids. The seminal paper by Bohn, Caramanis and Schweppe (1984) show (in their Result 2) that shadow prices of energy (the equivalent to our CMg_i^*) in an optimal dispatch with an AC transmission grid with losses, increase strictly in the direction of the flow: Proposition I is valid for optimal flows in AC grids.

3.4 Bilateral price agreements for spot trade in the region

The ways trade is done currently or has been done in the past in the region, in the framework of bilateral trade, are interesting precedents to be taken into account in the design of a new multilateral set of rules. Therefore the following is a simplified description of the price setting arrangements used in spot bilateral trade in the region.

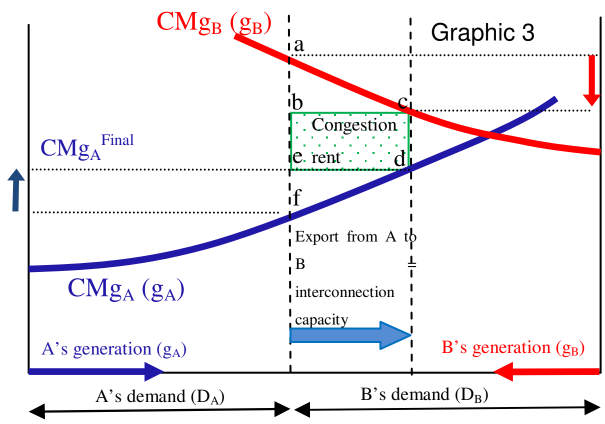
3.4.1 Price setting criteria

3.4.1.1 Nodal prices and split of congestion rents

This is the system established by Resolution 536 of the Andean Community of Nations for international energy trade, and has been applied to trade between Ecuador and Colombia. It is also the proposal in CIER (2011) in a study about the subject sponsored by CIER (the association of electricity utilities in South America).

The selling country receives for the energy its own marginal cost after trade (including other system charges) plus its share of congestion rents, if present. The buyer pays for the energy its own marginal cost after trade and receives the other part of congestion rents. The proportions to divide congestion rents are fixed beforehand. For simplicity we will suppose here that energy costs are the only relevant ones, excluding other costs and charges.

If the interconnection is big enough marginal costs after trade in both countries become practically equal, and congestion rents are negligible. If the interconnection is small, the marginal cost of the seller after trade remains lower than the buyer's and congestion rents are important.



Graphic 3 shows an example of optimal power trade between countries A and B when the interconnection is used at its maximum capacity.

The horizontal axis measures energy or power produced, demanded and traded. Country A is the seller. A's generation increases from left to right and B's generation increases from right to left. Both demands are D_A and D_B. CMg_A and CMg_B, increasing functions in the respective generations, are the marginal costs of production in the two countries, assumed to be twice differentiable. Let us denote by C the amount of energy traded.

A receives a price $CMg_A^{Final} = CMg_A(D_A + C)$ per unit of energy, plus its share of the congestion rent. B pays $CMg_B^{Final} = CMg_B(D_B - C)$, per unit of energy and receives the rest of the congestion rent. The total unit average price implicit in the transaction is $p_{AB} = CMg_A^{Final} + \alpha_A(CMg_B^{Final} - CMg_A^{Final})$, where α_A is the fraction of the congestion rent that goes to A. In the graphic: A obtains a benefit in trade equal to area **edf**, plus its share of the total congestion rent **bcde**. B obtains **acb** plus the rest of the congestion rent. Let us call respectively B_i^{BR} and B_i^T the benefit for country i before the distribution of congestion rent, and the total benefit in trade. Assuming the marginal cost functions are twice differentiable, it holds:

$$B_A^{BR} = \int_0^C [\,CMg_A(D_A + C) - CMg_A(D_A + x)]dx =$$

$$= \int_0^C \left[\frac{dCMg_A(D_A+C)}{dg}(C - x) - \frac{1}{2}\frac{d^2CMg_A(D_A+\theta_{A1}(x)\,C)}{dg^2}(C - x)^2\right]dx, \quad \text{with } \theta_{A1}(x) \in (0,1)$$

And therefore: $$B_A^{BR} < \frac{dCMg_A(D_A+C)}{dg} \cdot \frac{C^2}{2} - K_A^{inf}(D_A, D_A + C) \cdot \frac{C^3}{6} \qquad (3)$$

where $K_A^{inf}(D_A, D_A + C)$ is a lower bound of $\frac{d^2CMg_A}{dg^2}$ in $(D_A, D_A + C)$.

Similarly we can prove: $$\frac{dCMg_B(D_B-C)}{dg} \cdot \frac{C^2}{2} + K_B^{inf}(D_B - C, D_B) \cdot \frac{C^3}{6} < B_B^{BR} \qquad (4)$$

where $K_B^{inf}(D_B - C, D_B)$ is a lower bound of $\frac{d^2CMg_B}{dg^2}$ in $(D_B - C, D_B)$.

Let us assume that it holds:

$$\frac{dCMg_A}{dg}(D_A + \theta C) < \frac{dCMg_B}{dg}(D_B - \theta C) \quad \forall\, \theta \epsilon [0,1] \; and \; 0 \leq C < D_B \qquad (5)$$

When condition (5) holds it is reasonable to call country A the large country and country B the small country, as any power transaction changes more the marginal cost in the small country than in the large one. It is possible to proof two propositions about the advantage of small countries in trade.

Proposition III

If both marginal cost functions are convex, and (5) holds (A is the large country and B the small country) then $B_A^{BR} < B_B^{BR}$.

Proof

As both marginal cost functions are convex K_A^{inf} and K_B^{inf} are nonnegative and (5) holds, then using (3) and (4): $B_A^{BR} < \frac{dCMg_A(D_A+C)}{dg} \cdot \frac{C^2}{2} < \frac{dCMg_B(D_B-C)}{dg} \cdot \frac{C^2}{2} < B_B^{BR}$ ■

Proposition IV

If the following hypothesis hold, (H1) functions $\frac{\frac{d^2CMg_i}{dg^2}}{\frac{dCMg_i}{dx}}$ for $i = A, B$ are bounded in absolute value (these ratios are measures of the concavity of marginal cost curves) and (H2) inequality (5) holds with sufficient slack (that is, the difference in size between the countries is large enough), then $B_A^{BR} < B_B^{BR}$.

Proof

(H1) and (H2) ensure the following inequality holds:

$$-\left[K_A^{inf}(D_A, D_A + C) + K_B^{inf}(D_B - C, D_B)\right] \cdot \frac{C}{3} < \frac{dCMg_B(D_B - C)}{dg} - \frac{dCMg_A(D_A + C)}{dg}$$

Therefore:

$$\frac{dCMg_A(D_A + C)}{dg} \cdot \frac{C^2}{2} - K_A^{inf}(D_A, D_A + C) \cdot \frac{C^3}{6} < \frac{dCMg_B(D_B - C)}{dg} \cdot \frac{C^2}{2} + K_B^{inf}(D_B - C, D_B) \cdot \frac{C^3}{6}$$

Using (3) and (4) we obtain $B_A^{BR} < B_B^{BR}$. ■

Similar propositions can be proved if the large country is buying.

In conclusion, in bilateral international trade using nodal pricing the smaller country tends to receive a bigger share of the benefits of trade, before the distribution of the congestion rent.

Let us consider two extreme situations, an interconnection with an arbitrarily small capacity (I^{ε}), and another with infinite capacity (I^{∞}) between a large and a small country. Let us call α_{Large} and α_{Small} the respective shares of the congestion rent. In the case (I^{ε}) the change in both marginal costs as a result of trade is negligible and therefore the benefits consist almost exclusively in the shares of the congestion rents, so $\frac{B^T_{Small}}{B^T_{Large}} \cong \frac{\alpha_{Small}}{\alpha_{Large}}$. In the case (I^{∞}) there is no congestion rent and therefore $B^T_i = B^{BR}_i$ for both countries.

If both shares in congestion rent are equal and either Proposition III or Proposition IV holds, $B^T_{Large} = B^T_{Small}$ for I^{ε} , and $B^T_{Large} < B^T_{Small}$for I^{∞}. A conjecture that arises is that if $\alpha_{Large} = \alpha_{Small} = \frac{1}{2}$, under reasonable hypothesis the ratio $\frac{B^T_{Small}}{B^T_{Large}}$ will increase with the capacity of the interconnection link.

The nodal price regime has a resemblance with the result of a single competitive market integrating both electric systems, if the use of the interconnection is auctioned. The congestion rent would be the price in that auction.

Some of the possible criteria to define the shares for both countries in the congestion rent would be:

a) Divide it in the same proportion of the contributions made by the countries in the investment to build the interconnection, suggested in CIER (2011).
b) Share it in halves, as in the case of the interconnection between Colombia and Ecuador.
c) Divide it in the same proportion of the demands involved, the whole demand of the selling country and the energy purchases of the importing country. This was the criterion initially applied between Ecuador and Colombia.

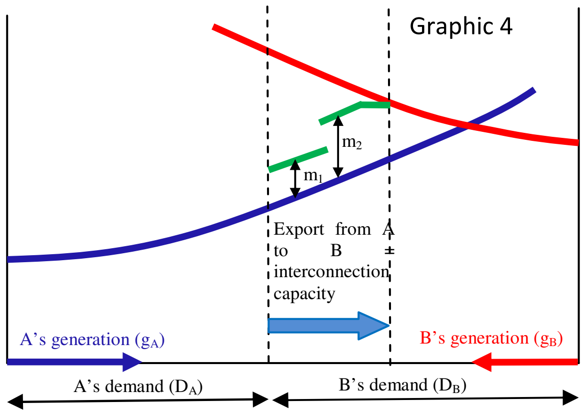
3.4.1.2 Share in halves of the benefits of trade

In this mechanism a price for the energy, or one price for each of the resources used incrementally by the seller are determined, so that the benefits of trade result equal for both countries. This is one of the regimes included in the Interconnection Agreement between Argentina and Uruguay, called "substitution".

3.4.1.3 *Price of each resource equal to its incremental cost plus a margin*

In this mechanism the price for each of the resources used incrementally by the seller is equal to the incremental cost for the seller plus a margin. The margins are limited to maximum values, one for each type of resource, determined beforehand. The regimes called "power" and "emergency" in the Interconnection Agreement between Argentina and Uruguay belong to this kind. In that case, margins were initially fixed equal to the average fixed cost of every resource, assuming a high load factor, including a return on the investment. With this criterion, the prices of the resources are equal to the total average costs and tend to be greater for peak units than for base load units.

Graphic 4 shows the share of benefits. Margins m_1 and m_2 correspond to two different kind of resources.



Selling prices are the addition of marginal costs of the units and margins. Let us suppose marginal cost curves are continuous, and margin $m(.)$ is increasing with the quantity sold. It may happen that the seller has to accept a reduction in the margin to sell the optimal quantity C (as shown in Graphic 4). This happens when:

$$CMg_A(D_A + C) < CMg_B(D_B - C) < CMg_A(D_B + C) + m(C)$$

3.4.1.4 *Other price regimes in the region*

Some other price regimes have been used or conceived in the region:

- The seller offers a fixed price for every resource, valid for a long period, for instance a semester, without other restrictions. This was the regime for international trade in Argentinean regulation in the 90´s.
- The seller offers a price, is dispatched according to this price but receives the spot price (the marginal cost) of the buying country. This is one of the regimes for imports in the Uruguayan regulation, to be applied only if spot trade with the neighbor country approximates to market integration, a condition never reached yet.

3.4.2 Transfer of rights to a third country

The following trade situation has happened in Mercosur. The first country A has energy surpluses to sell and has a trade agreement with a second country B. B does not require imports and transfers its rights and obligations in spot transactions resulting from the trade agreement with A, to a third country C. In the period 2006-2010, a few times Brazil played the role of the seller, and Argentina transferred its rights to buy energy to Uruguay.

3.4.3 Loan with hydraulic reserves

In this regime a country receives energy generated incrementally by another country using hydraulic reserves, and the receiving country commits to return the energy before a predetermined date. In the Mercosur region Brazil has acted as energy lender a few times (CNPE, 2008), and Argentina and Uruguay as borrowers.

3.4.4 Value of transactions as a function of seller's and buyer's costs

In the methods we called here nodal pricing, share in halves and cost plus margin, the total amount T of the transaction is determined by the set V of resources used incrementally by the seller and the set S of resources of the buyer substituted as a result of trade. Let us call C the amount of energy sold.

In nodal pricing:

$$T = C \times [CMg(V, C) \; + \; \alpha\,(CMg(S, 0) \; - \; CMg(V, C))]$$

where α is the share of the seller in the congestion rent.

This is a result of $CMg(V, C)$ being equal to the marginal cost of the seller and $CMg(S, 0)$ being equal to the marginal cost of the buyer, after trade.

In cost plus margin:

$$T = CT(V, C) + \sum_{r \in V} \int_{0}^{P_r} Min\big(m_r, CMg(S, 0) - f_r(x)\big)dx$$

where m_r is the margin for resource r of the seller.

In share in halves of the benefits:

$$T = CT(V,C) + \frac{1}{2}\,(\,CT(S,C) - \,CT(V,C)\,)$$

As the limitation of market power of the selling country is a concern, it is essential for a sustainable agreement that the unit costs of resources in V used to determine T, be the same as the costs used in the selling country in the local optimum dispatch. An exception could be made if the internal cost of the resources in V in the selling country is affected by subsidies to benefit the local consumers.

3.5 Multilateral trade

3.5.1 Problems arising with multilateral trade

When three or more countries are linked by interconnections of significant capacity, something has to be done to make the preexisting bilateral regimes compatible, or a new system must be designed, as the mere aggregation of bilateral agreements leave some problems unresolved, for instance:

- The problem of competition between buyers: when two or more countries are willing to buy energy from a selling country ¿how much of every resource of the seller has to be sold to every buyer?
- The problem of energy transits through third countries, and the possibility of intermediation in energy trade: ¿how much of the energy incoming to a country can be bought by it to be resold with profit, and how much has to be considered a transit of energy with destination to another country?

We can informally define transit as the situation when in the optimal flow with trade, a country A generates more energy than in the situation with no trade, and the opposite happens to country B, and the flow from A reaches B not only through the direct interconnection A-B, but also indirectly through at least another country C.

The total amount of energy transits through one country is usually defined as the minimum between the total amount of energy inflows to the country and the total amount of energy outflows from the country. The identification of the countries to be considered origins and destinations of the flows provoking the transit is a non-trivial problem, and a vast literature addresses the subject. For instance, in the European Union transits must be estimated to determine compensations for the use of the grid to countries hosting the transits (FSR, 2005). In South America, within the institutional framework of country bargains to divide the benefits of power trade, the determination of the responsibility for energy transits can be even more important, as the countries hosting transits could claim the right to intermediate in energy trade, receiving more than a compensation for the use of their grids.

3.5.2 Nodal pricing applied in a multilateral setting

Nodal pricing with distribution of the congestion rent is the mechanism proposed in the Mercado Eléctrico Regional (MER) of Central America, the solution chosen in Resolución 536 of the Andean Community of Nations (but only applied in bilateral transactions until now) and is the proposal in CIER (2011). It is apparently the simplest solution for multilateral spot trade. At first it seems the neatest solution to the problem of determining spot prices in international trade. However, we will show the problem is not so simple, and it is worthwhile to explore a more general family of agreements for trade.

If we assume that only energy costs are relevant, as a result of trade each country i has a net income T_i from the whole set of transactions with his neighbors:

$$T_i = CMg_i^* \left(\sum_{j \in suc(i)} t_{i,j}^* - \sum_{j \in pred(i)} t_{j,i}^* \right) + \sum_{j \in suc(i) \cup pred(i)} \alpha_{i,j}^i \left| CMg_i^* - CMg_j^* \right|$$

Where:

- CMg_i^* is the marginal cost of country i with optimal trade, as defined in before.
- $suc(i)$ and $pred(i)$ are respectively the set of nodes receiving energy from node (country) i in the optimal flows (the set of node i's successors in $\mathcal{G}$) and the set of nodes sending energy to node i (the set of node i's predecessors in $\mathcal{G}$).
- $t_{i,j}^*$ is the optimal energy flow from i to j
- $\alpha_{i,j}^i$ is the fraction of the congestion rent in the interconnection between countries i and j that goes to country i.

That result is equivalent to a set of transactions, one for each interconnection where the flow goes from i to j, in which country i sells the energy flow to country j at a price:

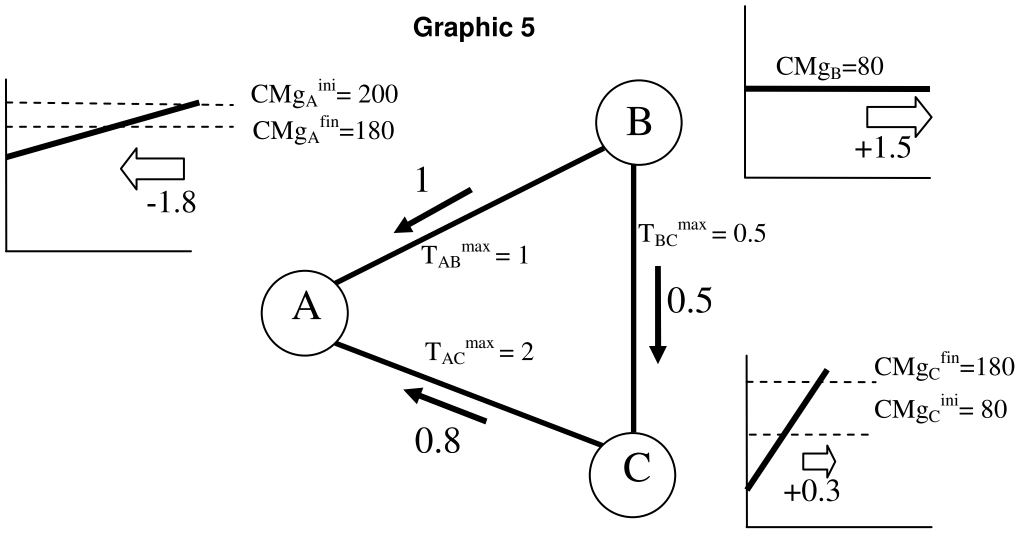
$$p_{i,j} = CMg_i^* + \alpha_{i,j}^i (CMg_i^* - CMg_j^*).$$

In section 4 we proved Propositions III and IV, about the shares of benefits in bilateral nodal pricing spot trade. Here we try to investigate whether those propositions still hold when trade with nodal pricing involves three or more countries. A general analytical proof seems quite difficult but we can resort to numerical simulation. Annex II shows the results of a simulation, where the two following propositions hold[3]:

- In multilateral trade, given two of the countries with different sizes, if the topology of the grid, interconnection capacities and levels of marginal costs curves are chosen randomly, the benefit per unit of energy traded is bigger for the smaller country.
- In the same context, if congestion rents are shared in halves in every interconnection, an increase in the interconnection capacity tends to favor more the smaller countries.

Another very important feature of nodal pricing in multilateral trade is that countries can make a very significant profit from the energy transits they host in their grids. Graphic 5 shows an example. The graphic presents the topology of the interconnection grid, with the maximum capacity T_{ij} of each link, the marginal cost curves of each country, the levels of marginal cost before and after optimal trade, and the optimal flows. Losses are supposed equal to zero. Congestion rents are shared in halves in each interconnection.

Graphic 5



Country B increases its generation in 1.5 units; let us suppose its marginal cost remains constant equal to 80. Country C increases its generation in 0.3 units and its marginal cost increases from 80 to 180, as a result of trade. Country A reduces its generation in 1.8 and its marginal cost is reduced from 200 to 180. Lines BA and BC are loaded to their

[3] A more precise formulation for both propositions is presented in Annex II

maximum capacities. Line AC has a remaining capacity so marginal costs in A and C are both equal to 180.

Net income for country C as a result of trade is:

(0.8-0.5) × 180 (value of incoming and outgoing flows at the marginal cost in C)

+ 0.5 × (180-80) ×0.5 (share of congestion rent in line BC)

$-\Delta G_C$ (increase in generation cost in C)

This means that country C, besides the net benefit from selling to A its own incremental generation ($0.3{\times}180-\Delta G_C$), receives one half of the congestion rent in BC for the transit of 0.5 units of energy flowing from B to A.

It is not obvious countries would be willing to agree on a mechanism like nodal pricing conceding such gains to intermediation, and an advantage to smaller countries, as shown above.

3.5.3 The definition of bilateral transactions consistent with optimal multilateral flows

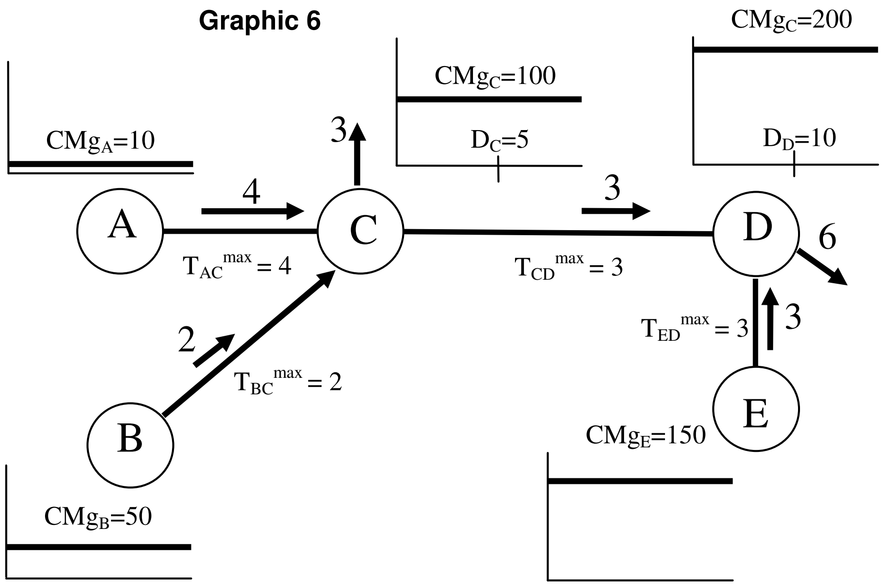
The goal of this section is to define a more general family of trade regimes, by allowing two degrees of freedom: a) the extent of the advantages a country has due to its location in the interconnection grid and b) the way benefits of trade are shared in each transaction.

In each regime, economically meaningful bilateral transactions are defined based on optimal flows. Each transaction trades generation resources, including those resulting from incremental generation in net exporting countries due to trade. In 3.5.3.1 an introductory example is shown. In 3.5.3.2 algorithms to define bilateral transactions are presented. In 3.5.3.3 the subject is the rules to share benefits in the transactions.

3.5.3.1 *Advantages due to location in the grid and energy intermediation*

Advantages due to location and intermediation are illustrated by the following example, presented in Graphic 6.

Graphic 6



Let us assume there is a trade regime of the cost plus margin kind. Energy is sold by exporting countries at a price equal to marginal cost plus a fixed margin m. When intermediation of energy takes place, the intermediating country earns a margin m' between the buying and selling price. All countries have constant marginal cost curves. The capacities of interconnection lines T_{ij}^{max} are shown beside each line. Arrows show the direction and magnitude of optimal trade flows. In the optimum flows: country C generates only 2 of the 5 units it demands (D_C) and imports 3; country D generates only 4 and imports 6 to supply its demand ($D_D = 10$). All lines are at their maximum capacity. Let us assume $m < 50$ and $m + m' < 150$.

There is more than one way to define bilateral transactions where net exporters A, B and E sell, and net buyers C and D buy. Both C and D would rather buy first from A, the cheapest seller (at price 10+ m), then from B (at price 50+ m), and only in the last place from D (at price 150+ m).

One way to split the cheapest energy is a proportional distribution, without considering location in the grid: C and D buy energy to A, B and E with proportions 4/9 from A, 2/9 from B and 3/9 from E, those of the total exports of the three countries. Flows in the grid are not considered. This criterion can be discarded right away as it would lead to the absurd of a country (in the example country C) buying at a price (150+ m) greater than its own marginal cost to another country (in the example country E).

A way to define economically meaningful transactions is to build them following the directed graph of optimal flows $\mathcal{G}$. In these transactions:

- The node of the selling country is always a direct or indirect predecessor in $\mathcal{G}$ of the buying country.
- The flow through each interconnection is assigned a composition where the components are flows originated in each of the net exporting countries.
- At each node, every component's balance must close.
- Let us convene that all the flows outgoing from a node have the same composition.

Two questions that must be answered to define such bilateral transactions are:

(Q1) ¿How much of the energy flowing into its node can a country buy?

(Q2) ¿Can a country buy the cheapest inflowing energy with priority (Option Q2.1), or should it always buy the same proportion of all components of the inflowing energy (Option Q2.2)?

Let us assume for (Q2) that Option Q2.1 holds. At least three options are possible for question (Q1).

Option Q1.1 – A country buys only the net energy it extracts from the grid. Only net importers in the optimal flows buy energy, and they buy precisely the amount of their net imports.

Option Q1.2 – A country buys to supply its own demand at minimum cost. Every country, either a net importer or not, can buy energy inflowing into its node, and buys precisely the resources that together with its own ones, allow the minimum cost supply of the country's demand. As a result the country can have surpluses of its own resources generating at the optimal solution, which can be exported.

