UNIVERSIDAD DE LA REPUBLICA URUGUAY

FACULTAD DE INGENIERÍA

Modelos de tiempo continuo para el scheduling de crudo y gases licuados de petróleo en procesos de refinación

Bernardo Zimberg

Programa de Posgrado en Ingeniería Eléctrica
Facultad de Ingeniería
Universidad de la República

Montevideo – Uruguay
Diciembre de 2025

UNIVERSIDAD DE LA REPUBLICA
URUGUAY

FACULTAD DE INGENIERÍA

Modelos de tiempo continuo para el scheduling de crudo y gases licuados de petróleo en procesos de refinación

Bernardo Zimberg

Tesis de Doctorado presentada al Programa de Posgrado en Ingeniería Eléctrica, Facultad de Ingeniería de la Universidad de la República, como parte de los requisitos necesarios para la obtención del título de Doctor en Ingeniería Eléctrica.

Director de tesis:
Ph.D. Eduardo Camponogara

Codirector:
Dr. Pablo Monzón

Director académico:
Dr. Pablo Monzón

Montevideo – Uruguay
Diciembre de 2025

Zimberg, Bernardo

Modelos de tiempo continuo para el scheduling de crudo y gases licuados de petróleo en procesos de refinación / Bernardo Zimberg. - Montevideo: Universidad de la República, Facultad de Ingeniería, 2025.

XVI, 130 p.: il.; 29, 7cm.

Director de tesis:

Eduardo Camponogara

Codirector:

Pablo Monzón

Director académico:

Pablo Monzón

Tesis de Doctorado – Universidad de la República, Programa de Ingeniería Eléctrica, 2025.

Referencias bibliográficas: p. 109 – 114.

1. Scheduling en procesos de refinación, 2. Ductos, 3. Modelo de tiempo continuo, 4. Programación entera, 5. Gestión de inventarios. I. Camponogara, Eduardo *et al.* II. Universidad de la República, Programa de Posgrado en Ingeniería Eléctrica. III. Título.

INTEGRANTES DEL TRIBUNAL DE DEFENSA DE TESIS

Dra. Mariana Corengia

Dr. Leandro Magatão

Dr. Mario Vignolo

Montevideo – Uruguay
Diciembre de 2025

Agradecimientos

Agradezco a Eduardo Camponogara
el gran aporte técnico y apoyo durante este largo período
Pablo Monzón
la contribución y el fuerte respaldo
Mario Vignolo
la gestión académica
Enrique Ferreira, Carlos Testuri, Germán Ferrari y Claudio Risso
el conocimiento y los proyectos
Valentina Levy
el trabajo en scheduling
Mi familia
la paciencia, las noches largas y los días cortos
El árbol que crece
Las piscinas azules
Agradezco
a quienes están en el recuerdo.

No hay caminos
Hay que caminar
Caminar soñando
Soñar caminando

RESUMEN

La aplicación de modelos de programación a los procesos de refinación de crudo es un área en desarrollo. Este trabajo presenta dos nuevos modelos de programación: el primero aborda la recepción de crudo en una terminal, incluyendo la mezcla y entrega a la refinería; el segundo se enfoca en la producción, mezcla y distribución de gas licuado de petróleo (GLP).

Ambos modelos utilizan una representación en tiempo continuo, que ofrece una descripción más realista del proceso, aunque más compleja que los modelos en tiempo discreto. En el modelo de crudo, la recepción, mezcla, entrega y procesamiento en la refinería se representan y validan utilizando un enfoque en tiempo continuo basado en espacios prioritarios (priority slots) y secuenciación de múltiples operaciones (MOS). Las calidades de los crudos a refinar coinciden con las mezclas de la terminal, lo que simplifica la formulación y elimina términos bilineales en las restricciones del modelo del oleoducto. El objetivo es determinar un programa optimizado que cumpla con todas las restricciones, formulado y resuelto como un problema de Programación No Lineal Entera Mixta (MINLP, por sus siglas en inglés).

Para el problema de programación del GLP, se aplica un marco similar en tiempo continuo. El modelo incorpora restricciones para operaciones que se superponen temporalmente y asigna dinámicamente el almacenamiento para la mezcla y la entrega en la terminal. El objetivo del programador es optimizar la secuencia de calidades intermedias transferidas a través del gasoducto que conecta la refinería con la terminal, así como determinar las mezclas finales óptimas entregadas al mercado, cumpliendo con diversas restricciones operativas.

Se aplican técnicas de ruptura de simetrías y heurísticas para mejorar los tiempos de solución considerando un programa de producción realista.

Palabras claves:
Scheduling en procesos de refinación, Ductos, Modelo de tiempo continuo, Programación entera, Gestión de inventarios.

ABSTRACT

The application of scheduling models to crude oil refining processes is a developing area. This work presents two new scheduling models: the first addresses crude oil reception at a terminal, including blending and delivery to the refinery; the second focuses on the production, blending, and distribution of liquefied petroleum gas (LPG).

Both models employ a continuous-time representation, which provides a more realistic depiction of the process but is more complex than discrete-time models. In the crude oil model, reception, blending, delivery, and processing at the refinery are represented and validated using a continuous-time approach based on priority slots and multi-operation sequencing (MOS). The refinery's target crude qualities are defined as pure qualities that match those of the terminal mixtures, which simplifies the formulation and eliminates bilinear terms in the pipeline model. The objective is to determine an optimized schedule that satisfies all constraints, formulated and solved as a Mixed-Integer Nonlinear Programming (MINLP) problem.

For the LPG scheduling problem, a similar continuous-time framework is applied. The model includes constraints for temporally overlapping operations and dynamically allocates storage for blending and delivery at the terminal. The scheduler's objective is to optimize the sequence of intermediate qualities transferred through the gas pipeline connecting the refinery to the terminal and to determine the optimal final blends delivered to the market, while meeting various operational constraints.

Symmetry-breaking techniques and heuristics are applied to improve solution times while considering a realistic production schedule.

Keywords:
Scheduling in refining processes, Pipelines, Continuous-time modeling, Integer programming, Inventory management.

Lista de figuras

Lista de tablas

Tabla de contenidos

Capítulo 1

An introduction to decision-making in petroleum refining processes

1.1. Main aspects of the decision-making process

Decision-making in petroleum refining production processes typically follows a structured sequence: Planning, Scheduling and Work Orders.

1.1.1. Planning

In production planning, the main objective is to maximize the economic profit of the process, that is, the difference between revenues from product sales, expenses from raw material purchases, and operational costs. This optimization must comply with a set of constraints related to the operational process (e.g., upper and lower bounds on flow rates, thermal consumption, inter-stream dependencies), market demands (product requirements), and quality specifications (for both intermediate and final products).

The resulting model is a nonlinear programming (NLP) problem that is non-convex. In some cases, it may include a few constraints involving binary variables, in which case it becomes a mixed-integer nonlinear programming (MINLP) model. In such formulations, guaranteeing a global optimum is not possible.

Initial linear approach solutions emerged in the 50s following Dantzig's development of the simplex algorithm. Recursive techniques began to be applied in the 80s. Since the 1990s, nonlinear problems have been addressed using the Successive Linear Programming (SLP) heuristic with distributive recursion (Khor and Varvarezos, 2017), often in combination with barrier algorithms and multiple starting points. Today, a variety of commercial software packages is available to support different refinery configurations, whether single-site or multi-site, with or without distribution logistics, and for evaluations over single or multiple time periods.

The operational plan outlines the main decisions for the time periods under consideration. Model backcasting is performed sequentially to validate its performance against actual operational results. Physical simulators are valuable tools for calibrating planning and scheduling models for processing units.

1.1.2. Scheduling

The scheduling process translates the production plan into detailed time sequences. Scheduling models are combinatorial in nature and are typically formulated as MINLPs. These models involve a significant number of variables and constraints. The decisions usually involve several possibilities, and many constraints are both nonlinear and non-convex.

Considering the sequence of events, the scheduling problem seeks to uncover benefits that the production plan may only capture on average, or approximately, over a multi-period evaluation. For example, consider the frontier between naphtha and light kerosene. If the plan tends to maximize the transfer of naphtha to kerosene, the scheduling of operations will aim to follow and further refine that guideline.

The optimization of these problems at an industrial scale remains highly complex and limited, despite the computational power currently available, due to their NP-hard nature. The optimal scheduling of crude oil operations has been studied since the 1990s (Mouret et al., 2009).

Numerous studies in the literature address the scheduling problem through certain modules, such as crude-oil reception, blending and transfer to the refinery, or blending within the refinery. Others focus on production as well as the blending and delivery of final-quality products. Some contributions treat planning and scheduling as a single integrated problem, whereas others concentrate on the scheduling of the entire refinery.

When adopting a modular approach to the scheduling problem, the literature includes models for operations involving either raw materials (e.g., crude oil) (Zimberg et al., 2015) or finished products (e.g., gasoline and other fuels) (Li et al., 2010; Castillo-Castillo and Mahalec, 2016). The specific configuration of each process strongly influences both the model structure and the solution methodology employed.

Works such as Maravelias and Sung (2009) address the joint optimization of planning and scheduling problems, while the impact of uncertainty on decision-making is examined in Li et al. (2020).

The study by Franzoi et al. (2024) proposes several strategies to improve the tractability of large-scale refinery applications. These include tight relaxations using piecewise McCormick or normalized multiparametric disaggregation, cluster decomposition, and Lagrangian decomposition. Such methods often introduce simplifications in process-unit modeling, narrow the scope of the process network, and restrict the search space explored by optimization algorithms.

Consequently, commercial software packages typically provide a wide range of alternatives, from simulation tools to optimization modules for specific processes, rather than refinery-wide solutions. These tools often rely on heuristics to generate acceptable solutions within reasonable time frames. Depending on the approach, planning and scheduling models for processing units can exhibit similar structures.

In practice, many refineries still perform scheduling manually, often supported by spreadsheet-based simulation tools. Planners generally employ scheduling to forecast inventories and product qualities in the medium term, whereas operators focus on short-term scheduling with greater operational and resource-level detail.

1.1.3. Work Order

The work order represents the final step in the decision-making chain, where planning and scheduling decisions are translated into actual operations. A work order includes details such as the task subject, execution timing, personnel involved, and safety considerations.

1.1.4. Main aspects of scheduling in refining processes

- Building a scheduling model
  There is no universal model for the scheduling of process systems; however, three core components are always present: the assignment of tasks to equipment, the sequencing of activities, and the timing of equipment and resource utilization by these processing tasks, (Reklaitis, 1996), (Shah, 1996). A review of scheduling methodologies for process industries is presented in Harjunkoski et al. (2014). The scheduling models discussed in this report correspond to a network-based approach, in which different materials are blended and moved. Materials flow between resources through specific tasks. For

such a network model, Kondili et al. (1993) proposed the State-Task Network (STN) approach. This approach is represented as a directed graph with two types of nodes: state nodes, denoted by circles, which represent materials; and task nodes, denoted by rectangular boxes, which represent operations. In the work of Pantelides (1994), the Resource-Task Network (RTN) was introduced, in which processing equipment, storage, material transfer, and utilities are considered resources, while operations are modeled either as special nodes or, in later works, as edges connecting the resources. The latter approach is adopted in this thesis report. Examples of resources considered in refining problems include storage tanks, vessels, trucks, production units, splitters, and blending manifolds.
Pipelines are unique resources that often require a spatial and temporal representation of queues for both volumes and qualities. Only a few studies have examined the detailed operation of this type of resource, such as Zhang and Xu (2015); Cafaro et al. (2019).

- Linearity
  Formulations can be either linear or nonlinear. When product quality blending is included, the model becomes nonlinear and nonconvex due to bilinear terms. Several methodologies have been proposed to obtain acceptable solutions while improving computational efficiency. Typically, the resulting formulation corresponds to a MINLP problem.

- Time representation
  Time representation can be classified into discrete-time and continuous-time formulations. Discrete-time formulations divide the time horizon into intervals of equal or varying length, with all events constrained to occur at the boundaries of these intervals. In systems with processing subsystems operating on different time scales, multiple grids may be used. One of the earliest formulations was introduced by Kondili et al. (1993).
  Continuous-time formulations, on the other hand, represent only actual event points, with the length of each time interval being variable and not predefined. These models typically require fewer binary variables than discrete-time formulations. Decisions occur at the time instants that separate two consecutive events. Two types of time grids are commonly used: a single time grid for all resources or a unit-specific time grid, where each resource operates

on its own independent timeline. To define the grid, a predetermined number of time slots must be specified in advance. These models are generally more compact in formulation but more complex than uniform discrete-time models, and they tend to produce weaker relaxations. Early formulations include the works of Zhang and Sargent (1996) and Schilling and Pantelides (1996).
The work by Mouret et al. (2009) introduces the concept of priority time-slot models, in which operations are assigned to a limited number of time slots, known as the Single-Operation Sequencing (SOS) model. In Mouret et al. (2011), the authors propose four formulations of priority time-slot models. Among these, the Multi-Operation Sequencing (MOS) model is more compact, requires fewer variables, and can be solved more efficiently, primarily because overlapping operations can share the same slot. This formulation helps reduce the total number of slots required for a given solution.

- Objective
  Unlike planning problems, the objective function in scheduling is not always clearly defined. What constitutes an optimal sequence in a production problem? The answer depends on the specific characteristics of the process. Typically, the objective reflects operational best practices, such as minimizing makespan, emptying or filling tanks, and maintaining consistent operating sequences where possible. It can also be modeled as a maximum profit problem, where, given the set of available resources and the values of input and output streams, the optimal schedule corresponds to the highest overall profit within a specified time horizon.
  The costs associated with scheduling problems are often arbitrary compared to those in planning, which are based on market values. In scheduling, the cost terms in the objective function represent the relative importance of decisions and are therefore highly process-dependent.

- Solving the model
  Scheduling problems often exhibit degeneracies or symmetries that degrade computational performance as the number of time intervals or events increases. Solution times can be improved using Lagrangian decomposition or less rigorous heuristics such as Genetic Algorithms, Rolling Horizon strategies, and Relax-and-Fix methods. Symmetry reduction involves adding additional constraints to reduce the search space.

1.1.5. Decision-making process in ANCAP

Planning tools have been used at ANCAP for the past thirty years. The production planning model is implemented using the commercial software Refinery and Petrochemical Modeling System (RPMS) by Honeywell, which is regularly maintained. These planning models are applied in short, medium, and long-term studies to evaluate crude oil purchases, crude oil and product contracts, production plans, and investment decisions.

Scheduling tools are based on in-house applications and are used to simulate the evolution of crude oil and product inventories on a daily basis over a one-year horizon. The main model provides a simplified representation of operations related to logistics, production, and product storage. Crude oil and product inventories are usually modeled with limited spatial detail. This model enables planners to represent an actual plan as a sequence of events, considering initial inventories, production, and demand in order to align the plan as closely as possible with a daily schedule. It is also used to assess the impact of maintenance on storage capacities or production units shutdowns in the evolution of inventories. This scheduling tool offers great flexibility for quickly responding to uncertain events.

Another scheduling model focuses on the evolution of inventories at the crude oil terminal, determining time windows for crude oil reception and optimizing crude quality blends to avoid undesired mixtures during unloading.

A third scheduling model, integrated with blending optimization, helps track the evolution of inventories and product qualities of LPG at both the refinery and the terminal for blending and final product distribution.

These scheduling models also help define initial inventories, raw materials, and intermediate products to be processed during the planning horizon. Results from both planning and scheduling models support pricing decisions for products between the production and commercial areas.

Optimization techniques are commonly applied for blending but are not typically used for scheduling. Applying optimization techniques to scheduling is viewed as the next step, once the basic aspects of the problem and its rules have been modeled as an objective function with a set of constraints. The goal of optimization is to find a good, ideally the best sequence of operations within the given constraints and initial conditions, within a reasonable amount of processing time.

As a consequence of uncertainty, the best sequence obtained on one day may remain feasible only for a short period. Therefore, optimization often needs to be reassessed daily. The quality of input data and the accuracy of production unit models significantly affect performance. For these reasons, there is some skepticism about the practicality of applying optimization, as it may be more effective to rely on simulation combined with technicians' expertise to identify good solutions. The company is currently analyzing commercial alternatives.

1.2. Thesis Contributions and Organization

The work presented in this thesis is structured into two main contributions, each addressing a key problem in the production process, and is preceded by an introduction in Chapter 2 to the main concepts of time representation and the MOS model, which are applied throughout this study.

Two new scheduling models are presented. The first addresses the problem of crude oil reception at a terminal, including blending and delivery to the refinery. The second focuses on LPG production, blending, and distribution.

The primary scheduling problem to be evaluated for optimization is the reception of crude at the terminal, unloading, mixing, and delivery to the refinery. It links medium-term planning evaluations (crude oil purchasing) with short-term goals (monthly production planning). It helps determine the best way to follow the planned optimal mixtures on a daily basis until the next crude oil purchase. It also allows for determining the evolution of inventories in tanks and in the main pipeline that transports crude from the terminal to the refinery.

The first problem contains all the typical components of a scheduling model with operations and resources: decisions (binary variables), time horizon definition, formulation of objectives and constraints, time representation, blending and nonlinearities (bilinear terms), and equivalent solutions (symmetries). The work in Zimberg et al. (2015) considered the terminal as the scope of the problem and used a discrete-time approach. As a contribution of this thesis, the problem was extended to include the main pipeline and the refinery's storage capacity. To reduce the problem size, a continuous-time approach was selected.

The second work addresses the problem of LPG production, blending, and delivery. It models and solves the integrated process of LPG production, storage at the refinery, transport to a terminal for blending, storage, and delivery of final products.

Both problems involve resources and operations, but the networks, constraints, modeling structures, and challenges differ. One problem relates to the beginning of the production chain, while the other concerns the final stage of the process for a specific set of products.

In the first problem, blending of qualities is performed at the reception terminal, from where the product is transported to the refinery through a pipeline. In the second problem, blending to produce multiple final qualities takes place at the delivery terminal, where the assignment of intermediate and final resources is not predefined. Both terminals can store intermediate materials, but final products are stored only at the LPG terminal. Dynamic assignment of products to resources is common to both problems; however, formulation and complexity are particularly critical in the LPG model.

In the first problem, the main pipeline and the sequence of packets play a significant role in the model formulation. In the second problem, the modeling of resources with overlapping operations is key. Both problems use the same model for the small pipeline, but with different interpretations: a subsea pipeline for crude oil unloading to the terminal, or a gas pipeline for delivering LPG to the terminal.

Non-convex constraints and their associated challenges are addressed differently in each case. In the crude oil model, the manifold and quality constraints allow mixtures to be replaced by pure components. In the LPG model, mixtures are tracked using a distinct manifold model and a different set of quality constraints.

The continuous-time and multi-operation slot approach is applied in both models due to its suitability for representing operations with fewer binary variables compared to other continuous or discrete-time approaches.

Symmetry is present in both models. Symmetry-breaking constraints are proposed to address this issue. Different subsets of operations are considered in each problem, and additional constraints are introduced in the second model.

Both models are implemented in the AMPL language and solved using similar methodologies with CPLEX. Heuristics are proposed and tuned differently for each model.

1.2.1. Crude oil scheduling. Main contributions

Chapter 3, presents the work "A continuous-time formulation for scheduling crude oil operations in a terminal with a refinery pipeline", (Zimberg et al., 2023b).

The comprehensive development of all components of the model significantly contributes to the overall impact of this thesis. In particular, the primary contributions can be summarized as follows:

- *General outline of the problem and network:* Crude oil reception, unloading through a subsea pipeline, storage at the terminal, blending, and delivery to the refinery through the main pipeline, followed by storage and consumption.