Option Q1.3 – A country buys all the inflows to its node and intermediates energy. Every country buys all the energy flows arriving at its node. With these purchases plus its own resources, the country supplies its demand at minimum cost. Surpluses are sold. As a result the country can be an intermediary.

Table 1 shows the application of the three criteria to the example presented above. As a result we will find the net cost in trade for both net importers C and D (showed in bold type in the table).

In all cases the transaction in which country E sells D three units at price $150+m$ is omitted in the table, as it is performed under all three options. As a result the net imports of C and D (from A and B) are both equal to three units.

In all three options C pays a smaller net amount than D for the same quantity of energy (3 units), so C has a smaller import unit cost, as a result of its proximity in the grid to exporting countries A and B.

For C option Q1.3 is better than Q1.2 and the latter is better than Q1.1, and exactly the opposite happens to D, as a result of the increasing value given to a favorable location in the grid (understood as proximity to cheap exporters). In the three options the addition of C's and D's total net import costs equals $140+6m$, the addition of the incomes required by A and B to export. (If for question (Q2) option Q2.2 is taken instead of Q2.1, C's costs would increase and D's would decrease, but we will not consider this option).

As generation costs in every country are equal in the three options, only the net costs resulting from trade are relevant.

Table 1 – Transactions and net costs in trade with option Q2.1

Option	Country C Energy	 Net cost	Country D Energy	 Net cost
Q1.1	Purchases from A = 3 (3 = net imports of C)	3 (10+m) **30 + 3m**	Purchases from A = 1 Purchases from B = 2	1 (10+m) 2 (50+m) **110 + 3m**
Q1.2	Purchases from A = 4 Purchases from B = 1 (4+1=5= C's total demand) Sales to D of C's own energy = 2 (C covers its entire demand with imports and sells 2 of its own generation to D)	4 (10+m) 1 (50+m) -2 (100+m) **-110 + 3m**	Purchases from B = 1 (Of the 6 units entering C, only 1 unit originated in B is available for D) Purchases from C = 2	1 (50+m) 2 (100+m) **250 + 3m**
Q1.3	Purchases from A = 4 Purchases from B = 2 (C buys all the incoming energy) Sales to D of C's own energy = 2 Sales to D of B´s energy= 1 (C sells the surplus of its own generation and 1 unit from B, not used for its own demand)	4 (10+m) 2 (50+m) -2 (100+m) -1 (50+m+m') **-110+3m–m'**	Purchases from C = 2 Purchases from B = 1	2 (100+m) 1 (50+m+m') **250+ 3m + m'**

3.5.3.2 An algorithm to define bilateral transactions

In what follows an algorithm to define bilateral transactions is presented. The resulting transactions are consistent with the optimal flows. Option Q.2.1 is taken: a country has the priority in the use of cheap resources flowing into its node. We will assume there are no losses, otherwise some corrections should be done as in Bialek (1996). Under the no losses hypothesis optimal flows in each interconnection verify: $t^{i\,*}_{i,j} = t^{j\,*}_{i,j} = t^{*}_{i,j}$.

Let us define:

- $acc(i)$ the set of all nodes which are directly or indirectly accessible from node i following arcs in graph $\mathcal{G}$. It is the union of set $suc(i)$ of i´s successors, with the sets of the successors of every node in $suc(i)$, and so on.

- Source**,** as a node with at least one outgoing arc and no inflowing arcs. Every finite digraph without cycles has at least one source.
- $DO(G,D)$ the optimal dispatch function, given a demand D and a set of generation resources $G = \{(i_r, c_r, P_r, f_r), r = 1,..M\}$, so that $\sum_{r=1}^{M} P_r \geq D$, is the function giving the set of resources allowing the supply of demand D at minimum cost:

  $$DO(G,D) = \{(i_r, c_r, \pi_r, f_r), \quad for\ every\ r \ \ / \pi_r > 0\}$$

  where $\pi_r, r = 1,..M$, are the powers minimizing in the problem:

  $$Min_{\{\pi_r\}} \left[\sum_{r=1}^{M} \int_{0}^{\pi_r} f_r(x) dx \right]$$

  $$s.a: \sum_{r=1}^{M} \pi_r \geq D \ ; \quad 0 \leq \pi_r \leq P_r, \qquad for\ r = 1, \ldots M$$

- $EX(G,D)$ the excedentary resources function given a demand D and the set of generation resources $G = \{(i_r, c_r, P_r, f_r), r = 1,..M\}$, is the function giving the set of excess resources after using resources from G in the optimal dispatch of demand D:

  $$EX(G,D) = \{(i_r, c_r, P_r - \pi_r, g_r) : g_r(x) = f(x + \pi_r), for\ r \ \ / \pi_r < P_r\}$$

  Where π_r are the powers in $DO(G,D)$

- $\alpha.r$, where $r = (i_r, c_r, P_r, f: [0, P_r^{max}])$ and α is a real number with $0 \leq \alpha \leq 1$, is another resource $(i_r, c_r, \alpha P_r, g(x) = f(x/\alpha)$

- $RD_i(Q_1, Q_2)$ displaced resources function in a country i, given two set of resources $Q_1 = \{(i, c, P_c^{Q_1}, f_c) : c \in C_i\}$ and $Q_2 = \{(i, c, P_c^{Q_2}, f_c) : c \in C_i\}$
  such that $P_c^{Q_1} \geq P_c^{Q_1}\ \forall c \in C_i$, is the set of resources

  $\{r = (i, c, P_c^{Q_1} - P_c^{Q_2}, f_r) : c \in C_i / P_c^{Q_1} > P_c^{Q_2}\}$ where

  $f_r: [0, P_c^{Q_1} - P_c^{Q_2}] \rightarrow R$, $f_r(\pi) = f_c(\pi + P_c^{Q_2})$.

  It holds: $\quad P[RD_i(Q_1, Q_2)] = P(Q_1) - P(Q_2) \qquad (6)$

The following algorithm defines transactions with generation resources by defining for every pair $i, j \in N$ the sets:

- V_{ij} resources that i sells to j
- S_i resources from generation units in i, which are used by the country if there is no trade, and are displaced as a result of international trade
- E_i resources that node i receives from its predecessors in the digraph of optimal flows
- F_i resources that node i sends to its successors

G_i^*, G_i^{SC}, D_i, d_i^* which are also used in the algorithm, are already defined.

The algorithm is intended to generalize the procedures followed to construct transactions in the example of the preceding section, defined by Graphic 6.

Start of the algorithm

Set $\mathscr{G}^\nu = \mathscr{G}$, ν=1

Set $E_i = \emptyset$ for every node $i \in N$, $V_{ij} = \emptyset$, for every pair of nodes $i,j \in N$, $j \neq i$

Step 1

For every source i in graph $\mathscr{G}^\nu$:

Step 1.1

With option Q1.1:

If $i \in N_X$ (i is a net exporter):

Let us define $F_i = EX(G_i^*, D_i) \cup E_i$

If $i \in N_M$ (i is a net importer):

Let us define $L_i = DO(E_i, d_i^*)$, $F_i = EX(E_i, d_i^*)$, $S_i = RD_i(G_i^{SC}, G_i^*)$

For every $r = (i_r, c_r, P_r, f_r) \in L_i$, r is considered sold by node i_r to node i, that is, r is added to $V_{i_r i}$. (it holds $i_r \neq i$ for every $r \in L_i$)

With option Q1.2:

Let us define $L_i = DO(E_i \cup G_i^*, D_i)$, $F_i = EX(E_i \cup G_i^*, D_i)$, $L_i^i = \{r = (i_r, c_r, P_r, f_r) \in L_i : i_r = i\}$, $S_i = RD_i(G_i^{SC}, L_i^i)$.

For every resource $r = (i_r, c_r, P_r, f_r) \in L_i$ so that $i_r \neq i$, r is considered sold by node i_r to node i, that is, r is added to $V_{i_r i}$.

With option Q1.3:

Let us define L_i=DO(E_i∪G_i*, D_i), F_i=EX(E_i∪G_i*, D_i), L_i^i={r=(i_r,c_r,P_r,f_r)ϵL_i:i_r=i}, S_i=RD_i(G_i^{SC}, L_i^i).

For every node k ϵ pred(i), all resources in F_{ki} are considered sold by k to i, that is, they are added to V_{ki}.

Step 1.2

For every node jϵ suc(i) let us define F_{ij}={α_j.r: for every rϵF_i} where $\alpha_j = \frac{t_{i,j}^*}{\sum_{j \in suc(i)} t_{i,j}^*}$.

For every node jϵ suc(i) all resources in F_{ij} are added to E_j.

Step 2

Let us define graph $\mathscr{G}^{sig}$ resulting of the deletion in $\mathscr{G}^{\nu}$ of all sources and all the arcs starting in sources.

If $\mathscr{G}^{sig}$ has at least one node:

ν is incremented by 1

Let us set $\mathscr{G}^{\nu} = \mathscr{G}^{sig}$

Go to Step 1.

If $\mathscr{G}^{sig}$ has no nodes the algorithm finishes.

The algorithm has the following features:

- It is finite and ends after a number $Niter$ of iterations, as a consequence of the inexistence of cycles and the finite number of nodes in $\mathscr{G}$.
  Graphs $\mathscr{G}^{\nu}$, for $\nu = 1,..Niter$, don't have cycles either, as they result from the deletion of nodes and arcs in graphs with no cycles.

- Any resource in set F_{ij} (resources node i sends to the set E_j of one of its successors j in Step 1.2), comes from G_i^*, or from E_i. Then $V_{ij} \neq \emptyset$ only if $j \in acc(i)$.

- As a consequence using Proposition I, for any resource r sold by node i to node j, with marginal cost $f_r(P_r)$ it holds:
  $f_r(P_r) \leq CM_i^* \leq CM_j^*$ (7)

- As in the optimal solution the resources in set S_j are not used, then:

$CM_j^* \leq CMg(S_j, 0)$ (8)

- In Step 1.1:
  - With Q1.1: in the net importing nodes (the buyers):

    $\sum_{k \in N} P(V_{ki}) = P(L_i) = d_i^*$ and $P(S_i) = P[RD_i(G_i^{SC}, G_i^*)]$.

    Using (6): $P(S_i) = P\left(G_i^{SC}\right) - P(G_i^*) = D_i - P(G_i^*) = d_i^*$.
  - With Q1.2:

    $$\sum_{k \in N} P(V_{ki}) = P(L_i) - P\left(L_i^i\right) = D_i - P(L_i^i)$$

    and $P(S_i) = P[RD_i\left(G_i^{SC}, L_i^i\right)]$.

    Using (6) $P(S_i) = P\left(G_i^{SC}\right) - P\left(L_i^i\right) = D_i - P\left(L_i^i\right)$.

As a result both with Q1.1 and Q1.2, for every net importing node the amount of power it buys is equal to the amount of power displaced by trade:

$$\sum_{k \in N} P(V_{ki}) = P(S_i)$$

On the contrary with Q1.3: $V_{ki} = F_{ki}$, and $\sum_{k \in N} P(V_{ki}) = \sum_{k \in N} P(F_{ki}) = P(E_i)$, which can be greater than $P(S_i) = P\left(G_i^{SC}\right) - P\left(L_i^i\right) = D_i - P\left(L_i^i\right)$. In the numerical example shown above $P(E_C) = 6$ exceeds $P(S_C) = 5$ by the amount of energy intermediated by country C, equal to 1.

3.5.3.3 Rules to share the gains from trade

With each of the options for Q1, the algorithm defines $\{V_{ij}, S_j\}$ for $i \in N$, $j \in N$, $j \neq i$. Let us call $C_{ij} = \sum_{r \in V_{ij}} P_r$, the power sold by i to j.

The amount of money T_{ij} paid by j to i, for the energy V_{ij}, depends on the method to define transaction prices. We here generalize to the multilateral case the three definitions presented in 3.4.4 in a bilateral context:

With nodal pricing:

$$T_{ij} = C_{ij}[CMg(V_{ij}, C_{ij}) + \alpha'_{ij} \left(CMg(S_j, 0) - CMg(V_{ij}, C_{ij})\right)]$$

Where:

- $j \in acc(i)$ and not necessarily $j \in suc(i)$

- A generalized congestion rent can be defined as $CMg(S_j, 0) - CMg(V_{ij}, C_{ij})$, for $j \in acc(i)$, and the relevant parameters are α'_{ij}, share of seller i in the generalized rent with country j, to be negotiated between i and j. Using (7) and (8) results $CMg(S_j, 0) - CMg(V_{ij}, C_{ij}) \geq 0$.

With cost plus a margin method:

$$T_{ij} = CT\left(V_{ij}, C_{ij}\right) + \sum_{r \in \mathrm{V_{ij}}} \int_0^{P_r} f_r(x) + Min\left(m_r, CMg(S_j, 0) - f_r(x)\right) dx$$

The parameters are the margins m_r for every resource of the seller. P_r is the power of resource $r \in \mathrm{V_{ij}}$.

With share in halves method:

$$T_{ij} = CT\left(V_{ij}, C_{ij}\right) + \frac{1}{2}\left(CT(S_j, C_j) \frac{C_{ij}}{\sum_{i \in N} C_{ij}} - CT\left(V_{ij}, C_{ij}\right) \right)$$

where $C_j = \sum_i C_{ij}$

In this case, the assumption is made that each seller i substitutes the same proportion $\frac{C_{ij}}{\sum_{i \in N} C_{ij}}$ of every resource in S_j. Option Q.1.3 is meaningless for this method as the total power bought by j can exceed the country's demand D_j.

3.6 Conclusions

The paper shows the diversity of possible agreements for international power spot trade, when countries negotiate rules to set energy prices and share the gains from trade, which is the institutional framework in South America.

A family of methods is described to define economically meaningful bilateral transactions, based on the digraph of optimal flows through the interconnections, which has no cycles.

The algorithm to build those transactions iteratively follows the optimal flows. In each iteration the source nodes (countries) of the graph are processed. For each of these nodes the resources bought from the preceding nodes and the resources sent to the following nodes are determined. At the end of each iteration, those source nodes are removed and a new digraph with no cycles results. Three methods to define the transactions are presented that differ in two features: the ability of one country to use with priority the cheaper resources entering its node, and the possibility of intermediating resources.

Once the bilateral transactions are defined, prices for the energy can be determined. Three different criteria to set prices are described, generalizing the methods used for bilateral trade in the recent past in the region: nodal pricing, share in halves of the benefits and cost plus margin.

3.7 Annexes to Chapter 3

3.7.1 Annex I – Two properties of optimal flows

3.7.1.1 Monotonicity of CMgᵢ following a path in the graph of optimal flows – Proposition I*

CMg_i^* is the dual variable associated to constraint (2a), demand supplied at node i.

We will prove that given two nodes i and j, linked by an interconnection, so that at the optimum $t_{i,j}^{i\,*} > 0$, $t_{i,j}^{j\,*} > 0$, then $CMg_i^* \leq CMg_j^*$ holds. Let us call CT^* the total cost in O^*, the optimal solution of problem (2), and $T(t) = t - r_{ij}t^2$ the function giving the power received at j when power t is injected in i.

It holds $T'(t) = 1 - 2r_{ij}t < 1,\ T''(t) < 0$.

Let us suppose $CMg_j^* < CMg_i^*$. We can choose $\varepsilon > 0$ arbitrarily small and taking problem (2) as a base, define a new problem $PP^{\varepsilon j}$ perturbed in j, which in node j has a demand $D_j + \varepsilon$. By the definition of the dual variable of the demand restriction in j, the optimal solution $O^{\varepsilon j}$ of problem $PP^{\varepsilon j}$, has a cost:

$$CT^{*\varepsilon j} = CT^* + CMg_j^*.\varepsilon + \theta^2(\varepsilon) \qquad \text{(A6)}$$

where $\theta^2(\varepsilon)$ is an infinitesimal of order greater than 1.

Assuming the continuity of optimal solutions respect to the demands, in problem $PP^{\varepsilon j}$ the flows from i to j measured at i (t^{ε}) and measured at j ($T(t^{\varepsilon})$) are both positive.

Let us define a second perturbed problem $PP^{\delta i}$, with demand D_j at j, and demand $D_i + \delta(\varepsilon)$, at i, where $\delta(\varepsilon)$ is defined by: $T(t^{\varepsilon} - \delta) = T(t^{\varepsilon}) - \varepsilon$. (A7)

It holds:

- $\delta(\varepsilon) > \varepsilon$, as a result of $\varepsilon = T(t^{\varepsilon}) - T(t^{\varepsilon} - \delta) = T'(t^{\varepsilon} - \alpha\delta)\delta$, with $\alpha \in [0,1]$ and $T' < 1$.
- From (A7) $\delta = t^{\varepsilon} - T^{-1}(T(t^{\varepsilon}) - \varepsilon)$ and then as both T and T^{-1} are continuous at t^{ε}, then $\delta(\varepsilon) \to 0$, for $\varepsilon \to 0$.

A feasible solution $S^{\delta i}$ of $PP^{\delta i}$ is formed by taking the optimal variables from $O^{\varepsilon j}$ (optimal solution of $PP^{\varepsilon j}$) except the flow in arc $i-j$, which is taken equal to $t^{\varepsilon}-\delta$, measured at i. Then the flow measured at j is taken equal to $T(t^{\varepsilon})-\varepsilon$. Those flows are feasible as by hypothesis t^{ε} and $T(t^{\varepsilon})$ are both positive and ε and δ can be chosen arbitrarily small.

The cost $CTS^{\delta i}$ of $S^{\delta i}$ is the same as the cost $CT^{*\varepsilon j}$ of $O^{\varepsilon j}$, as in both of them the generation in every node is identical.

Using (A6): $\quad CTS^{\delta i} = CT^{\varepsilon j} = CT^{*} + CMg_j^{*}.\varepsilon + \theta^2(\varepsilon)$

Therefore as $\delta(\varepsilon) > \varepsilon$, it holds:

$$CTS^{\delta i} < CT^{*} + CMg_j^{*}.\delta + \theta^2(\delta) \qquad \text{(A8)}$$

By the definition of dual variable CMg_i^{*}, the optimal solution $O^{\delta i}$ of problem $PP^{\delta i}$ has cost: $\quad CT^{*\delta i} = CT^{*} + CMg_i^{*}.\delta + \theta_2^2(\delta) \qquad$ (A9)

By (A8) and (A9):

$$CT^{*\delta i} - CTS^{\delta i} > \left(CMg_i^{*} - CMg_j^{*}\right)\delta + \theta_3^2(\delta)$$

If δ is taken sufficiently small it would hold $CT^{*\delta i} - CTS^{\delta i} > 0$. The feasible solution $S^{\delta i}$ of problem $PP^{\delta i}$ would have a smaller cost than the optimal solution $O^{\delta i}$. Therefore $CMg_i^{*} > CMg_j^{*}$ cannot be true.

Let us observe that this reasoning applies regardless of arc $i-j$ having or not spare capacity at the optimal flows.

3.7.1.2 Non-existence of cycles in the digraph of optimal flows – Proposition II

Let us call: $O^{*} = (\{P_c^{*}\}, \{t_{i,j}^{i\,*}\}, \{t_{i,j}^{j\,*}\})$ the optimal solution to problem (2).

Let us suppose there is a cycle of arcs $(a_1, a_2, \dots a_K)$, with flows outgoing from their respective initial nodes $(s_{a_1}, \dots\dots s_{a_K})$ and incoming into their respective final nodes $(f_{a_1}, \dots\dots f_{a_K})$, and constants $(r_{a_1}, \dots\dots r_{a_K})$ determining their losses. Node s_{a_1}, the initial node of arc a_1, has a positive generation g_1. By (2c) it holds for every arc a_k:

$$f_{a_k} = s_{a_k} - r_{a_k}\left(s_{a_k}\right)^2 =: T_k\left(s_{a_k}\right), \quad k = 1, \dots K \qquad \text{(A1)}$$

Function $T_k(.)$ gives the power received at the final node of arc a_k, as a function of the power injected in the initial node of the arc. It holds: $T_k'(s_{a_k}) = 1 - 2\, r_{a_k} e_{a_k} < 1$.

Taking $\varepsilon > 0$ and sufficiently small, another set of flows through the cycle can be determined, with outgoing flows $\sigma = (\sigma_{a_1}, \ldots \ldots \sigma_{a_K})$, and incoming flows

$\varphi = (\varphi_{a_1}, \ldots \ldots \varphi_{a_K})$ such that:

$$s_{a_1} - \sigma_{a_1} = \varepsilon \qquad \text{(A2)}$$

$$\varphi_{a_k} = T_k(\sigma_{a_k}), for\ k = 1, \ldots K \qquad \text{(A3)}$$

$$s_{a_k} - \sigma_{a_k} = f_{a_{k-1}} - \varphi_{a_{k-1}} \text{ for } k = 2, \ldots K \qquad \text{(A4)}$$

From (A1) and (A3) we have, for $k = 1, \ldots K$:

$$f_{a_k} - \varphi_{a_k} = T_k(s_{a_k}) - T_k(\sigma_{a_k}) = T'_k(\theta_{a_k})(s_{a_k} - \sigma_{a_k})\ with\ \theta_{a_k} \in (\sigma_{a_k}, s_{a_k}) \qquad \text{(A5)}$$

And by (A4): $f_{a_k} - \varphi_{a_k} = T'_k(\theta_{a_k})(f_{a_{k-1}} - \varphi_{a_{k-1}})$, for $k = 2, \ldots K$

Applying (A4) for $k = 2, \ldots K$, (A5) for $k = 1$, and (A2) we arrive at:

$$f_{a_K} - \varphi_{a_K} = T'_K(\theta_{a_K}) \ldots T'_1(\theta_{a_1})\varepsilon_1 =: \varepsilon_{K,1} < \varepsilon_1$$

Those flows and the rest of the variables in O^*, fulfill all the equations (2b), (2c) and (2d) and equations (2a) in every node except s_{a_1}, where there is now a surplus power $\varepsilon_1 - \varepsilon_{K,1} > 0$. If the power generated in s_{a_1} is reduced by that amount, the new generation at the node $\gamma_{K,1} := g_1 - (\varepsilon_1 - \varepsilon_{K,1})$, the flows in σ and φ and the other values in O^* are a feasible solution of the problem, with a smaller generation cost, which contradicts the optimality of O^*.

3.7.2 Annex II – Numerical simulations of the nodal pricing method

The goal of this annex is to describe the numerical simulations to test in a particular case the following propositions:

- Given two countries of different size, if the topology of the interconnection grid, the capacities of the lines and the levels of the marginal cost curves are chosen randomly, the benefits per unit of energy traded are greater, the smaller the country´s power system.

- In the same context, if congestion rents are shared in halves in every interconnection, an increase in the interconnection capacity tends to favor more the smaller countries.

A series of 1000 problems of optimal trade were resolved, all with four countries with demands 1, 5, 10 and 30 GW. An interconnection grid was defined for every problem. The existence of every one of the six possible lines linking the four countries was determined randomly, with probability 0.3 of existence. Configurations with non-connected graphs were rejected, and new draws were performed in those cases. The capacity of every line $\{i, j\}$ was chosen as $f_d.Min(D_i, D_j)$ in both directions. For f_d three values were used: 0.25, 0.5, 1.

The marginal cost function for country i without trade $CMg_i(g_i)$ was defined by:

$$CMg_i = c\left(\frac{g_i}{D_i}\right) \times \mu_i = \left[0.100\left(\frac{g_i}{D_i}\right) + 0.050\left(\frac{g_i}{D_i}\right)^2\right] \times \mu_i$$

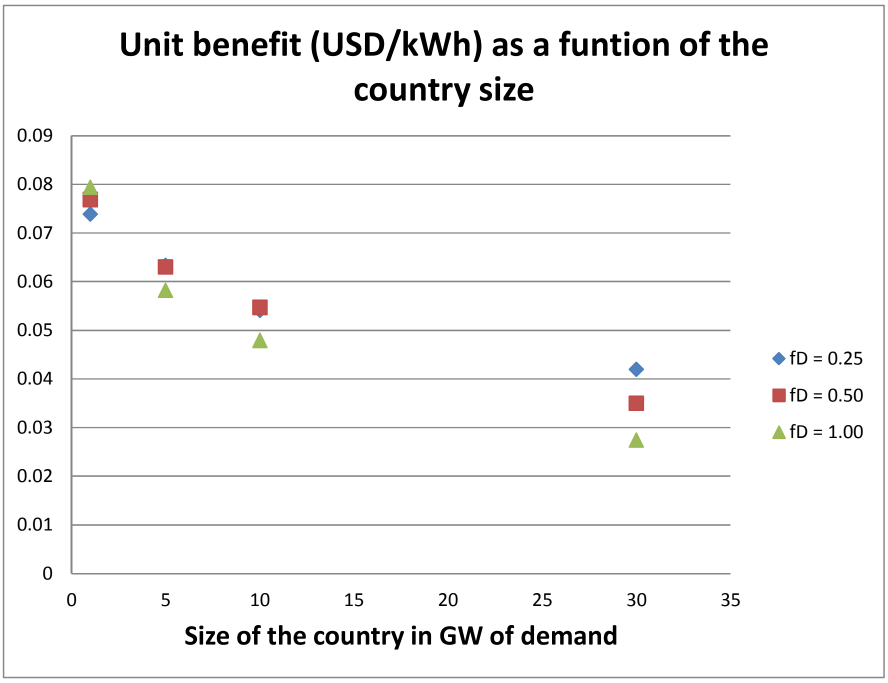
where μ_i was the result of a draw with uniform distribution in [0,2]

g_i and D_i are expressed in GW and CMg_i in USD/kWh.