- *Subsea pipeline:* Represents the delivery of crude oil through a relatively small pipeline that ensures complete displacement of the initial inventory.
- *Initial conditions structure for the main pipeline:* An extension of the MOS model to represent the initial state of the pipeline.
- *Manifold model:* Terminal crude oil qualities are blended in the manifold before being pumped into the pipeline. The manifold models the transformation of terminal mixtures into refinery pure qualities according to a set of specifications. This approach ensures that only pure qualities, meeting specified requirements, are delivered through the pipeline, thereby suppressing nonlinearities.
- *Time delays between operations:* Time delays are modeled between certain non-overlapping operations to represent times required for water settling and removal or for analysis.
- *Maintenance operations:* During a given time period, three types of restrictions are considered for resources: limited to their maximum capacity, minimum capacity, or maintained at a previous intermediate state. Maintenance operations are not mandatory; they are carried out only if there is an economic incentive and no conflict with other operations.
- *Code development:* Development of the heuristics model and its implementation code, including the design and implementation of structures required to establish dependencies between volumes and time through slot dependencies, which are necessary for the final solution report.

1.2.2. Liquefied Petroleum Gas scheduling: Main contributions

Chapter 4 discusses the article "A continuous-time scheduling model for Liquefied Petroleum Gas production, blending and delivery". Initial results, "Liquefied Petroleum Gas Scheduling, Blending, and Optimization Model," were presented at the IFORS 2023 meeting, (Zimberg et al., 2023a). The final work was published in Zimberg et al. (2025).

Each component of the model has been carefully developed to enhance the overall contribution of this thesis. The key innovations are outlined below:

- Conceptualization of the scheduling problem for LPG production, blending, and market delivery; network design.

- Introduction of new constraints to address overlapping operations in resources. Each resource is represented as a pair of virtual spheres, with appropriate volume and time equations ensuring feasibility.
- A manifold model is developed for blending at a the terminal sphere with contributions from the gas pipe or by mixing products from spheres. Each sphere at the terminal can store either intermediate or final products, as determined during the optimization process.
- Bilinear terms representing concentration requirements are needed to model the qualities of products delivered to the blending manifold and the final product delivery. Quality constraints are imposed to ensure the proper transfer of mixtures between resources.
- Problem size is reduced by adopting the common real-world practice of considering specific subsets of resources to deliver particular final products and prioritizing certain spheres over others for the purpose of blending. Depending on the problem instance, some spheres may also be excluded.
- New time delay constraints are introduced
- Heuristics model, implementation code and final solution report.

1.2.3. Conclusions and future work

Chapter 5 presents the conclusions of the thesis and outlines the main directions for future work.

Capítulo 2

Time representation. The MOS model

2.1. Discrete-time models

In the discrete-time approach, the time horizon is divided into a series of uniform time intervals, the number of which is an input to the problem. The beginning and end of each task are aligned with these intervals. This representation provides a common time grid for all operations across resources, allowing constraints to be formulated in a straightforward manner. Since time evolves continuously in real-world processes, the duration of each interval represents a compromise between solution quality and the computational effort required to solve the combinatorial problem. To improve quality for a given solution time, more than one time grid can be defined for specific subsets of resources. Two types of time grids are commonly used: a single time grid for all resources or a unit-specific time grid, where each resource operates on its own independent timeline.

Figure 2.1 shows a resource $r \in R$, an inlet operation $w^t_{j,r} \in \mathcal{W}_j$ and an outlet operation $w^t_{r,i} \in \mathcal{W}_r$ at time $t \in T$, as well as the transport of volume and quality at time t.

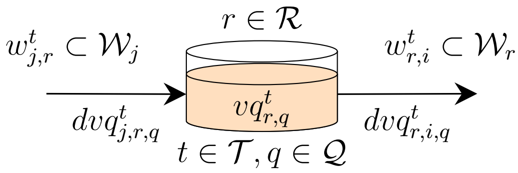

Figure 2.1: Diagram of the storage tank model's variables and parameters.

In the following example (2.1), the binary variable $w_{r,i}^t$ represents an operation from resource r to resource i at time $t \in \mathcal{T}$

$$\sum_{i \in \mathcal{W}_r} w_{r,i}^t \leq 1, \quad \forall r \in \mathcal{R}, t \in \mathcal{T} \tag{2.1}$$

The volume balance of quality $q \in \mathcal{Q}$ in resource $r \in \mathcal{R}$ over the interval $[t-1, t]$ is expressed in (2.2), where $vq_{r,q}^t$ denotes the volume of quality q in resource r at time t, and the last two terms represent the total inlet and outlet material associated with all operations involving resource r.

$$vq_{r,q}^t = vq_{r,q}^{t-1} + \sum_{\substack{i \in \mathcal{R} \\ r \in W_i}} dvq_{i,r,q}^{t-1} - \sum_{\substack{i \in \mathcal{R} \\ i \in W_r}} dvq_{r,i,q}^{t-1}, \quad \forall q \in \mathcal{Q}, r \in \mathcal{R}, t \in 2 \ldots \mathcal{T} \tag{2.2}$$

2.2. Continuous-time models

2.2.1. Time-slot model

This approach is based on the concept of time slots, with the number of slots defined by the user. In global event-based models, the time representation consists of a set of events applied to all tasks. This definition can be extended by introducing event points on a resource basis, allowing operations associated with the same event point but occurring on different resources to take place at different times. This variant is referred to as a unit-specific event-based model.

Figure 2.2 compares the time representations of the discrete- and continuous-time approaches for the single time grid and the global event-based models, respectively. The duration of the time slots is determined by the solution of the model, whereas the step size of the discrete approach and the number of continuous-time slots are specified in advance.

0 — 1 — 2 — 3 — 4 — 5 — 6 — 7 — 8 ⟶

0 ———— 1 — 2 —— 3 —— 4 — 5 ⟶

Figure 2.2: Discrete (above) and continuous (below) time representations

As mentioned by Floudas and Lin (2004), the most important variables in this formulation include:

- t^k: the continuous time of event k;
- $w_{r,i}^k$: a binary variable indicating an operation from r to i starting at t^k;
- $x_{r,i}^{k,k'}$: a binary variable indicating whether operation $w_{r,i}^k$ starts at t^k and ends at $t^{k'}$. All times t^k are monotonically increasing.

Constraint (2.3) ensures that each operation is defined between two time slots, while Constraint (2.4) specifies that the duration of a task, $t_{r,i}^k$, spanning resources r and i and starting at t^k, is given by the sum of successive slot times. This nonlinear expression can be replaced by a set of linear constraints.

$$w_{r,i}^k = \sum_{k'>k} x_{r,i}^{k,k'}, \qquad \forall i \in \mathcal{W}_r, r \in \mathcal{R}, k \in \mathcal{K} \tag{2.3}$$

$$t_{r,i}^k = \sum_{k'>k} x_{r,i}^{k,k'} \left(t^{k'} - t^k\right), \qquad \forall i \in \mathcal{W}_r, r \in \mathcal{R}, k \in \mathcal{K} \tag{2.4}$$

2.2.2. Multi-Operation Sequencing (MOS) model

In this formulation, the number of slots is also specified in advance. A priority slot is a position i in the sequence of operations, having a higher scheduling priority than other priority slots $j > i$. In this formulation, overlapping operations can be assigned to the same priority slot, and the corresponding start and end times of such operations are assigned to that slot. Operations are modeled as elements of a general set, with subsets corresponding to the inlet and outlet operations for each resource.

Key aspects of this formulation include:

- In the Multi-Operation Sequencing (MOS) model, given two non overlapping operations w_1 and w_2, where w_1 has priority over w_2, and time slots i and j, with $i < j$, the priority mechanism is applied as follows: operation w_1 (assigned to slot i) must end before operation w_2 (assigned to slot j) begins. Non overlapping operations are scheduled so that increasing slot numbers reflect increasing execution times.
- Overlapping operations may share the same slot, with the solution assigning them distinct start and end times within that slot.

- Symmetry increases solution time. Adding symmetry-breaking constraints to the model helps reduce the number of assignment possibilities, for example, by preferring the lowest available slot.

The mutual exclusivity of simultaneous operations can be represented through cliques of operations. A clique $c \subseteq \mathcal{W}$ denotes a subset of operations that cannot be carried out concurrently within any slot i. Accordingly, the following constraints specify that, in a given slot i, no more than one operation w from a clique c may be executed, as shown in Eq. (3.1).

$$\sum_{w \in c} z_w^i \leq 1, \quad \forall i \in \mathcal{S}, c \in \mathcal{C} \tag{2.5}$$

where z_w^i is a binary variable that takes on value 1 if operation w is performed in slot $i \in \mathcal{S}$, otherwise it takes on value 0, and $\mathcal{C}$ is the set of cliques.

Regarding the material balance, Equation (2.6) specifies how the inventory of quality q in resource r evolves over slot i:

$$vq_{r,q}^i = Vq0_{r,q} + \sum_{\substack{j \in \mathcal{S}, w \in \mathcal{I}_r \\ j < i}} dvq_{w,q}^j - \sum_{\substack{j \in \mathcal{S}, w \in \mathcal{O}_r \\ j < i}} dvq_{w,q}^j, \quad \forall i \in \mathcal{S}_1, r \in \mathcal{R}, q \in \mathcal{Q} \tag{2.6}$$

where $Vq0_{r,q}$ is a parameter with the initial volume of quality q in resource r. This constraint specifies that the volume of quality q in resource r at slot i, denoted by $vq_{r,q}^i$, is obtained by adding the initial volume and the volumes transferred through all inlet operations $w \in \mathcal{I}_r$, and subtracting the volumes withdrawn through all outlet operations $w \in \mathcal{O}_r$, executed in slots j prior to i.

Equation (2.7) enforces that operations within a clique $c \in \mathcal{C}$ cannot overlap, requiring their execution in distinct time slots i_1 and i_2.

$$\sum_{w \in c} te_w^{i_1} + \sum_{\substack{j \in \mathcal{S}, w \in c \\ i_1 < j < i_2}} td_w^j \leq \sum_{w \in c} ts_w^{i_2} + T\left(1 - \sum_{w \in c} z_w^{i_2}\right), \quad \forall i_1, i_2 \in \mathcal{S}, i_1 < i_2, c \in \mathcal{C} \tag{2.7}$$

Figure 2.3 presents three Gantt charts for a sequence of operations involving eight resources: cargo, pipeline, production units 1 and 2, and tanks 1 to 4. The time horizon is 10 days, and operations are labeled along the left axis.

Non overlapping operations are grouped into subsets:

- Pairs (201,202), (203,204), (201,301), (202,302), (203,303), (204,304).
- Set 101, 301, 302, 303, 304.

The figure compares the schedules under three different time representations. The first diagram corresponds to a discrete-time approach with a single time grid and a time step of 0.2 days; a 0.4-day step is shown to simplify the scheme. A total of 50 time points are considered.

The second diagram illustrates a continuous-time, global event-based model with 11 slots. Event times are displayed in the top bar, and slot assignments are indicated within each operation bar.

For the MOS approach, 7 priority slots are required, resulting in a comparatively smaller problem size. The assignment of operations to slots requires symmetry-breaking constraints, which tend to assign the lowest slot to a given operation. Symmetry allows alternative slot assignments; for example, operations 203 and 204 can occupy slots 2, 3, and 4 instead of 1, 2, and 4. Operation times must not overlap within the same subset, including any new instances of an operation.

Example of a continuous operation:

- Unit1 delivers to Tank2 (slot 1).
- Unit1 delivers to Tank1 (slot 2).
- Unit1 delivers to Tank2 (slot 3).

Examples of overlapping operations:

- Unit1 delivers to Tank2 and Unit2 delivers to Tank3 (slot 1).
- Unit1 to Tank1 and Unit2 to Tank4 (slot 2).
- Unit1 to Tank1 and Tank2 to Pipeline (slot 2).

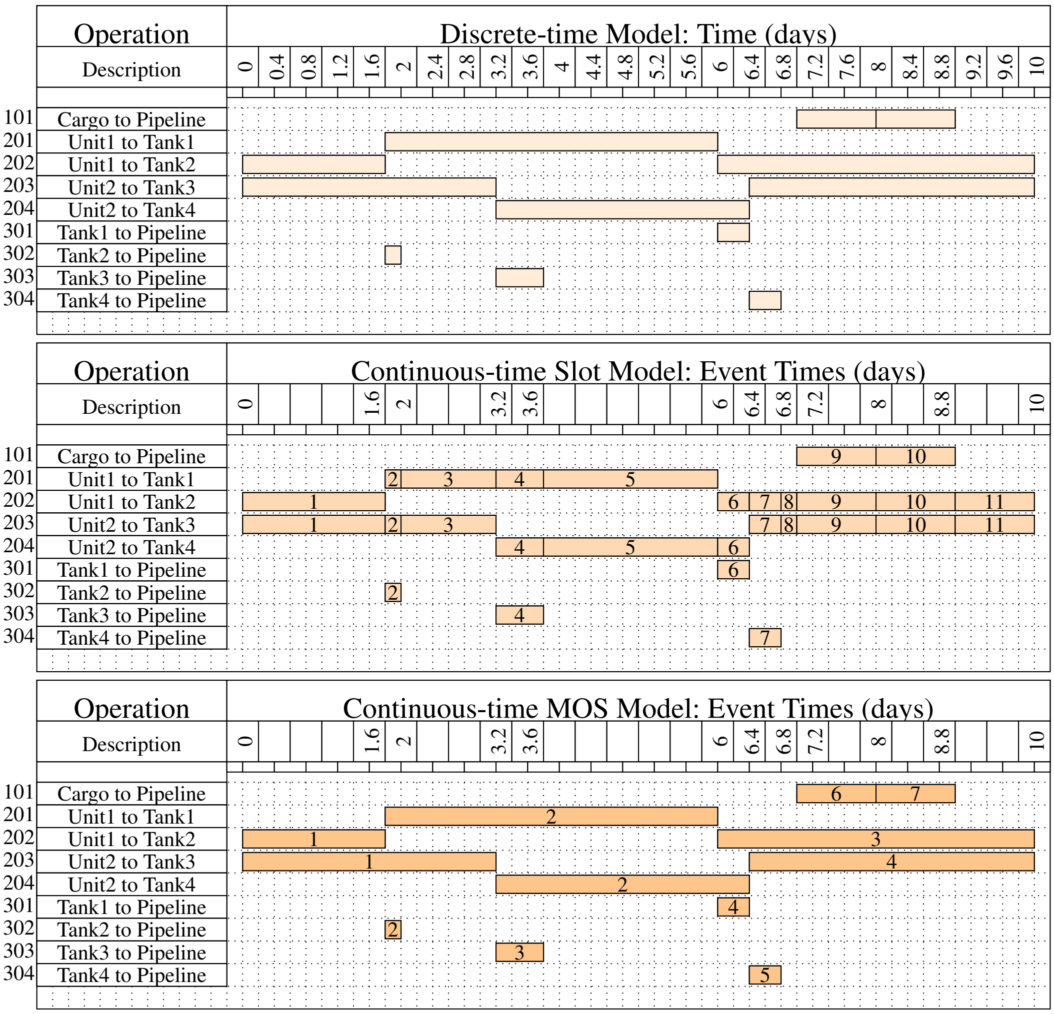

Figure 2.3: Gantt charts of a schedule in three time representations

Capítulo 3

A continuous-time formulation for scheduling crude oil operations in a terminal with a refinery pipeline

ABSTRACT

The reception, mixture, delivery, and processing of crude oil at a refinery are modeled and validated. We employ a continuous-time model based on priority-slots and a multi-operation sequencing approach. The refinery's desired qualities are defined as equivalent to the terminal mixtures, thus streamlining the formulation and eliminating any bilinear terms in the equations related to pipeline delivery. The goal is to find an optimized schedule that satisfies all constraints, for which an MINLP model is proposed and solved. However, by ruling out mixtures in tanks, the formulation is reduced to a manageable MILP model. Also, the Relax & Fix strategy can be used to determine an optimized schedule of operations over a time horizon in a timely and cost-effective manner. This work presents an innovative solution to the complex challenge of crude oil reception, mixture, pipeline delivery, and refinery processing.

Keywords:
Scheduling in refining processes, Pipelines, Continuous-time modeling, Integer programming, Inventory management, Crude oil scheduling, Pipeline, Continuous-time modeling, Integer programming, Inventory management.

3.1. Introduction

Refining involves the reception of crude oil, blending qualities, and processing. Refineries usually receive crude oil from storage tanks located at terminals. Cargoes are unloaded at the terminal at planned times. Mixtures are made at the terminal or the refinery. Finally, crude blends are transferred from the charging tanks to the distillation units.

Planning requirements determine which are the best qualities to be processed at the refinery. For scheduling purposes, these blends can be modeled with a set of specifications to approximate the proposed plan. In addition, some qualities, *e.g.* when processing specific mixtures such as asphaltic crude, should be processed on specific time periods due to demand constraints. It is necessary to determine if the proposed crude oil slate can be satisfied or not according to capacity, maintenance, quality, and other constraints and, as a result, which schedule arises from the whole set of constraints.

There are two categories of crude scheduling formulations: discrete-time and continuous-time formulations. Continuous-time models are more complex, but good solutions are achieved in lower computational times compared to discrete-time models (Moro and Pinto, 2004). Formulations can fall in the class of Mixed-Integer Linear Programming (MILP) or Mixed-Integer Non-Linear Programming (MINLP). MINLP models arise from bilinear terms in the formulation, because the concentration of the outlet flow from a storage tank must be equal to the concentration inside the tank. The calculation of the concentrations and other properties of the crude oil blends in linear models are approximated by linear constraints (Oddsdottir et al., 2013; Zimberg et al., 2015).

Other authors, Mouret et al. (2009, 2011) consider a two-step MILP-NLP procedure to solve the MINLP model. The MILP approximation, without bilinear terms, is solved. Binary variables are fixed at the values computed in the first step, bilinear terms included, and then the non-linear model is solved as a second step. According to Zhao et al. (2017), although the composition concentration discrepancy can be avoided in the previous models, a solution may not be found for some cases even if a feasible one exists. They propose a valid inequality in the formulation to reduce the composition concentration discrepancy and solve iteratively a series of successive MILP problems.

Regarding time representation, Yadav and Shaik (2012) propose a state-task-network (STN) based formulation using a unit-specific event-based continuous-time representation. The MINLP model is relaxed to a MILP by dropping the bilinear terms. Results are checked to verify the absence of composition discrepancies.

Li et al. (2011) develop a unit specific event-based continuous-time MINLP formulation. Their work considers crude blending. Computation time is improved by applying piecewise-linear underestimation of bilinear terms to globally optimize the MINLP problem (Karuppiah and Grossmann, 2006).

Castro and Grossmann (2014) introduce a resource-task network formulation. The MINLP model is solved with a two-step MILP-NLP algorithm and the multi-parametric disaggregation technique proposed by Kolodziej et al. (2013).

Mouret et al. (2011) develop the concept of priority-slot models, where operations are assigned to a given number of slots. The paper presents three continuous-time models and one discrete-time model as follows: Multi-operation sequencing (MOS), Multi-operation sequencing with synchronized start times (MOS-SST), Multi-operation sequencing with fixed start times (MOS-FST), and Single operation sequencing (SOS). The number of priority-slots has to be postulated a priori which increases in the order (MOS) $\leq$ (MOS - SST) $\leq$ (MOS - FST) and (MOS - SST) $\leq$ (SST). The MOS model is more compact, requires fewer variables and can be solved in reduced times. The main reason is that in MOS models, operations that overlap in time can be assigned to the same slot.

The work by Zhao et al. (2017) applies the SOS model. Zhang and Xu (2015) consider a unit-specific time-event (a slot based approach) defined for different sets of operations: crude unloading, transferring, and charging. Sequencing constraints are added to represent time priorities between operations.

A pipeline is modeled in the works by Zhang and Xu (2015) and Cafaro et al. (2019). Zhang and Xu (2015) model packets of crude inside the pipeline (pipeline slots) for each time event. Cafaro et al. (2019) consider four types of operations: unloading, transferring to the pipeline, transferring from the pipeline to the refinery tanks, and charging of distillation units. The model makes use of global precedence sequencing variables to determine the order of operations. Cafaro et al. (2019) propose a methodology to estimate the minimum number of lots for each set of operations. The representation, based on sets of lots, resembles an SOS methodology.

Below follows the main contributions of our work:

- We extend the MOS formulation to include a subsea pipeline and a main pipeline connecting vessels to storage tanks in a terminal and the terminal to the refinery respectively. The fact that one vessel can unload to several tanks adds an additional layer of complexity to the optimization problem, since there are now multiple potential pathways for inputs to flow through the system.

- We reduce the need for bilinear term constraints by defining the refinery's desired qualities as equivalent to the terminal mixtures, thus streamlining the formulation and eliminating any bilinear terms in the equations related to pipeline delivery. The manifold model is a way to map the input mixtures at the terminal to the desired quality specifications at the refinery. By using a manifold model, the optimization problem can be more accurately tailored to the specific requirements of the refinery.
- Our work also includes modeling maintenance operations and settling times between different operations. By modeling maintenance time, the optimization problem can take into account the fact that certain components may be out of commission for a period of time, and therefore adjust the flow of inputs and outputs accordingly. Settling time, on the other hand, refers to the time required for certain inputs to settle or separate before they can be processed further. By modeling settling time, the optimization problem can ensure that the system is not overloaded with inputs that are not yet ready to be processed.
- In addition, symmetry breaking equations are formulated to allow feasibility for all operations. By doing so, the overall system can be made more robust and reliable.

In the next section, the problem is defined. Section 3.3 presents the model in detail. Section 3.4 discusses the heuristic strategies used to solve the problem, and Section 3.5 presents the results of experiments conducted to test the model and strategies. Finally, Section 3.6 summarizes the findings and draws conclusions about the effectiveness of the proposed solution. Appendix 6 provides an overview of the notation used throughout the paper.

3.2. Problem Statement

In this work, the goal is to optimize the operations of crude oil reception, unloading, blending, delivery to the refinery, storage, and final processing at a distillation unit (CDU). Based on a desirable schedule established from previous linear modeling solutions, the volumes and qualities of crude oils to be processed are determined. The demand requirements and inventory levels of certain products drive the processing of certain crude oil qualities at specific times, while other refinery qualities are left open for determination.

The final plan of mixtures must abide by a set of constraints, some of which are hard, such as storage capacity and non-compatible qualities at the terminal and refinery, and some are soft, such as desired volumes and qualities, continuous operation of resources when possible, minimum tank levels during reception, maximum capacity during delivery, minimum demurrage times, and maintenance requirements.

The problem involves the terminal, main pipeline, and refinery operations, and the objective is to find the best operations that meet the desired schedule while ensuring that the quality of the mixtures is within the constraints. The objective function takes into account the costs associated with the differences in volume and quality of the mixtures delivered to the CDU compared to the desired quantities, as well as the costs of differences in volumes and qualities that remain in the pipeline and refinery at the end of the evaluation period. Other costs include operating requirements during tank reception, number of total operations, number of packets delivered by the pipeline, number of mixtures in storage and refinery tanks, demurrage and start times.

The mathematical model follows these considerations:

- Cargo arrival times are known in advance.
- When vessel unloading starts, the subsea pipeline volume is completely flushed to a tank.
- Vessel can only unload to one tank at a given time.
- When mixing is allowed, new and old crude qualities are completely mixed in the tank after vessel unloading.
- Resource capacity limitations must be satisfied.
- Tank inlet and outlet operations cannot be performed simultaneously.
- There is a time delay between inlet and outlet operations.
- The number of storage tanks that deliver crude to the pipeline simultaneously is bounded.
- There is a given type of mixtures allowed to be processed at the refinery.
- Volume and qualities of the desired mixtures are known in advance.
- Delivery of crude oil to the CDU is continuous.

3.3. Mathematical Programming Model

This work describes a detailed MILP model for the problem of planning the daily operations in a terminal where vessels unload different crude oil types in tanks. Certain qualities are blended and the required mixture is delivered through a pipeline and received in refinery tanks. The mixture feeds a Crude Distillation Unit (CDU) and the products are delivered to other processes. Figure 3.1 represents the model. Vessels arrive at certain times and unload crude oil through a subsea pipeline to terminal tanks. The mixture of qualities is made in a manifold and from there goes through the main pipeline to the refinery tanks. Crude oil is processed in the CDU and the output goes downstream. The model follows a continuous-time formulation (Mouret et al., 2009) using the MOS approach (Mouret et al., 2011), where the operations performed are grouped into slots according to their priorities and interactions between each other. The model designed for the main pipeline is inspired by the works of Cafaro et al. (2019) and extended here to integrate with the proposed continuous-time formulation.

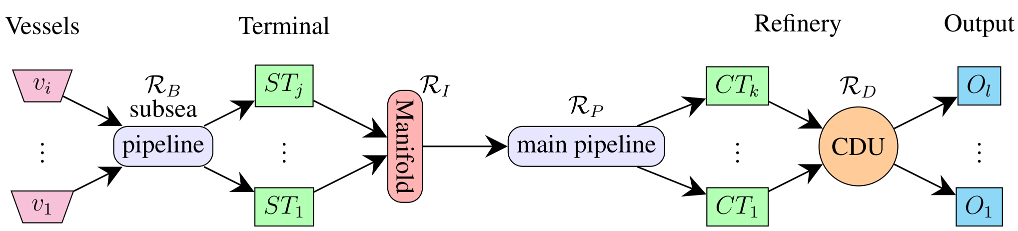

Figure 3.1: Diagram of the general model. Crude qualities are supplied to the Terminal tanks by the vessels, mixed in the Manifold to generate the final qualities sent to the charging tanks through the main pipeline which afterwards will be delivered to the distillation unit and output resources.

Constraints are grouped as follows: Operations and resources constraints in Subsection 3.3.1, time assignments and sequencing in 3.3.2, Maintenance constraints in 3.3.3, quality concentration relationships in 3.3.4, symmetry breaking in 3.3.5, and objective in 3.3.6.

Unless otherwise mentioned, constraints are valid for the following sets: slots $\mathcal{S}$, operations $\mathcal{W}$, resources $\mathcal{R}$, and crude oil qualities $\mathcal{Q}$. Please refer to Appendix 6 for the detailed description of sets, indexes, symbols, and variables.

3.3.1. Operations and Resources

This section introduces the mathematical models proposed to describe the relationships between transfer operations, resources, and crude qualities.

Basic Structures and Properties

Cliques

A clique $C \subseteq \mathcal{W}$ is a subset of operations that cannot be performed simultaneously during any priority-slot i. The constraints that at most one operation w belonging to a clique C can be performed in a slot i is given by Eq. (3.1):

$$\sum_{w \in C} z_w^i \leq 1, \quad \forall i \in \mathcal{S}, C \in \mathcal{C} \tag{3.1}$$

Volume Consistency

Resource volumes at time slot i, final volumes, and transfer operation volumes variables must be consistent with the relationships between total and quality volumes, Eqs. (3.2):

$$\begin{cases} v_r^i = \sum_{q \in \mathcal{Q}_r} vq_{r,q}^i, & \forall i \in \mathcal{S}_2, r \in \mathcal{R} \\ dv_w^i = \sum_{q \in \mathcal{Q}_T} dvq_{w,q}^i, & \forall i \in \mathcal{S}, w \in \mathcal{W}_V \cup \mathcal{W}_B \cup \mathcal{W}_T \cup \mathcal{W}_{PI} \\ dv_w^i = \sum_{q \in \mathcal{Q}_C} dvq_{w,q}^i, & \forall i \in \mathcal{S}, w \in \mathcal{W}_{PO} \cup \mathcal{W}_D \cup \mathcal{W}_O \\ vfq_{r,q} = vq_{r,q}^{N+1}, & \forall q \in \mathcal{Q}_r, r \in \mathcal{R}_T \cup \mathcal{R}_C \cup \mathcal{R}_P \\ vf_r = v_r^{N+1}, & \forall r \in \mathcal{R}_T \cup \mathcal{R}_C \cup \mathcal{R}_P \\ \sum_{r \in \mathcal{R}_C} vf_r \geq \underline{Vfc} \end{cases} \tag{3.2}$$

Resource Limits

Resource limits are given by Eqs. (3.3):

$$\begin{cases} \underline{V_r} \leq v_r^i \leq \overline{V_r}, & \forall i \in \mathcal{S}_2, r \in \mathcal{R} \\ vq_{r,q}^i \leq \overline{V_r} \cdot xvq_{r,q}^i, & \forall i \in \mathcal{S}_2, r \in \mathcal{R}_T \cup \mathcal{R}_C, q \in \mathcal{Q}_r \end{cases} \tag{3.3}$$

Operations Bounds

Operation bounds are expressed by Eqs. (3.4):

$$\begin{cases} \underline{DV}_w \cdot z_w^i \leq dv_w^i \leq \overline{DV}_w \cdot z_w^i, & \forall i \in \mathcal{S}, w \in \mathcal{W} \\ dv_w^i \leq \overline{V_r} \cdot z_w^i, & \forall i \in \mathcal{S}, r \in \mathcal{R}_T \cup \mathcal{R}_P \cup \mathcal{R}_C, w \in \mathcal{O}_r \\ \underline{VF}_w \cdot td_w^i \leq dv_w^i \leq \overline{VF}_w \cdot td_w^i, & \forall i \in \mathcal{S}, w \in \mathcal{W} \\ \sum_{i \in \mathcal{S}} \sum_{w \in \mathcal{W}_{PI}} dvq_{w,q}^i \leq \overline{Vq}_q, & \forall q \in \mathcal{Q}_C \end{cases} \tag{3.4}$$

Vessel Operations

The total crude cargo from vessels $r \in \mathcal{R}_V$ must be unloaded. It is assumed that each cargo transports only one crude oil quality, Eq. (3.5):

$$\sum_{i \in \mathcal{S}} \sum_{w \in \mathcal{O}_r} dv_w^i = V0_r, \forall r \in \mathcal{R}_V \tag{3.5}$$

Material Balance

A schedule consists of the daily operations defined over a time horizon T. Eq. (3.6) establishes the dynamics of the inventory of crude quality q, in resource r, before operations take place at slot i. Volume additivity assumes similarity in the chemical composition of the components.

$$vq_{r,q}^i = Vq0_{r,q} + \sum_{\substack{j \in \mathcal{S}_1, j<i \\ w \in \mathcal{I}_r}} dvq_{w,q}^j - \sum_{\substack{j \in \mathcal{S}_1, j<i \\ w \in \mathcal{O}_r}} dvq_{w,q}^j, \forall i \in \mathcal{S}_2, r \in \mathcal{R}, q \in \mathcal{Q}_r \tag{3.6}$$

Figure 3.2 illustrates the material balance and notation for the case of a terminal tank.

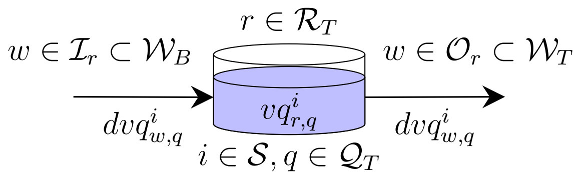

Figure 3.2: Diagram explaining the storage tank model variables and parameters.

In the case of the buoy shown in Figure 3.3, the volume transferred to this resource in an unloading operation is given by the following constraints, Eq. (3.7):

$$
\begin{cases}
dvq^i_{w,q} \leq vq^i_{r,q} + dvbq^i_{w,q} + \overline{Vs} \cdot (1 - z^i_w), \\
dvq^i_{w,q} \geq vq^i_{r,q} + dvbq^i_{w,q} - \overline{Vs} \cdot (1 - z^i_w), \\
dvbq^i_{w,q} \leq \overline{Vs} \cdot z^i_w,
\end{cases}
\quad \begin{array}{c} \forall i \in \mathcal{S}, r \in \mathcal{R}_B, \\ \forall w \in \mathcal{O}_r, q \in \mathcal{Q}_T \end{array} \tag{3.7}
$$

For every vessel $r \in \mathcal{R}_V$ and subsea pipeline $b \in \mathcal{R}_P$, an outlet operation w from the vessel into the pipeline has a minimum volume $\underline{DV}_w$ equal to the maximum pipeline volume $\overline{V}_b$ in order to always flush the subsea pipeline.

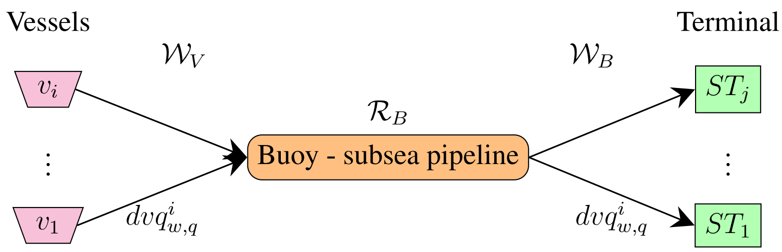

Figure 3.3: Diagram explaining the buoy and subsea pipeline variables and parameters.

Observing that the volume of these specific resources remain constant, Eq. (3.8) implies that

$$V0_r = \underline{V_r} = \overline{V_r} = v^i_r, \forall i \in \mathcal{S}, r \in \mathcal{R}_B \cup \mathcal{R}_P \cup \mathcal{R}_I \cup \mathcal{R}_D, q \in \mathcal{Q}_r. \tag{3.8}$$

Adding up the material balance equation (3.6) for each resource $r \in \mathcal{R}$, over all qualities $q \in \mathcal{Q}$, notice that the sum $\sum_{q \in \mathcal{Q}} Vq0_{r,q} = V0_r$ and $\sum_{q \in \mathcal{Q}} vq^i_{r,q} = v^i_r = \underline{V_r} = \overline{V_r} = V0_r$ for all $r \in \mathcal{R}_B \cup \mathcal{R}_P \cup \mathcal{R}_I \cup \mathcal{R}_D$. Furthermore, by analyzing the resulting equations for all $i > 1$, the following relation, Eq. (3.9), results in:

$$\sum_{w \in \mathcal{I}_r} dv^i_w = \sum_{w \in \mathcal{O}_r} dv^i_w, \quad \forall i \in \mathcal{S}, r \in \mathcal{R}_B \cup \mathcal{R}_P \cup \mathcal{R}_I \cup \mathcal{R}_D, \tag{3.9}$$

meaning that the total volume that enters such resources is precisely the outlet volume, in order to ensure a fixed stored volume.

By the definition of the problem, for resources $r \in \mathcal{R}_B \cup \mathcal{R}_P$, at most one input operation $w_1 \in \mathcal{I}_r$ can be performed in any slot, and likewise for output operation $w_2 \in \mathcal{O}_r$. It follows then that:

$$dv^i_{w_1} = dv^i_{w_2}, w_1 \in \mathcal{I}_r, w_2 \in \mathcal{O}_r | z^i_{w_1} = z^i_{w_2} = 1, \quad \forall i \in \mathcal{S}, r \in \mathcal{R}_B \cup \mathcal{R}_P \cup \mathcal{R}_D \tag{3.10}$$

Considering that for the resources $r \in \mathcal{R}_I \cup \mathcal{R}_D$ the total initial volume $V0_r = 0$, quality volumes must go through these resources, so the following relations on the transfer volumes of operations also hold:

$$\sum_{w \in \mathcal{I}_r} dvq^i_{w,q} = \sum_{w \in \mathcal{O}_r} dvq^i_{w,q}, \quad \forall i \in \mathcal{S}, r \in \mathcal{R}_I \cup \mathcal{R}_D, q \in \mathcal{Q}_r \tag{3.11}$$

Quality

The maximum number of crude types allowed in tanks is given by:

$$\begin{aligned} \sum_{q \in \mathcal{Q}_r} xvq^i_{r,q} &\leq \overline{Q_T}, \quad \forall i \in \mathcal{S}_2, r \in \mathcal{R}_T \\ \sum_{q \in \mathcal{Q}_C} xvq^i_{r,q} &\leq \overline{Q_C}, \quad \forall i \in \mathcal{S}_2, r \in \mathcal{R}_C \end{aligned} \tag{3.12}$$

Crude incompatibilities in storage and charging tanks are enforced:

$$\begin{cases} xvq^i_{r,q_1} + xvq^i_{r,q_2} \leq 1, & \forall i \in \mathcal{S}_2, r \in \mathcal{R}_T, \ (q_1, q_2) \in \mathcal{QQ}_T \\ xvq^i_{r,q_1} + xvq^i_{r,q_2} \leq 1, & \forall i \in \mathcal{S}_2, r \in \mathcal{R}_C, \ (q_1, q_2) \in \mathcal{QQ}_C \end{cases} \tag{3.13}$$

Manifold Model

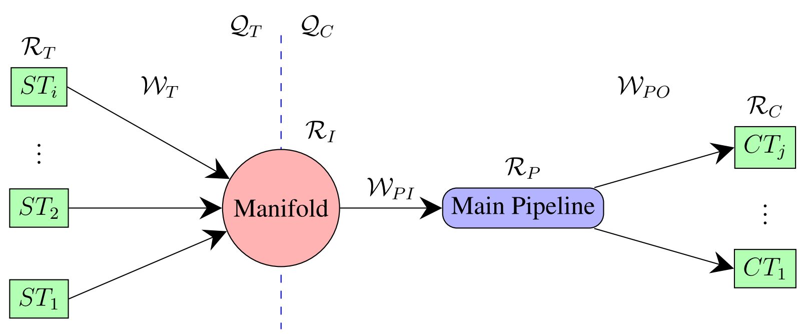

Figure 3.4: Schematics of the manifold-pipeline arrangement from the storage tanks ST_i to the charging tanks CT_j. Crude qualities $q \in \mathcal{Q}_T$ in storage tanks are mixed in the Manifold to obtain crude types $q \in \mathcal{Q}_C$ to be delivered to the charging tanks that feed the refinery.

Terminal crude oil qualities are blended in the manifold before being pumped into the pipeline as shown in the schematics of Figure 3.4.

At most $\overline{R}$ tanks can inject crude into the manifold simultaneously:

$$\sum_{w \in \mathcal{W}_T} z^i_w \leq \overline{R}, \forall i \in \mathcal{S} \tag{3.14}$$

Eq. (3.15) models the transformation of the terminal mixtures into refinery pure qualities according to a set of specifications. This approach guarantees that only pure qualities are delivered through the pipeline, with given quality specifications. As a result, in a given slot, the quality of the material that enters a resource $r \in \mathcal{R}_C$ is equal to the composition of that material in the pipeline. Unlike the works of de Assis et al. (2017, 2019), with this modeling strategy, nonlinear relations are not needed to express the condition that a quality fraction in the volume of a resource r, has to be the same fraction in the outlet volume.

$$
\begin{aligned}
\underline{Q_{q_1,q_2}} \cdot dv_w^i/100 - \overline{Vs} \cdot (1 - yvq_{r_1,q_1}^i) \leq dvq_{w,q_2}^i \\
\leq \overline{Q_{q_1,q_2}} \cdot dv_w^i/100 + \overline{Vs} \cdot (1 - yvq_{r_1,q_1}^i), \\
\forall i \in \mathcal{S}, w \in \mathcal{O}_r, r \in \mathcal{R}_I, r_1 \in \mathcal{R}_P, q_1 \in \mathcal{Q}_C, q_2 \in \mathcal{Q}_T
\end{aligned}
\tag{3.15}
$$

$$
\begin{cases}
\sum_{q \in \mathcal{Q}_C} yvq_{r_1,q}^i \leq 1, & \forall i \in \mathcal{S}, r_1 \in \mathcal{R}_P \\
dv_w^i \leq \overline{Vs} \cdot \sum_{q \in \mathcal{Q}_C} yvq_{r_1,q}^i, & \forall i \in \mathcal{S}, w \in \mathcal{O}_r, r \in \mathcal{R}_I, r_1 \in \mathcal{R}_P.
\end{cases}
\tag{3.16}
$$

Equation (3.17) evaluates the volume of the refinery quality.

$$
\begin{cases}
dvq_{w,q}^i \leq dv_w^i + \overline{V_{r_1}} \cdot (1 - yvq_{r_1,q}^i), & \\
dvq_{w,q}^i \geq dv_w^i - \overline{V_{r_1}} \cdot (1 - yvq_{r_1,q}^i), & \forall i \in \mathcal{S}, r_1 \in \mathcal{R}_P, \\
\sum_{w_1 \in \mathcal{I}_{r_1}} dvq_{w_1,q}^i \leq \overline{V_{r_1}} \cdot yvq_{r_1,q}^i, & \forall w \in \mathcal{I}_{r_1}, q \in \mathcal{Q}_C
\end{cases}
\tag{3.17}
$$

Pipeline Model

A main pipeline $r \in \mathcal{R}_P$ connects the terminal with the refinery, transferring the crude mixed in the terminal manifold to charging tanks in the refinery. Variable $vp^{j,i}$ constitutes the volume of the packet that enters the pipeline in slot $j < i$ and remains in the resource in slot i.