The variable μ_i can be interpreted as representing a random shock due to availability in primary sources for generation. Function $c\left(\frac{g_i}{D_i}\right)$ determines the form of the marginal cost curve before that random shock. The dependency of $\frac{g_i}{D_i}$ is a way of defining marginal cost curves before random shocks that differ only as a consequence of the scale of the country, as the proportions between different kinds of generation resources in all the countries are the same.

The results of the simulation are presented in the following graphic, showing the benefit per unit of net energy traded, expressed in USD/kWh. The net energy traded by country i is defined as: $CN_i = \left|\sum_{j \in A_i^-} t_{j,i} - \sum_{k \in A_i^+} t_{i,k}\right|$.

The three series of four points correspond to the three values chosen for f_d, each one corresponding to a different level of average interconnection capacity.



The results show the two properties we wanted to test: i) the unit benefits are smaller the larger the size of the country, and ii) as the average interconnection capacity increases (f_d is greater) the relative advantage of the smaller countries increases.

4 THE EFFECT OF INTERNATIONAL POWER TRADE ON THE DESIGN OF THE OPTIMAL GENERATION PORTFOLIO

4.1 Introduction

The subject under analysis is the effect of variations in the prices of international energy trade on the optimal design of a country's power generation system, and particularly the determination of conditions leading to imports to be a complement or a substitute of wind capacity.

The optimal design of a generation system is a problem of practical importance since in many countries, even with competitive markets for generation, the authorities conduct planning processes to shape the power system. The goal is to find the amount of capacity to be installed for every available kind of generation unit, assuming that capacity will be used optimally. As investments in power plants are irreversible, the problem is dynamic, since present investment decisions affect the future optimal short run performances of the system. In this paper we will use a simplified static model, with two kinds of local generation resources:

- One thermal plant technology, with unit size small enough to assume the available power probability distribution to be concentrated, as a large number of units with independent outages are installed. This technology will be represented by a single fixed available power.
- Wind power, with random available power. The dispersion of available power is a feature in common with other renewable sources as photovoltaic solar energy. In a simplified way we will assume two levels of wind power availability corresponding to two types of days, with and without wind.

Additionally there is the possibility of international trade. The country can import energy form neighbor countries without restrictions, and export wind energy surpluses. In the model the energy imports are fully characterized by a single constant price.[4]

[4] Besides neglecting the variability of prices, the treatment given here to imports does not take into account the perception of risk from an excessive dependence on imported energy. These considerations could be included in the model by imposing restrictions to the problem of optimal design, to ensure enough installed capacity is installed locally.

This chapter is organized as follows. In section 2 some relevant literature is briefly discussed and the model is presented in detail. The two nested problems are formulated: the determination of the optimal operation (usually called the economic dispatch) given the installed capacity of each generation resource, and the determination of the portfolio with optimal installed capacities when optimal operation is assumed. The different possible cases of optimal solutions are discussed and necessary conditions for the parameters are found for the validity of each case. Section 3 analyzes the trajectories of optimal generation portfolios when imports price increases from zero, showing the transitions between the different cases of optimal solution found in section 2. Three different classes of trajectories are found. Special attention is given to the problem of determining whether imports are a substitute or a complement of wind installed capacity. This general analysis is illustrated with a numerical example, for the particular case of demand with linear load duration curve. Section 4 presents the conclusions of the chapter, and section 5 contains annexes with the detailed proofs of the results from sections 2 and 3, and some properties of the objective function and the optimal generation portfolios.

4.2 Modelling of generation and demand

4.2.1 Relevant literature

The analytical modelling of demand and supply of the power generation sector is the subject of a vast literature.

There is a large number of papers analyzing the problem of optimal tariffs in the electric sector, the most significant of them reviewed by Joskow (1977). To solve this problem it is necessary to determine, in a very simplified way, the optimal design of the generation portfolio. Therefore simplified models for demand and supply are required.

The most frequent representation for the supply is a set of generation resources with a fixed cost per unit of installed capacity, a constant variable cost and a maximum capacity.

Another solution could be to increase import prices to include the expected outage costs derived from that excessive dependence.

Demand is generally modelled by a probability distribution, as in Chao (1983), representing both the seasonal and daily variation patterns and the randomness. In the language of the power sector this distribution is often called the load duration curve (Vardi and others, 1977), and is represented with the y-axis measuring the amount of power demanded, and the x-axis measuring the probability or the amount of hours with demand exceeding a certain power.

If all generation resources have constant maximum capacities and no additional constraints exist on the amount of energy that each of them can supply, then the their optimal operation can be intuitively described as the result of filling the load duration curve, from bottom to top, using the generation resources in the increasing order of their variable cost (Crew and Kleindorfer, 1977). In this way the available power of resources with lesser variable costs is used during the longest possible period. This is the simplest form of the economic dispatch, also applied sometimes in computational models used in practice as the WASP (IAEA, 2001). Models to develop optimal tariffs can add more complexity as multiple demand periods, outage costs and uncertainty in the availability of generation units, as in Kleindorfer and Fernando (1993).

On the other side, the increasing importance of wind energy for power systems has given place to many economic studies, putting aside the numerous technical papers elaborated mainly from the point of view of engineering.

Kennedy (2006) using load duration curves and simplified economic dispatches estimates the social benefits of the introduction of wind energy in a power system, without considering international energy trade. Some papers study the role of wind energy using portfolio analysis, where wind power installed capacity is an asset that can reduce cost volatility resulting from fuel costs uncertainty. Doherty, Outhred, and O'Malley (2006) determine the effect of the installation of wind power on the efficient portfolio frontier in Ireland. Huang and Wu (2008) study the impact of renewable energies on the power system in Taiwan, considering an objective function which includes the average and the variance of supply costs.

The interaction of international trade and a high share of wind energy in generation is analyzed by Özdemir and others (2013). In the European energy spot markets, as the participation of intermittent wind energy supplies is increasing, there is a need for firm generation capacity to fill the gaps. If one of the countries, in this case Germany, creates a market to remunerate installed capacity and exports energy when the neighbor countries experience an increase in net demand as a result of wind shortage, then these countries will act as free riders. Meibom and Sbrensen (1999) study the international trade resulting from wind power in the Nordic Power Pool, and obtain numerical results, but do not attempt to obtain general analytical results.

The aim of the present chapter is to develop an analytical model to study the effect of international energy trade on the optimal amount of wind installed capacity in a power system. The subject is relevant for countries with both a high share of wind power in their generation systems, and a large installed capacity in interconnections with other power systems. Uruguay is perhaps the better example of this situation. The country´s energy policy has determined an expansion in wind capacity to reach 25% of energy supply from wind energy by 2017 (DNE, 2013). Besides, the interconnection capacity with Argentina is 2000 MW and with Brazil will increase to 570 MW by 2015. These capacities should be compared with a maximum demand of 1800 MW.

4.2.2 Modelling of the resources and demand and hypotheses on the parameters

Let us consider a period, for instance a year, with duration H measured in hours. We will call N the number of days in the period, so $H = N.24$.

Let P_e and P_t be the installed capacities measured in MW of two technologies: wind and thermal.

Wind capacity has a yearly fixed cost per MW equal to f_e, expressed in USD/MW, and has null variable cost. The amount of energy generated in a given day by wind capacity P_e is not controllable. With probability π_1 the day is windy and available power is $\alpha_1 P_e$, and with probability $\pi_2 = 1 - \pi_1$ the available power is equal to $\alpha_2 P_e$, with $\alpha_1 > \alpha_2$. Let us call $P_e^i = \alpha_i P_e$, with i=1,2. In both cases wind power is assumed constant during the day. We can then speak of type 1 and type 2 days, depending on the wind power available. Let us assume $\alpha_2 = 0$.

Thermal capacity has a yearly fixed cost per MW equal to f_t and variable energy cost c_t. We will assume the thermal installed capacity P_t is always entirely available.[5] The values of f_t , f_e and c_t are strictly positive.

The surpluses of wind energy can be exported receiving a price v_{exp} expressed in USD/MWh. Energy can be imported at price c_{imp} expressed in USD/MWh, with no restrictions, and importing is always preferable to rationing the demand.

The assumption of constant import and export prices would be a gross simplification in terms of a realistic calculation model, but is reasonable for the goal of this model, if the local country is small enough and its supply of wind energy is not correlated with the generation costs in the neighbor country.[6]

The demand function $\mathscr{P}(t):[0,24]\rightarrow R$, determines the power demanded at each instant of the day, with continuous time measured in hours. For simplicity, we will assume all days of the year have the same demand curve.

As mentioned before, a convenient representation of the demand used in analytical and numerical models is the load duration curve, which we will denote by $D(P)$. The supply cost is the same with $D(P)$ and with $\mathscr{P}(t)$, if dynamic phenomena are neglected. We define $D(P): [0,\ P_{max}] \rightarrow [0,24]$ as the measure expressed in hours of the set $T(P)=\{t: \mathscr{P}(t)\geq P\}$. In words, $D(P)$ is the total duration of the periods of the day when the demand $\mathscr{P}(t)$ is greater or equal to power P expressed in MW.

Graphic 2.1 shows the appearance of both curves. P_{max} and P_{min} are the maximum and minimum power demanded during the day. $D(P) = 24$ for $P < P_{min}$.

Let us assume that $D(P)$ is differentiable, that for $P > P_{min}$ it is strictly monotonically decreasing, and that $D(P_{max}) = 0$.

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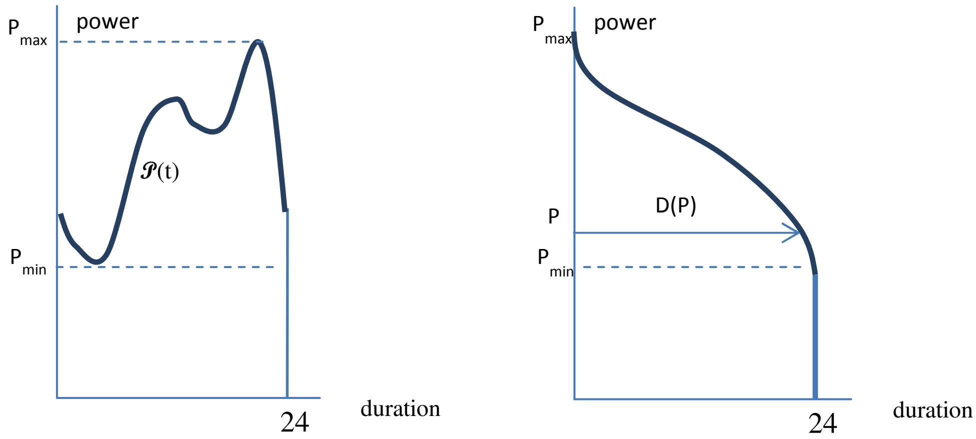
[5] No constrains are imposed on the operation of thermal plants such as minimum power or maximum power ramps. If outages were taken into account P_t could be interpreted as a net available power, and f_t should be increased in consequence.

[6] The hypothesis does not hold if the neighbor country has a strong wind power generation and its winds are strongly correlated with the local country's.

Graphic 2.1

Demand and load duration curves



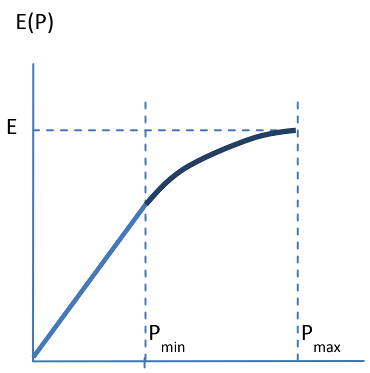
Let us call $P(d): [0,24) \rightarrow (P_{min}, P_{max}]$ the inverse of function $D(P)$, also strictly decreasing. It holds $P_{max} = P(0)$.

We define $P_{min} = P(24)$, the minimum value of the demand.

Let us suppose $P_{max} > P_{min} > 0$.

We will call base of the load duration curve the rectangle below P_{min}.

Graphic 2.2



We define $E(P)$, as: $E(P) = \int_0^P D(x)dx$

Graphic 2.2 shows the appearance of $E(P)$.

The total amount of energy to be supplied daily in each day of the year (E) is:

$$E = \int_0^{P_{max}} D(x)dx$$

We will assume that the unit costs of generation resources and the prices of trade result from the sampling of random variables with continuous densities, and therefore we will omit the analysis of some cases with null probability.

We will assume the following hypotheses:

- $v_{exp} < c_t$ (2.1)

  This means that thermal plants will never be used to export.

- The amount of imported energy is not upper bounded, the country can import as much energy as it needs to supply the demand.[7] This hypothesis is not unrealistic for a small country with very strong interconnection links with its big neighbors.

- $\frac{f_e}{\alpha_1} > H\pi_1 v_{exp}$ (2.2)

  The costs of wind energy and the export price are such that it is not convenient to install wind capacity with the sole purpose of exporting energy in type 1 days. It is immediate to conclude that the optimal portfolios fulfill: $P_e \alpha_1 \leq P_{max}$.

- Even if $c_{imp} < v_{exp}$, it is not possible to import energy to make profits from arbitrage, re-exporting the imported energy. All imported energy is used to supply the local demand.

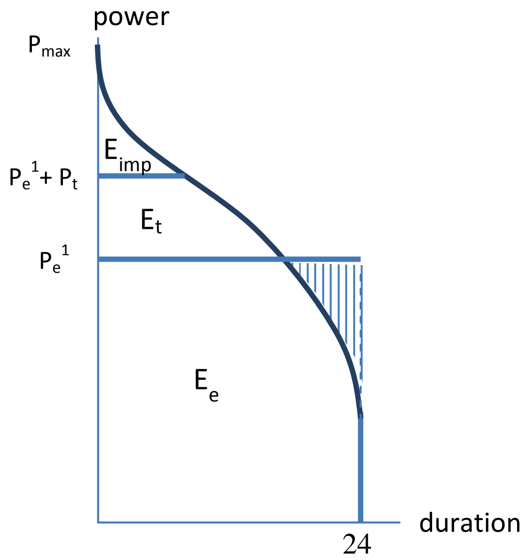
The objective of the mathematical problem is to minimize the total expected cost in a year, which consists of the investment costs of both technologies (thermal and wind) plus the net expected variable cost. The net variable cost is the addition of the variable cost of thermal plants and the import costs, minus the income from exports.

The problem of finding the operation in the short run of the fixed installed capacities, that yields the minimum net variable cost in a day, is called the optimal load dispatch and is addressed in part 2.3. The problem of determining the optimal capacities to be installed of each type of technology, assuming they will be used optimally is the subject of part 2.4

4.2.3 Optimal dispatch given the installed capacities

[7] This hypothesis is not unrealistic for a small country with very strong interconnection links with its big neighbors, like Uruguay.

Graphic 2.3



4.2.3.1 Optimal dispatch if $c_t<c_{imp}$

Under the hypothesis $c_t < c_{imp}$ wind energy is dispatched first, then thermal generation, and last the imports.

Graphic 2.3 shows the dispatch in a type 1 day, if $P_e^1 > 0, P_t > 0$ and $P_e^1 + P_t < P_{max}$.

$P_e^1 = \alpha_1 P_e$ is the available wind power during all the hours in a type 1 day.

Wind energy with zero variable cost, supplies the demand below $P = P_e^1$, and generates a surplus represented by the vertically hatched area, exported at a price v_{exp}.

The amount of exported energy is: $E_{exp} = 24P_e^1 - E(P_e^1)$

The energy from thermal plants supplies the area between $P = P_e^1$ and $P = P_e^1 + P_t$; the thermal plants generate at full load during $D(P_e^1 + P_t)$ hours, and have non zero generation during $D(P_e^1)$ hours. The amount of energy supplied by thermal plants in the whole day is equal to $E(P_e^1 + P_t) - E(P_e^1)$.

Above the level $P_e^1 + P_t$ the demand is supplied with imports. The amount of imported energy is equal to a $E - E(P_e^1 + P_t)$.

The problem of optimal dispatch is the same for all days of the same type. Therefore the total net expected variable cost CO in a year is equal to:

$$CO = \pi_1 N\, CO_1 + \pi_2 N\, CO_2$$

where CO_i is the optimal net variable cost in a day of type i.

Let us call p_{t1} and p_{t2} the maximum power from thermal plants in type 1 and type 2 days respectively.

The optimal dispatch problem in a type i day, given the installed capacities P_t and P_e can be formulated as follows.

The net variable cost in a type i day is:

$$CO_i(P_e, P_t, p_{ti}) = [E - E(P_e\alpha_i + p_{ti})]c_{imp} + [E(P_e\alpha_i + p_{ti}) - E(P_e\alpha_i)]c_t$$
$$-[24P_e\alpha_i - E(P_e\alpha_i)]v_{exp} \qquad (2.3)$$

where p_{ti} is subject to: $P_t > p_{ti},\ p_{ti} \geq 0, p_{ti} \leq P_{max} - P_e\alpha_i$.

The optimal dispatch problem is:

$$Min_{p_{ti}} CO_i(P_e, P_t, p_{ti})$$

subject to the same three constraints.

The constraint $p_{ti} \leq P_{max} - P_e\alpha_i$ can be omitted without changing the problem, because any solution with $p_{ti} \geq P_{max} - P_e\alpha_i$ has a value of the objective $CO_i(P_{max} - P_e\alpha_i, P_e, P_t)$, as a result of $E(P) = E(P_{max})$ for every $P \geq P_{max}$. Only the solution with $p_{ti} = P_{max} - P_e\alpha_i$ is economically meaningful.

The constraint $p_{ti} \geq 0$, can also be omitted as:

$$\frac{\partial CO_i}{\partial p_{ti}} = -D(P_e\alpha_1 + p_{ti})(c_{imp} - c_t) \leq 0 \qquad (2.4)$$

The optimal dispatch problem can be formulated then as:

$$Min_{p_{ti}} CO_i(P_e, P_t, p_{ti})$$

s.t. $\quad P_t - p_{ti} \geq 0$

Applying the Kuhn-Tucker conditions, using $E' = D$, and calling λ_i the dual variable associated to the constraint, the following are necessary conditions for optimality:

$$D(P_e\alpha_i + p_{ti})(c_{imp} - c_t) = \lambda_i \qquad (2.5)$$

$$\lambda_i \geq 0$$

$$\lambda_i(P_t - p_{ti}) = 0$$

At the optimum:

- Either $\lambda_i = 0$ and $D(P_e\alpha_i + p_{ti}) = 0$, and therefore $P_e\alpha_i + p_{ti} \geq P_{max}$, and in particular $P_e\alpha_i + p_{ti} = P_{max}$.

  This means thermal power supplies all the demand above the level of wind generation.
- Or $\lambda_i > 0$, $p_{ti} = P_t$, and $\lambda_i = D(P_e\alpha_i + p_{ti})(c_{imp} - c_t)$. Thermal plants are dispatched at their maximum capacity but imports are still necessary, as $D(P_e\alpha_i + P_t) > 0$ and therefore $P_e\alpha_i + P_t < P_{max}$

<u>Proposition 2.1</u>: Let us assume $\alpha_1 > \alpha_2 = 0$. Then $p_{t2} \geq p_{t1}$.

Proof:

- If $\lambda_1 = 0$ and $\lambda_2 = 0$, then $P_e\alpha_1 + p_{t1} = p_{t2} = P_{max}$. As $P_e\alpha_1 > 0$, then $p_{t2} \geq p_{t1}$.
- If $\lambda_1 = 0$ and $\lambda_2 > 0$, then $p_{t1} \leq P_t$ and $p_{t2} = P_t$, so the proposition holds.
- If $\lambda_1 > 0$, $p_{t1} = P_t$ and $\lambda_2 = 0$, $p_{t2} < P_t$, then using (2.5) for $i = 1$, it would hold D$(P_e\alpha_1 + p_{t1}) > 0$, $P_e\alpha_1 + P_t < P_{max}$. Using (2.5) for $i = 2$ would yield $D(p_{t2}) = 0$, $p_{t2} \geq P_{max}$. As a result $P_e\alpha_1 + P_t \leq p_{t2}$, which is not possible as $\alpha_1 > 0$ and $P_t \geq p_{t2}$.
- If $\lambda_1 > 0$, $\lambda_2 > 0$ then $p_{t1} = P_t$ and $p_{t2} = P_t$, so the proposition holds. ■

Therefore it holds $p_{t2} \geq p_{t1}$: the maximum thermal power in type 2 days is always greater or equal the maximum thermal power in type 1 days.

4.2.3.2 Optimal dispatch if $c_t \geq c_{imp}$

When imports are cheaper than thermal energy, then the optimal dispatch has no thermal energy.

In the particular case when $c_t = c_{imp}$, there are optimal dispatches with thermal energy but as thermal capacity has a non-zero fixed cost, the optimal portfolio will have no thermal plants.

Then the net variable cost in each type i day is:

$$CO_i(P_e, 0, 0) = [E - E(P_e\alpha_i)]c_{imp} - [24P_e\alpha_i - E(P_e\alpha_i)]v_{exp} \qquad (2.6)$$

for any installed capacity P_t.

4.2.4 Optimal capacities

Without loss of generality we can assume that at the optimum $p_{t2} = P_t$, as $p_{t2} \geq p_{t1}$ by Proposition 2.1. Otherwise if $P_t > p_{t2}$ there would be a surplus of unused thermal capacity.

The following table summarizes the formulas for net variable costs found above:

Type of day	Expected number of days	Net variable cost in a day when $c_t<c_{imp}$	Net variable cost in a day when $c_t \geq c_{imp}$, with $P_t=p_{ti}=0$
1	$\pi_1 N$	$CO_1(P_e,P_t,p_{t1}) = [E - E(P_e\alpha_l+p_{t1})]c_{imp}$ $+ [E(P_e\alpha_l+p_{t1}) - E(P_e\alpha_l)]c_t$ $- [24.P_e\alpha_l - E(P_e\alpha_l)]\, v_{exp}$	$CO_1(P_e, 0, 0) = [E - E(P_e\alpha_l)]c_{imp}$ $- [24.P_e\alpha_l - E(P_e\alpha_l)]\, v_{exp}$
2	$\pi_2 N$	$CO_2(P_e, P_t, P_t) = [E - E(P_t)]c_{imp} + E(P_t)\, c_t$	$CO_2(P_e, 0, 0) = E\, c_{imp}$

In what follows the Kuhn-Tucker conditions are applied to the problem of optimal capacities, searching for local minima, which will be candidates to solve the global problem. Depending on the values of the parameters, different cases for local minima are possible. We will determine necessary conditions on the parameters for each case, and explore the unicity of the local minimum. We will call those local minima K-T solutions.

4.2.4.1 *Problem when $c_t<c_{imp}$*

The objective to minimize, the total yearly cost including fixed capacity costs, can be expressed as:

$$CT(P_e, P_t, p_{t1}) = P_e f_e + P_t f_t + \pi_1 N\, CO_1(P_e, P_t, p_{t1}) + \pi_2 N\, CO_2(P_e, P_t, p_{t1})$$

The optimal design problem is then:

$$Min_{P_e,P_t,p_{t1}} CT(P_e, P_t, p_{t1})$$

s.t.

$P_t - p_{t1} \geq 0$ dual variable λ

$P_e \geq 0$ dual variable ν

$P_t \geq 0$ dual variable μ

Where:

$$CT(P_e, P_t, p_{t1}) = P_e f_e + P_t f_t +$$

$$\pi_1 N\{[E - E(P_e\alpha_1 + p_{t1})]c_{imp} + [E(P_e\alpha_1 + p_{t1}) - E(P_e\alpha_1)]c_t - [24P_e\alpha_1 - E(P_e\alpha_1)]v_{exp}\} +$$

$$\pi_2 N\{[E - E(P_t)]c_{imp} + E(P_t)c_t\} \quad (2.7)$$

Using the same arguments as in 2.3 constraints $p_{ti} \leq P_{max} - P_e\alpha_i$ and $p_{ti} \geq 0$ can be omitted.

The necessary Kuhn-Tucker conditions are:

$$P_e) \quad f_e - \alpha_1\pi_1 N\{D(P_e\alpha_1 + p_{t1})(c_{imp} - c_t) + D(P_e\alpha_1)(c_t - v_{exp}) + 24v_{exp}\} = \nu \quad (2.8)$$

$$P_t) \quad f_t - \pi_2 ND(P_t)(c_{imp} - c_t) = \lambda + \nu \quad (2.9)$$

$$p_{t1}) \quad \pi_1 ND(P_e\alpha_1 + p_{t1})(c_{imp} - c_t) = \lambda \quad (2.10)$$

$\lambda, \nu, \mu \geq 0$

$\lambda(P_t - p_{t1}) = 0$

$\mu P_t = 0$

$\nu P_e = 0$

The qualification of constraints conditions hold, as the constraints are linear and the associated normal vectors are linearly independent.