The initial conditions for the volume of packets in the pipelines, for all qualities, are imposed by:

$$
vpq_{r,q}^{j,1} = Vpq0_{r,q}^j, \quad \forall j \in \mathcal{S}_0, r \in \mathcal{R}_P, q \in \mathcal{Q}_C \tag{3.18}
$$

In order to represent past events, the initial conditions are given by values of $j < 0$.

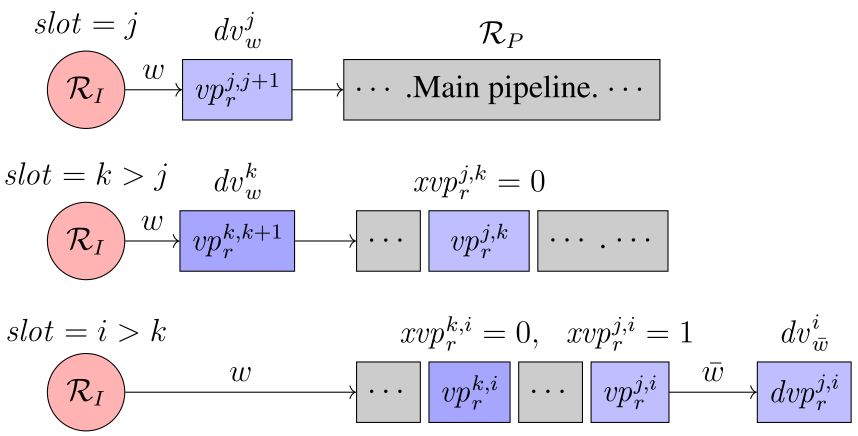

Figure 3.5: Main pipeline functional description at 3 different priority time slots. At time slot j a packet is entering the pipeline $r \in \mathcal{R}_P$, which moves through the pipeline at time slots $k > j$ and finally leaves the pipeline at time slot $i > k$.

The volume of quality q that enters the pipeline by operation w in slot $j < i$, constitutes the packet $vpq_{r,q}^{j,j+1}$, that will remain in the pipeline in slot $j+1$, being imposed by:

$$vpq_{r,q}^{j,j+1} = \sum_{w \in \mathcal{I}_r} dvq_{w,q}^{j}, \qquad \forall j \in \mathcal{S}_1, r \in \mathcal{R}_P, q \in \mathcal{Q}_C \tag{3.19}$$

Variable $dvpq_{r,q}^{j,i}$ constitutes the amount of volume of packet $vpq_{r,q}^{j,j+1}$ that leaves the pipeline in slot i. The following equations constitute the material balance for a packet:

$$\begin{cases} vpq_{r,q}^{j,i+1} = vpq_{r,q}^{j,i} - dvpq_{r,q}^{j,i}, & \forall j \in S_1, i \in S_2, r \in \mathcal{R}_P, q \in \mathcal{Q}_C \\ vp_r^{j,i} = \displaystyle\sum_{q \in \mathcal{Q}_C} vpq_{r,q}^{j,i}, & \forall j \in S_1, i \in S_2, r \in \mathcal{R}_P \end{cases} \tag{3.20}$$

Below follows the relation between the volume of a packet that leaves the pipeline and the volume of the corresponding output operations,

$$\begin{cases} \displaystyle\sum_{w \in \mathcal{O}_r} dvq_{w,q}^{i} = \sum_{\substack{j \in \mathcal{S}_1 \\ j < i}} dvpq_{r,q}^{j,i}, & \forall i \in \mathcal{S}, r \in \mathcal{R}_P, q \in \mathcal{Q}_C \\ dvp_r^{j,i} = \displaystyle\sum_{q \in \mathcal{Q}_C} dvpq_{r,q}^{j,i}, & \forall i \in \mathcal{S}, j \in \mathcal{S}_1, r \in \mathcal{R}_P \end{cases} \tag{3.21}$$

Considering that the pipeline volume remains constant, it follows from Eq. (3.9) that the total inlet volume equals the total outlet volume. Combining this equation with Eqs. (3.2) and (3.21), it can be seen that a volume of product will only leave the pipeline in case it is pushed, Eq. (3.22), implying that:

$$\sum_{w \in \mathcal{I}_r} \sum_{q \in \mathcal{Q}_C} dvq^i_{w,q} = \sum_{\substack{j \in \mathcal{S}_1 \\ j<i}} \sum_{q \in \mathcal{Q}} dvpq^{j,i}_{r,q} = \sum_{w \in \mathcal{O}_r} \sum_{q \in \mathcal{Q}_C} dvq^i_{w,q}, \ \forall i \in \mathcal{S}, r \in \mathcal{R}_P \quad (3.22)$$

Binary variable $xvp^{j,i}_r$ flags when a whole or a portion of a packet that enters the pipeline in slot j leaves it in slot i. The following constraints determine such logic conditions:

$$\begin{cases} dvp^{j,i}_r \geq \underline{DVp} \cdot xvp^{j,i}_r, \\ dvp^{j,i}_r \leq \overline{V_r} \cdot xvp^{j,i}_r, \end{cases} \quad \forall i \in \mathcal{S}, j \in \mathcal{S}_1, j < i, r \in \mathcal{R}_P \quad (3.23)$$

The structure of the model and the initial conditions imply that $xvp^{j,i}_r = 0$ for all $j \geq i$. The following constraint is needed to assure the FIFO model for the packets,

$$dvp^{j,i}_r \leq \sum_{w \in \mathcal{O}_r} dv^i_w - \sum_{\substack{j_1 \in \mathcal{S}_1 \\ j_1<j}} vp^{j_1,i}_r + \overline{V_r}(1 - xvp^{j,i}_r), \ \forall i \in \mathcal{S}, j \in \mathcal{S}_1, r \in \mathcal{R}_P \quad (3.24)$$

Best Practices

It is desired to try to fill a storage or charging tank $r \in \mathcal{R}_T \cup \mathcal{R}_C$ as much as possible. The variable vr^i_r keeps track of the remaining spare volume in a tank $r \in \mathcal{R}_T \cup \mathcal{R}_C$, after filling the tank. This variable, evaluated in slot i, measures the difference between the maximum capacity and the actual volume of the resource in slot $i + 1$. The spare volume is calculated by the following constraints:

$$\begin{cases} vr^i_r \leq \overline{V_r} - v^{i+1}_r, \\ vr^i_r \geq \overline{V_r} - v^{i+1}_r - \overline{V_r} \cdot (1 - \sum_{w \in \mathcal{I}_r} z^i_w), \end{cases} \quad \forall i \in \mathcal{S}, i < N, r \in \mathcal{R}_T \cup \mathcal{R}_C \quad (3.25)$$

Notice that the variable vr^i_r assumes the spare volume if an inlet operation is performed (i.e., $z^i_w = 1$ for some w), otherwise vr^i_r is only bounded from above but will be driven to zero because its value is penalized in the objective.

To prioritize crude loading into an empty tank, the variable vm^i_r keeps track of the actual volume stored in a tank $r \in \mathcal{R}_T \cup \mathcal{R}_C$, whenever it is being filled.

$$\begin{cases} vm_r^i \leq v_r^i + \overline{V_r} \cdot \Big(1 - \sum_{w \in \mathcal{I}_r} z_w^i\Big), \\ vm_r^i \geq v_r^i - \overline{V_r} \cdot \Big(1 - \sum_{w \in \mathcal{I}_r} z_w^i\Big), \end{cases} \qquad \forall i \in \mathcal{S}, r \in \mathcal{R}_T \cup \mathcal{R}_C. \tag{3.26}$$

For storage tanks $\mathcal{R}_T$, variable vm_r^i assumes the value of the stored volume if an inlet operation is performed (i.e., $z_w^i = 1$ for some w). Otherwise, vm_r^i will be forced to zero because it is penalized in the objective. Thus, loading is promoted into an empty tank because a penalty cost will not be incurred.

$$\begin{cases} vm_r^i \leq \overline{V_r} - v_r^i + \overline{V_r} \sum_{w \in \mathcal{I}_r} z_w^i, \\ vm_r^i \geq \overline{V_r} - v_r^i - \overline{V_r} \sum_{w \in \mathcal{I}_r} z_w^i, \end{cases} \qquad \forall i \in \mathcal{S}, r \in \mathcal{R}_C. \tag{3.27}$$

For charging tanks $\mathcal{R}_C$, loading is also promoted into an empty tank by way of Eq. (3.26) and the penalty cost on the volume vm_r^i stored in the tank. However, unlike storage tanks, a penalty cost is imposed through Eq. (3.27) on the difference between the tank capacity and volume stored, which drives the charging tanks to be full when inlet operations are not performed.

3.3.2. Time and Sequencing

Relations that induce the desired behavior regarding time and sequencing of operations are introduced below. Equation (3.28) establishes the relationships between initial and end times and time extensions. Equation (3.29) relates to the continuous operation of the distillation column.

Time Relationships

$$\begin{cases} te_w^i = ts_w^i + td_w^i, \\ te_w^i \leq T \cdot z_w^i, \end{cases} \qquad \forall i \in \mathcal{S}, w \in \mathcal{W}_1 \tag{3.28}$$

Continuous Distillation Constraints

$$\sum_{i \in \mathcal{S}} \sum_{w \in \mathcal{I}_r} td_w^i = T, \qquad \forall r \in \mathcal{R}_D \tag{3.29}$$

Notice that the clique $C = \mathcal{I}_r$ prevents simultaneous inlet operations into the distillation column r.

Sequencing Constraints

A delay in a vessel's unloading and refinery distillation storage is allowed for all time slots i, when applicable:

$$ts_w^i \geq Ts_r \cdot z_w^i, \qquad \forall r \in \mathcal{R}_V, w \in \mathcal{O}_r \tag{3.30a}$$

$$ts_w^i \geq Ts_r \cdot z_w^i, \qquad \forall r \in \mathcal{R}_O, w \in \mathcal{I}_r \tag{3.30b}$$

$$te_w^i \leq Te_r + dt_r, \qquad \forall r \in \mathcal{R}_V, w \in \mathcal{O}_r \tag{3.30c}$$

$$te_w^i \leq Te_r + dt_r, \qquad \forall r \in \mathcal{R}_O, w \in \mathcal{I}_r \tag{3.30d}$$

$$dt_r \leq Dt_r, \qquad \forall r \in \mathcal{R}_V \cup \mathcal{R}_O \tag{3.30e}$$

Being nonnegative, dt_r defines the cargo unloading or product delivery time delay, which is bounded by Dt_r.

The crude cargo of vessel r_1 must be fully unloaded into the terminal, before the unloading is started from another vessel r_2 that arrives later at the terminal.

$$\max_{\substack{i \in \mathcal{S}, \\ w \in \mathcal{O}_{r_1}}} te_w^i \leq \min_{\substack{i \in \mathcal{S}, \\ w \in \mathcal{O}_{r_2}}} \left(ts_w^i + T \cdot (1 - z_w^i) \right), \quad \forall r_1, r_2 \in \mathcal{R}_V, Ts_{r_1} < Ts_{r_2} \tag{3.31a}$$

$$\sum_{\substack{j \in \mathcal{S}, j < i \\ w \in \mathcal{O}_{r_1}}} z_w^j \geq \sum_{\substack{j \in \mathcal{S}, j < i \\ w \in \mathcal{O}_{r_2}}} z_w^j, \quad \forall i \in \mathcal{S} \tag{3.31b}$$

The first equation is linearized by means of variables $\overline{te_{r_1}}$ and $\underline{ts_{r_2}}$ which bound the arguments of the respective $\max$ and $\min$ operators, as follows:

$$te_w^i \leq \overline{te_{r_1}}, \qquad \forall i \in \mathcal{S}, r_1 \in \mathcal{R}_V, w \in \mathcal{O}_{r_1}, \tag{3.32a}$$

$$\underline{ts_{r_2}} \leq \left(ts_w^i + T \cdot (1 - z_w^i)\right), \qquad \forall i \in \mathcal{S}, r_2 \in \mathcal{R}_V, w \in \mathcal{O}_{r_2}, \tag{3.32b}$$

$$\overline{te_{r_1}} \leq \underline{ts_{r_2}}, \qquad \forall r_1, r_2 \in \mathcal{R}_V,\, Ts_{r_1} < Ts_{r_2} \tag{3.32c}$$

The priorities of operations associated with output resources must be addressed by the production plan. If a resource r_1 delivers a crude mix before resource r_2, then all the input operations for the first resource must be assigned to priority slots preceding the first slot in which an operation is assigned for resource r_2. This condition aims to reduce symmetry as implemented below:

$$\sum_{\substack{j \in \mathcal{S}, j < i,\\ w \in \mathcal{I}_{r_1}}} z_w^j \geq \sum_{\substack{j \in \mathcal{S}, j < i,\\ w \in \mathcal{I}_{r_2}}} z_w^j, \quad \forall i \in \mathcal{S}, r_1, r_2 \in \mathcal{R}_O,\, Ts_{r_1} < Ts_{r_2} \tag{3.33}$$

In order to mix qualities, more than one input operation is allowed into the manifold, in which case they all have the same starting time and duration. However, the system establishes that only one output operation can be performed from the manifold at any given priority slot, which is established by a clique. The starting time and duration of the output operation is the same as the input operations. These conditions are given by the following constraints:

$$\begin{cases} \sum\limits_{w_2 \in \mathcal{O}_r} ts_{w_2}^i - T \cdot (1 - z_{w_1}^i) \leq ts_{w_1}^i \leq \sum\limits_{w_2 \in \mathcal{O}_r} ts_{w_2}^i + T \cdot (1 - z_{w_1}^i), \\ \sum\limits_{w_2 \in \mathcal{O}_r} td_{w_2}^i - T \cdot (1 - z_{w_1}^i) \leq td_{w_1}^i \leq \sum\limits_{w_2 \in \mathcal{O}_r} td_{w_2}^i + T \cdot (1 - z_{w_1}^i), \end{cases}$$
$$\forall i \in \mathcal{S}, w_1 \in \mathcal{I}_r,\ r \in \mathcal{R}_I \tag{3.34}$$

Input and output operations in the resources $r \in \mathcal{R}_B \cup \mathcal{R}_P \cup \mathcal{R}_D$ must be performed simultaneously. Because the cliques for these resources ensure at most one inlet (outlet) operation is performed in each slot, the requirement that inlet and outlet operations are simultaneous is given by:

$$\begin{cases} \sum\limits_{w \in \mathcal{I}_r} ts_w^i = \sum\limits_{w \in \mathcal{O}_r} ts_w^i, \\ \sum\limits_{w \in \mathcal{I}_r} td_w^i = \sum\limits_{w \in \mathcal{O}_r} td_w^i, \end{cases} \forall i \in \mathcal{S}, r \in \mathcal{R}_B \cup \mathcal{R}_P \cup \mathcal{R}_D \tag{3.35}$$

Operations in the same clique C must not overlap, even when they are executed in different time slots $i_1 < i_2$, as imposed by Eq. (3.36).

$$\sum_{w \in C} te_w^{i_1} + \sum_{\substack{j \in \mathcal{S}, \\ i_1 < j < i_2}} \sum_{w \in C} td_w^j \leq \sum_{w \in C} ts_w^{i_2} + T \cdot (1 - \sum_{w \in C} z_w^{i_2}),$$

$$\forall i_1, i_2 \in \mathcal{S}, i_1 < i_2, C \in \mathcal{C} \quad (3.36)$$

Now, considering distinct operations w_1 and w_2 of the same clique, Eq. (3.37) ensures the settling time Tt_c between these operations.

$$te_{w_1}^{i_1} + Tt_C \cdot z_{w_1}^{i_1} \leq ts_{w_2}^{i_2} + (T + Tt_C)(1 - z_{w_2}^{i_2})$$

$$\forall i_1, i_2 \in \mathcal{S}, i_1 < i_2, w_1, w_2 \in C, w_1 \neq w_2, C \in \mathcal{C} \quad (3.37)$$

3.3.3. Maintenance Operations

For resources $r \in \mathcal{R}_T \cup \mathcal{R}_C$, three types of operations $w \in \mathcal{W}_M$ are considered. According to the value of Mo_w, the operations can *i)* restrict the volume of resource r to be fixed at its maximum capacity $\overline{V_r}$ ($Mo_w = 1$), *ii)* restrict r to its minimum capacity $\underline{V_r}$ ($Mo_w = 0$), or *iii)* keep the volume level at the level of the last operation performed in the resource ($Mo_w \notin \{0, 1\}$). If a maintenance operation $w \in \mathcal{W}_M$ is performed, then it should extend from time TMs_w to TMe_w, as modeled by the following equations:

$$\sum_{i \in \mathcal{S}} z_w^i \leq 1, \qquad \forall w \in \mathcal{W}_M \qquad (3.38a)$$

$$\begin{cases} ts_w^i = TMs_w \cdot z_w^i, \\ te_w^i = TMe_w \cdot z_w^i, \end{cases} \qquad \forall i \in \mathcal{S}, w \in \mathcal{W}_M \qquad (3.38b)$$

Maintenance operations are not mandatory. These operations are executed only if there is an economic incentive and there is no conflict with operations that must be continuously executed, such as crude oil delivery to a CDU.

Maintenance operations and the corresponding input and output operations for a given resource must be included in a clique. If the maintenance operation is performed during a time interval, then no transfer of crude in or out of the resource can be performed, keeping the volume at its level for the duration of the maintenance.

Regarding the maintenance operations $w \in \mathcal{W}_M$, which keep the volume of the resource r at the previous level ($Mo_w \notin \{0,1\}$), the clique structure suffices to impose the desired behavior.

The cases when the operation $w \in \mathcal{W}_M$ keeps the volume of resource r at its minimum ($Mo_w = 0$) and maximum ($Mo_w = 1$) values are enforced by the following constraints:

$$\begin{aligned}&\Big(\overline{V_r} \cdot Mo_w + \underline{V_r} \cdot (1 - Mo_w)\Big) \cdot z_w^i \leq \\ &\qquad v_r^i \leq \Big(\overline{V_r} \cdot Mo_w + \underline{V_r} \cdot (1 - Mo_w)\Big) \cdot z_w^i + \overline{V_r} \cdot (1 - z_w^i), \\ &\qquad\qquad \forall i \in \mathcal{S}, r \in \mathcal{R}_T \cup \mathcal{R}_C, w \in \mathcal{W}_r, Mo_w \in \{0,1\} \end{aligned} \tag{3.39}$$

Thus, for the latter two types of maintenance operations to be performed, the volume of the resource should be previously driven to the required level by the respective operation.

3.3.4. Concentration Relationships

Concentration of crude types q in the outlet volume from storage and charging tanks must be equal to the concentration inside those resources. More precisely, the ratio of the volume $vq_{r,q}^i$ (of quality q) stored in a tank r to the total volume v_r^i, in a given slot i, must be the same ratio of the outlet volume $dvq_{w,q}^i$ (of quality q) to the total outlet dv_w^i yielded by an output operation w. These are non linear relationships in the variables involved, as established by Eq. (3.40):

$$\frac{vq_{r,q}^i}{v_r^i} = \frac{dvq_{w,q}^i}{dv_w^i}, \qquad \forall i \in \mathcal{S}, r \in \mathcal{R}_T \cup \mathcal{R}_C, q \in \mathcal{Q}_r, w \in \mathcal{O}_r. \tag{3.40}$$

Such relationships can be expressed as the bilinear equations:

$$vq_{r,q}^i \cdot dv_w^i = v_r^i \cdot dvq_{w,q}^i, \qquad \forall i \in \mathcal{S}, r \in \mathcal{R}_T \cup \mathcal{R}_C, q \in \mathcal{Q}_r, w \in \mathcal{O}_r. \tag{3.41}$$

As mentioned in Section 3.3.1, bilinear constraints are not required to model transfer operations between pipeline and charging tanks. Despite being required to model operations between terminal tanks and the manifold and between charging tanks and the distillation unit, these nonlinear constraints may not be required in the terminal and refinery because the extent of the mixtures is limited by Eqs. (3.12) and (3.13), along with the specifications defined in problem data in terms of the sets $\mathcal{QQ}_C$ and $\mathcal{QQ}_T$. In the refining industry, it is considered a good practice to not allow the mixture of qualities, because it provides a better control of the qualities delivered to the CDU. In the end, depending on the case under consideration, the concentration relationships are ensured by the restrictions on the mixtures without bilinear constraints.

3.3.5. Additional Assignment Constraints

In order to reduce the assignment choices and speed up the optimization process, new constraints are introduced to establish a relationship between the variables that assign operations to the first available slot. As a result, several solutions with similar costs will not be considered by the optimization algorithm. This strategy is called symmetry breaking, as referred to by Mouret et al. (2009). Mouret et al. (2011) defined these constraints considering a set $\mathcal{C}$ of non overlapping operations. Their formulation can not assure feasibility when operations in a given resource can not be assigned to the earliest slot at any point of the schedule, *i.e.* when there is a crude quality that is consumed in a later time period and cannot be assigned to an early priority slot due to other model constraints.

We adapt the formulation of Mouret et al. (2011) to include at least the pipeline inlet operations, and extend the relations to allow the assignment of operations to prior and later slots. The set $\mathcal{A}$ consists of pairs of different non overlapping operations from $\mathcal{C}$ and pairs of these operations with the inlet pipeline operation.

Operation w can be assigned to slot i, *i.e.* $z_w^i = 1$, if it was active in a previous slot $i-1$ or if at least one operation w_1 of the pair $(w, w1)$ or $(w1, w)$ is active in slots i or $i-1$. This condition is expressed by the constraint:

$$z_w^i \leq z_w^{i-1} + \sum_{\substack{w_1 \in \mathcal{W}_1, \\ (w,w_1) \vee (w_1,w) \in \mathcal{A}}} \left(z_{w_1}^i + z_{w_1}^{i-1} \right), \quad \forall i \in \mathcal{S}, i < N, w \in \mathcal{W}_1 \tag{3.42}$$

Similarly, the next equation refers to slot $i+1$.

$$z_w^i \leq z_w^{i+1} + \sum_{\substack{w_1 \in \mathcal{W}_1, \\ (w,w_1) \vee (w_1,w) \in \mathcal{A}}} \left(z_{w_1}^i + z_{w_1}^{i+1} \right), \quad \forall i \in \mathcal{S}, i < N, w \in \mathcal{W}_1 \tag{3.43}$$

3.3.6. Objective Function

The objective minimizes a weighted sum of terms accounting for deviations from the refinery schedule, quality of the crudes delivered to refinery, number of operations performed, and adherence to maintenance operations, among others. Cost terms are adimensional and a normalization factor is added to all terms to scale them between 0 and 1 before applying the weightings to avoid a bias in the optimization towards larger quantities. The contribution of each term is driven by the cost parameter. For instance, the magnitude of the volume and quality costs is typically higher than that of other costs. In what follows, the cost terms considered in the objective function are presented according to their categories.

Volume and Quality

The terms cv and cvq penalize the difference between the required and actual volumes ($dvol_r$), and required and actual quality volumes ($dqual_{r,q}$) delivered to the refinery as a percentage of the respective required volumes (Vr_r and $Vqr_{r,q}$).

$$\begin{cases} cv = \sum_{r \in \mathcal{R}_O} Cv_r \dfrac{dvol_r}{Vr_r} \\ cvq = \sum_{q \in \mathcal{Q}_C} \sum_{r \in \mathcal{R}_O} Cvq_{r,q} \dfrac{dqual_{r,q}}{(1 + Vqr_{r,q}) \cdot |\mathcal{Q}_C|} \end{cases} \tag{3.44}$$

where $|\cdot|$ gives the cardinality of a countable set and the auxiliary variables are defined below:

$$\begin{cases} -dvol_r \leq Vr_r - \sum_{i,w \in \mathcal{I}_r} dv_w^i \leq dvol_r, \\ -dqual_{r,q} \leq Vqr_{r,q} - \sum_{i,w \in \mathcal{I}_r} dvq_{w,q}^i \leq dqual_{r,q}, \end{cases} \quad \forall r \in \mathcal{R}_O, q \in \mathcal{Q}_C. \tag{3.45}$$

The terms cvf and $cvqf$ penalize the difference between the final and remaining volumes in charging tanks and pipeline ($dvolf_r$), and likewise the difference between final and remaining quality volumes ($dqualf_{r,q}$) as a percentage of the volume capacity of each resource.

$$\begin{cases} cvf = \sum_{r \in \mathcal{R}_C} Cvf \dfrac{dvolf_r}{|\mathcal{R}_C| \cdot \overline{V_r}} \\ cvqf = \sum_{q \in \mathcal{Q}_C} \sum_{r \in \mathcal{R}_C \cup \mathcal{R}_P} Cvqf \dfrac{dqualf_{r,q}}{|\mathcal{Q}_C| \cdot |\mathcal{R}_C \cup \mathcal{R}_P| \cdot \overline{V_r}} \end{cases} \tag{3.46}$$

where:

$$\begin{cases} -dvolf_r \leq Vf_r - vf_r \leq dvolf_r, & \forall r \in \mathcal{R}_C \\ -dqualf_{r,q} \leq Vqf_{r,q} + vqf_{r,q} \leq dqualf_{r,q}, & \forall r \in \mathcal{R}_C \cup \mathcal{R}_P, q \in \mathcal{Q}_r \end{cases} \tag{3.47}$$

Best Practices

The objective also accounts for the costs associated with fulfillment of best practices. Specifically, cvr penalizes the remaining volume percentage in the terminal and charging tanks, since vr_r^i is the remaining spare volume in resource r if an operation is performed in slot i, and zero otherwise. Likewise, cvm adds cost proportionally to the volume percentage of crude stored in resource r or its spare capacity depending on whether there is an inlet operation to that resource or not. This cost term encourages inlet operations to be performed in an empty tank, but also penalizes for not performing an inlet operation when a tank is not full.

$$\begin{cases} cvr = \sum_{\substack{i \in \mathcal{S}, \\ r \in \mathcal{R}_T \cup \mathcal{R}_C}} Cvr \dfrac{vr_r^i}{\overline{V_r}} \\ cvm = \sum_{\substack{i \in \mathcal{S}, \\ r \in \mathcal{R}_T \cup \mathcal{R}_C}} Cvm \dfrac{vm_r^i}{\overline{V_r}} \end{cases} \tag{3.48}$$

Operations

In order to reduce symmetry and improve computation time, it is desirable to minimize the number of operations executed (cz) and the number of packets per output operations performed in the pipeline ($cxvp$).

$$\begin{cases} cz = \sum_{i\in\mathcal{S},w\in\mathcal{W}} Cz\dfrac{z_w^i}{N\cdot|\mathcal{W}|} \\ cxvp = \sum_{\substack{i\in\mathcal{S},j\in S_1,\\ j<i,r\in\mathcal{R}_P}} Cxvp\dfrac{xvp_r^{j,i}}{|\mathcal{R}_P|\cdot N(N-1)/2} \end{cases} \tag{3.49}$$

Quality

The objective favors the reduction in the number of qualities in the terminal and charging tanks.

$$cxvq = \sum_{\substack{i\in\mathcal{S},\\ r\in\mathcal{R}_T\cup\mathcal{R}_C}} \sum_{q\in\mathcal{Q}_r} Cxvq\frac{xvq_{r,q}^i}{N\cdot|\mathcal{Q}|\cdot(|\mathcal{R}_T|+|\mathcal{R}_C|)} \tag{3.50}$$

Maintenance

There is a cost associated with the non fulfillment of maintenance operations.

$$cm = \sum_{w\in\mathcal{W}_M} Cm_w\left(\frac{1-\sum_{i\in\mathcal{S}} z_w^i}{\mathcal{W}_M}\right) \tag{3.51}$$

Time

The term cdt refers to demurrage cost which penalizes the delay dt_r for cargo unloading from a vessel r. Finally, the term cts minimizes the starting time of operations performed in vessels.

$$\begin{cases} cdt = \sum_{r\in\mathcal{R}_V} Cd_r\dfrac{dt_r}{Dt_r|\mathcal{R}_V|} \\ cts = \sum_{i\in\mathcal{S},w\in\mathcal{W}_V} Cts\dfrac{ts_w^i}{T\cdot N|\mathcal{W}_V|} \end{cases} \tag{3.52}$$

Total cost

The total cost results from the addition of the above stated terms:

$$
\begin{aligned}
cost = \; & ctv + cvq + cvf + cvqf + cvr + cvm + cz \\
& + cxvp + cxvq + cm + cdt + cts. \qquad (3.53)
\end{aligned}
$$

The values of the weighted cost terms allow the planners to express the relative impact of the requirements, *e.g.* keeping a tank empty during a specific time span or fulfilling the volumes and qualities required by the refinery.

3.3.7. Compact Formulation

Having introduced the notation, objective, and constraints, we can state the problem in a compact form as follows:

$$
\begin{aligned}
\text{mín} \;\; & cost \\
\text{s.t. } & \text{Eqs. (3.1)-(3.7)}, \\
& \text{Eqs. (3.9), (3.11)-(3.21)}, \\
& \text{Eqs. (3.23)-(3.31)}. \\
& \text{Eqs. (3.33)-(3.39)}. \\
& \text{Eqs. (3.41)-(3.53)}.
\end{aligned} \qquad (3.54)
$$

3.3.8. Bounds for the Number of Slots

According to Mouret et al. (2011), the number of slots that will yield a global optimum is not known before hand, but sometimes bounds can be derived for the number of slots based on the problem structure and data. A global optimum may be obtained with a sufficiently large number of priority slots, which typically renders the optimization problem intractable considering the number of variables and constraints involved. Thus, a trade-off between solution quality and tractability should be reached. Here we present lower bounds for the number of priority slots.

Although the formulation of the model allows sets $\mathcal{R}_B$, $\mathcal{R}_P$, and $\mathcal{R}_D$ to contain more than one resource, the following equations assume a configuration with one subsea pipeline, main pipeline, and CDU which is the existing configuration in ANCAP, Uruguay.

Considering the refinery side of the network, crudes with the given specification must be continuously delivered to the CDU. Let V_{dm} be the total volume required by the refinery over the planning horizon, and let $\overline{V_C} = \text{máx}\{\overline{V_r} : r \in \mathcal{R}_C\}$ be the maximum capacity among the charging tanks. Then, the minimum number of inlet operations into the CDU can be estimated as the number of transfers, with volume $\overline{V_C}$ or smaller, that correspond to the volume V_{dm}. Also, we need to factor in the transfer of crudes from the pipeline into the charging tanks to meet the demands. The minimum volume pumped from the pipeline is V_{dm} discounted the crudes stored in the charging tanks at initial conditions, added by the total volume of crudes that must remain in charging tanks at termination ($\underline{Vfc}$). Further considering that inlet and outlet operations can take place in parallel for distinct tanks, we arrive at the following lower bound for the number of priority slots:

$$lb_C^{slots} = \text{máx}\left\{ \left\lceil \frac{V_{dm}}{\overline{V_C}} \right\rceil, \left\lceil \frac{V_{dm} - \sum\limits_{r \in \mathcal{R}_C} V0_r + \underline{Vfc}}{\overline{V_C}} \right\rceil \right\} \tag{3.55}$$

Regarding the farm of storage tanks at the terminal, let the total volume of crude to be offloaded as being $V_{cargo} = \sum_{r \in \mathcal{R}_V} V0_r$. Let $\overline{V_T} = \text{máx}\{\overline{V_r} : r \in \mathcal{R}_T\}$ be the maximum volume capacity among storage tanks. An operation can pump during a batch the maximum volume $\overline{V_T}$ from the vessels into a storage tank, which leads to a lower bound on the number of transfer operations from vessels. We should consider the operations that transfer crude from storage tanks into the manifold, in order to make room for crude being offloaded from the cargo and to supply the demand. The volume transferred from the storage tanks into the manifold is the demand V_{dm}, or the cargo volume V_{cargo} discounted the spare capacity in the storage tanks, whichever is the largest. Let $\overline{R}$ be the maximum number of operations that can be performed in parallel from the storage tanks into the manifold. Then, the maximum volume that can be transferred from storage tanks to the manifold, by parallel operations in a single slot, is $\overline{V_T} \cdot \overline{R}$. According with this discussion, we can obtain a lower bound on the number of priority slots:

$$\begin{aligned} lb_T^{slots} = \text{máx}\Bigg\{ & \left\lceil \frac{V_{cargo}}{\overline{V_T}} \right\rceil, \\ & \left\lceil \frac{V_{dm} - \sum\limits_{r \in \mathcal{R}_C} V0_r + \underline{Vfc}}{\overline{V_T} \cdot \overline{R}} \right\rceil, \left\lceil \frac{V_{cargo} - \sum\limits_{r \in \mathcal{R}_T} (\overline{V_r} - V0_r)}{\overline{V_T} \cdot \overline{R}} \right\rceil \Bigg\} \end{aligned} \tag{3.56}$$

In addition, the minimum number of slots could be determined by the number of pipeline operations required to deliver crude oil to the refinery. We define $\overline{V_P}$ as the maximum volume of pipeline, which leads to a lower bound on the number of slots,

$$lb_P^{slots} = \text{máx}\left\{ \left\lceil \frac{V_{dm} - \sum_{r \in \mathcal{R}_C} V0_r + \underline{Vfc}}{\overline{V_P}} \right\rceil, \left\lceil \frac{V_{cargo} - \sum_{r \in \mathcal{R}_T} (\overline{V_r} - V0_r)}{\overline{V_P}} \right\rceil \right\} \tag{3.57}$$

By combining the three lower bounds given above, we arrive at a lower bound on the number of priority slots:

$$lb^{slots} = \text{máx}\{lb_C^{slots}, lb_T^{slots}, lb_P^{slots}\} \tag{3.58}$$

Considering that differences in tank capacities could increase the required number of slots, slots can be estimated by replacing maximum capacities in the above equations with the respective average values. For charging tanks, the maximum volume $\overline{V_C}$ would be replaced by the average charging tank capacity, given by:

$$V_C^* = \frac{\sum_{r \in \mathcal{R}_C} \overline{V_r}}{|\mathcal{R}_C|} \tag{3.59}$$

Similarly, the average value of the storage tank capacity would be used for computing an approximate number of priority slots.

The additional number of slots required could be higher than the number of maintenance operations. Also, slots could be shared with other operations depending on the clique structure without requiring additional slots, rendering complex the estimation of the influence of the maintenance operations in the bounds on the number of slots.

3.4. Heuristic Strategies

The Relax & Fix strategy optimizes the original time horizon of the problem associated to a set of slots and generates several solutions. As mentioned by Zimberg et al. (2019), initially, a variant of the model is solved, where a subset of the binary variables is relaxed for slots $i = \Delta + 1, \ldots, N$, where the step parameter Δ is provided by the user. Follows an iteration for slots $i = 1, \ldots, N - \Delta$, where:

i) binary variables are fixed with values of the last solution;

ii) integrality is recovered in slot $i + \Delta$; and

iii) the new variant is solved.

The algorithm is depicted in Algorithm 2. This approach assumes that decisions, in terms of binary variables, are mainly affected by the next $\Delta - 1$ slots and variables can be relaxed in subsequent slots. It assumes that increasing times are mostly associated with increasing the number slots. It involves the solution of a series of partially relaxed problem variants, each with a number of binary variables that is small enough to be quickly and optimally solved by conventional branch-and-cut methods.

Figure 3.6 depicts the strategy for a problem case with 7 slots and $\Delta = 5$, where the sequence of algorithm steps is represented in the figure by the sequence of insets (a), (b), and (c). At each node, characters B, R, and F identify specific slots where variables remain binary, relaxed, and fixed respectively.

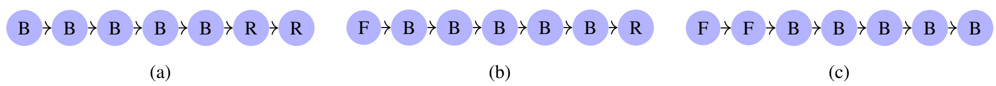

Figure 3.6: Example of relax-and-fix strategy with 7 slots and $\Delta = 5$, where algorithm steps are represented by insets (a), (b), and (c).

Data: Δ : number of slots where integrality is kept
Data: $Model$: model instance
Result: Solution of a restricted $Model$
Relax binary variables for $i = \Delta + 1, \ldots, N$;
Solve;
for $i = 1, \ldots, N - \Delta$ **do**
    Fix binary variables in slot i;
    Restore integrality of relaxed binary variables in slot $i + \Delta$;
    Solve;
end

Algorithm 1: Relax-and-fix strategy

3.5. Computational Experiments

This section aims to assess the flexibility and performance of the continuous-time model for scheduling crude oil operations. The experiments are carried out

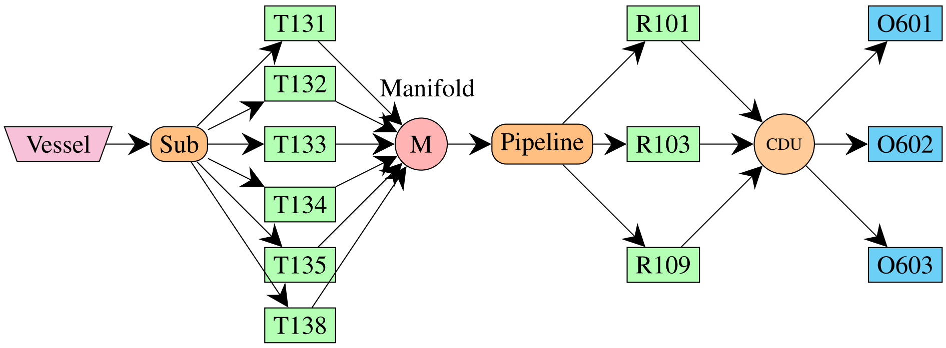

Figure 3.7: Model for cases 1 to 7

considering the operations in the terminal of ANCAP, Uruguay. The section starts by presenting the data and structure of the terminal that characterize the problem instance. Then it follows the details of how the model was implemented and solved. Finally, the results from computational experiments of the model applied to the ANCAP terminal are reported and discussed.

3.5.1. Experimental Set-up

Figure 3.7 presents the graph for the set of cases that capture typical operations performed in the ANCAP terminal. It comprises one cargo reception, six storage tanks at the terminal, and three charging tanks at the refinery. The system composed by the buoy and subsea pipeline allows the discharge of crude oil to the terminal. Terminal qualities are blended in the manifold yielding refinery qualities that are sent to the charging tanks through the main pipeline. Batches of different crude oil qualities are delivered to the distillation unit.

In order to present the capabilities of the model, the following cases were evaluated. The set of cases represents a problem with a time horizon of 17 days with 3 terminal qualities, 4 allowed refinery qualities, and 2 required refinery qualities. There is one cargo reception and 7 slots are considered for the model. For cases 1, 2, and 7 the model is optimized with the Branch & Bound algorithm. There is also a nonlinear step but it does not contribute because bilinear terms do not appear in the formulation, since quality mixtures are not allowed in the terminal and refinery. The composition of the refinery quality Q2 is open bounded in cases 1, 3, 5, and 7 to let the model obtain the best composition. For all cases, minimum and maximum flowrates are tightened in order to smooth the delivery of crude to the CDU. The model is evaluated with the Relax & Fix heuristics in cases 3, 4, 5, and 6. The composition of quality Q2 is fixed in cases 2, 4, and 6. Case 7 evaluates the feasibility of two maintenance operations.

Case 1: Quality Q2 is open bounded. Branch & Bound evaluation followed by nonlinear optimization.

Case 2: Q2 is fixed. Branch & Bound evaluation followed by nonlinear optimization.

Case 3: Q2 is open bounded. Relax & Fix. Step 6 followed by nonlinear optimization.

Case 4: Q2 is fixed. Relax & Fix. Step 6 followed by nonlinear optimization.