From (2.9) and (2.10) it holds:

$$f_t - [\pi_1 ND(P_e\alpha_1 + p_{t1}) + \pi_2 ND(P_t)](c_{imp} - c_t) = \mu \quad (2.11)$$

The following table summarizes the cases and describes briefly the solutions in each of them. A detailed analysis is presented in Annex 1 of this chapter.

Cases	Dual variables			Features of the solution	
	Marginal value of installed thermal capacity	Marginal value of installed wind capacity	Thermal power in type 1 days constraint	Optimal controls	Other relevant features
A1	μ=0	ν=0	λ=0	$P_t \geq 0$, $P_e \geq 0$, $P_t \geq p_{t1}$	Imports occur only in type 2 days.
A2			λ >0	$P_t \geq 0$, $P_e \geq 0$, $P_t = p_{t1}$	Imports in both types of day. Thermal capacity is used at full load in both types of day.
B1		ν>0	λ=0	$P_t \geq 0$, P_e=0, $P_t \geq p_{t1}$	Demand in both types of day is supplied purely with thermal energy.
B2			λ >0	$P_t \geq 0$, P_e=0, $P_t = p_{t1}$	Thermal capacity is used at full load in both types of day. There is no wind capacity. Optimal dispatch is identical in both types of day.
C1	μ>0	ν=0	λ=0	P_t=0; $P_e \geq 0$	No thermal capacity is installed. In type 1 days wind supplies the whole demand, and in type 2 the whole supply is imported.
C2			λ >0	P_t=0; $P_e \geq 0$, p_{t1} = 0	No thermal capacity is installed. In type 2 days the whole demand is imported. Type 1 days also have imports.
D1		ν>0	λ=0	$P_t = 0$, $P_e = 0$	Annex 1 proves no solutions of this kind can exist for any parameter values.
D2			λ >0	$P_t = 0$, $P_e = 0$	No thermal or wind capacity is installed. The whole demand in both types of day is imported.

The values of dual variables ν and μ at the optimum have an intuitive interpretation as the marginal values of installed wind and thermal capacity:

(-ν) is the net saving at the optimum from having a marginal amount of additional wind capacity, displacing thermal energy and imports, when P_t and p_{t1} are fixed.

(-μ) is the net saving at the optimum from having a marginal amount of additional thermal capacity, displacing imported energy, when P_e and p_{t1} are fixed.

4.2.4.2 Problem when $c_t \geq c_{imp}$

Under this hypothesis we know the optimal thermal capacity to be installed is zero, as imported energy is cheaper and has no fixed capacity cost. The objective of the problem is:

$Min_{P_e} CT^*(P_e)$
s.t.

$P_e \geq 0$ dual variable ν

Where:

$$CT^*(P_e) = P_e f_e + \pi_1 N\{[E - E(P_e\alpha_1)]c_{imp} - [24P_e\alpha_1 - E(P_e\alpha_1)]v_{exp}\} + \pi_2 N c_{imp} \tag{2.12}$$

Neglecting the part of the objective which does not depend on the control variable we obtain the following new simplified objective:

$$Min_{P_e} \quad P_e f_e - \pi_1 N\{E(P_e\alpha_1)(c_{imp} - v_{exp}) + 24\alpha_1 P_e v_{exp}\}$$

Kuhn-Tucker necessary conditions are:

$$P_e) \quad f_e + \pi_1 N\{-\alpha_1 D(P_e\alpha_1)(c_{imp} - v_{exp}) - 24\alpha_1 v_{exp}\} = \nu \tag{2.13}$$

$$\nu \geq 0$$

$$\nu P_e = 0$$

The following table presents the resulting possible cases. A detailed analysis is presented in Annex 2 of this chapter.

Case	Marginal value of installed wind capacity	Features of the solution	
		Optimal controls	Other relevant features
E1	$\nu=0$	$P_e \geq 0$, $P_t=0$	In type 1 days both wind and imports are used. In type 2 days the whole demand is imported.
E2	$\nu>0$	$P_e=0$, $P_t=0$	The whole demand in both types of day is imported.

4.2.5 Summary of necessary conditions and features of the K-T solutions in the different cases

The detailed analysis in Annexes 1 and 2 obtains the Kuhn-Tucker necessary conditions for each possible case of local minima, and the constraints in the parameters for each case to occur.

Some of the constraints in the parameters and its logical complements appear repeatedly and in the exposition are denoted as follows:

$(*\,1)\qquad f_t \leq H(c_{imp} - c_t)$

$(No * 1)\qquad f_t > H(c_{imp} - c_t)$

$(*\,2)\qquad f_e/\alpha_1 \leq \pi_1 H c_{imp}$

$(No * 2)\qquad f_e/\alpha_1 > \pi_1 H c_{imp}$

$(*\,3)\qquad f_e/\alpha_1 \leq \pi_1 H c_t$

$(No * 3)\qquad f_e/\alpha_1 > \pi_1 H c_t$

$(*\,4)\qquad f_t \leq \pi_2 H(c_{imp} - c_t)$

$(No * 4)\qquad f_t > \pi_2 H(c_{imp} - c_t)$

$(*5) \qquad f_e/\alpha_1 > \pi_1(f_t + Hc_t)$

$(*6) \qquad f_t > \left[\frac{f_e}{\alpha_1 \pi_1 N} - 24v_{exp}\right]\frac{(c_{imp}-c_t)}{(c_{imp}-v_{exp})}N\pi_1 + \pi_2 H(c_{imp} - c_t)$

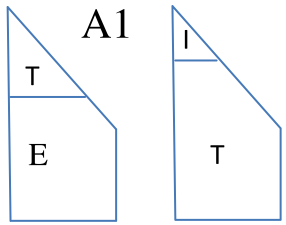
The following graphics summarize the possible optimal cases, and show:

- The necessary conditions in the parameters to allow the existence of minima in each case
- The unicity or multiplicity of optimal solutions
- The variation of optimal controls P_e and P_t as functions of c_{imp} y v_{exp}.

For each case the optimal dispatch is represented with two graphics, one for each type of day (type 1 on the left and type 2 on the right).

Sub-case A2-b, is the particular case of A2 when $\alpha_1 P_e \leq P_{min}$, which is useful to analyze separately as will be shown in Annex 3.

A1



$c_{imp} \geq c_t$

(*4) $f_t \leq \pi_2 H(c_{imp}-c_t) \rightarrow$ (*1)

(*3) $f_e/\alpha_1 \leq \pi_1 Hc_t \rightarrow$ (*2) y (No*5)

If $f_t = \pi_2 H(c_{imp}-c_t)$ infinite solutions with $P_t \leq P_{min}$

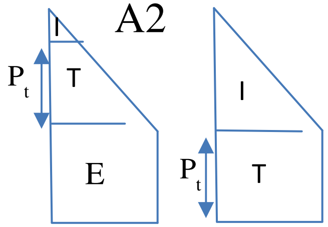
If $f_t < \pi_2 H(c_{imp}-c_t)$ unique solution in the case with $P_t > P_{min}$, with P_t increasing in c_{imp} and independent of v_{exp}

If $f_e/\alpha_1 = \pi_1 Hc_t$, infinite solutions with $P_e\alpha_1 \leq P_{min}$

If $f_e/\alpha_1 < \pi_1 Hc_t$ unique solution in the case with $P_e\alpha_1 > P_{min}$, P_e indep. of c_{imp} and increasing in v_{exp}

lim $P_t = P_{max}$, when $c_{imp} \rightarrow +\infty$

A2



$c_{imp} \geq c_t$

(*1) $f_t \leq H(c_{imp}-c_t)$

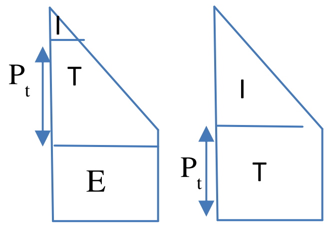
(*2) $f_e/\alpha_1 \leq \pi_1 Hc_{imp}$

Each of the following is a sufficient condition for unicity of the solution in the case:

- $P_{max} > P_e\alpha_1 > P_{min}$ and $P_{max} > P_t > P_{min}$
- $P_e\alpha_1 > P_{min}$ and $P_e\alpha_1 + P_t < P_{max}$
- $P_t > P_{min}$ and $P_e\alpha_1 + P_t < P_{max}$

P_e increasing in v_{exp}

A2-b (particular case of A2 when $\alpha_1 P_e < P_{min}$)



$c_{imp} \geq c_t$

(*1) $f_t \leq H(c_{imp}-c_t)$

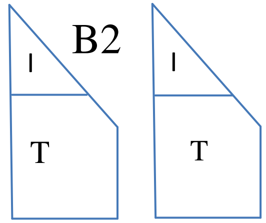
$\pi_1 Hc_t \leq f_e/\alpha_1 \leq \pi_1(f_t + Hc_t)$

$f_e/\alpha_1 \leq \pi_1 Hc_{imp}$

$f_t + \pi_1 Hc_t \leq f_e/\alpha_1 + \pi_2 H(c_{imp}-c_t)$

lim $P_t = P_{max}$, when $c_{imp} \rightarrow +\infty$

B2



$c_{imp} \geq c_t$

(*1) $f_t \leq H(c_{imp}-c_t)$

(*5) $f_e/\alpha_1 > \pi_1(f_t + Hc_t) \rightarrow$ (No*3)

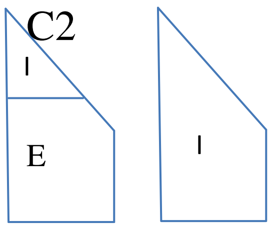
$P_t > P_{min}$

P_t increasing in c_{imp} and indep. of v_{exp}

If $f_t = H(c_{imp}-c_t)$ infinite solutions with range $0 \leq P_t \leq P_{min}$

If $f_t < H(c_{imp}-c_t)$ unique solution with $P_t > P_{min}$ in the case, with P_t increasing in c_{imp} and independent of v_{exp}

lim $P_t = P_{max}$, when $c_{imp} \rightarrow +\infty$



$c_{imp} \geq c_t$

(*6) $f_t > \left[\frac{f_e}{\alpha_1 \pi_1 N} - 24 v_{exp}\right] \frac{(c_{imp} - c_t)}{(c_{imp} - v_{exp})} N\pi_1 + \pi_2 H(c_{imp} - c_t)$

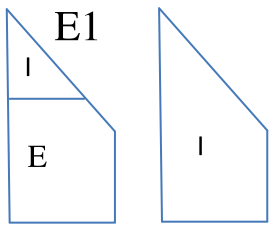
→ (No*4)

(*2) $f_e/\alpha_1 \leq \pi_1 H c_{imp}$

If $f_e/\alpha_1 = \pi_1 H c_{imp}$ infinite solutions with $0 \leq \alpha_1 P_e \leq P_{min}$

If $f_e/\alpha_1 < \pi_1 H c_{imp}$ unique solution with $\alpha_1 P_e > P_{min}$

P_e increasing in v_{exp} and in c_{imp}



$c_{imp} < c_t$

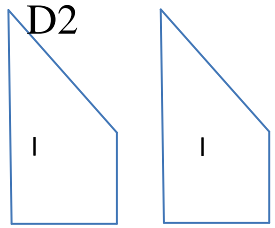
(*2) $f_e/\alpha_1 \leq \pi_1 H c_{imp}$

$c_{imp} > v_{exp}$

If $f_e/\alpha_1 = \pi_1 H c_{imp}$ infinite solutions in the range $0 \leq \alpha_1 P_e \leq P_{min}$

If $f_e/\alpha_1 < \pi_1 H c_{imp}$ unique solution in the case with $\alpha_1 P_e > P_{min}$

P_e increasing in v_{exp} and in c_{imp}

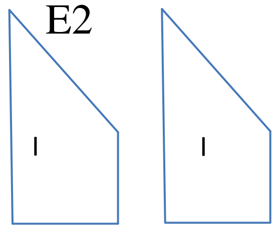


$c_{imp} \geq c_t$

(No*2) $f_e/\alpha_1 > \pi_1 H c_{imp}$

(No*1) $f_t > H(c_{imp} - c_t)$

Unique solution in the case



$c_{imp} < c_t$

(No*2) $f_e/\alpha_1 > \pi_1 H c_{imp}$

(No*1) $f_t > H(c_{imp} - c_t)$

Unique solution in the case

4.3 Trajectories of the optimal design solution when c_{imp} increases

The main goal of the chapter is to present the effect of changes in the prices of international trade on the optimal design of the generation portfolio, more precisely on the optimal values of P_e and P_t. Then a reasonable way to describe the optimal solutions is to analyze the different types of trajectories of the solutions for increasing values of c_{imp}, starting at zero.

4.3.1 Upper hemicontinuity of the optimal solution trajectories

To determine the trajectories of P_e and P_t when c_{imp} changes, we use Proposition 5.25, from Annex 4, about the upper hemicontinuity of the solution set: for a given value c of the import cost c_{imp}, if there is a single optimal solution $x(c_{imp})$ for every $c_{imp} \neq c$ in a neighborhood of c, and $x(c_{imp}) \rightarrow x^c$ for $c_{imp} \rightarrow c$, then x^c is one (possibly the only) optimal solution at $c_{imp} = c$.

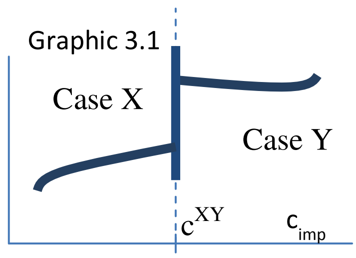
We also use Proposition 5.23 from Annex 4, which affirms that the optimal solutions for any given values of the parameters form a convex set.

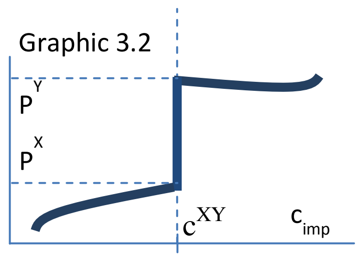
To describe the trajectories we will use the expression: the solution "moves" or "passes" from case X to case Y, which means more precisely that there is a $c^{XY} \in R^+$ so that:

- For every $c_{imp} < c^{XY}$ the optimal solutions belong to case X
- For every $c_{imp} > c^{XY}$ the optimal solutions belong to case Y
- For $c_{imp} = c^{XY}$ the solutions (or the single solution) belong to any of these cases, or to both of them.

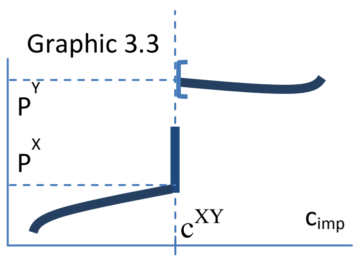
If at c^{XY} the solution moves from case X to case Y, and in a neighborhood of c^{XY} which does not contain it, there is a single solution for every c_{imp}, the graphics of P_e and P_t can have one of the following appearances:

- If at c^{XY} there is a single solution, the solutions are continuous curves in c^{XY}
- If at c^{XY} the solution is not unique, then the solutions belong to a convex set, and as a result of upper hemicontinuity the solutions have the appearance of Graphic 3.1





The detailed discussion in Annexes 1 to 3, shows that in the situations of multiple solutions at the c_{imp} between cases, the appearance of the solutions is that of Graphic 3.2, where the solutions $P_t(c^{XY})$ and $P_e(c^{XY})$ for c^{XY}, are line segments $[P^X, P^Y]$, where P^X and P^Y are the respective one-sided limits of the solutions, when c_{imp} approaches c^{XY} from the left and from the right.



Upper hemicontinuity of the solutions rules out the possibility of passage between cases with trajectories like the one in Graphic 3.3, where for $c_{imp} = c^{XY}$ the solution belongs to case X, but the one-sided limit when c_{imp} approaches c^{XY} from the right in case Y, is not a solution in case X for $c_{imp} = c^{XY}$. (Obviously the symmetric case with a discontinuity from the left is also excluded).

4.3.2 Summary of the possible trajectories

Annex 3 analyzes exhaustively the trajectories and the passages of the optimal solutions between cases when c_{imp} increases starting at zero. Graphic 3.4 summarizes the results. There are three kinds of trajectories, depending on the values of the parameters, depicted with differently coloured arrows. The conditions on the parameters originating each kind of trajectory are also presented. Let us call the three kinds of trajectories Fin A1, Fin A2 and Fin B2, according to the final case reached when c_{imp} tends to infinity. The qualitative description of these three possible behaviors of the trajectories is presented in the following sections.

4.3.2.1 First behavior of the trajectories (Fin A1)

This behavior occurs when wind capacity is so cheap that supplying with wind energy the base of the demand in type 1 days is less costly than the variable cost of thermal energy for the same purpose. This happens when $f_e/\alpha_1 < \pi_1 H c_t$. In Graphic 3.4 this behavior is represented with black fill arrows.

For very low values of c_{imp}, so that $c_{imp} < \frac{f_e}{\alpha_1 \pi_1 H} =: c_{imp}^2$ the whole demand is supplied with imports (cases D2 and E2).

When c_{imp} exceeds c_{imp}^2, the solution moves to cases C2 or E1 and includes an amount of wind capacity, increasing with c_{imp}; in other words wind capacity and imports are substitutes.

With a further increase in c_{imp}, when this price exceeds c_{imp}^4, (solution of equation 5.53 from Annex 3, $f_t = \left[\frac{f_e}{\alpha_1 \pi_1 N} - 24 v_{exp}\right] \frac{(c_{imp} - c_t)}{c_{imp} - v_{exp}} N \pi_1 + \pi_2 H (c_{imp} - c_t)$), the trajectory of optimal solutions moves to case A2. The optimal solutions have both nonzero wind and thermal capacities, and this thermal capacity is fully used during some period in type 1 as well as in type 2 days.

This case A2 allows the possibility of wind power and imports being complementary. The intuition is that the mix of wind capacity in type 1 days plus imports in type 2 days competes with thermal capacity to fill the optimal portfolio. When imports price increases thermal capacity becomes more competitive and the optimal wind capacity decreases. This complementarity does not necessarily occur, as a result of the indetermination in the sign of (5.18) from Annex 1, but depends on the parameters and the form of the load duration curve $D(P)$. Proposition 5.6 from Annex 1 proves that if $D(P)$ is linear, complementarity in case A2 in fact occurs.

With further increases of c_{imp}, exceeding the value of c_{imp}^5, (solution of equation 5.54 from Annex 2, $\frac{f_t}{\pi_2 N (c_{imp} - c_t)} = D(P_t) = D\left(P_{max} - P\left(\frac{\frac{f_e}{\alpha_1 \pi_1 N} - 24 v_{exp}}{c_t - v_{exp}}\right)\right)$), the optimal solution moves to case A1.

At the optimum there are now both positive wind and thermal capacities. Thermal capacity is not fully used in any period of type 1 days. In type 1 days there are no imports. When c_{imp} increases, the optimal wind capacity remains constant, thermal capacity increases, and the share of imports in the supply of type 2 days decreases. When c_{imp} tends to infinity, the solution remains in case A1 indefinitely, and imports tend to zero.

4.3.2.2 Second behavior of the trajectories (Fin A2)

Trajectories are of this kind when the price of wind capacity is "intermediate", as condition $\pi_1 \mathrm{Hc_t} < \frac{f_e}{\alpha_1} < \pi_1(\mathrm{f_t} + \mathrm{Hc_t})$ holds. In Graphic 3.4 this behavior is represented with grey fill arrows.

The trajectories are similar to Fin A1, except that in Fin A2 they remain in case A2 indefinitely and never reach case A1. There is always an amount of imported energy, decreasing when $\mathrm{c_{imp}}$ grows, and tending to zero as $\mathrm{c_{imp}}$ tends to infinity.

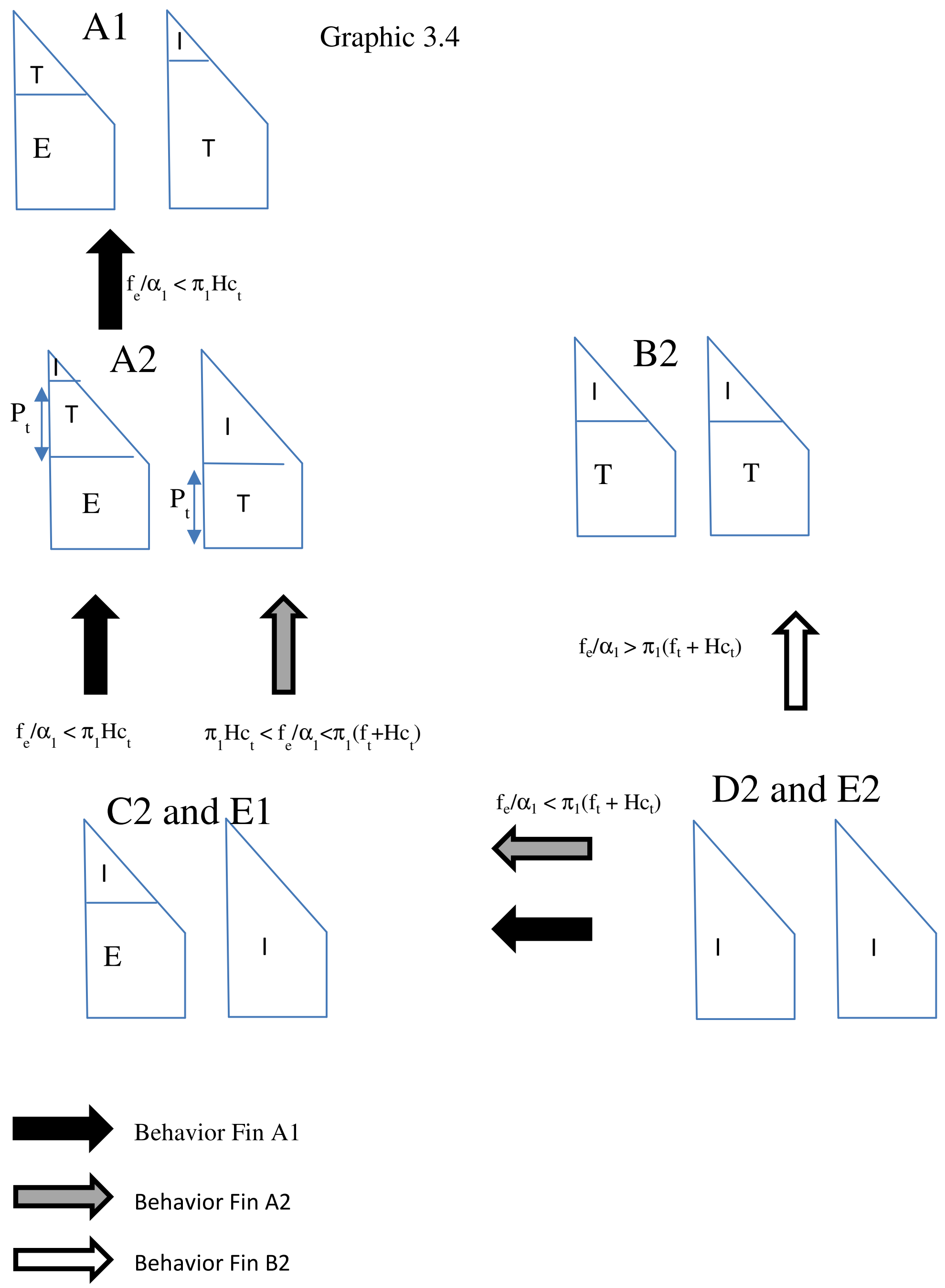
Regarding the possibility of complementarity between wind capacity and imports, the same reasoning of the preceding section is valid.

4.3.2.3 Third behavior of the trajectories (Fin B2)

Trajectories are of this third kind when the fixed cost of wind capacity is relatively high, as $\frac{f_e}{\alpha_1} > \pi_1(f_t + Hc_t)$ holds. In Graphic 3.4 this behavior is represented with white fill arrows. The optimal wind installed capacity is zero, for all import prices. The way demand is supplied is identical in both types of day.

If imports are cheap enough and $c_{imp} < \frac{f_t}{H} + c_t =: c_{imp}^1$ holds, the optimal solution consists in supplying with imports the whole demand in both types of day (cases D2 y E2). When c_{imp} exceeds c_{imp}^1, the solution moves to case B2, and includes a positive thermal capacity, increasing with c_{imp}, which is fully used during some period in both types of day. When c_{imp} tends to infinity, the solution remains indefinitely in case B2, thermal capacity tends to P_{max} and energy imports tend to zero.

Graphic 3.4



4.3.3 Effect of export prices

Whenever wind capacity participates in the optimal solution and $\alpha_1 P_e > P_{min}$ holds, so that exports occur, the optimal wind capacity is increasing in exports price.

Exports price does not affect the optimal portfolio in some situations when there is wind capacity at the optimum, but the restriction above does not hold; this happens in some combinations of parameters with null probability and in case A2-b when wind installed capacity is less than P_{min} and therefore exports are zero.

4.3.4 Examples of the three kinds of trajectories when D(P) is linear

The goal of this section is showing numerical examples for each one of the three possible behaviors the optimal trajectories can have.

As the load duration curve $D(P)$ is chosen linear, by Proposition 5.6 from Annex 1, when the solution trajectory belongs to case A2 there is complementarity between wind capacity and imports: when c_{imp} increases, the optimal wind capacity P_e decreases.