Case 5: Q2 is open bounded. Relax & Fix. Step 5 followed by nonlinear optimization.

Case 6: Q2 is fixed. Relax & Fix. Step 5 followed by nonlinear optimization.

Case 7: Maintenance operations. Branch & Bound evaluation followed by nonlinear optimization.

Table 3.1 presents the initial inventories for all the resources of the model. Terminal qualities are T1 (Vasconia), T2 (Qua) and T3 (Nemba), whereas refinery qualities are named Q1, Q2, Q3 and Q4. The initial qualities in the pipeline are shown in Table 3.2. There are two packets. Packet -1 contains quality Q1 whereas packet -2, the one closest to the refinery, contains quality Q2.

Table 3.1: Cases 1 to 7. Initial inventories and qualities (10^3 m^3).

	Quality						
Resource	T1	T2	T3	Q1	Q2	Q3	Q4
subsea		5					
Vessel			144				
t131		28.4					
t132		56					
t133	9.5						
t134			53				
t135			4.6				
t138		3.3					
r101					4.8		
r103					18.5		
r109					7.0		
Manifold							
Pipeline				3.4	16.8		
Distillation							
o601							
o602							
o603							

Tables 3.3 and 3.4 present the required final inventories, volume and quality deliveries, respectively. These requirements are not mandatory.

Table 3.2: Cases 1 to 7. Pipeline. Initial packet qualities (10^3 m^3).

	Quality			
Packet	Q1	Q2	Q3	Q4
-1	3.4			
-2		16.8		

Tables 3.5 and 3.6 depict the penalties for each objective term. Volume and quality penalties are defined one order higher than other parameters.

Table 3.3: Cases 1 to 7. Required final inventories (10^3 m^3).

	Quality			
Resource	Q1	Q2	Q3	Q4
r101		30		
r103		15		
r109		0		
Pipeline		20		

Table 3.4: Cases 1 to 7. Required volume and quality for each delivery (10^3 m^3), Time (day).

	Quality							
Required	Q1	Q2	Q3	Q4	T3	T start	T end	Dem
Vessel					144	12	14	1
o601		45				0	6	
o602	41.4					6	12	
o603		36.1				12	17	

Maintenance penalties regarding case 7 are shown in Table 3.7. Two operations are proposed in this case. Operation 1 requires tank T132 not to be operated between days 10 and 12. Operation 2 requires tank T138 to be empty between days 12 and 17. The table also displays the costs for not fulfilling the requirements.

Table 3.8 shows the composition of the refinery qualities expressed as percentage of crude oil terminal qualities. The composition of the refinery quality Q2 is open bounded for cases 1, 3, 5 and 7.

Table 3.5: Cases 1 to 7. Objective penalties (adimensional)

	Volume	Quality Delivered			
Resource	Delivered	Q1	Q2	Q3	Q4
O601	50	50	20	15	10
O602	50	50	20	15	10
O603	50	50	20	15	10

Table 3.6: Cases 1 to 7. Objective penalties. Additional terms (adimensional)

Vol. Final	Qual. Final	Vol. Max.	Vol. Min.	Operation	Pipe Op.	Mixture	Demurrage	Start Time
10	20	0.5	0.5	0.1	0.1	0.1	1	0.5

Table 3.7: Case 7. Maintenance operations. Time (day), Penalty (adimensional)

Resource	Oper	Type	T start	T end	Penalty
T132	1	Fix	10	12	50
T138	2	Min	12	17	50

Table 3.8: Composition of refinery qualities ($\mathcal{Q}_C$) as percentage of terminal qualities ($\mathcal{Q}_T$).

	$\mathcal{Q}_T$ Composition					
	Cases 1,3,5,7			Cases 2,4,6		
$\mathcal{Q}_C$ Quality	T1	T2	T3	T1	T2	T3
Q1	20		80	20		80
Q2		60-80	20-40		70	30
Q3		100			100	
Q4			100			100

3.5.2. Implementation

The problem instances were formulated in the algebraic modeling language AMPL (Fourer et al., 2002) and then solved with CPLEX using its command line.

In order to resolve the bilinear terms, the IPOPT solver (Wächter and Biegler, 2006) was used in a second step, fixing the binary assignment variables from the CPLEX solution of the first step. Such a strategy for dealing with MINLP problems is known as MILP-NLP decomposition, which has been proven to be effective in a number of problems (de Assis et al., 2019).

Instead of optimizing for the original time horizon of the problem, given by T, the relax-and-fix approach proposed in Section 3.4 was implemented.

3.5.3. Results

A sufficient number of slots is estimated by means of Eq. (3.55) to (3.58), whereby the average capacity V^* is used in place of the maximum capacity $\overline{V}$, like in Eq. (3.59) for charging tanks. The results with the estimates for the minimum number of slots are shown in Table 3.9. Seven slots are recommended to evaluate the proposed cases.

Table 3.9: Estimation of the number of slots

Resource	Refinery		Terminal			Pipeline	
Estimation	Demand	Ref.Transf.	Cargo	Ref.Transf.	Cargo Transf.	Ref.Transf.	Cargo Transf.
7	6	6	1	1	0	7	0

Table 3.10 displays, for each case, the size of the model after the first presolve step in terms of the number of rows (constraints), columns (variables), nonzeros, and binary variables, and also the solution time, the objective obtained by the optimization strategy, and the respective optimum. Reported time values include processing times for linear and non linear steps. Notice that the Relax & Fix (R&F) approach is applied only when the parameter Delta is smaller than the number of slots, otherwise the optimization strategy coincides with the optimization over the entire planning horizon. Thus, in the former approach the objective may be worse than the optimum. The results from Table 3.10 lead to the following remarks:

- Recall that bounds are not imposed in quality Q2 for cases 1, 3, and 5, whereas Q2 is fixed to the values reported in Table 3.8 for cases 2, 4 and 6. The comparison of cases 1 and 2 (Exact Solution), 3 and 4 (R&F with Delta 6), and 5 and 6 (R&F with Delta 5) show that fixing quality Q2 has a small impact on the objective cost.

- Solution time is reduced when applying the Relax & Fix approach in cases 3 and 5 compared to the exact solution of case 1 (when Q2 is not fixed), and also for cases 4 and 6 compared to the exact solution of case 2 (when Q2 is fixed). The objective cost does not change significantly by applying R&F.
- Case 7 extends case 1 adding two maintenance operations. The solution must decide whether or not each maintenance operation is performed, a decision that depends on the cost incurred in the operations and otherwise the penalty for not performing maintenance. The final cost is higher compared to previous solutions, considering that only one maintenance operation is performed and a penalty is imposed on the other.

Table 3.10: Cases 1 to 7. Model size, solution time (s) and objective (adimensional)

Case	Slots	Delta	Rows	Columns	Non zeros	Binary	Time (s)	Objective	Optimum
1	7	7	4570	2179	20017	373	81	11.2876	11.2876
2	7	7	4570	2179	20017	373	138	11.6622	11.6622
3	7	6	4542	2138	19929	295	62	11.2882	11.2876
4	7	6	4542	2138	19929	295	61	11.6622	11.6622
5	7	5	4530	2112	19891	236	31	11.2876	11.2876
6	7	5	4530	2112	19891	236	32	11.6622	11.6622
7	7	7	4682	2188	20479	381	103	36.2882	36.2882

The contribution of the cost terms for each solution is presented in Tables 3.11 and 3.12. Required volumes and qualities are mostly fulfilled. Quality Q2 is present in the refinery tanks and pipeline as required but the final volumes are not the expected ones. Resulting costs are shown in columns 4 and 5 of Table 3.11. Tanks are not necessarily filled, i.e. tank T138 receives quality T2 from the subsea pipeline, as seen in Figure 3.11, tank T133 contains what remains of quality T1, Figure 3.14 and tank R109 is not empty, Figure 3.15. Crude oil volumes in tanks R101 and R103 vary between the maximum and the minimum capacity. The corresponding costs are shown in Table 3.11, columns labeled “Vol. Max.” and “Vol. Min.”

In the solution of case 7, operation 1 is fulfilled, whereas operation 2 remains inactive. The volume of tank T132 is fixed between times 10 and 12. Tank T138 is not kept empty between times 12 and 17. The corresponding maintenance cost is presented in Table 3.12, column labeled “Maintenance.”

Table 3.13 presents, for case 1, the composition of the refinery qualities which

Table 3.11: Cases 1 to 7. Solution. Objective costs (adimensional). Terms 1 to 6

Case	Vol. Deliv.	Qual. Deliv.	Vol. Final	Qual. Final	Vol. Max.	Vol. Min.
1	0.21	0.02	3.95	1.5	2.01	3.3
2	0	0	3.829	1.448	1.625	3.452
3	0.208	0.02	3.954	1.495	2.013	3.299
4	0	0	3.829	1.448	1.625	3.452
5	0.208	0.02	3.954	1.495	2.013	3.299
6	0	0	3.829	1.448	1.625	3.452
7	0.208	0.02	3.954	1.495	1.991	3.321

Table 3.12: Cases 1 to 7. Solution. Objective costs (adimensional). Terms 7 to 12

Case	Operation	Pipe.Op.	Mixture	Maintenance	Demurrage	Start time
1	0.031	0.048	0.012	0	0	0.207
2	0.032	0.048	0.012	0	1	0.216
3	0.032	0.048	0.012	0	0	0.207
4	0.032	0.048	0.012	0	1	0.216
5	0.031	0.048	0.012	0	0	0.207
6	0.032	0.048	0.012	0	1	0.216
7	0.032	0.048	0.012	25	0	0.207

are delivered by the pipeline. As per Table 3.8, quality Q1 is produced with 20 % T1 and 80 % T3. This mixture is delivered in slots 1, 2, and 3. As a result, the composition of the pipeline is 100 % Q1 in slots 2, 3 and 4, as shown in Figure 3.8. Similarly, quality Q2 is delivered from the terminal in slots 4, 5, and 6, being present in the pipeline in slots 5, 6, and 7 and at the end of the study. The composition of the mixture is bounded as defined in Table 3.8. Also, for cases 2, 4 and 6 the composition of Q2 is fixed to 70 % T2 and 30 % T3.

Table 3.13: Case 1. Quality of mixtures delivered to the refinery (%)

	Terminal Qualities		
Slot	T3	T2	T1
1	80		20
2	80		20
3	80		20
4	37	63	
5	23	77	
6	32	68	
7	33	67	

Table 3.14 depicts the flowrate for each delivery. The values are bounded by the data entries given by parameters $\underline{VF}_r$ and $\overline{VF}_r$.

Tables 3.15 and 3.16 show the diagram of operations and slots for the solution of case 1. Operations and main resources associated with each operation are presented

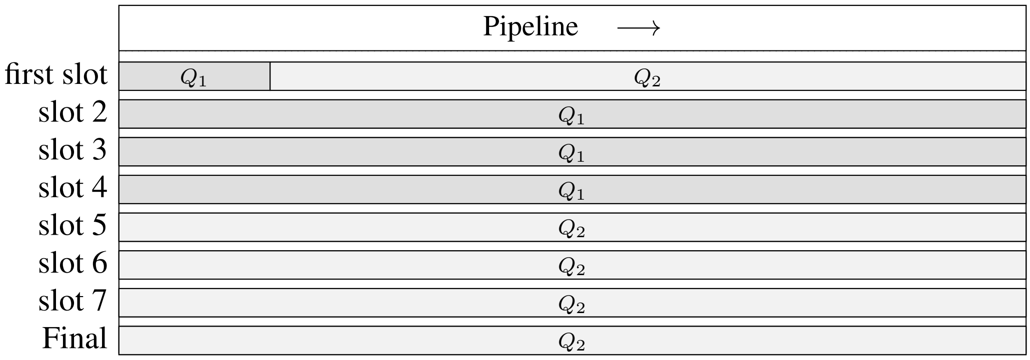

Figure 3.8: Case 1 results: Crude oil packets in the Pipeline.

Table 3.14: Case 1. Flowrate of the qualities delivered to the CDU (10^3 m^3 /day)

	Operations		
Slot	601	602	603
1	7.5		
2	7.5		
3	7.5		
4		6.9	
5		6.9	
6			7.0
7			7.5

in the first two rows. Operation 0 delivers material from the manifold to the pipeline, while 101 corresponds to the delivery of crude from the cargo to the subsea pipeline. Operations 201 to 206 connect the subsea pipeline to the terminal tanks. Operations 301 to 306 go from the terminal tanks to the manifold, 401 to 403 connect the pipeline to the refinery tanks, 501 to 503 to the CDU, and 601 to 603 are associated to three required deliveries.

Table 3.15: Case 1. Solution. Diagram of operations vs slots. Terminal

Slot/Oper	0 Pipe	101 Cargo	201 T131	202 T132	203 T133	204 T134	205 T135	206 T138	301 T131	302 T132	303 T133	304 T134	305 T135	306 T138
1	✓										✓	✓		
2	✓										✓	✓		
3	✓										✓	✓		
4	✓	✓						✓	✓			✓		
5	✓	✓				✓			✓				✓	
6	✓	✓	✓							✓		✓		
7	✓	✓					✓		✓	✓				

Table 3.16: Case 1. Solution. Diagram of operations vs slots. Refinery

Slot/Oper	401 R101	402 R103	403 R109	501 R101	502 R103	503 R109	601 O601	602 O602	603 O603
1	✓				✓		✓		
2		✓		✓			✓		
3	✓					✓	✓		
4	✓				✓			✓	
5		✓		✓				✓	
6	✓				✓				✓
7		✓		✓					✓

For better understanding of the diagram of operations and slots, the sets of cliques defined for cases 1 to 6, along with the contribution of maintenance operations for case 7, are presented in Figures 3.9 and 3.10. One unloading operation (clique 1) and one refinery tank reception (clique 8) are allowed per slot. Inlet and outlet operations can not overlap (cliques 2 to 7 and 9 to 11). One refinery outlet operation (clique 12) and one final delivery operation (clique 13) are allowed per slot. Maintenance operations are included in cliques 3 and 7 for the formulation of case 7.

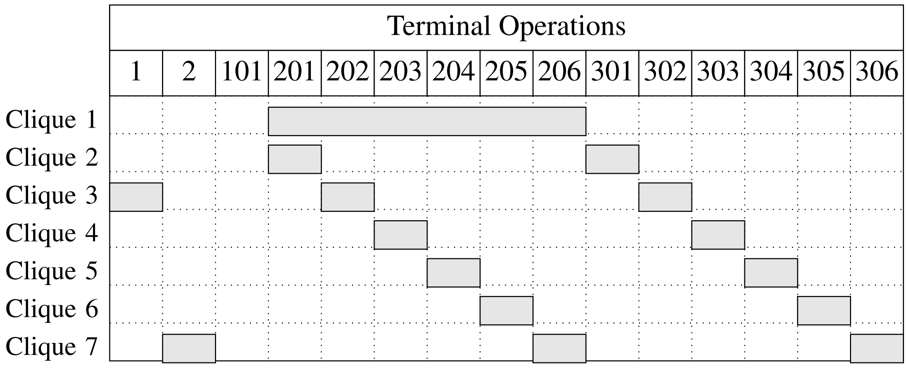

Figure 3.9: Cases 1 to 7. Clique sets. Terminal operations.

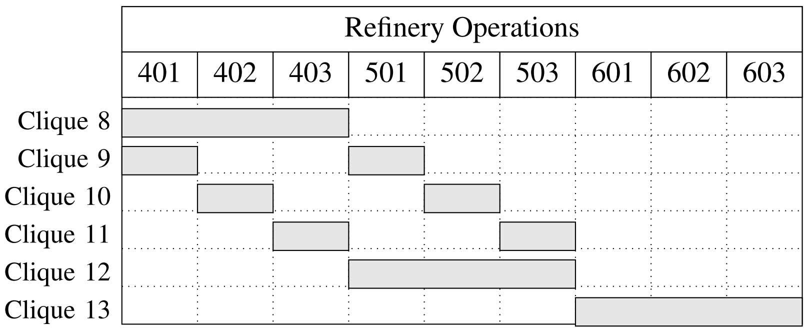

Figure 3.10: Cases 1 to 7. Clique sets. Refinery operations.

Operation 0 is active in each slot, as expected from the estimation of the required number of slots. Operations 303 and 304 are active in slot 1. There is a crude oil delivery from the terminal to the manifold, and then to the pipeline and refinery through operation 401. The CDU receives quality Q2 in operation 502 and from there to resource O601 by means of operation 601. Figure 3.11 depicts the start and end times of each operation (given in days).

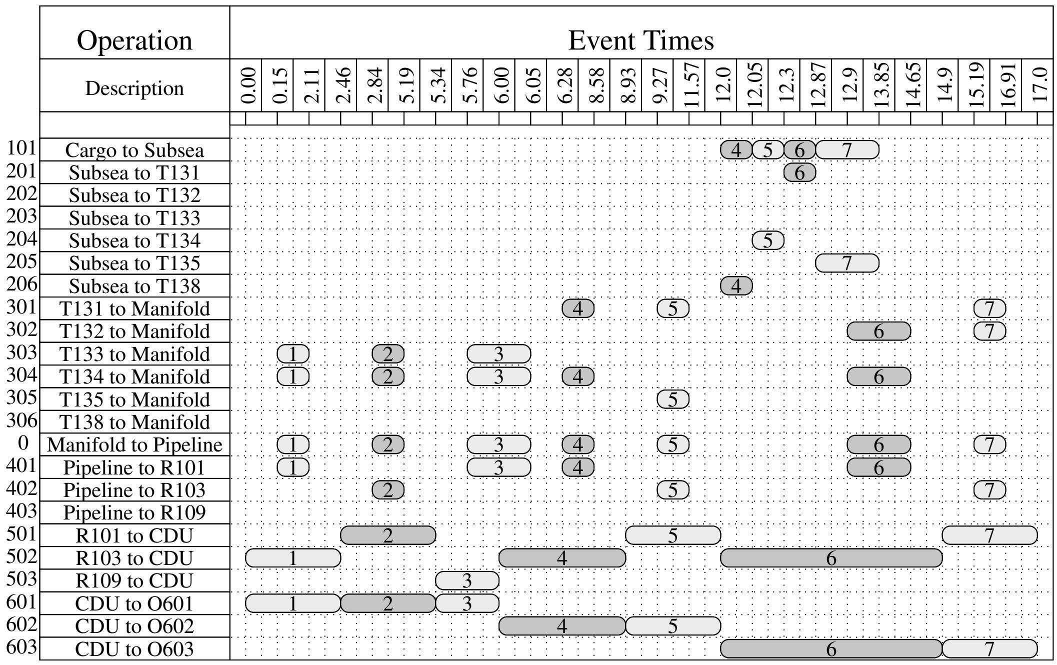

Figure 3.11: Case 1. Solution. Diagram of operations and times (given in days).

The delivery of crude oil from the terminal tanks to the refinery in slot 1 goes from time 0.15 to 2.11. The delivery from the refinery tanks to the CDU goes from time 0 to 2.46. At time 2.46, slot 2, resource O601 starts receiving a second packet through operation 601. Operations 601, 602 and 603 run continuously, where increasing times are associated with an increasing number of slots, as expected from Eqs. (3.29), (3.33), and (3.36). This behavior is actually present in most of the operations of the solution, with the exception of operations associated with cargo unloading in slots 4 and 5. Time sequencing and symmetry breaking Eqs. (3.42) and (3.43) contribute to these results. In addition, the MOS model allows the overlap of operations not belonging to the same clique. The time evolution of the volume delivered to the CDU is depicted in Figure 3.12.