4.3.4.1 Common data in the three examples

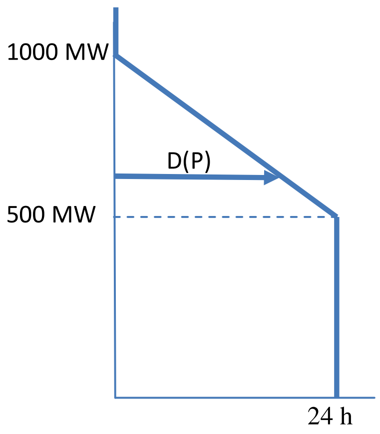
The examples show the numerical results of three series of problems, increasing c_{imp} by steps of 2 USD/MWh, and starting from zero.

The common data are the following:

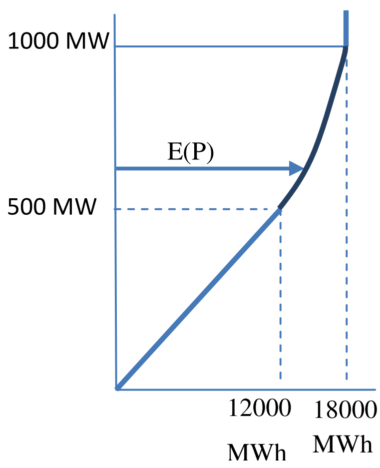
- $N = 365$ days
- P is expressed in MW, $D(P)$ in hours, $E(P)$ in MWh
- $H = 8760$ hours
- $P_{min} = 500$ MW
- $P_{max} = 1000$ MW
- $D(P) = 24$ hours for $P < 500$ MW
- $D(P) = 24[(1000 - P)/500]$ hours for $500 < P < 1000$ MW
- $D(P) = 0$ for $P > 1000$ MW
- $E(P) = 24P$ for $P \leq 500$ MW
- $E(P) = 12000 + (P - 500) * (24 + D(P))/2$ for $500 < P \leq 1000$ MW
- $E(P) = 18000$ for $P > 1000$ MW

The following graphics show the appearance of D and E.

Graphic 3-5



Graphic 3-6



The daily energy demand is 18 000 MWh.

- $\pi_1 = 0.8, \pi_2 = 0.2$
- $\alpha_1 = 0.5, \alpha_2 = 0.0$, and as a result the expected capacity factor for wind energy is equal to 0.8x0.5 = 0.4

The annual energy demand is equal to 6 570 000 MWh.

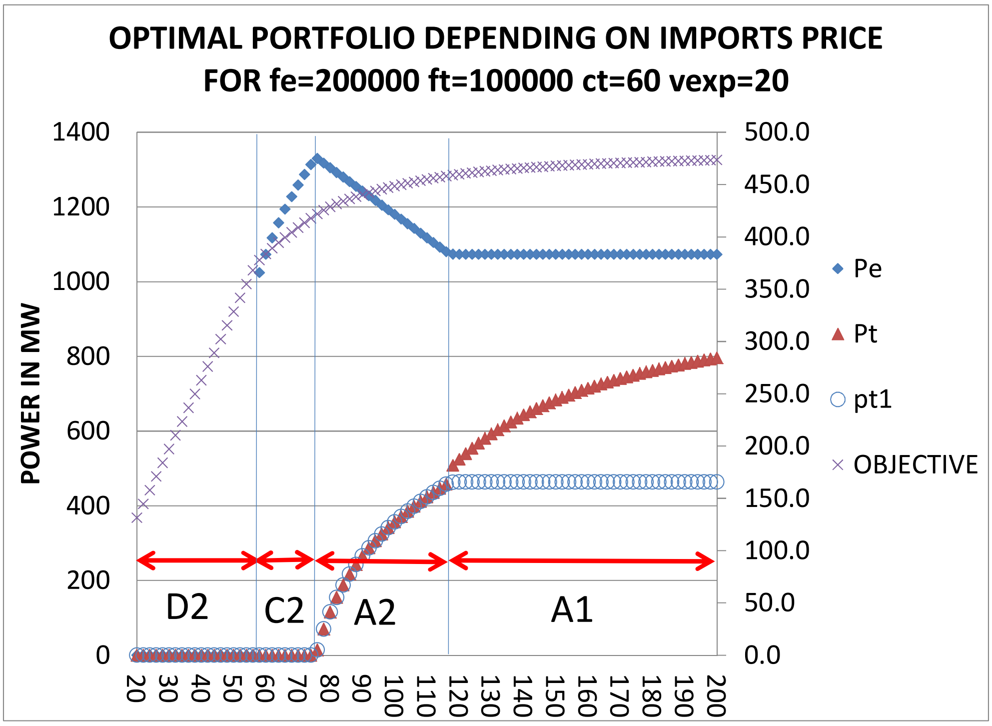
4.3.4.2 Results

Each of the following examples corresponds to one of the three possible behaviors of the trajectories. The graphic in each example shows the values of P_e, P_t and p_{t1} and the value of the objective, for increasing values of c_{imp}. The vertical axis on the left measures power in MW. The vertical axis on the right measures the value of the objective in MUSD per year.

4.3.4.2.1 Behavior Fin A1. Sequence of cases D2-C2-A2-A1

We assume: $f_e = 200000$ USD/MW, $f_t = 100000$ USD/MW, $c_t = 60$ USD/MWh, $v_{exp} = 20$ USD/MWh. It holds that: $\frac{f_e}{\alpha_1} = 400000 < \pi_1 H c_t = 420480$. The values of P_e, P_t, y p_{t1} and the objective are shown in the following graphic.

Graphic 3.7



As c_{imp} increases starting at zero in case E2/D2, and exceeds c_{imp}^2, the solution moves from E2/D2 to C2, with:

$$c_{imp}^2 = \frac{f_e}{\alpha_1 \pi_1 H} = \frac{200000}{0.8 \times 0.5 \times 8760} \approx 57.0777\ USD/MWh$$

Imports become costly enough to justify the use of wind energy to supply the entire base of the load duration curve in type 1 days, with $P_e = 1000$ MW, $P_e \alpha_1 = 500$ MW.

When c_{imp} increases in case C2, wind installed capacity also increases, as shown in Proposition 5.12, and thermal power remains null.

The transition from C2 to A2 occurs at c_{imp}^4 solution of (5.53), with $c_{imp}^4 \approx 75.5539$. When the trajectory enters case A2, increasing thermal capacities appear in the optimal solution, with $p_{t1} = p_{t2} = P_t$, while optimal wind capacity decreases, fulfilling Proposition 5.6. Wind capacity and imports are complementary with linear $D(P)$.

The solution passes from A2 to A1 at $c_{imp} = c_{imp}^5$, with:

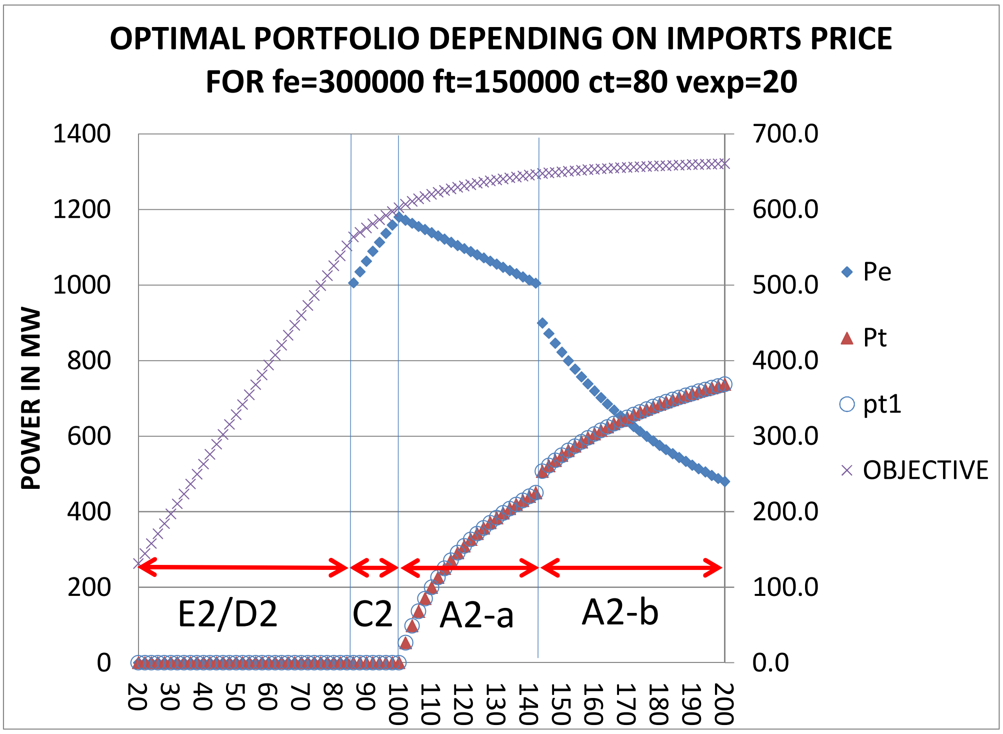
$$c_{imp}^5 = \frac{f_t}{\pi_2 H} + c_t = \frac{100000}{0.2 \times 8760} + 60 \approx 117.0776\, USD/MWh$$

After entering case A1, P_e and p_{t1} remain constant, and P_t is increasing, fulfilling Proposition 5.1. As the price of imports increases the amount of imports in type 2 days decreases, but the dispatch in type 1 days remains unchanged. The fixed capacity cost of wind power is low enough compared to thermal costs to keep optimal wind capacity in the range $\alpha_1 P_e > P_{min}$.

4.3.4.2.2 Behavior Fin A2. Sequence of cases D2-C2-A2

We assume: $f_e = 300000$ USD/MW, $f_t = 150000$ USD/MW, $c_t = 80$ USD/MWh, $v_{exp} = 20$ USD/MWh. It holds that: $\pi_1 H c_t = 560640 < \frac{f_e}{\alpha_1} = 600000 < \pi_1 (f_t + H c_t) = 680640$. The values of P_e, P_t, y p_{t1} and the objective are shown in the following graphic.

Graphic 3.8



Initially this trajectory shows the same features as the previous one.

At c_{imp}^2 solution passes from D2 to C2, with:

$$c_{imp}^2 = \frac{f_e}{\alpha_1 \pi_1 H} = \frac{300000}{0.8 \times 0.5 \times 8760} \approx 85.6164\ USD/MWh$$

At c_{imp}^4 solution passes from C2 to A2, with $c_{imp}^4 = 100$, solution of (5.53) in Annex 1.

The passage between subcases A2-a and A2-b occurs at a value of c_{imp} with multiple solutions for P_t. By equation (5.28) in Annex 1, this happens when:

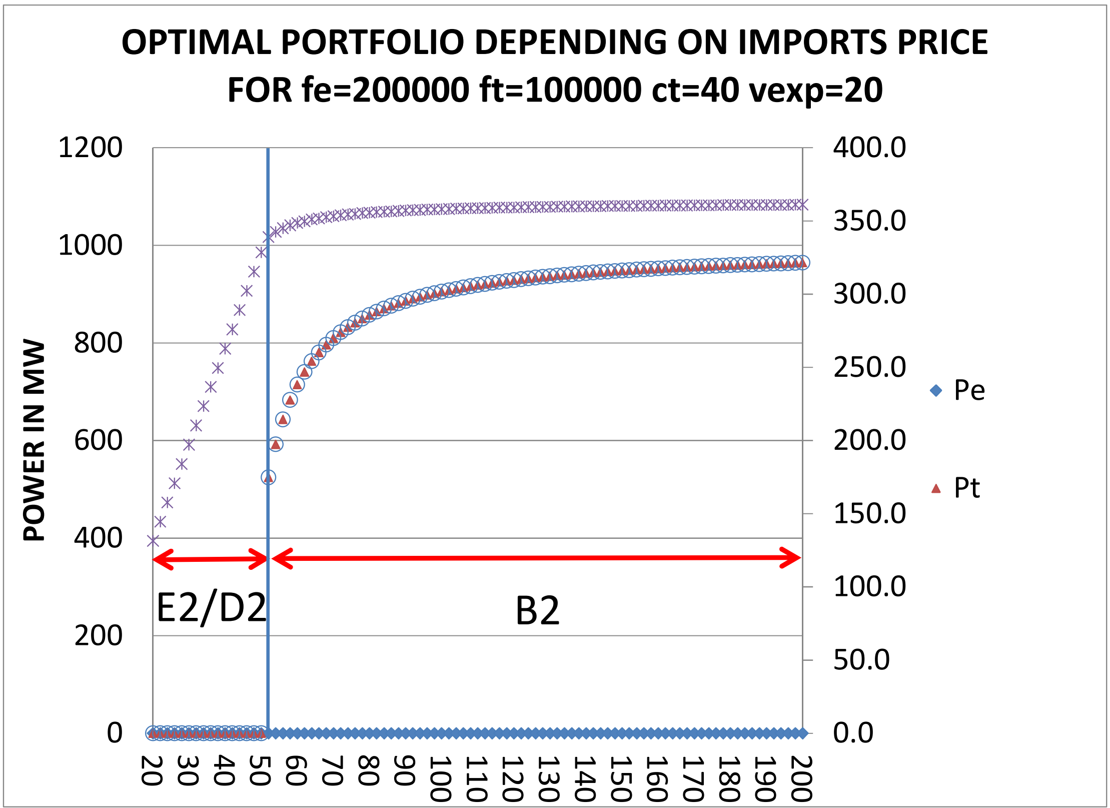
$$c_{imp} = c_t + \frac{f_t + H\pi_1 c_t - \frac{f_e}{\alpha_1}}{\pi_2 H} = 80 + \frac{150000 + 8760 \times 0.8 \times 80 - \frac{300000}{0.5}}{0.2 \times 8760} \approx 143.1507$$

At this point, the wind capacity supplies exactly the base of the load duration curve. If c_{imp} increases beyond that value, optimal wind capacity goes on decreasing. There is no passage to case A1. Imports decreases in both types of day.

4.3.4.2.3 Behavior Fin B2. Sequence of cases D2/E2-B2

We assume: $f_e = 200000$ USD/MW, $f_t = 100000$ USD/MW, $c_t = 40$ USD/MWh, $v_{exp} = 20$ USD/MWh. It holds that: $\pi_1 H c_t = 280320 < \pi_1(f_t + Hc_t) = 360320 < \frac{f_e}{\alpha_1} = 400000$. The values of P_e, P_t, y p_{t1} and the objective are shown in the following graphic.

Graphic 3.9



As c_{imp} increases and reaches the transition from E1/D2 to B2, thermal energy becomes more convenient than imports to supply the base of the load duration curve in both types of day. This happens when c_{imp} equals the value:

$$c_{imp}^{1} = \frac{f_t}{H} + c_t = \frac{100000}{8760} + 40 \approx 51.4951\ USD/MWh$$

The maximum thermal power in both types of day is $p_{t1} = p_{t2} = P_t$.

As c_{imp} increases imports decrease and tend to zero, and the solution remains indefinitely in case B2.

Wind capacity is so costly that it never participates in the optimal solution.

4.4 Conclusions

Using a simplified static model of supply and demand of power generation we have shown that imported energy can locally be either a substitute or a complement of wind capacity, or have no impact at all on it, in the optimal design of the generation portfolio. The actual behavior depends on the configuration of the optimal solution, which is a result of the relative costs of the different generation resources. This is a consequence of the random variability of the available power from wind power plants. For simplicity we have assumed two levels of power availability from wind: greater than zero in type 1 days and zero in type 2 days.

We analyzed the trajectories of the optimal solution when imports price increases starting at zero. The condition to have wind capacity in the optimal solution for some level of import costs is $\frac{f_e}{\alpha_1} < \pi_1(f_t + Hc_t)$, and the conclusions we present correspond to this case.

The local effect of increases in imports price c_{imp} on the optimal amount of wind capacity is not the same for all cases and values of the parameters: imported energy can be either a complement or a substitute of wind capacity, or imports price can have no effect at all.

If $c_{imp} \leq \frac{f_e}{\pi_1\alpha_1 H}$ the optimal solution is to import the whole demand and no wind capacity is needed.

As imports price increases and $\frac{f_e}{\pi_1\alpha_1 H} < c_{imp} < c_{imp}^4$ holds, the optimal solution has wind capacity and imports, in cases C2/E1. Thermal capacity does not participate in the optimal solution. Imported energy is a substitute of wind capacity: the greater the imports price the bigger the optimal wind installed capacity.

Further increases in imports price, so that $c_{imp} > c_{imp}^4$ holds, lead the solution to case A2, where thermal capacity participates in the optimal solution. Depending on the sign of (5.18), which is the result of both the relative costs of the resources and the form of the load duration curve $D(P)$, complementarity of wind capacity and imports can occur in case A2: as imports price increases, wind capacity at the optimum decreases. The intuition is that a mix of wind capacity together with imports compete against thermal capacity, with imports filling the gaps left by the absence of wind in type 2 days. The same argument can be applied to all intermittent non-conventional renewable energies.

If the fixed cost of wind capacity is cheap enough and $\frac{f_e}{\alpha_1} < \pi_1 H c_t$ holds (wind power is cheaper than the variable cost of thermal energy to supply the base of the load duration curve in type 1 days), an increase in c_{imp} beyond c_{imp}^4 to reach c_{imp}^5 leads the solution to case A1: optimal wind capacity does not depend on c_{imp} and remains constant as imports prices tend to infinity.

As can be expected, in all cases where wind energy surpluses are exported, an increase in the exports price increases the optimal wind capacity.

The qualitative conclusion of the chapter is that for a country with strong interconnections with its neighbors, the negotiation of agreements for international power trade can have a strong influence on the optimal design of the generation portfolio, and in particular on the amount of wind and other renewables capacity. For a small country an improvement in such agreements would lead to a decrease in the prices of its imports and an increase in the prices of its exports. We found conditions under which smaller import prices can lead to greater optimal wind capacities: an improvement in international power trade would then be favorable to the installation of renewable energies.

4.5 Annexes to Chapter 4

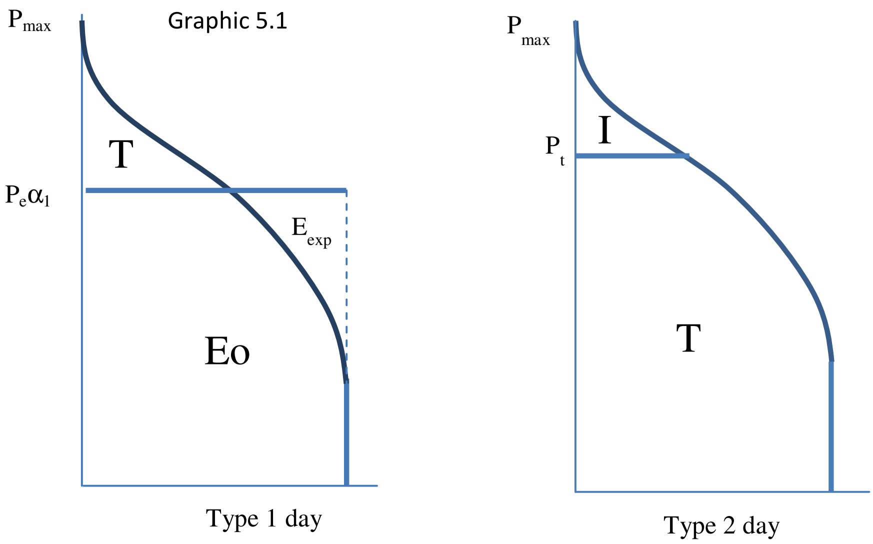
4.5.1 Annex 1 – Analysis of the possible cases of the optimal solution when $c_t<c_{imp}$

4.5.1.1 Case A1

It holds $\mu = 0, \nu = 0, \lambda = 0$; $P_t \geq 0, P_e \geq 0, P_t \geq p_{t1}$

After (2.10) it holds $D(P_e\alpha_1 + p_{t1}) = 0$, and therefore $P_e\alpha_1 + p_{t1} = P_{max}$; in type 1 days there are no imports and thermal capacity is never fully dispatched. In type 2 days the thermal capacity if fully dispatched during some period.

The next graphic represents the situation: Eo denotes wind energy, T thermal energy and I imports.



Graphic 5.1

4.5.1.1.1 Necessary conditions

From (2.9) results: $$f_t - \pi_2 ND(P_t)(c_{imp} - c_t) = 0 \qquad (5.1)$$

From (5.1) results: $D(P_t) = \frac{f_t}{\pi_2 N(c_{imp} - c_t)}$.

A necessary condition for this case to be possible is: $0 \leq \frac{f_t}{\pi_2 N(c_{imp}-c_t)} \leq 24$ which is equivalent to:

$c_{imp} > c_t$ that holds by hypothesis,

and $f_t \leq \pi_2 H\left(c_{imp} - c_t\right)$ (5.2)

If (5.2) holds as an equality, there are infinite solutions with $0 \leq P_t \leq P_{min}$.

If (5.2) holds as a strict inequality, there is a single value for:

$$P_t = P\left(\frac{f_t}{\pi_2 N(c_{imp}-c_t)}\right) \tag{5.3}$$

It also holds $P_t > P_{min}$.

As function $P(.)$ is a strictly decreasing function, P_t is a strictly increasing function of c_{imp}, besides it does not depend on the value of v_{exp}.

Under the hypothesis that the solution remains in this case:

$\lim_{c_{imp}\to+\infty} \ P_t = P(0) = P_{max}$

Using (2.8), and making $P_e\alpha_1 + p_{t1} = P_{max}$ it holds:

$f_e - \alpha_1\pi_1 N\{D(P_e\alpha_1)(c_t - v_{exp}) + 24v_{exp}\} = 0$ and therefore:

$$D(P_e\alpha_1) = \frac{\frac{f_e}{\alpha_1\pi_1 N} - 24v_{exp}}{c_t - v_{exp}} \ ; \tag{5.4}$$

Let us observe that P_e does not depend on the imports price.

A necessary condition for this case to be possible is:

$$0 \leq \frac{\frac{f_e}{\alpha_1\pi_1 N} - 24v_{exp}}{c_t - v_{exp}} \leq 24$$

The first inequality holds strictly by hypotheses (2.1) and (2.2).

The second inequality holds if $\frac{f_e}{\alpha_1 \pi_1 N} \leq 24c_t$ or its equivalent: $\frac{f_e}{\alpha_1} \leq \pi_1 H c_t$ (5.5)

This means that to supply the base of the load duration curve in type 1 days, wind capacity is less costly than the variable cost of thermal plants.

It also holds: $\frac{f_e}{\alpha_1} \leq \pi_1 H c_{imp}$ (5.6)

The case when the second member of (5.4) is equal to 24 and (5.5) holds as equality, has infinite solutions, with $P_e \alpha_1 \leq P_{min}$.

If (5.5) holds as inequality there is a single value:

$$P_e = P\left(\frac{\frac{f_e}{\alpha_1 \pi_1 N} - 24v_{exp}}{c_t - v_{exp}}\right) \times \frac{1}{\alpha_1} = P\big(F(v_{exp}, c_t)\big) \times \frac{1}{\alpha_1} \quad (5.7)$$

with $P_e \alpha_1 > P_{min}$, where $F(v, c)$ is defined as: $F(v, c) := \frac{\frac{f_e}{\alpha_1 \pi_1 N} - 24v}{c - v}$ (5.8)

It holds $\frac{\partial F(v_{exp}, c_t)}{\partial v_{exp}} = \frac{-24c_t + \frac{f_e}{\alpha_1 \pi_1 N}}{(c_t - v_{exp})^2}$, which by (5.5) is non-positive.

Therefore P_e is increasing in v_{exp} and does not depend on c_{imp}.

In summary the following proposition holds:

Proposition 5.1:

- The following are necessary conditions for the K-T solutions in case A1:

  $f_t \leq \pi_2 H\big(c_{imp} - c_t\big)$ and $\frac{f_e}{\alpha_1} \leq \pi_1 H c_t$
- If both inequalities hold strictly, then the K-T solutions in this case are unique for every set of parameters.

  If $f_t < \pi_2 H\big(c_{imp} - c_t\big)$ then $P_t > P_{min}$ and if $\frac{f_e}{\alpha_1} < \pi_1 H c_t$ then $P_e \alpha_1 > P_{min}$.
- The K-T solutions have:
  - P_t strictly increasing in c_{imp} and independent of v_{exp}
  - P_e increasing in v_{exp} and independent of c_{imp}.

4.5.1.1.2 Variation of the solutions as c_{imp} increases

Les us denote x^* a K-T solution in this case. If (5.2) and (5.5) hold as strict inequalities, we can find unique values for $P_e\alpha_1$ and P_t, both strictly greater than P_{min}.