The unloading of the cargo runs from time 12 to 13.85 (in days). Operation 101 is active in slots 4, 5, 6 and 7, where successive packets of crude are unloaded, as illustrated in Figure 3.13. At the same times and slots, operations 206 (subsea to T138), 204 (subsea to T134), 201 (subsea to T131) and 205 (subsea to T135) deliver crude volumes to the terminal tanks. When crude oil unloading starts, quality T2 is delivered from the subsea pipeline to tank T138, where an initial volume of quality T2 remains. A total amount of 144 10^3 m^3 of quality T3 is unloaded to tanks T134,

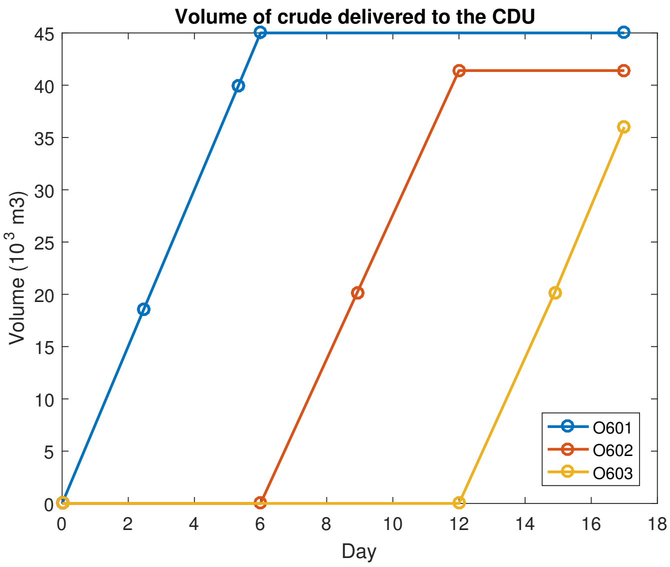

Figure 3.12: Case 1. Solution. Volume of crude delivered to the CDU (10^3 m^3).

T131 and T135. Considering that the maximum capacity of the terminal tanks is 57 10^3 m^3, Figure 3.14 shows that two tanks are filled with crude, as desired from Eq. (3.25) but only one tank remains filled at the end of the time horizon.

Time evolution of volume in refinery tanks is depicted in Figure 3.15. The maximum capacities of the tanks $(10^3\, m^3)$ are 32 (R101), 32 (R103) and 8 (R109). Initially, tank R101 receives quality Q2 from the pipeline, while tank R103 delivers Q2 to the CDU. Neither tank R101 nor tank R103 achieve full capacity but tanks are empty when reception starts. The volume in tank R109 is partially consumed. These results can be explained considering that volume and quality deliveries do not necessarily require the tanks to be completely filled.

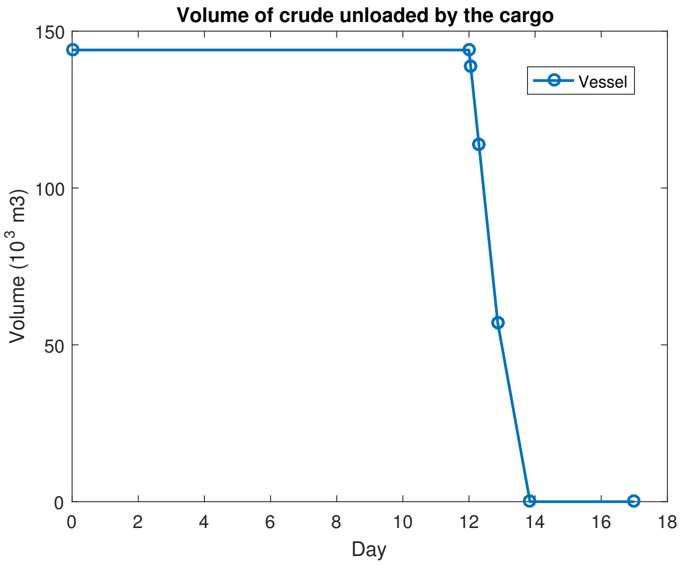

Figure 3.13: Case 1. Solution. Volume of crude in the cargo tanks (10^3 m^3).

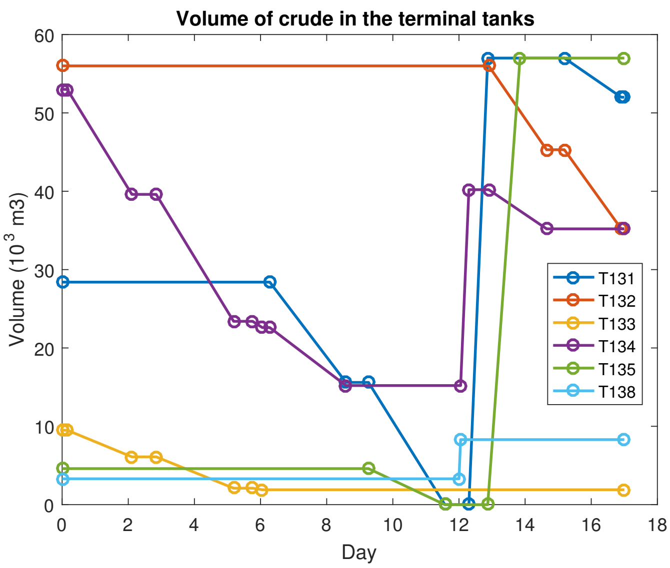

Figure 3.14: Case 1. Solution. Volume of crude in the terminal tanks (10^3 m^3).

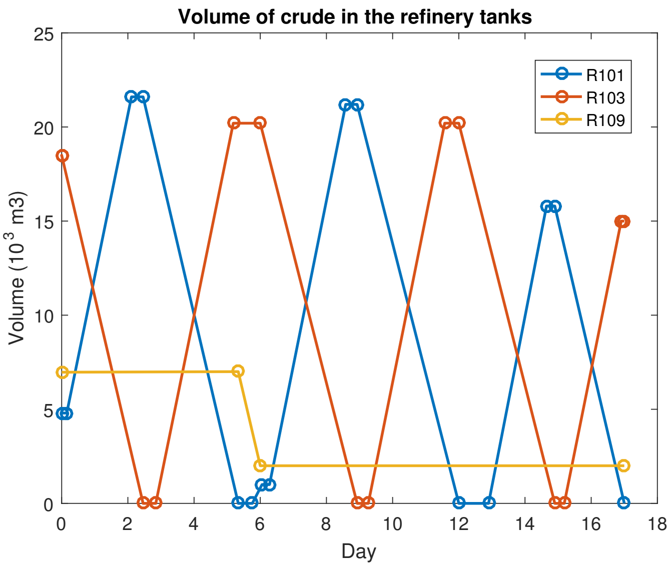

Figure 3.15: Case 1. Solution. Volume of crude in the refinery tanks (10^3 m^3).

3.5.4. Additional Experiments

Table 3.17 displays the size of the model for additional experiments regarding case 1. In the first section of the table, an exact solution for case 1 is evaluated while increasing the number of slots from 8 to 14. A one-hour limit is imposed on problem solving. The solution with 8 slots achieves a lower cost compared to case 1 evaluated with 7 slots. The other cases require processing times higher than one hour, which explains why the corresponding objective is not reported in the table (i.e., a solution was not found). In the second section of the table, the maximum processing time is increased to 5 hours. The solution with 9 slots presents an additional improvement in the objective.

Table 3.17: Case 1, exact solutions applying 7 to 14 slots. Model size, solution time (s), mipgap (%) and objective (adimensional). A maximum CPU time of one hour is imposed on the experiments of the upper section, while the limit for the lower section is five hours.

Case	Slots	Rows	Columns	Non zeros	Binary	Time	MIP gap	Objective
1	7	4570	2179	20017	373	81	0.098	11.29
1	8	5582	2580	25404	434	604	0.10	7.95
1	9	6673	2996	31490	496	3600	43	
1	10	7843	3426	38311	559	3600	69.9	
1	11	9095	3873	45898	623	3600	80.7	
1	12	10427	4335	54287	688	3600	79.6	
1	13	11838	4811	63513	754	3600	85.5	
1	14	13331	5304	73607	821	3600	98.6	
1	9	6673	2996	31490	496	6338	0.10	6.7
1	10	7843	3426	38311	559	18000	58.9	
1	11	9095	3873	45898	623	18000	67.1	
1	12	10427	4335	54287	688	18000	81.1	
1	13	11838	4811	63513	754	18000	76.8	
1	14	13331	5304	73607	821	18000	77.9	

A comparison of the cost of solutions for case 1 with 7, 8 and 9 slots is presented in Tables 3.18 and 3.19. There is an improvement in the fulfillment of the required volumes and qualities as the number of slots is increased from 7 to 9. In particular, the solutions with 8 and 9 slots show an improvement in the allocation of the final volume and quality, as shown in the columns "Vol. Final" and "Qual. Final" of Table 3.18 and detailed in Table 3.20. Additionally, there is also a better allocation of crude and a reduction in the term "Vol. Max." when compared to the planning with 7 slots.

All cases guarantee that quality Q2 is present in the refinery tanks and pipeline at the end of the time horizon. However, Table 3.20 shows that the distribution of quality Q2 in the refinery tanks and the pipeline becomes closer to the desired distribution given in Table 3.3, as the number of slots increases.

Considering the experiments with case 1 reported here, the results show that a general improvement is achieved by increasing the number of slots, in terms of the final distribution of quality Q2 and reduction in the objective cost. Nevertheless, the solution obtained with just 7 slots was satisfactory for being feasible with regards to the required crude deliveries over time, while demanding less computational effort.

Table 3.18: Comparison of case 1 solutions. Objective costs (adimensional). Terms 1 to 6.

Case	Slots	Costv Vol. Deliv.	Costq Qual. Deliv.	Costvf Vol. Final	Costvqf Qual. Final	Costvr Vol. Max.	Costvm Vol. Min.
1	7	0.21	0.02	3.95	1.5	2.01	3.30
1	8	0	0	2.18	0.83	1.25	3.42
1	9	0	0	1.16	0.45	1.34	3.52

Table 3.19: Comparison of case 1 solutions. Objective costs (adimensional). Terms 7 to 12.

Case	Slots	Costz Operation	Costxvp Pipe. Op.	Costxvq Mixture	Costm Maintenance	Costdt Demurrage	Costts Start time
1	7	0.03	0.05	0.01	0	0	0.21
1	8	0.03	0.04	0.01	0	0	0.18
1	9	0.03	0.04	0.01	0	0	0.16

Table 3.20: Comparison of case 1 solutions. Final volume in refinery tanks and pipeline (10^3 m^3).

			Refinery Tanks			
Case	Slots	Qual.	r101	r103	r109	pipeline
1	7	Q2	0	15	1.99	20
1	8	Q2	20.2	3.9	0	20
1	9	Q2	30	3.9	0	20

For case 1 evaluated with 8 slots, the diagram of operations and times is shown in Figure 3.16. The unloading of the cargo runs from time 12 to 13.93 (in days). Operation 101 is active in slots 5, 6, 7 and 8. Operations 202 (subsea to T132), 205 (subsea to T135), 206 (subsea to T138) and 201 (subsea to T131) deliver crude volumes to the terminal tanks. There is a resting time between the end of operation 301 in slot 7 and the start time of operation 201 in slot 8. As a consequence, there is a delay between the end of operation 206 in slot 7 and the start of operation 201 in slot 8. Figure 3.17 shows that two tanks at the terminal are filled with crude, but only one tank remains filled at the end of the time horizon.

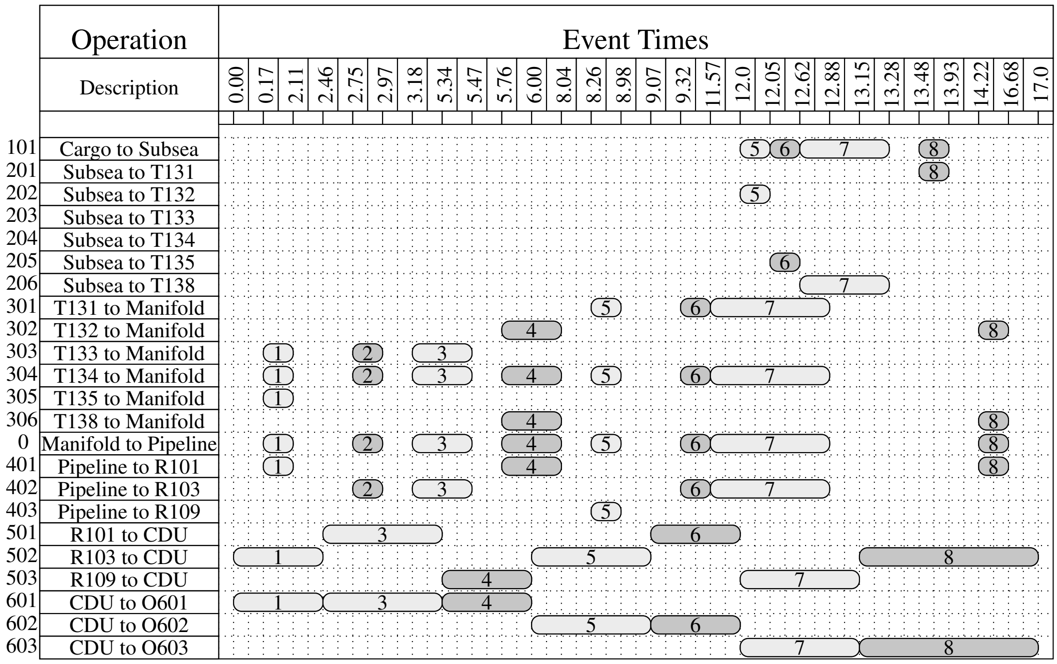

Figure 3.16: Case 1. Solution involving 8 slots. Diagram of operations and times (given in days).

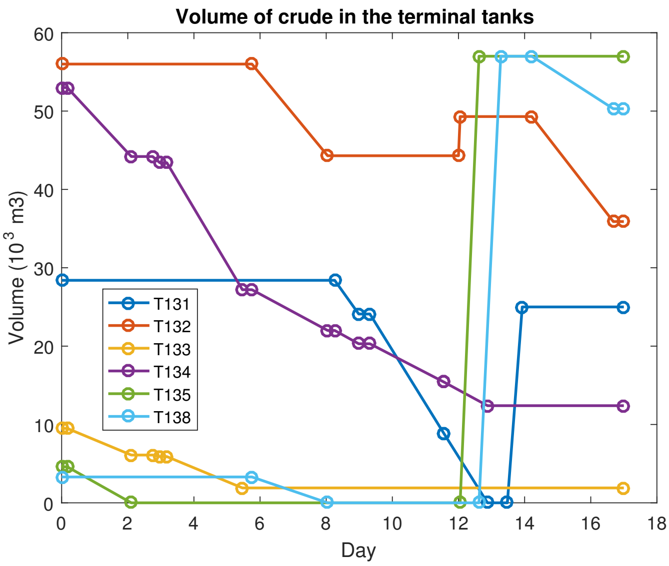

Figure 3.17: Case 1. Solution involving 8 slots. Volume of crude in the terminal tanks (10^3 m^3).

3.6. Conclusions

In this work, we introduce a cutting-edge model for scheduling the complex and integrated process of crude oil reception, blending, delivery through a pipeline, and processing at a refinery. Our continuous-time model, formulated as a MINLP problem, extends the concept of multi-operation sequencing to include the pipeline operations. This innovative approach not only allows us to suppress bilinear terms in the pipeline equations, but also offers greater flexibility in modeling the system. However, it is important to note that bilinear terms may still be present if mixtures are allowed at the terminal or refinery.

To further optimize the solving time, we considered symmetry breaking constraints and developed an efficient formulation. We demonstrated the practicality of our model by presenting and solving a real-world refinery problem. Additionally, we propose the application of the Relax & Fix strategy to further reduce processing times.

We also acknowledge the importance of considering planning requirements and provide the flexibility to modify the costs in the objective function. This allows for a customizable approach, where the priority can be given to either the crude oil schedule or maintenance operations, depending on the specific needs of the system.

It is worth mentioning that an exact solution is not always guaranteed. Depending on the problem, a heuristic approach may be required to obtain a solution in an affordable time. Exact solutions can be improved by increasing the number of slots. The solution obtained with the minimum number of slots could be acceptable regarding the required crude deliveries over time, while demanding less computational effort.

In conclusion, our model presents a comprehensive solution to the complex challenge of scheduling crude oil reception, blending, delivery, and processing at a refinery. The combination of its innovative formulation, efficient implementation, and flexible approach make it a valuable tool for optimizing the entire process.

Future work will explore additional symmetry-breaking rules, giving preference to certain operation sequences. In addition, a reduction in the number of terms in the objective will be analyzed. Regarding computation time, the application of problem decomposition techniques will be investigated.

Capítulo 4

A continuous-time scheduling model for Liquefied Petroleum Gas production, blending and delivery

ABSTRACT

Liquefied Petroleum Gases (LPG) constitute a set of leading products in the refining process. LPG primarily consists of a mixture of hydrocarbons with three and four carbons. Commercial grades commonly adhere to a set of specifications. In a configuration where blending and final product qualities are determined at a terminal and intermediate and final qualities can share capacity within a defined time horizon, the scheduler's objective is to optimize the sequence of intermediate qualities transferred through a gas pipeline connecting the refinery to the terminal. Additionally, the scheduler must determine the optimal final quality mixtures delivered to the market while adhering to various constraints. This work aims to develop a model that optimizes the sequencing of products in the pipeline, the allocation of resources at the terminal, and the blending and delivery of final product qualities. It aims to provide an optimal solution to a real-world problem within a reasonable timeframe.

Keywords:
Scheduling in refining processes, Pipelines, Continuous-time modeling, Integer programming, Inventory management, Crude oil scheduling, Pipeline, Continuous-time modeling, Integer programming, Inventory management, LPG scheduling, LPG blending, Continuous-time modeling, Integer programming, Inventory management.

4.1. Introduction

Liquefied Petroleum Gases (LPG) represent a key category of refining products. LPG is a mixture of hydrocarbons, primarily composed of three- and four-carbon molecules, with minor contributions from two- and five-carbon molecules. Various technologies enable the application of these hydrocarbons for the production of other products or for blending purposes. This discussion will focus on the latter. Commercial grades typically comply with defined specifications: Mixture, a blend of propanes and butanes, and Propane, which contains a high percentage of propanes. These products are primarily used for heating and cooking.

A refinery produces intermediate products daily, storing them on-site in spherical tanks known as *spheres*. The quality of these products depends on the specific refining processes. Typical streams include one product with a high percentage of propane and another consisting mostly of butane.

Depending on the process configuration, blending to achieve marketable qualities can be performed at the refinery before delivery to the market. Another configuration involves delivery through a gas pipeline to a distribution plant, where the final product is stored and sold. A third configuration involves transporting intermediate products to a blending and distribution terminal, where final product qualities are achieved and delivered to the market.

In this last configuration, intermediate and final products can be assigned to specific spheres, or a single sphere may occasionally function as either intermediate or final product storage. Blending alternatives range from receiving batches of refinery products through the gas pipeline into a designated sphere, followed by recirculation, analysis, and certification. Alternatively, the process can be supplemented by the transfer of products from other spheres at the terminal.

As we moved into the literature review of related topics, we found only one reference on this subject, to the best of our knowledge.

4.1.1. The scheduling problem

Most technical literature on scheduling and refining focuses on crude oil scheduling, reception, blending, and transfer, as well as gasoline blending and scheduling.

General classification

According to (Pinto and Grossmann, 1998), there is no absolute general model for the scheduling of process systems, and three central components are always present: assignment of tasks to equipment, sequencing of activities, and timing for utilization of equipment and resources by these processing tasks. The authors present an overview of assignment and sequencing models used for scheduling process operations. In Harjunkoski et al. (2014), existing scheduling methodologies developed for process industries are reviewed.

Resources, operations and objective formulations

The integrated problem of LPG production, delivery to a distribution plant via pipeline, blending, and inventory management is not widely addressed in the research literature. The study by Pinto and Moro (2000) presents a mixed-integer optimization model that incorporates production, storage, and delivery. In terms of resources, this research considers multiple production units, component separation, and storage for marketing purposes. Concerning the previously mentioned process configurations, it addresses a problem involving LPG components used both as raw materials for other products and for blending and delivery from the refinery. The objective function seeks to maximize sales revenue while accounting for the operating costs of each unit.

The work by Moro and Pinto (2004) addresses the problem of oil inventory management in a real-world refinery that receives several types of oil delivered through a pipeline. The resources and operations in this problem involve transfers from the pipeline to crude tanks, settling time constraints for separating brine from the oil, interface separation between different types of oil, blending, and feeding the crude distillation units. The sequence and timeline of the oil batches are known in advance. The study maximizes the feed flow rates to the distillation units while minimizing the cost of tank operations.