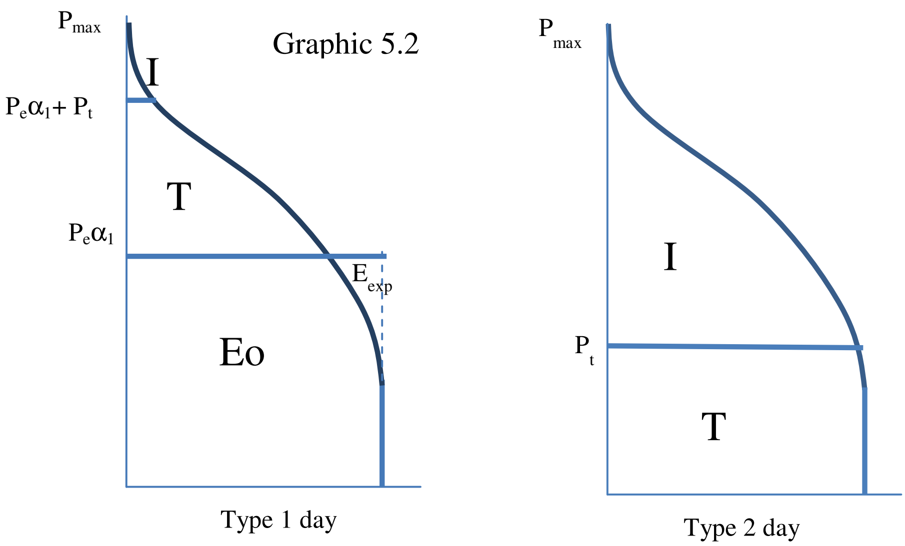
As c_{imp} increases, starting at the value corresponding to solution x^*, the necessary conditions for the solution to belong to this case still hold. The solutions have a constant P_e (by (5.4)) and increasing values of P_t (by (5.3)). P_t tends to P_{max} as c_{imp} tends to infinity.

Therefore the following proposition holds:

<u>Proposition 5.2</u>: If, starting at a unique solution in case A1, the value of c_{imp} is increased, the K-T solutions remain in case A1, are unique for each value of c_{imp}, and have a constant P_e and increasing values of P_t with limit P_{max}.

4.5.1.2 Case A2

It holds: $\mu = 0, \nu = 0, \lambda > 0$; $P_t \geq 0, P_e \geq 0, P_t = p_{t1}$



Using (2.10) we have: $D(P_e\alpha_1 + P_t) > 0$ and therefore $P_e\alpha_1 + P_t = P_{max}$; in type 1 days there are energy imports and the thermal capacity P_t is fully dispatched during some period.

In type 2 days thermal capacity is fully dispatched during some period, and there are energy imports. The maximum imported power is greater in type 2 than in type 1 days.

We can assume: $\frac{f_e}{\alpha_1} \leq \pi_1 H c_{imp}$

Otherwise it would be convenient to substitute imports for the whole wind capacity.

From (2.8) we have:

$$f_e - \alpha_1 \pi_1 N\{D(P_e\alpha_1 + P_t)(c_{imp} - c_t) + D(P_e\alpha_1)(c_t - v_{exp}) + 24v_{exp}\} = 0$$

$$D(P_e\alpha_1 + P_t)(c_{imp} - c_t) + D(P_e\alpha_1)(c_t - v_{exp}) = \frac{f_e}{\alpha_1\pi_1 N} - 24v_{exp} \tag{5.9}$$

From (2.11):

$$f_t - [\pi_1 ND(P_e\alpha_1 + P_t) + \pi_2 ND(P_t)](c_{imp} - c_t) = 0$$

$$\pi_1 D(P_e\alpha_1 + P_t) + \pi_2 D(P_t) = \frac{f_t}{N(c_{imp} - c_t)} \tag{5.10}$$

Using (5.10), and $\pi_1 + \pi_2 = 1$ we have:

$$\frac{f_t}{N(c_{imp} - c_t)} \leq 24 \tag{5.11}$$

The Jacobian M in P_e and P_t of the system of implicit equations (5.9) and (5.10) is:

$$M = \begin{bmatrix} [D'(P_e\alpha_1 + P_t)(c_{imp} - c_t) + D'(P_e\alpha_1)(c_t - v_{exp})]\alpha_1 & D'(P_e\alpha_1 + P_t)(c_{imp} - c_t) \\ \pi_1 D'(P_e\alpha_1 + P_t)\alpha_1 & \pi_1 D'(P_e\alpha_1 + P_t) + \pi_2 D'(P_t) \end{bmatrix} \tag{5.12}$$

$$\det(M) = [D'(P_e\alpha_1 + P_t)(c_{imp} - c_t) + D'(P_e\alpha_1)(c_t - v_{exp})]\alpha_1\pi_2 D'(P_t) + D'(P_e\alpha_1)(c_t - v_{exp})\alpha_1\pi_1 D'(P_e\alpha_1 + P_t) \tag{5.13}$$

As $c_{imp} > c_t$ in this case, and $c_t > v_{exp}$ by hypothesis (2.1), any of the following is a sufficient condition for det(M)>0:

- $P_{min} < P_e\alpha_1 < P_{max}$ and $P_{min} < P_t < P_{max}$ (5.14)

- $P_{min} < P_e\alpha_1$ and $P_e\alpha_1 + P_t < P_{max}$ (5.15)
- $P_{min} < P_t$ and $P_e\alpha_1 + P_t < P_{max}$ (5.16)

Therefore if any of these three conditions above hold, by the implicit function theorem there exist locally continuously differentiable functions determining the values of P_e and P_t, and the following proposition holds:

Proposition 5.3: If any of the conditions (5.14), (5.15) and (5.16) hold and there is a K-T solution in case A2, it is unique for every c_{imp}.

Differentiating (5.9) and (5.10) we have the following system of equations S in dP_e and dP_t:

$$\left[\mathrm{D}'(P_\mathrm{e}\alpha_1 + P_t)\left(c_{\mathrm{imp}} - c_t\right) + \mathrm{D}'(P_\mathrm{e}\alpha_1)\left(c_t - v_{\mathrm{exp}}\right)\right]\alpha_1 dP_e + \mathrm{D}'(P_\mathrm{e}\alpha_1 + P_t)\left(c_{\mathrm{imp}} - c_t\right)dP_t$$
$$= -D(P_\mathrm{e}\alpha_1 + P_t)dc_{\mathrm{imp}} + [D(P_\mathrm{e}\alpha_1) - 24]dv_{\mathrm{exp}}$$

$$\pi_1\mathrm{D}'(P_\mathrm{e}\alpha_1 + P_t)\alpha_1 dP_e + [\pi_1\mathrm{D}'(P_\mathrm{e}\alpha_1 + P_t) + \pi_2\mathrm{D}'(P_t)]\,\mathrm{d}P_t = -\frac{f_t}{N\left(c_{imp} - c_t\right)^2}dc_{imp}$$

Let us define the matrix:

$$M_{P_e} = \begin{bmatrix} -\mathrm{D}(P_\mathrm{e}\alpha_1 + P_t)\mathrm{d}c_{\mathrm{imp}} + [\mathrm{D}(P_\mathrm{e}\alpha_1) - 24]\mathrm{d}v_{exp} & \mathrm{D}'(P_\mathrm{e}\alpha_1 + P_t)(c_{imp} - c_t) \\ -[f_t/\mathrm{N}(c_{imp} - c_t)^2]\mathrm{d}c_{imp} & \pi_1\mathrm{D}'(P_\mathrm{e}\alpha_1 + P_t) + \pi_2\mathrm{D}'(P_t) \end{bmatrix}$$

It holds: $dP_e = \frac{\det(M_{P_e})}{\det(M)}$

$$dP_e = \left\{-\mathrm{D}(P_\mathrm{e}\alpha_1 + P_t)[\pi_1\mathrm{D}'(P_\mathrm{e}\alpha_1 + P_t) + \pi_2\mathrm{D}'(P_t)] + \mathrm{D}'(P_\mathrm{e}\alpha_1 + P_t)\frac{f_t}{\mathrm{N}(c_{imp} - c_t)}\right\}\frac{1}{\det(M)}dc_{imp}$$
$$+[\mathrm{D}(P_\mathrm{e}\alpha_1) - 24][\pi_1\mathrm{D}'(P_\mathrm{e}\alpha_1 + P_t) + \pi_2\mathrm{D}'(P_t)]\frac{1}{\det(M)}dv_{exp}$$

(5.17)

In particular, using (5.10), we have:

$$\frac{\partial P_e}{\partial c_{imp}} = \pi_2[\mathrm{D}'(P_\mathrm{e}\alpha_1 + P_t)\mathrm{D}(P_t) - \mathrm{D}(P_\mathrm{e}\alpha_1 + P_t)\mathrm{D}'(P_t)]\frac{1}{\det(M)} \quad (5.18)$$

The sign of this derivative cannot be determined beforehand, as it depends on the values of P_{e} and P_t, and the form of the load duration curve D.

As for the derivative respect to v_{exp}, if $P_{\mathrm{e}}\alpha_1 > 0$, as $\mathrm{D}'(P_{\mathrm{e}}\alpha_1 + P_t) < 0$ and $\mathrm{D}(P_{\mathrm{e}}\alpha_1) < 24$, the coefficient of dv_{exp} is positive and the following proposition holds:

<u>Proposition 5.4</u>: If $P_{\mathrm{e}}\alpha_1 > 0$, then P_{e} is increasing in v_{exp}.

Let us define the matrix:

$$M_{P_t} = \begin{bmatrix} [\mathrm{D}'(P_{\mathrm{e}}\alpha_1 + P_t)(c_{\mathrm{imp}} - c_t) + \mathrm{D}'(P_{\mathrm{e}}\alpha_1)(c_t - v_{\mathrm{exp}})]\alpha_1 & -\mathrm{D}(P_{\mathrm{e}}\alpha_1 + P_t)\mathrm{d}c_{\mathrm{imp}} + [\mathrm{D}(P_{\mathrm{e}}\alpha_1) - 24]\mathrm{d}v_{exp} \\ \pi_1\mathrm{D}'(P_{\mathrm{e}}\alpha_1 + P_t)\alpha_1 & -[f_t/\mathrm{N}(c_{imp} - c_t)^2]\mathrm{d}c_{imp} \end{bmatrix}$$

It holds that:

$$dP_t = \frac{\det(M_{P_t})}{\det(M)}$$

$$dP_t = \frac{1}{\det(M)}\left\{-[\mathrm{D}'(P_{\mathrm{e}}\alpha_1 + P_t)(c_{\mathrm{imp}} - c_t) + \mathrm{D}'(P_{\mathrm{e}}\alpha_1)(c_t - v_{\mathrm{exp}})]\alpha_1\left[\frac{f_t}{\mathrm{N}(c_{imp} - c_t)^2}\right]\mathrm{d}c_{imp} - \pi_1\mathrm{D}'(P_{\mathrm{e}}\alpha_1 + P_t)\alpha_1\{-\mathrm{D}(P_{\mathrm{e}}\alpha_1 + P_t)\mathrm{d}c_{\mathrm{imp}} + [\mathrm{D}(P_{\mathrm{e}}\alpha_1) - 24]\mathrm{d}v_{exp}\}\right\}$$

(5.19)

In particular, using (5.10) we have:

$$\frac{\partial P_t}{\partial c_{imp}} = \frac{1}{\det(M)}\alpha_1\left\{-\pi_2\mathrm{D}'(P_{\mathrm{e}}\alpha_1 + P_t)\mathrm{D}(P_t) - \mathrm{D}'(P_{\mathrm{e}}\alpha_1)(c_t - v_{\mathrm{exp}})\frac{f_t}{\mathrm{N}(c_{imp} - c_t)^2}\right\}$$

(5.20)

As $c_t > v_{exp}$ by hypothesis (2.1), any of the following is a sufficient condition for that derivative to be strictly positive:

- (5.15): $P_{min} < P_e\alpha_1$ and $P_e\alpha_1 + P_t < P_{max}$

- (5.16): $P_{min} < P_t$ and $P_e\alpha_1 + P_t < P_{max}$
- $P_{max} > P_e\alpha_1 > P_{min}$ (5.21)

Proposition 5.5: if any of the conditions (5.15), (5.16) or (5.21) hold, in the K-T solutions in case A2, the thermal capacity is strictly increasing in the imports price c_{imp}.

4.5.1.2.1 Linear load duration curve

When the load duration curve is linear, it holds that if $P_{max} > P_e\alpha_1 > P_{min}$, then $\mathrm{D}(P_e\alpha_1 + P_t) > \mathrm{D}(P_t) \geq 0$ and $\mathrm{D}'(P_e\alpha_1 + P_t) \leq \mathrm{D}'(P_t)$. If both inequalities are used with (5.18), we obtain the following proposition.

Proposition 5.6: In case A2, if D(P) is linear for $P > P_{min}$, and $P_{max} > P_e\alpha_1 > P_{min}$, holds, then P_e is decreasing in c_{imp}.

4.5.1.2.2 Necessary conditions for $P_e\alpha_1 \leq P_{min}$ (case A2-b)

As will be shown later, a further study of the K-T solutions when $P_e\alpha_1 \leq P_{min}$ is convenient. Let us call case A2-a that with $P_e\alpha_1 > P_{min}$ and case A2-b that with $P_e\alpha_1 \leq P_{min}$.

Proposition 5.7: $\pi_1 H c_t \leq \frac{f_e}{\alpha_1} \leq \pi_1(f_t + Hc_t)$ are necessary conditions for the existence in case A2 of K-T solutions with $P_e\alpha_1 \leq P_{min}$, (Case A2-b).

Proof:

As $P_e\alpha_1 \leq P_{min}$, $\mathrm{D}(P_e\alpha_1) = 24$ and using (5.9) it holds:

$$D(P_e\alpha_1 + P_t) = \frac{1}{(c_{imp} - c_t)}\left[\frac{f_e}{\alpha_1\pi_1 N} - 24c_t\right] \tag{5.22}$$

For $D(P_e\alpha_1 + P_t) \geq 0$ it must hold:

$$\frac{f_e}{\alpha_1\pi_1 N} \geq 24c_t, \text{ which is equivalent to } \quad \frac{f_e}{\alpha_1} \geq \pi_1 H c_t \tag{5.23}$$

For $D(P_e\alpha_1 + P_t) \leq 24$ it must hold:

$$\frac{f_e}{\alpha_1\pi_1 N} - 24c_t \leq 24(c_{imp} - c_t), \text{ which is equivalent to } \quad \frac{f_e}{\alpha_1} \leq \pi_1 H c_{imp} \tag{5.24}$$

Let us observe that if c_{imp} is big enough then (5.24) is not restrictive.

From (5.10) and (5.22) we have:

$$\frac{\pi_1}{(c_{imp}-c_t)}\left(\frac{f_e}{\alpha_1\pi_1 N}-24c_t\right)+\pi_2 D(P_t)=\frac{f_t}{N(c_{imp}-c_t)}$$

$$D(P_t)=\frac{f_t-\frac{f_e}{\alpha_1}+c_t\pi_1 H}{\pi_2 N(c_{imp}-c_t)} \tag{5.25}$$

For $(P_t)\geq 0$, it must hold:

$$\frac{f_e}{\alpha_1}\leq f_t+H\pi_1 c_t \tag{5.26}$$

For $(P_t)\leq 24$, it must hold:

$$f_t-\frac{f_e}{\alpha_1}+H\pi_1 c_t\leq \pi_2 H(c_{imp}-c_t)$$

$$f_t+H\pi_1 c_t\leq \frac{f_e}{\alpha_1}+\pi_2 H(c_{imp}-c_t) \tag{5.27}$$

Let us observe that if c_{imp} is big enough (5.27) is not restrictive.

The value of c_{imp} when $D(P_t)=24$ and (5.27) holds as equality, leads to infinite solutions for P_t , and is equal to:

$$c_{imp}=c_t+\frac{f_t+H\pi_1 c_t-\frac{f_e}{\alpha_1}}{\pi_2 H} \tag{5.28}$$

If condition $D(P_t)\geq D(P_e\alpha_1+P_t)$ is imposed, so that the K-T solution is physically meaningful, and using (5.22) and (5.25), we have:

$$\frac{1}{(c_{imp}-c_t)}\left[\frac{f_e}{\alpha_1\pi_1 N}-24c_t\right]\leq\frac{f_t-\frac{f_e}{\alpha_1}+24c_t\pi_1 N}{\pi_2 N(c_{imp}-c_t)}\text{, that yields}\qquad \frac{f_e}{\alpha_1}\leq \pi_1(f_t+Hc_t) \tag{5.29}$$

This condition is more restrictive than (5.26). In summary (5.23) and (5.29) are the necessary conditions. ■

If the optimal solution remains in this case A2-b) as c_{imp} increases, using (5.25) we have that P_t grows and the following proposition holds:

<u>Proposition 5.8</u>:

If the optimal solution remains in case A2-b) as c_{imp} grows, then:

$$\lim_{c_{imp}\to+\infty} P_t = P(0) = P_{max}$$

4.5.1.2.3 Variation of the K-T solutions in case A2-b as c_{imp} increases

Let us denote by x^* a K-T solution in case A2-b. If (5.23), (5.24), (5.26), (5.27) and (5.29) hold as strict inequalities, there are unique values for $P_e\alpha_1$ and P_t.

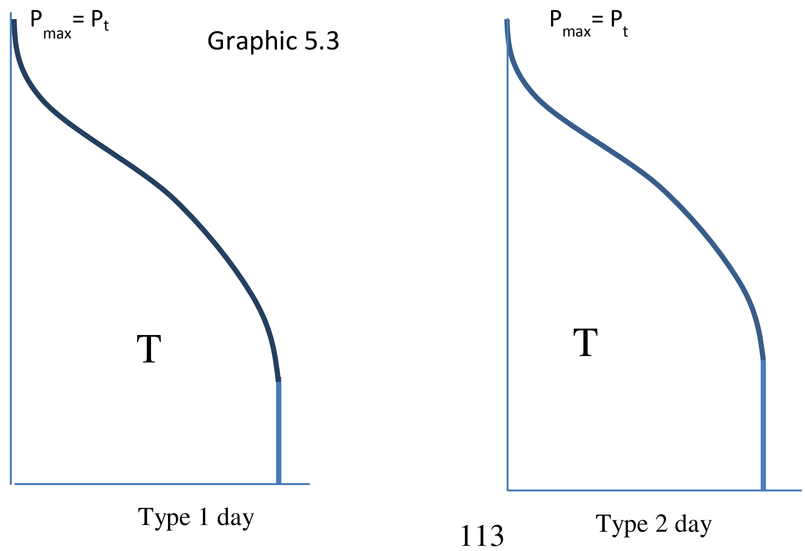
As c_{imp} grows, the necessary conditions for the solution to belong to case A2-b still hold. Therefore the following proposition holds:

<u>Proposition 5.9</u>: If for a set of parameters there is a unique K-T solution in case A2-b, increases of c_{imp} lead to unique solutions in the same case A2-b.

4.5.1.3 Case B1

It holds: $\mu = 0, \nu > 0, \lambda = 0\ ;\ P_t \geq 0, P_e = 0, P_t \geq p_{t1}$

By (2.10) it holds: $\pi_1 ND(p_{t1})(c_{imp} - c_t) = 0$



Graphic 5.3

As by hypothesis $c_{imp} > c_t$, then $D(p_{t1}) = 0$ and $p_{t1} = P_{max}$.

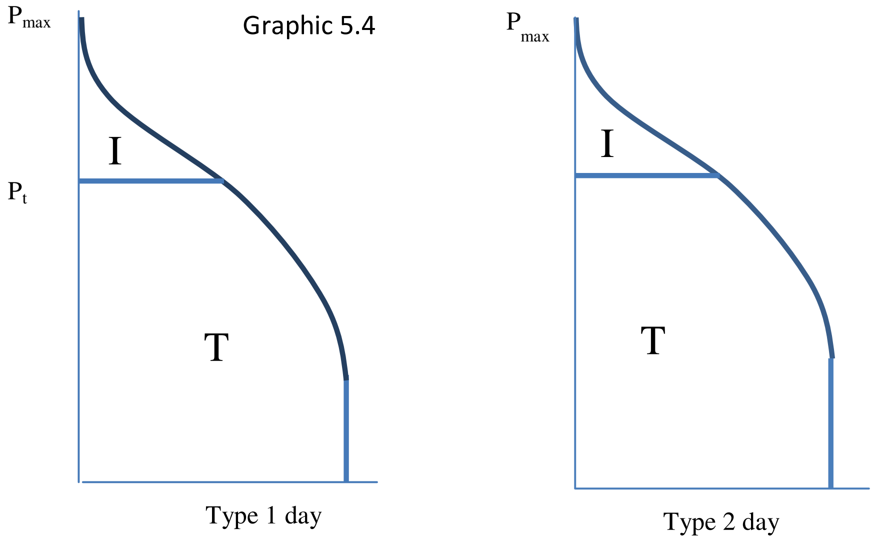
As $P_e = 0$ both types of day have identical dispatches and $p_{t1} = p_{t2} = P_{max}$.

Using (2.9) and $P_t = P_{max}$, we conclude that $f_t = 0$, which contradicts the hypotheses. Therefore there can be no solutions in this case.

4.5.1.4 Case B2

It holds: $\mu = 0, \nu > 0, \lambda > 0$; $P_t \geq 0, P_e = 0, P_t = p_{t1}$

As $P_e = 0$ both types of day have identical dispatches.



4.5.1.4.1 Necessary conditions

Using (2.8) we have:

$$f_e - \alpha_1 \pi_1 N\{D(P_t)(c_{imp} - c_t) + 24c_t\} = \nu > 0$$

$$\frac{f_e}{\alpha_1} > \pi_1 N\{D(P_t)(c_{imp} - c_t) + 24c_t\} \tag{5.30}$$

From (2.11) it holds: $f_t = ND(P_t)(c_{imp} - c_t)$

$$D(P_t) = \frac{f_t}{N(c_{imp} - c_t)} \tag{5.31}$$

For this equality to hold it is necessary that $c_{imp} > c_t$, which is true by hypothesis, and

$D(P_t) = \frac{f_t}{N(c_{imp} - c_t)} \leq 24$, which is equivalent to:

$$f_t \leq H(c_{imp} - c_t) \tag{5.32}$$

If (5.32) holds as equality, there exist infinite optimal solutions with $P_t \leq P_{min}$. If (5.32) holds as strict inequality then $P_t > P_{min}$ and there is a unique solution P_t, increasing in c_{imp}:

$$P_t = P\left(\frac{f_t}{N(c_{imp} - c_t)}\right) \tag{5.33}$$

If $f_t < H(c_{imp} - c_t)$ and the solution remains in case B2, then $\lim_{c_{imp} \to +\infty} P_t = P(0) = P_{max}$

Using (5.31) and (5.30) we have, as necessary condition for this case:

$$\frac{f_e}{\alpha_1} > \pi_1(f_t + Hc_t) \tag{5.34}$$

In summary the following proposition holds:

Proposition 5.10

- If $f_t = H(c_{imp} - c_t)$, there are infinite optimal solutions with $P_t \leq P_{min}$
- If $f_t < H(c_{imp} - c_t)$, the K-T solution is unique for every c_{imp}, $P_t > P_{min}$ and P_t is increasing in c_{imp}
- If $f_t < H(c_{imp} - c_t)$ and the solution remains in case B2, $\lim_{c_{imp} \to +\infty} P_t = P(0) = P_{max}$

4.5.1.4.2 Variation of the solutions as c_{imp} grows

If (5.32) holds as a strict inequality, there is a unique P_t, strictly greater than P_{min}.

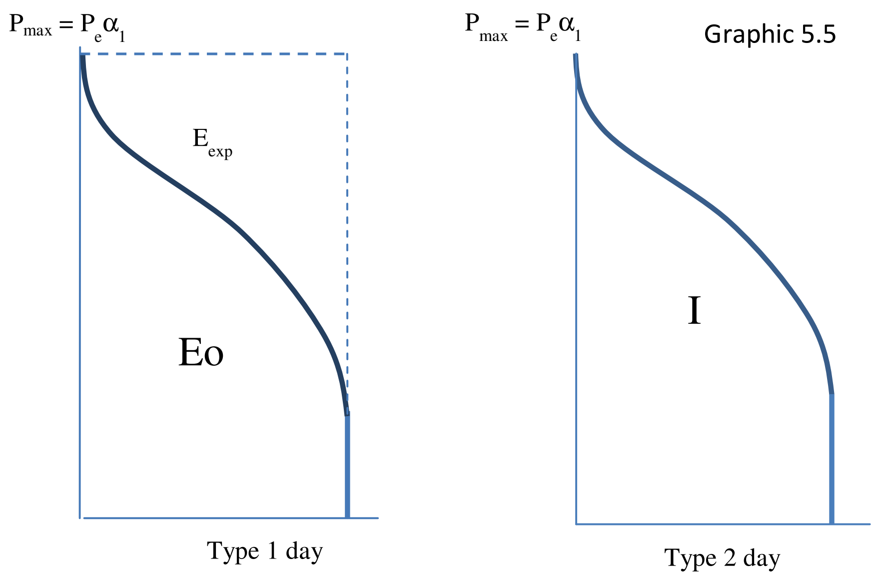
Let us denote by x^* a unique K-T solution. As c_{imp} grows, the necessary conditions for the solution to belong to this case still hold, and unique values for P_t are found, increasing in c_{imp} (by (5.33), so the following proposition holds.

Proposition 5.11: If for a set of parameters there is a unique K-T solution in case B2, increases of c_{imp} lead to unique solutions in the same case B2, with increasing values of P_t and $\lim_{c_{imp}\to+\infty} P_t = P(0) = P_{max}$.

4.5.1.5 Case C1

It holds: $\mu > 0, \nu = 0, \lambda = 0$; $P_t = p_{t1} = 0, P_e \geq 0$

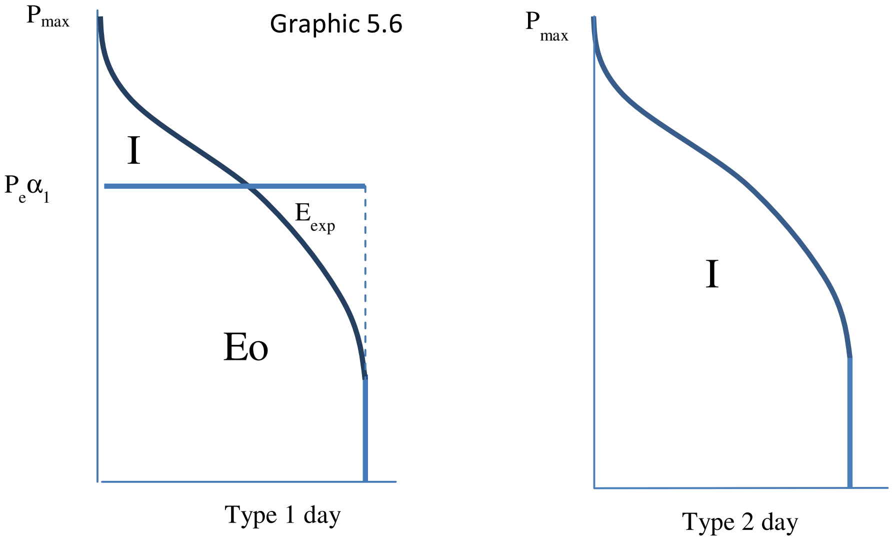
By (2.10) it holds $\pi_1 ND(P_e\alpha_1)(c_{imp} - c_t) = 0$. As by hypothesis $c_{imp} > c_t$, then $D(P_e\alpha_1) = 0$ and $P_e\alpha_1 = P_{max}$.



Using (2.8) and $P_e\alpha_1 = P_{max}$ yields: $f_e - \alpha_1\pi_1 Hv_{exp} = 0$, which contradicts hypothesis (2.2). Therefore there can be no solutions in this case.

4.5.1.6 Case C2

It holds: $\mu > 0, \nu = 0, \lambda > 0$; $P_t = 0, P_e \geq 0, p_{t1} = 0$



Using (2.8) we have:

$$f_e - \alpha_1 \pi_1 N\{D(P_e\alpha_1)(c_{imp} - c_t) + D(P_e\alpha_1)(c_t - v_{exp}) + 24v_{exp}\} = 0$$

$$f_e - \alpha_1 \pi_1 N\{D(P_e\alpha_1)(c_{imp} - v_{exp}) + 24v_{exp}\} = 0$$

By (2.1) $v_{exp} < c_t$. As in this case $c_t < c_{imp}$, then $c_{imp} > v_{exp}$ holds and then:

$$D(P_e\alpha_1) = \frac{\frac{f_e}{\alpha_1 \pi_1 N} - 24v_{exp}}{(c_{imp} - v_{exp})} \qquad (5.35)$$

For this case to be possible, it is necessary that: $\frac{\frac{f_e}{\alpha_1 \pi_1 N} - 24v_{exp}}{(c_{imp} - v_{exp})} > 0$

The numerator is positive by (2.2), and also the denominator as $c_{imp} > v_{exp}$.

It must also hold: $\frac{\frac{f_e}{\alpha_1 \pi_1 N} - 24v_{exp}}{(c_{imp} - v_{exp})} \leq 24$, and then:

$$\frac{f_e}{\alpha_1} \leq \pi_1 H c_{imp} \qquad (5.36)$$

If (5.36) holds as equality, there are infinite solutions with $0 \leq P_e\alpha_1 \leq P_{min}$. If (5.36) holds as strict inequality there is a unique solution for each c_{imp}, with $P_e\alpha_1 > P_{min}$.

By (2.11):

$$f_t - [\pi_1 ND(P_e\alpha_1) + \pi_2 N24](c_{imp} - c_t) = \mu > 0$$

$$f_t > [\pi_1 ND(P_e\alpha_1) + \pi_2 N24](c_{imp} - c_t) \tag{5.37}$$

Using (5.35) and (5.37) we have:

$$f_t > \left[\frac{f_e}{\alpha_1\pi_1 N} - 24v_{exp}\right]\frac{(c_{imp}-c_t)}{(c_{imp}-v_{exp})}N\pi_1 + \pi_2 H(c_{imp} - c_t) \tag{5.38}$$

If (5.36) holds as strict inequality from (5.35) there is a unique P_e:

$$P_e = \frac{1}{\alpha_1}P\left(\frac{\frac{f_e}{\alpha_1\pi_1 N}-24v_{exp}}{c_{imp}-v_{exp}}\right) = \frac{1}{\alpha_1}P\big(F(v_{exp}, c_{imp})\big) \quad \text{with } F \text{ defined in (5.8)}$$

$$\frac{\partial F(v_{exp}, c_{imp})}{\partial v_{exp}} = \frac{-24c_{imp}+\frac{f_e}{\alpha_1\pi_1 N}}{(c_{imp}-v_{exp})^2}$$, which in this case is negative by (5.36).

$$\frac{\partial F(v_{exp}, c_{imp})}{\partial c_{imp}} = -\frac{\frac{f_e}{\alpha_1\pi_1 N}-24v_{exp}}{(c_{imp}-v_{exp})^2}$$, which is always negative by (2.2).

As $P(d)$ is strictly decreasing, the optimal P_e is increasing in v_{exp} and also in c_{imp}.

In summary the following proposition holds:

Proposition 5.12:

In case C2:

- If $\frac{f_e}{\alpha_1} = \pi_1 Hc_{imp}$ there are infinite solutions for each c_{imp}, in the range $0 \leq P_e\alpha_1 \leq P_{min}$.
- If $\frac{f_e}{\alpha_1} < \pi_1 Hc_{imp}$ there is a unique solution for each c_{imp}, with $P_e\alpha_1 > P_{min}$.
- The optimal P_e in increasing both in v_{exp} and in c_{imp}.

By hypothesis (2.1) $v_{exp} < c_t$ and therefore $c_{imp} > v_{exp}$.

By hypothesis (2.2) $\frac{f_e}{\alpha_1 \pi_1 N} - 24v_{exp} > 0$. So the first addend of the second member of (5.38) is greater than zero and therefore the following proposition holds.

<u>Proposition 5.13</u>:

The values of c_{imp} in case C2 must hold:

$$c_{imp} < \frac{f_t}{\pi_2 H} + c_t \tag{5.39}$$

4.5.1.7 Case D1

It holds: $\mu > 0, \nu > 0, \lambda = 0\ ;\ P_t = p_{t1} = 0, P_e \geq 0$

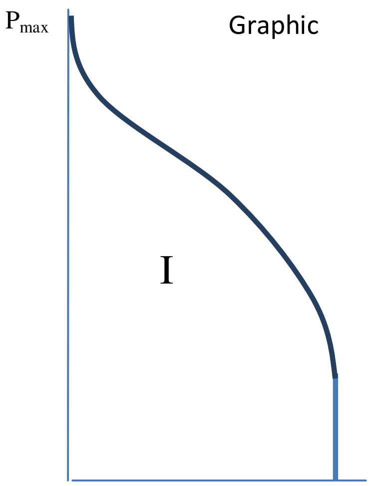
By (2.10): $\pi_1 ND(0)\left(c_{imp} - c_t\right) = 0$

This case is impossible unless $c_{imp} = c_t$, which contradicts the hypotheses.

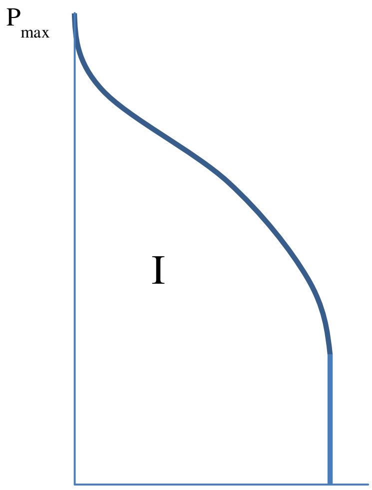
4.5.1.8 Case D2

It holds: $\mu > 0, \nu > 0, \lambda > 0\ ;\ P_t = p_{t1} = 0, P_e = 0$.

Graphic



Type 1 day



Type 2 day

By (2.8) for this case to be possible it must hold:

$f_e - \alpha_1 \pi_1 H c_{imp} = \nu > 0$

$\frac{f_e}{\alpha_1} > \pi_1 H c_{imp}$ (5.40)

From (2.11) results:

$f_t - H(c_{imp} - c_t) = \mu > 0$

$f_t > H(c_{imp} - c_t)$ (5.41)

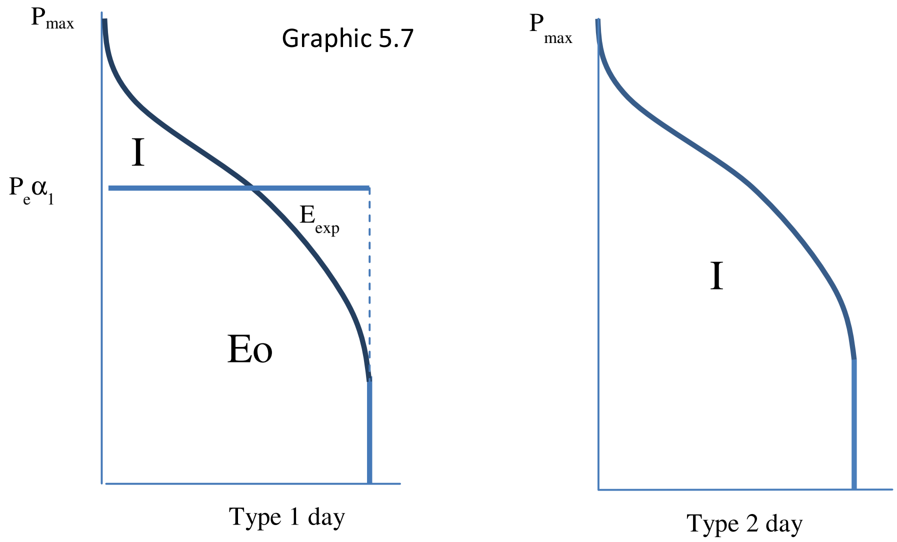
4.5.2 Annex 2 – Analysis of the possible cases of the optimal solution when $c_t \geq c_{imp}$

Under the hypothesis $c_t \geq c_{imp}$, we know beforehand that the optimal solution has:

$P_t = p_{t1} = 0.$

4.5.2.1 *Caso E1*

It holds: $\nu = 0$; $P_t = p_{t1} = 0, P_e \geq 0$



Using (2.13) we have:

$$f_e + \pi_1 N\{-\alpha_1 D(P_e\alpha_1)(c_{imp} - v_{exp}) - 24\alpha_1 v_{exp}\} = 0$$

$$D(P_e\alpha_1) = \frac{\frac{f_e}{\alpha_1\pi_1 N} - 24v_{exp}}{(c_{imp} - v_{exp})} \quad (5.42)$$

Then it must hold: $0 \leq \frac{\frac{f_e}{\alpha_1\pi_1 N} - 24v_{exp}}{(c_{imp} - v_{exp})} \leq 24$

The numerator is positive by (2.2) so a necessary condition for the existence of solutions in this case is:

$$c_{imp} > v_{exp} \quad (5.43)$$

It must also hold $\frac{f_e - \pi_1 H \alpha_1 v_{exp}}{\pi_1 N \alpha_1 (c_{imp} - v_{exp})} \leq 24$, and therefore:

$$\frac{f_e}{\alpha_1} \leq \pi_1 H c_{imp} \tag{5.44}$$

If condition (5.44) holds as equality there are infinite solutions with $P_e \alpha_1 \leq P_{min}$.

If (5.44) holds as a strict inequality there is a single value for P_e, with $P_e \alpha_1 > P_{min}$.

Using the definition of $F(v, c)$ from (5.8) results:

$$P_e = P\left(\frac{\frac{f_e}{\alpha_1 \pi_1 N} - 24 v_{exp}}{c_{imp} - v_{exp}}\right) \times \frac{1}{\alpha_1} = P\big(F(v_{exp}, c_{imp})\big) \times \frac{1}{\alpha_1}$$

As $P(.)$ is a decreasing function, P_e is increasing in c_{imp}.

It holds $\frac{\partial F(v_{exp}, c_{imp})}{\partial v_{exp}} = \frac{-24 c_{imp} + \frac{f_e}{\alpha_1 \pi_1 N}}{(c_{imp} - v_{exp})^2}$ which by (5.44) is non-positive.

Therefore P_e is increasing in v_{exp}.

In summary, the following proposition holds:

Proposition 5.14:

In case E1:

- $c_{imp} > v_{exp}$ and $\frac{f_e}{\alpha_1} \leq \pi_1 H c_{imp}$ are necessary conditions for the case
- The optimal P_e is increasing in c_{imp} and in v_{exp}.
- If $\frac{f_e}{\alpha_1} = \pi_1 H c_{imp}$ then there are infinite solutions with $P_e \alpha_1 \leq P_{min}$, and if $\frac{f_e}{\alpha_1} < \pi_1 H c_{imp}$ there is a single solution with $P_e \alpha_1 > P_{min}$.

4.5.2.2 Case E2

It holds: $v > 0$; $P_t = p_{t1} = 0, P_e = 0$

In this case, the whole demand is supplied with imports.

By (2.13):

$$f_e + \pi_1 N(-\alpha_1 24 c_{imp}) = \nu > 0$$

$$\frac{f_e}{\alpha_1} > \pi_1 H c_{imp}$$

If $c_{imp} \leq v_{exp}$, using (2.1) it holds $\frac{f_e}{\alpha_1} > \pi_1 H v_{exp}$, and therefore $\frac{f_e}{\alpha_1} > \pi_1 H c_{imp}$.

4.5.3 Annex 3 – Analysis of the optimal solution trajectories with increasing values of c_{imp}

4.5.3.1 Origin of the trajectories in D2/E2 for a null c_{imp}

If $c_{imp} = 0$, the problem has a single optimal solution with $P_e = 0$ and $P_t = 0$, in case E2.

As c_{imp} increases, the optimal solution remains unique for each value of c_{imp}, and belongs to case D2/E2, with $P_e = 0$ and $P_t = 0$ as long as the following conditions hold:

(No*1) $f_t > H(c_{imp} - c_t)$ which is equivalent to: $c_{imp} < \frac{f_t}{H} + c_t =: c_{imp}^1$ (5.45)

(No*2) $f_e/\alpha_1 > \pi_1 H c_{imp}$ which is equivalent to: $c_{imp} < \frac{f_e}{\alpha_1 \pi_1 H} =: c_{imp}^2$ (5.46)

No K-T solutions exist in other cases, as they require as necessary conditions either (*1) or (*2), the logical complements of the conditions above. Therefore the single solution in case D2/E2 is the unique solution of the problem of optimal dispatch.

The trajectory of the solution should move to another case when any of the conditions (*1) and (*2) become true as c_{imp} increases.

- If $\frac{f_e}{\alpha_1} < \pi_1(f_t + Hc_t)$, then $c_{imp}^2 < c_{imp}^1$ holds and (*2) becomes true before (for a smaller c_{imp}) than (*1).

  Condition (No*1) holds in a neighborhood to the right of c_{imp}^2, therefore the optimal solution cannot move at c_{imp}^2 to cases A1, A2 or B2, which have (*1) as a necessary condition.

  The only feasible possibility is the passage from case D2/E2 to C2/E1. For $c_{imp} = c_{imp}^2$, case C2/E1 has infinite solutions, in the range $0 \le P_e\alpha_1 \le P_{min}$, with $P_t = 0$. For values of c_{imp} greater than and arbitrarily close to c_{imp}^2, by Proposition 5.12 we have single K-T solutions in the case, with $P_e\alpha_1 > P_{min}$, which are also the unique solutions of the general problem of optimal dispatch, as solutions in other cases are not possible.

- If $\frac{f_e}{\alpha_1} > \pi_1(f_t + Hc_t)$ then $c_{imp}^1 < c_{imp}^2$ and (*1) becomes true before than (*2) when c_{imp} increases.

  Condition (No*2) holds in a neighborhood to the right of c_{imp}^1, therefore the optimal solution cannot move to cases C2/E2, A2 or A1, which have (*2) as a necessary condition.

  The only remaining possibility is the passage at c_{imp}^1 from case D2/E2 to case B2, and the permanence of the solution at case B2 in a neighborhood to the right of c_{imp}^1. By Proposition 5.10, at c_{imp}^1, case B2 has infinite solutions in the range $0 \leq P_t \leq P_{min}$. For values of c_{imp}, greater than and arbitrarily close to c_{imp}^1, we have unique solutions in case B2, with $P_t > P_{min}$, which are also the unique solutions of the general problem of optimal dispatch, as solutions in other cases are not possible.

- The event $\frac{f_e}{\alpha_1} = \pi_1(f_t + Hc_t)$ has null probability and will not be considered.

In summary, starting at D2/E2, as c_{imp} grows the trajectories of the optimal solutions:

- Move to case C2/E1 , if $\frac{f_e}{\alpha_1} < \pi_1(f_t + Hc_t)$.
- Move to case B2 , if $\frac{f_e}{\alpha_1} > \pi_1(f_t + Hc_t)$.

With further increases in c_{imp} solutions cannot return to case D2/E2, as either (*1) or (*2) hold indefinitely. The same reasoning applies in the following cases, eliminating the possibility of a return to D2/E2.

4.5.3.2 Trajectories in C2/E1 originated in D2/E2

The precedent analysis showed that trajectories originated in D2/E2 move to C2/E1 and for $c_{imp} > c_{imp}^2$ and close enough to c_{imp}^2, have unique solutions with $P_e\alpha_1 > P_{min}$.

By Proposition 5.12, as c_{imp} grows, the optimal P_e in case C2/E1 also increases and therefore it holds $P_e\alpha_1 > P_{min}$.

If a trajectory is in case C2/E2 after arriving from D2/E2, then $\frac{f_e}{\alpha_1} < \pi_1(f_t + Hc_t)$ holds. Therefore (*5) does not hold, and the optimal solution cannot move to case B2 with further increases in c_{imp}.

As a trajectory cannot return to case D2/E2, the only remaining possibilities are moving to cases A2 or A1. This can only happen if c_{imp} exceeds the value c_t, so only C2 can be the origin of such a passage, and not E1.

Let us define $c_{imp}^3 := \frac{f_t}{\pi_2 H} + c_t > c_{imp}^1$

The solutions in case A1 fulfill condition (*4) and therefore the following proposition holds:

<u>Proposition 5.15</u>: For any solution in case A1 holds the condition: $c_{imp} \geq \frac{f_t}{\pi_2 H} + c_t = c_{imp}^3$

Let us define: $$R(c_{imp}) := \left[\frac{f_e}{\alpha_1 \pi_1 N} - 24v_{exp}\right] \frac{(c_{imp} - c_t)}{(c_{imp} - v_{exp})} N\pi_1 + \pi_2 H\big(c_{imp} - c_t\big) \tag{5.47}$$

The solutions in case C2 fulfill condition (*6)

$$f_t > \left[\frac{f_e}{\alpha_1 \pi_1 N} - 24v_{exp}\right] \frac{(c_{imp} - c_t)}{(c_{imp} - v_{exp})} N\pi_1 + \pi_2 H\big(c_{imp} - c_t\big) = R(c_{imp}) \tag{5.48}$$

and also $c_{imp} \geq c_t$.

By hypothesis (2.1) $c_t > v_{exp}$ and therefore $c_{imp} > v_{exp}$.

By hypothesis (2.2) $\frac{f_e}{\alpha_1 \pi_1 N} - 24v_{exp} > 0$. Then the first addend of the second member in inequality (5.48) is positive.

It also holds:

$$\frac{dR}{dc_{imp}} = \left[\frac{f_e}{\alpha_1 \pi_1 N} - 24v_{exp}\right] \frac{(c_t - v_{exp})}{(c_{imp} - v_{exp})^2} N\pi_1 + \pi_2 H > 0 \tag{5.49}$$

so R is strictly increasing in c_{imp}.

$$R(c_{imp}^3) = \left[\frac{f_e}{\alpha_1 \pi_1 N} - 24v_{exp}\right]\frac{(c_{imp}^3 - c_t)}{(c_{imp}^3 - v_{exp})} N\pi_1 + f_t = k + f_t \tag{5.50}$$

Where $k > 0$ is a constant.

Therefore, for every c_{imp} leading to solutions in case C2, by (5.48) and (5.50), the following condition holds:

$$R(c_{imp}^3) = k + f_t > f_t > R(c_{imp}) \tag{5.51}$$

As R is continuous and strictly increasing, the following proposition holds:

Proposition 5.16: There exists a constant $\delta > 0$, so that for every c_{imp} leading to solutions in case C2, the following holds: $c_{imp}^3 > \delta + c_{imp}$

By propositions 5.15 and 5.16 it is impossible for the trajectory of optimal solutions to move directly from case C2 to case A1 as c_{imp} grows.

As f_e, f_t and c_t are finite, when $c_{imp} \to +\infty$, the solution cannot remain in case C2/E1 indefinitely. Otherwise costs would exceed any arbitrary value, as a fixed amount of energy in type 2 days would be supplied at cost c_{imp}.

The only remaining possibility is that a trajectory in case C2/E1, originated in D2/E2, moves afterwards to case A2 as c_{imp} grows.

4.5.3.3 Trajectories in A2 resulting from the sequence D2/E2 – C2/E1

4.5.3.3.1 Trajectory entry in case A2 from case C2

As $P_t = 0$ in every solution in case C2, and the solutions are upper hemicontinuous, it is necessary to investigate the solutions in case A2 which have $P_t = 0$, or arbitrarily close to zero in the vicinity of the passage from C2 to A2. A passage to other solutions in A2 without that feature would violate the upper hemicontinuity.

Using (5.9) and (5.10) (the necessary conditions for case A2) and making $P_t = 0$, we have:

$$D(P_e\alpha_1)(c_{imp}-v_{exp}) = \frac{f_e}{\alpha_1\pi_1 N} - 24v_{exp}$$

$$\pi_1 D(P_e\alpha_1) + 24\pi_2 = \frac{f_t}{N(c_{imp}-c_t)}$$

These conditions hold if:

$$D(P_e\alpha_1) = \frac{\frac{f_e}{\alpha_1\pi_1 N}-24v_{exp}}{c_{imp}-v_{exp}} = \frac{f_t}{N\pi_1(c_{imp}-c_t)} - 24\frac{\pi_2}{\pi_1} \tag{5.52}$$

The first equality is the same as (5.35) which is fulfilled by the optimal values of P_e in case C2 solutions.

The second equality is equivalent to:

$$f_t = \left[\frac{f_e}{\alpha_1\pi_1 N} - 24v_{exp}\right]\frac{(c_{imp}-c_t)}{c_{imp}-v_{exp}}N\pi_1 + \pi_2 H(c_{imp}-c_t) \tag{5.53}$$

Using the definition of $R(c_{imp})$ from (5.47), equation (5.53) can be rewritten as $f_t = R(c_{imp})$.

Let us call c_{imp}^4 the solution of c_{imp} in (5.53), which is unique as R is strictly increasing by (5.49).

The equality (5.53) is the border condition of inequality (*6) which is a necessary condition of case C2.

In summary we have found a solution x^4 in case A2, for $c_{imp} = c_{imp}^4$, which is the limit of the solutions in C2 (unique for every c_{imp}) when $c_{imp} \to c_{imp}^4$. This solution x^4 is the passage from C2 to A2. By the upper hemicontinuity no other solutions in A2 could be the destination of such passage.

The solution x^4 in case A2 has $P_e\alpha_1 > P_{min}$, as the solutions in C2 in a neighborhood of c_{imp}^4 also fulfill this condition and have strictly increasing values of P_e.

Therefore in x^4 it holds $P_{min} < P_e\alpha_1$ and $P_e\alpha_1 + P_t < P_{max}$, which is the sufficient condition (5.15) of Proposition 5.3. As a consequence x^4 is a unique K-T solution in case A2 and moreover is the unique solution of the general problem, as no K-T solutions in other cases can exist.

4.5.3.3.2 Exit or permanence in A2

As the trajectory of optimal solutions enters A2, condition (5.53) (which is a particular case of (No*6)) is fulfilled. Condition (No*6) remains true as c_{imp} grows, as $R(c_{imp})$ is strictly increasing. Therefore the trajectory cannot return to C2, where (*6) holds.

We have proved before that the solution cannot return to D2/E2 either.

As $\frac{f_e}{\alpha_1} < \pi_1(f_t + Hc_t)$, the solution cannot move to case B2, which has $\frac{f_e}{\alpha_1} > \pi_1(f_t + Hc_t)$ as a necessary condition.

Therefore as c_{imp} grows, the only possibilities for an optimal trajectory are either remaining indefinitely in case A2, or moving to case A1.

As we have seen, the solutions in case A2 have the following as necessary conditions:

(5.9) $D(P_e\alpha_1 + P_t)(c_{imp} - c_t) + D(P_e\alpha_1)(c_t - v_{exp}) = \frac{f_e}{\alpha_1\pi_1 N} - 24v_{exp}$

(5.10) $\pi_1 D(P_e\alpha_1 + P_t) + \pi_2 D(P_t) = \frac{f_t}{N(c_{imp} - c_t)}$

The following proposition can be proved:

<u>Proposition 5.17</u>: If $D(P_e\alpha_1 + P_t) > 0$ and $P_e\alpha_1 > P_{min}$ in a solution in case A2, and the solution remains in case A2 as c_{imp} grows, then $D(P_e\alpha_1 + P_t)$ is strictly decreasing in c_{imp} , and therefore $P_e\alpha_1 + P_t$ is strictly increasing.

Proof:

Let us denote "strictly increasing in c_{imp}" by the sign ↑ and "strictly decreasing in c_{imp}" by the sign ↓.

Using (5.9), as $D(P_e\alpha_1 + P_t) > 0$, $c_t - v_{exp} > 0$, $c_{imp} - c_t > 0$, the second member of (5.9) does not depend on c_{imp}, and $c_{imp} - c_t\uparrow$, then if the solution remains in case A2 at least one of the following is true: $D(P_e\alpha_1 + P_t)\downarrow$ or $D(P_e\alpha_1)\downarrow$.

Using (5.10), as $\pi_1 > 0$, $\pi_2 > 0$ and $\frac{f_t}{N(c_{imp}-c_t)}\downarrow$, then if the solution remains in case A2, at least one of the following is true: $D(P_e\alpha_1 + P_t)\downarrow$ or $D(P_t)\downarrow$.

Let us assume that $D(P_e\alpha_1 + P_t)\downarrow$ is not true. Then $D(P_e\alpha_1)\downarrow$ and $D(P_t)\downarrow$ must hold, as long as the trajectory remains in case A2. But then $P_e\alpha_1\uparrow$ and $P_t\uparrow$, and as a consequence $P_e\alpha_1 + P_t\uparrow$. As from the beginning $P_e\alpha_1 > P_{min}$ holds, then $D(P_e\alpha_1 + P_t)\downarrow$, which is absurd. ■

As the trajectory of optimal solutions comes from a sequence of cases D2/E2 – C2/E1, then $\frac{f_e}{\alpha_1} < \pi_1(f_t + Hc_t)$ holds. Let us consider two subcases:

- Subcase A2-i: $\frac{f_e}{\alpha_1} < \pi_1 Hc_t$ holds, wind capacity is so cheap that wind energy costs less than the pure variable cost of thermal plants to supply the base of the load duration curve in type 1 days.
- Subcase A2-ii: $\pi_1 Hc_t < \frac{f_e}{\alpha_1} < \pi_1(f_t + Hc_t)$

The event when $\frac{f_e}{\alpha_1} = \pi_1 Hc_t$ holds, has null probability and will not be considered.

4.5.3.3.2.1 Subcase A2-i, $f_e\alpha_1 < \pi_1 Hc_t$

By Proposition 5.7, $\pi_1 Hc_t \leq \frac{f_e}{\alpha_1}$ is a necessary condition for solutions in case A2 to have $P_e\alpha_1 \leq P_{min}$. Therefore in the Subcase A2-1 we are considering here, as the solutions do not fulfill that necessary condition, they all have $P_e\alpha_1 > P_{min}$.

Therefore for any solution with $P_e\alpha_1 + P_t < P_{max}$, it holds $D(P_e\alpha_1 + P_t) > 0$ and the hypothesis of Proposition 5.17 are fulfilled. Then $D(P_e\alpha_1 + P_t)$ is strictly decreasing and $P_e\alpha_1 + P_t$ is strictly increasing in c_{imp}, as long as the solution remains in case A2.

As $P_e\alpha_1 > P_{min}$ and $P_e\alpha_1 + P_t < P_{max}$, then condition (5.15) holds and using Proposition 5.3 for each c_{imp} the K-T solution is unique in case A2. No solutions can exist in other cases, so the solution is also unique for the general problem.

As (5.15) holds, the hypothesis of Proposition 5.5 are fulfilled and therefore P_t is increasing with c_{imp}.

Let us assume the solution remains indefinitely in case A2 (in subcase A2-i) as c_{imp} grows.

Using equation (5.10) results $\lim D(P_e\alpha_1 + P_t) = 0$ and $\lim D(P_t) = 0$ when $c_{imp} \to +\infty$. Therefore $\lim P_e\alpha_1 + P_t = P_{max}$ and $\lim P_t = P_{max}$. Then we have $\lim P_e\alpha_1 = 0$. But this is not possible as the solutions in subcase A2-i all have $P_e\alpha_1 > P_{max}$. The contradiction comes from assuming the trajectory of optimal solutions remain in case A2 indefinitely; therefore we conclude the solutions move to case A1.

Using $\frac{f_e}{\alpha_1} < \pi_1 H c_t$ and Proposition 5.1, we conclude that $P_e\alpha_1 > P_{min}$ for all the solutions in case A1 and that the optimal wind capacity does not depend on c_{imp}. Using (5.4) we have:

$D(P_e\alpha_1) = \frac{\frac{f_e}{\alpha_1\pi_1 N} - 24v_{exp}}{c_t - v_{exp}}$ and therefore as $P_e\alpha_1 > P_{min}$:

$$P_e\alpha_1 = \frac{1}{\alpha_1} P\left(\frac{\frac{f_e}{\alpha_1\pi_1 N} - 24v_{exp}}{c_t - v_{exp}}\right) =: P_e^5 \quad .$$

Let us impose the condition $P_e\alpha_1 + P_t = P_{max}$ to solutions in case A1.

$P_e = P_e^5$, $P_t = P_{max} - P_e^5\alpha_1$ is a solution in case A1, for the value of c_{imp} that fulfills the following equation, which comes from (5.1):

$$\frac{f_t}{\pi_2 N(c_{imp} - c_t)} = D(P_t) = D\left(P_{max} - P\left(\frac{\frac{f_e}{\alpha_1\pi_1 N} - 24v_{exp}}{c_t - v_{exp}}\right)\right) \tag{5.54}$$

Let us call c_{imp}^5 the solution of (5.54).

As the second member of this inequality is less than or equal to 24, it holds:

$\frac{f_t}{\pi_2 N(c_{imp}^5 - c_t)} \le 24$, and then $f_t \le \pi_2 H(c_{imp}^5 - c_t)$. (5.55)

Let us observe that:

$$c_{imp}^5 \ge \frac{f_t}{\pi_2 H} + c_t > \frac{f_t}{H} + c_t = c_{imp}^1 > c_{imp}^2 \quad (5.56)$$

For $c_{imp} = c_{imp}^5$, the values $P_e = P_e^5$ and $P_t = P_{max} - P_e^5\alpha_1$, are solutions in case A2, as they fulfill conditions (5.9) y (5.10):

(5.9): $0.\left(c_{imp}^5 - c_t\right) + D(P_e^5\alpha_1)\left(c_t - v_{exp}\right) = \frac{f_e}{\alpha_1\pi_1 N} - 24v_{exp}$, is fulfilled by the definition of P_e^5.

(5.10): $\pi_1 . 0 + \pi_2 D(P_t) = \frac{f_t}{N(c_{imp}^5 - c_t)}$, is fulfilled by the definition of c_{imp}^5.

In summary we have found a common solution, shared by cases A2 and A1, for the value $c_{imp} = c_{imp}^5$. This solution is then the passage from case A2 to case A1.

4.5.3.3.2.2 Subcase A2-ii, $f_e\alpha_1 > \pi_1 H c_t$

As $\frac{f_e}{\alpha_1} > \pi_1 H c_t$, the condition (*3) which is necessary for solutions in case A1, does not hold. Then the trajectories of optimal solutions cannot move from case A2 to case A1. We have proved that the transitions to every other case are not possible, therefore we conclude the solutions remain indefinitely in case A2 as c_{imp} grows.

From equation (5.10) we conclude that: $\lim D(P_e\alpha_1 + P_t) = 0$ and $\lim D(P_t) = 0$ when $c_{imp} \to +\infty$. Therefore $\lim P_e\alpha_1 + P_t = P_{max}$ and $\lim P_t = P_{max}$. Then we have $\lim P_e\alpha_1 = 0$, so the solution remain in subcase A2-b.

4.5.3.4 Trajectories in A1 resulting from the sequence D2/E2 – C2/E1 – A2 (subcase A2-i, $f_e\alpha_1<\pi_1Hc_t$)

At $c_{imp} = c_{imp}^5$, the trajectory of optimal solutions (a unique solution for each c_{imp}), moves from case A2 to A1, and the following conditions hold: $P_e\alpha_1 > P_{min}$ and $f_t \leq \pi_2 H(c_{imp}^5 - c_t)$. As c_{imp} grows beyond c_{imp}^5 the trajectories:

- Cannot move to D2/E2 as $c_{imp} > c_{imp}^5 > c_{imp}^2$, so (No*2) does not hold.
- Cannot move to C2 as $c_{imp} > c_{imp}^5 \geq \frac{f_t}{\pi_2 H} + c_t$, while in C2 by (5.39) $c_{imp} < \frac{f_t}{\pi_2 H} + c_t$
- Cannot move to B2 as $\frac{f_e}{\alpha_1} < \pi_1 H c_t$ by hypothesis, so (*5) does not hold.

Finally, we will prove in what follows that the trajectories cannot return to A2.

By Proposition 5.2, as c_{imp} exceeds c_{imp}^5, P_e remains constant and P_t is strictly increasing. At $c_{imp} = c_{imp}^5$, it holds $P_e + P_t = P_{max}$. Then for any $c_{imp} > c_{imp}^5$, there is a constant k^5 so that the solutions in A1 fulfill the condition: $P_e + P_t > P_{max} + k^5$.

By the upper hemicontinuity of the solutions, the trajectory cannot move to case A2, where the solutions fulfill the condition $P_e + P_t \leq P_{max}$. Therefore the trajectory must remain in case A1, with constant P_e and with P_t increasing in c_{imp} and tending to P_{max} as $c_{imp} \to \infty$.

4.5.3.5 Trajectories in B2 originated in D2/E2

If a trajectory of optimal solutions is in case B2, and has come from case D2/E2, then $\frac{f_e}{\alpha_1} > \pi_1(f_t + Hc_t)$ (denoted by (*5)) must hold.

We have shown in section 5.3.1 that a trajectory originated in D2/E2 and entering case B2, fulfills $P_t > P_{min}$, in a vicinity to the right of c_{imp}^1, and contains unique solutions of the general problem.

We have also shown that as c_{imp} grows the trajectory cannot return to D2/E2.

The trajectory cannot move to A1, as this case requires the necessary condition $\frac{f_e}{\alpha_1} < \pi_1 H c_t$, which contradicts (*5) that holds by hypothesis.

All solutions in case B2 have $P_e = 0$. If the trajectory of optimal solutions moved from B2 to A2, at a value c_{imp}^6 of imports price, then by the upper hemicontinuity of the solutions, the right hand limit of P_e for $c_{imp} \rightarrow {c_{imp}^6}^+$ in case A2, would be $P_e \rightarrow 0$. Then the solutions would be in case A2-b, as $P_e < P_{min}$. But the solutions in A2-b must fulfill the necessary condition $\frac{f_e}{\alpha_1} \leq \pi_1(f_t + H c_t)$, which contradicts the hypothesis. We conclude that the passage from B2 to A2 is not possible.

The trajectory cannot move to case C2. If such a passage existed, by the hemicontinuity of the solutions, as $P_t > P_{min}$ in case B2, there would exist a solution in case C2 with $P_t \geq P_{min}$, which is not possible.

In summary the trajectories originated in D2/E2 that move to case B2, remain indefinitely in case B2, as c_{imp} grows.

4.5.4 Annex 4- Properties of the objective function and the solutions of the optimal portfolio problem

By (2.12) the objective function to minimize in the optimal portfolio problem when $c_{imp} \leq c_t$, is:

$$CT^*(P_e) = P_e f_e + \pi_1 N\{[E - E(P_e\alpha_1)]c_{imp} - [24P_e\alpha_1 - E(P_e\alpha_1)]v_{exp}\} + \pi_2 NEc_{imp}$$

By (2.7) the objective function when $c_{imp} > c_t$, is:

$$CT(P_e, P_t, p_{t1}) = P_e f_e + P_t f_t + F_1(P_e, p_{t1}) + F_2(P_t)$$

Where:

$$F_1(P_e, p_{t1}) = \pi_1 N\{[E - E(P_e\alpha_1 + p_{t1})]c_{imp} + [E(P_e\alpha_1 + p_{t1}) - E(P_e\alpha_1)]c_t - [24P_e\alpha_1 - E(P_e\alpha_1)]v_{exp}\}$$

$$F_2(P_t) = \pi_2 N\{[E - E(P_t)]c_{imp} + E(P_t)c_t\}$$

The linear constraints of the problem are:

$$P_t - p_{t1} \geq 0$$

$$P_e \geq 0$$

$$P_t \geq 0$$

4.5.4.1 Existence of an optimal solution

In both cases $c_{imp} \leq c_t$ and $c_{imp} > c_t$ the objective functions are continuous in the control variables. The constraints determine a compact set. By the Weierstrass theorem there is at least one optimal solution.

4.5.4.2 Convexity of the objective function

4.5.4.2.1 Convexity of the objective CT to be minimized when $c_{imp} > c_t$

Our problem is to find sufficient conditions for the convexity of the objective CT to minimize, or equivalently for the concavity of $-\mathrm{CT}$, therefore we will analyze the concavity of $-\mathrm{F}_1$ and $-\mathrm{F}_2$.

4.5.4.2.1.1 Concavity of $-F_1$ and $-F_2$

Proposition 5.18

$-F_1(P_e, p_{t1})$ is non-strictly concave when $c_{imp} > c_t$, $c_t > v_{exp}$, conditions which are fulfilled by hypothesis, and moreover is strictly concave at points where $P_{min} < P_e\alpha_1 + p_{t1} < P_{max}$.

The partial derivatives of F_1 are:

$$F_{1P_e} = -\alpha_1\pi_1 N\{D(P_e\alpha_1 + p_{t1})(c_{imp} - c_t) + D(P_e\alpha_1)(c_t - v_{exp}) - 24\}$$

$$F_{1P_eP_e} = -\alpha_1{}^2\,\pi_1 N\{D'(P_e\alpha_1 + p_{t1})(c_{imp} - c_t) + D'(P_e\alpha_1)(c_t - v_{exp})\}$$

$$F_{1P_ep_{t1}} = -\alpha_1\pi_1 ND'(P_e\alpha_1 + p_{t1})(c_{imp} - c_t)$$

$$F_{1p_{t1}} = -\pi_1 ND(P_e\alpha_1 + p_{t1})(c_{imp} - c_t)$$

$$F_{1p_{t1}p_{t1}} = -\pi_1 ND'(P_e\alpha_1 + p_{t1})(c_{imp} - c_t)$$

The Hessian matrix of $-F_1$, which we will denote by H_1 is:

$$H_1 = \pi_1 N\begin{bmatrix} \alpha_1^2 D'(P_e\alpha_1{+}p_{t1})(c_{imp} - c_t) + \alpha_1^2 D'(P_e\alpha_1)(c_t - v_{exp}) & \alpha_1 D'(P_e\alpha_1{+}p_{t1})(c_{imp} - c_t) \\ \alpha_1 D'(P_e\alpha_1{+}p_{t1})(c_{imp} - c_t) & D'(P_e\alpha_1{+}p_{t1})(c_{imp} - c_t) \end{bmatrix}$$

For $-F_1$ to be non-strictly concave, H_1 must be negative semidefinite, which has the following necessary and sufficient condition:

$$\alpha_1^2 D'(P_e\alpha_1{+}p_{t1})(c_{imp} - c_t) + \alpha_1^2 D'(P_e\alpha_1)(c_t - v_{exp}) \le 0 \tag{5.57}$$

$$\begin{vmatrix} \alpha_1^2 D'(P_e\alpha_1{+}p_{t1})(c_{imp} - c_t) + \alpha_1^2 D'(P_e\alpha_1)(c_t - v_{exp}) & \alpha_1 D'(P_e\alpha_1{+}p_{t1})(c_{imp} - c_t) \\ \alpha_1 D'(P_e\alpha_1{+}p_{t1})(c_{imp} - c_t) & D'(P_e\alpha_1{+}p_{t1})(c_{imp} - c_t) \end{vmatrix} \ge 0 \tag{5.58}$$

For the strict concavity to hold, both inequalities must hold strictly.

The value of the determinant of the matrix in (5.58) is:

$$\alpha_1^2 D'(P_e\alpha_1)D'(P_e\alpha_1{+}p_{t1})(c_{imp} - c_t)(c_t - v_{exp})$$

For both inequalities (5.57) and (5.58) to hold non-strictly it is sufficient that: $c_{imp} \ge c_t$ and $c_t \ge v_{exp}$. Both conditions are fulfilled by hypothesis so $-F_1$ is non-strictly concave and therefore non-strictly quasiconcave.

For both inequalities to hold strictly, ensuring the strict concavity, the following set of conditions is sufficient: $c_{imp} > c_t$ and $c_t > v_{exp}$, (both true by hypothesis), and $P_{min} < P_e\alpha_1 + p_{t1} < P_{max}$.

Proposition 5.19

$-F_2(P_t)$ is non-strictly concave when $c_{imp} \geq c_t$, and strictly concave if this inequality holds strictly and additionally $P_t > P_{min}$.

The second derivative of $-F_2$ is:

$$-F_2''(P_t) = \pi_2 ND'(P_t)(c_{imp} - c_t)$$

This derivative is non-negative if $c_{imp} \geq c_t$ and is strictly positive if $c_{imp} > c_t$ and additionally $P_{min} < P_t < P_{max}$.

4.5.4.2.1.2 Concavity of -CT

Let us call:

$$F = -CT = -P_e f_e - P_t f_t - F_1(P_e, p_{t1}) - F_2(P_t)$$

The non-strict concavity of F requires that for all $x = (P_e^x, P_t^x, p_{t1}^x)$, $y = (P_e^y, P_t^y, p_{t1}^y)$, and all $\alpha \in [0,1]$:

$$F(\alpha x + (1-\alpha)y) \geq \alpha F(x) + (1-\alpha)F(y)$$

Using the definition of F in the second member of the inequality we have:

$$\alpha F(x) + (1-\alpha)F(y) = \alpha[-P_e^x f_e - P_t^x f_t - F_1(P_e^x, p_{t1}^x) - F_2(P_t^x)]$$

$$+(1-\alpha)\left[-P_e^y f_e - P_t^y f_t - F_1\left(P_e^y, p_{t1}^y\right) - F_2\left(P_t^y\right)\right] =$$

$$= -\left[\alpha P_e^x + (1-\alpha)P_e^y\right]f_e - \left[\alpha P_t^x + (1-\alpha)P_t^y\right]f_t$$

$$-\left[\alpha F_1(P_e^x, p_{t1}^x) + (1-\alpha)F_1\left(P_e^y, p_{t1}^y\right)\right] - \left[\alpha F_2(P_t^x) + (1-\alpha)F_2(P_t^y)\right]$$

If $-F_1$ and $-F_2$ are concave then:

$$\alpha F(x) + (1-\alpha)F(y) \le -\left[\alpha P_e^x + (1-\alpha)P_e^y\right]f_e - \left[\alpha P_t^x + (1-\alpha)P_t^y\right]f_t$$

$$-F_1\left[\alpha P_e^x + (1-\alpha)P_e^y, \alpha p_{t1}^x + (1-\alpha)p_{t1}^y\right]$$

$$-F_2\left[\alpha P_t^x + (1-\alpha)P_t^y\right]$$

$$= F[\alpha x + (1-\alpha)y\,]$$

for all $\alpha \in [0,1]$, x, y.

Therefore the following proposition holds:

Proposition 5.20: If $c_{imp} \ge c_t$ and $c_t \ge v_{exp}$ (and this conditions hold by hypothesis), $-CT$ is concave, CT is convex and therefore quasiconvex.

If $-F_1$ and $-F_2$ are strictly concave, the inequality required by the concavity holds strictly for any x, y. Using Propositions 5.18 and 5.19, we conclude in the following proposition:

Proposition 5.21: If $c_{imp} > c_t$ and $c_t > v_{exp}$ hold (and this conditions hold by hypothesis), and additionally the following conditions $P_{min} < P_e\alpha_1 < P_{max}$ and $P_{min} < P_t < P_{max}$ hold, then $-CT$ is strictly concave and the objective CT is strictly convex.

4.5.4.2.2 Convexity of the objective CT*(Pe) to be minimized when $c_{imp} \le c_t$

Derivating in (2.12) we get:

$$\frac{dCT^*}{dP_e} = f_e - \alpha_1\pi_1 N\left[D(P_e\alpha_1)c_{imp} + (24 - D(P_e\alpha_1))v_{exp}\right]$$

$$\frac{d^2CT^*}{dP_e^2} = -\alpha_1^2\pi_1 ND'(P_e\alpha_1)(c_{imp} - v_{exp}) \qquad (5.59)$$

If $c_{imp} \ge v_{exp}$, then $\frac{d^2CT^*}{dP_e^2} \ge 0$, the objective function CT^* is non-strictly convex and therefore quasiconvex in $[0, P_{max})$.

If $c_{imp} < v_{exp}$ then:

$\frac{dCT^*}{dP_e} > f_e - \alpha_1 \pi_1 H v_{exp} > 0$ by hypothesis (2.2). This means that CT^* is monotonically increasing and therefore quasiconvex.

Then the following proposition holds:

Proposition 5.22: For any values of the parameters of the problem CT^* is cuasiconvex.

4.5.4.3 Properties of the solutions

As a result of Propositions 5.20 and 5.22, the objective of the problem of optimal design is quasiconvex for any values of the parameters that fulfill the hypotheses.

If the objective function of a minimization problem is non-strictly quasiconvex and the constraints of the problem determine a convex feasible set, then the set of optimal solutions is convex.

The constraints of our problem of optimal design are linear and therefore determine a convex set. Therefore the following proposition holds:

Proposition 5.23: For any set of parameters that fulfill the hypotheses, the set of solutions of the optimal design problem is convex.

If the objective function of a minimization problem is strictly quasiconvex and the constraints determine a convex feasible set, then the optimal solution is unique.

Proposition 5.24: If a solution of the optimal design problem fulfills the following conditions $P_{min} < P_e \alpha_1 < P_{max}$ and $P_{min} < P_t < P_{max}$, then this solution is unique.

Proof:

If $c_{imp} > c_t$ the objective function is CT. Let us call x^* one of the optimal solutions of the problem. By Proposition 5.23, the set of optimal solutions is convex. Let us assume there exists another solution y^*.

Every point in the segment $x^* y^*$ that joins x^* and y^*, is also an optimal solution, as the set of solutions is convex.

As x^* is interior to the set defined by $P_{min} < P_e\alpha_1 < P_{max}$ and $P_{min} < P_t < P_{max}$, there exist an optimal solution y' contained in x^*y^* and a segment x^*y', the points of which are optimal solutions and also fulfill both conditions.

In the segment x^*y' the objective CT is strictly convex by Proposition 5.21. Therefore, there exists a solution y'' interior to the segment x^*y', and $CT(y'') < CT(y') = CT(x^*)$ holds, against the hypothesis of optimality of x^*. Therefore the solution x^* must be unique.

If $c_{imp} \leq c_t$ the objective function is CT^*. By Proposition 5.14, if a solution fulfills condition $P_{min} < P_e\alpha_1 < P_{max}$, it is unique.

4.5.4.4 Upper hemicontinuity of the optimal solutions

The problem of optimal design of the generation portfolio has control variables P_e, P_t, p_{t1}.

Assuming π_1, π_2, α_1 and $D(.)$ are fixed, the parameters of the problem are the costs and prices f_e, f_t, c_t, c_{imp} and v_{exp}.

Let us call $S(.)$, the correspondence that for every set of parameters, gives the set of optimal solutions. This set of optimal solutions is contained in the space of controls P_e, P_t, p_{t1}.

The problem of optimal design fulfills the hypotheses of the maximum theorem. The objective function is continuous both in the parameters and the control variables. The set of feasible points is delimited by linear constraints, independent of the parameters, which is a particular form of continuous dependence.

The image in S of any set of parameters is bounded, as the feasible solutions of the problem are always bounded and so the optimal solutions.

By the maximum theorem the correspondence S is upper hemicontinuous and the value of the objective is a continuous function of the parameters.

The upper hemicontinuity means in this problem that for any sequence $\{x_m\}_{m=1}^{\infty}$ in the space of parameters, and any sequence $\{s_m\}_{m=1}^{\infty}$ in the space of controls, such that $s_m \in S(x_m)$ (s_m

is one of the optimal solutions for x_m), $x_m \to x$ and $s_m \to s$, then $s \in S(x)$ (s is an optimal solution for the parameters x). Then the following proposition holds:

Proposition 5.25: Let us consider fixed all other parameters so that only c_{imp} can change. If there is a unique solution $x(c_{imp})$ for every $c_{imp} \neq c$, so that $x(c_{imp}) \to x^c$ when $c_{imp} \to c$, then x^c is one (possibly the unique) solution of the problem for $c_{imp} = c$.

In particular if c_{imp} belongs to a segment (c_1, c_2) where the solutions are unique, upper hemicontinuity is equivalent to continuity.

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