While the first problem considers multiple production units and final products, the second focuses on the reception of raw materials, their mixture, and processing through a single production unit. It models the pipeline as a source of fixed batches of crude oil. Both problems follow similar time representations. The objective function in the first problem is market-driven, whereas the second is based on operational practices.

Modeling a trunk pipeline and crude oil blending requires representing a queue of volumes and qualities and the corresponding constraints to track a packet in the pipeline. Only some works have focused on the detailed operation of this resource. Works by Zhang and Xu (2015); Cafaro et al. (2019) formulate a slot-based continuous-time model for delivering crude oil mixtures through a trunk pipeline. In Zimberg et al. (2020, 2023b), a continuous-time formulation and multiple operations per slot approach is adopted, thereby reducing the required number of slots.

4.1.2. Discrete and continuous-time models

Mathematical modeling implies the definition of mixed-integer models with continuous and integer (binary) variables. According to (Moro and Pinto, 2004), a key aspect of mixed-integer scheduling models is time representation, which can be classified into two categories: discrete- and continuous-time formulations.

Discrete-time formulations discretize the time horizon into several intervals of equal or different lengths. In this case, all events are forced to coincide with one of the interval boundaries. This approach invariably requires a large number of decision variables because the time span of all tasks must be a multiple of the discretization interval Lee et al. (1996); Zimberg et al. (2015). Multi and nonuniform discrete-time grid formulations are the subject of the work by (Velez and Maravelias, 2013). These authors mentioned that more than one grid can be applied in facilities with processing subsystems that have different time scales. They propose using nonuniform grids for problems with a need for particular time points and problems with long scheduling horizons.

Continuous-time formulations consider actual event points only, and the length of each time interval is unknown. They require fewer binary variables compared to the previous discrete-time formulation. Two classes of grids are used: a grid for all resources (single time-grid model) or a grid in which each resource is handled independently (unit-specific time-grid model) (Chen et al., 2012). The grids consist of sets of time slots, where the intervals between slots are determined through optimization. In Moro and Pinto (2004), a single time-grid model is applied for a crude oil inventory management problem. The work by Pinto and Moro (2000) also follows this approach for the LPG problem. These models are more complex in their formulation than single and uniform discrete-time approaches, and the solution rests on several assumptions or specialized algorithmic techniques. In Jia and Ierapetritou (2004), a unit-specific model is considered for a problem that includes crude oil unloading and blending, production unit operations, product blending, and delivery. The formulations have the advantage of requiring fewer event points but the disadvantage of involving more complicated balance constraints(Moro and Pinto, 2004).

In the work of (Li et al., 2010), the authors develop a single time-grid model that integrates the blending of intermediate gasoline components to produce and deliver various qualities of finished gasoline products. The horizon is divided into a given number of slots of variable length, common to tanks and blenders. In (Li and Karimi, 2011), the authors extend their previous work by defining a unit-specific time-grid model with an equal number of time slots for intermediate tanks, blenders, and finished product tanks. The work by Castillo-Castillo and Mahalec (2016) further improves the previous formulation while maintaining the existing time slot definition.

In Mouret et al. (2009), the continuous-time approach is reformulated as a Single-Operation Sequencing (SOS) model. It differs from previous formulations as it requires postulating the number of priority-slots in which operations take place instead of specifying the number of time intervals or event points used in the schedule. In this approach, one event corresponds to one slot. In a follow-up work, Mouret et al. (2011) introduces four time representations. In particular, Multi-Operation Sequencing (MOS) allows more than one event associated with a given slot. In a recent study, Wang et al. (2024) applies a MOS model and introduces the concept of restricted overlapping operations, wherein certain operations are permitted to overlap, but only a limited number may do so at any given time.

4.1.3. Linear and nonlinear models

Formulations can be linear or nonlinear. When a mixture of product qualities, *i.e.*, crudes or LPG, is considered in the formulation, the resulting model becomes nonconvex and nonlinear due to bilinear terms related to blending. Several methodologies have been proposed to find an acceptable solution while improving computation time. Authors Karuppiah and Grossmann (2006) apply piecewise-linear underestimation of bilinear terms within a branch-and-bound algorithm to optimize the MINLP problem globally. The work by Mouret et al. (2009) builds a MINLP model to obtain an optimal schedule through a sequence of operations with a continuous-time formulation. They use a two-step MILP–NLP procedure to solve the MINLP model. In Castro and Grossmann (2014), a resource-task network formulation is developed. The resulting MINLP model, which includes blending constraints, is solved with a two-step MILP–NLP algorithm and the multiparametric disaggregation technique proposed by (Kolodziej et al., 2013). The work by Zimberg et al. (2015) applies linearization of the bilinear terms, resulting in linear constraints with an adjustment term for composition discrepancies and objective costs that force the concentration in a tank to be equal to the concentration of the outlet volume. Rigorous relaxation approaches have the advantage of computing a rigorous lower bound on the optimal solution, *i.e.*, McCormick envelopes and piecewise McCormick envelopes (McCormick, 1976; Karuppiah and Grossmann, 2006; Kolodziej et al., 2013; Castro, 2015; de Assis et al., 2017). However, rigorous methodologies often require reduced-size problems to find a solution in an affordable time.

4.1.4. Model size and solving techniques

Solving times for large scale configurations and small time horizons can be improved by applying Lagrangian decomposition (Shah and Ierapetritou, 2015). For larger configurations, heuristics tend to be more efficient but less rigorous. Examples include Genetic algorithms (Oliveira et al., 2011), MINLP-NLP decompositionsde Assis et al. (2019), Rolling Horizon methods (Zimberg et al., 2015), Relax & Fix strategies (Zimberg et al., 2020, 2023b), clusterization of system resources de Assis et al. (2021) and sequences of uniform discrete-time models (Chen et al., 2022). More recently, a reinforcement learning strategy has been proposed to address large-scale, long-term refinery production scheduling problems (Chen et al., 2024). In (Ikonen et al., 2025), the authors work on accurately predicting the optimal objective function value before solving the actual scheduling instance. The work by Franzoi et al. (2024) presents several strategies aimed at making large-scale refinery applications more tractable. These include tight relaxations using piecewise McCormick or normalized multiparametric disaggregation, cluster decomposition, and Lagrangian decomposition. Such methods often introduce simplifications in process unit modeling, reduce the scope of the process network, and limit the search space in optimization algorithms. In (Chen and Jiang, 2024), the authors present a refinery scheduling model based on the variable-driven modeling approach, reducing the number of variables and constraints. The work by He et al. (2025) is based on a hybrid optimization algorithm that combines mathematical programming with particle swarm optimization to solve a refinery scheduling problem.

4.1.5. Synthesis of literature

Table 4.1 summarizes the contributions of the above-mentioned works.

4.1.6. Main contributions

In this work, we apply a continuous-time model with multi-operation sequencing to address a scheduling problem involving the production, blending, and delivery of LPG. This type of representation has not been previously explored in the literature for either LPG or gasoline problems. Our model does not focus on blending at the production site but rather on the transportation of intermediate products through a pipeline to a blending facility. Unlike previous works (Li et al., 2010; Li and Karimi, 2011; Castillo-Castillo and Mahalec, 2016), intermediate and final storage resources are not pre-defined. Instead, our problem assumes that the storage spheres at the terminal are dynamically assigned to different tasks based on the timing of reception, blending, and delivery. We address a real-world problem that incorporates several operational complexities and is inspired by the process at the national refinery of Uruguay.

The main contributions are in the following topics:

- *Innovative Modeling Approach for Handling Simultaneous Operations in Resources:* We propose a novel approach for modeling resource conflicts by introducing constraints based on the abstraction of virtual spheres. This representation is a more precise and flexible characterization of overlapping operations, with volume and temporal consistency ensured through tailored mathematical formulations.
- *Adaptive Blending Model for Terminal Operations Optimization:* We introduce a flexible modeling framework for managing complex blending operations at terminals, where materials can originate from pipelines or inter-sphere transfers. Our approach features a novel blending manifold model that dynamically determines the role of each storage unit—whether for intermediate or final products—within the optimization process.
- *Quality-Ensured Blending via Bilinear Concentration Modeling:* We develop a model incorporating bilinear formulations to capture the interplay between product concentrations and blending operations. By embedding these constraints, the model ensures accurate and consistent transfer of mixtures across resources to meet final product specifications.

Table 4.1: Comparison of Scheduling Models for Refinery Operations

Refs	Resources	Time Representation	Solution Methods	Objective
Pinto and Grossmann (1998)	General process equipment	Discrete, continuous	MILP and MINLP	Cost, profit, task allocation and resource utilization
Harjunkoski et al. (2014)	Process industries in general	Discrete, continuous	MILP, MINLP, Hybrid	Problem-specific
Pinto and Moro (2000)	LPG production units, storage	Continuous (single time-grid)	MILP	Maximize profit
Moro and Pinto (2004)	Crude oil, tanks, pipeline, CDU	Continuous (single time-grid)	MINLP, MILP	Maximize feed rate, Minimize operating cost
Zhang and Xu (2015)	Crude oil, tanks, trunk pipeline system	Slot-based continuous-time	MINLP, outer-approximation based decomposition method	Minimize cost
Cafaro et al. (2019)	Crude oil, tanks, trunk pipeline system	Slot-based and general precedence continuous-time	MINLP, MILP-NLP	Minimize cost
Zimberg et al. (2020, 2023b)	Crude oil, tanks, trunk pipeline system	Continuous, MOS	MINLP, MILP-NLP with Relax & Fix	Minimize cost
Lee et al. (1996)	Crude oil, tanks	Discrete	MILP, several techniques	Minimize cost
Zimberg et al. (2015)	Crude oil, tanks	Discrete	MILP, linearized blending	Minimize cost, enforce compositions
Velez and Maravelias (2013)	Chemical processes	Discrete, non-uniform multi-grid	MILP, various techniques	Minimize cost, maximize profit
Chen et al. (2012)	Crude oil, tanks	Continuous, single time grid, unit-specific time grid, MOS	MINLP, MILP-NLP	Minimize cost
Jia and Ierapetritou (2004)	Refinery, crude oil unloading, production, blending	Continuous, unit-specific	MINLP	Minimize cost
Li et al. (2010)	Gasoline, tanks, blenders	Continuous, single time-grid	MINLP	Minimize cost
Li and Karimi (2011)	Gasoline, tanks, blenders	Continuous, unit-specific time-grid	MINLP	Minimize cost
Castillo-Castillo and Mahalec (2016)	Gasoline, tanks, blenders	Continuous, unit-specific time-grid	MINLP	Minimize cost
Mouret et al. (2009)	Crude oil, tanks, CDU	Continuous, SOS	MINLP, MILP–NLP	Maximize profit
Mouret et al. (2011)	Crude oil, tanks, CDU	Continuous, including SOS, MOS	MINLP	Maximize profit
Wang et al. (2024)	Refinery	Continuous, MOS, restricted overlapping operations	MINLP, MILP–NLP	Maximize profit
Karuppiah and Grossmann (2006)	Water systems		Generalized Disjunctive Program, Piecewise McCormick, Branch and Bound, global optimization	Minimize flows
Castro and Grossmann (2014)	Crude oil, process units, tanks, blending systems	Continuous	MINLP, multiparametric disaggregation	Maximize profit, Minimize cost
Kolodziej et al. (2013)	Test problems		Piecewise McCormick, multiparametric disaggregation, MILP, MINLP, global optimization	Problem-specific
McCormick (1976)	Test problems		Relaxation of bilinear terms	Problem-specific
Castro (2015)	Test problems		Piecewise McCormick, partition-dependent bounds, MILP	Problem-specific
de Assis et al. (2017)	Crude oil, process units, tanks, blending systems	Discrete	Piecewise McCormick, domain tightening, MINLP, MILP-NLP	Minimize cost
Shah and Ierapetritou (2015)	Refinery, large-scale scheduling network	Continuous	MILP, Lagrangian decomposition	Minimize cost
Oliveira et al. (2011)	Refinery, multi-product	Discrete	MILP, Genetic algorithms	Minimize unfulfilled demand
de Assis et al. (2019)	Crude oil supply and processing	Discrete	MINLP, MILP-NLP, piecewise McCormick, domain tightening	Maximize delivery
de Assis et al. (2021)	Crude oil supply and processing	Discrete	MINLP, MILP-NLP, clustering heuristics with relaxations	Maximize delivery, quality
Chen et al. (2022)	Refinery, long horizon	Discrete	MILP, LP, knowledge transfer-based algorithm	Minimize cost
Chen et al. (2024)	Refinery, long horizon	Discrete	Reinforcement learning	Minimize cost
Ikonen et al. (2025)	Batch processing	Discrete	MILP, machine learning	Minimize cost, minimize makespan
Franzoi et al. (2024)	Refinery	Discrete	Advanced process-unit modeling, MINLP, decomposition, heuristics	Maximize profit
Chen and Jiang (2024)	Refinery	Discrete	Variable-driven modeling, MILP	Minimize cost
He et al. (2025)	Refinery	Discrete	Particle swarm optimization, MILP	Minimize cost

- *Symmetry Breaking and Resource Blocking Strategies for Problem Size Reduction:* We introduce a symmetry breaking and resource blocking techniques inspired by real-world operational practices, which selectively focuses on key resources and prioritizes specific storage units for blending. This approach reduces problem complexity and enhances computational efficiency, while maintaining solution quality.

4.1.7. Paper organization

The paper continues with Section 4.2, which provides a detailed description of the problem. Section 4.3 describes the Mathematical Programming Model. Section 4.4 outlines the heuristic strategies employed to solve the problem. Section 4.5 discusses the implementation details, experiments, and results, highlighting the behavior of the model and strategies. Finally, Section 4.6 summarizes the main findings and presents conclusions about the model. Appendix 7 presents the notation used throughout the paper.

4.2. Problem description

4.2.1. Process configuration

This work focuses on the configuration shown in Figure 4.1. Intermediate products are first stored at the refinery before being transported through a gas pipeline to a terminal, where they can be stored in individual spheres or blended to achieve the desired final qualities. Production can also be supplemented with imported products. Final qualities can be achieved either by blending intermediate components in a terminal sphere after reception or by mixing with products from other spheres at the terminal. Blending is represented as a loop. The diagram considers two final products: one is delivered to a facility where it is stored in cylinders for retail purposes, while the other is loaded into trucks at the terminal and transported to the market.

What we present is not intended to be limiting; it represents a subset of a real-world facility designed to address the meaningful operations commonly found in real-world operational problems.

In particular, Figure 4.2 presents a diagram that depicts an instance of the general problem, showing resources and operations. We will focus on the following

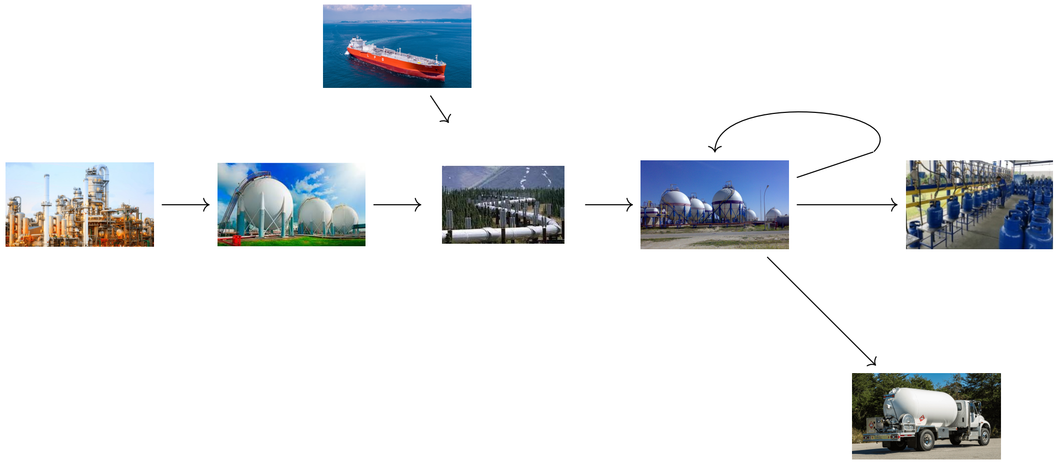

Figure 4.1: Generic diagram of LPG production, transport, blending and delivery at a terminal.

configuration details.

Production consists of streams of propane (C3R), butane (C4R), and propane-butane mixture (C3C4R) that are stored at the refinery in spheres as shown in the diagram. The suffix R refers to refinery-produced components. These products are pumped from the spheres through the main pipeline (Gas pipe) to the terminal. The refinery storage spheres are labeled E1 to E4. In this particular configuration, The Gas pipe capacity is small compared to the transport volumes, and it can be assumed that during operation, the entire initial volume is displaced. In addition, spheres E2 and E3 can simultaneously perform reception and delivery operations, while overlapping operations are not possible for spheres E1 and E4.

Additionally, depending on seasonal demand, propane (C3I) and butane (C4I) are imported. The suffix I denotes imported components. Imported materials are pumped from the vessel to the blending terminal through the Gas pipe. For technical reasons, C4I can only be pumped after all C3I has arrived at the distribution terminal. At any given time, only one quality of product can be pumped into the pipeline.

Terminal spheres E12 to E15 and E18 to E21 receive the products. Each sphere can receive LPG either from the gas pipeline or from other spheres. Final product qualities are typically achieved by mixing LPG batches from the pipeline or by combining the contents of two spheres, with the latter option preferred to reduce processing times.

These final qualities are then delivered to the market. At any given time, some spheres contain intermediate-quality products, while others store final-quality

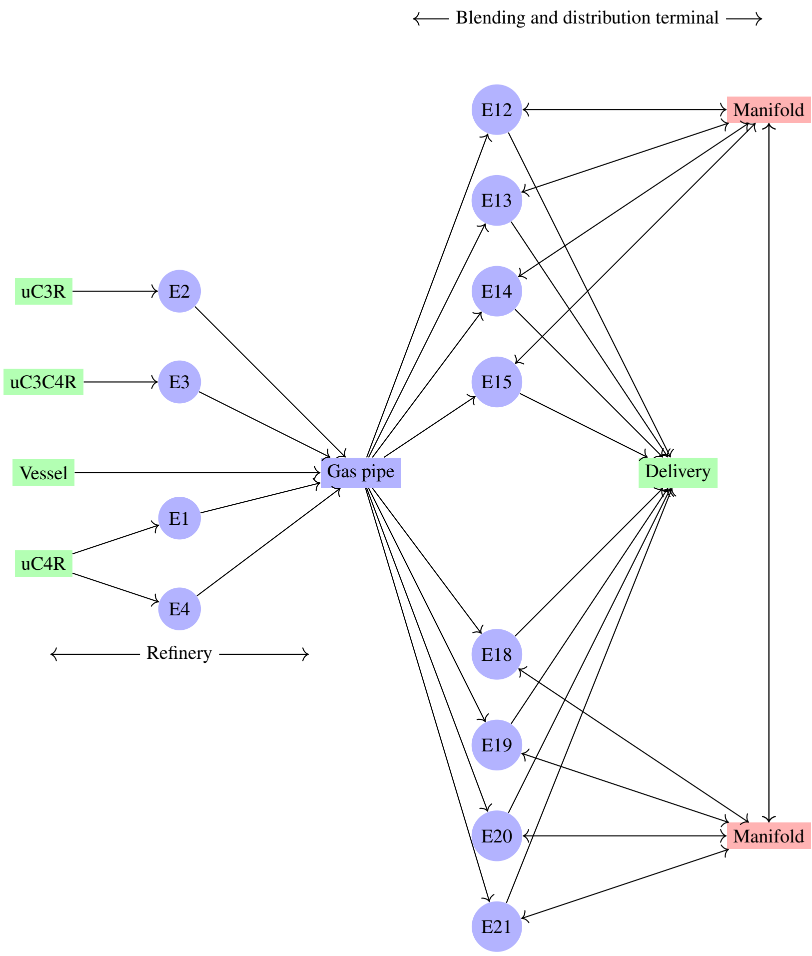

Figure 4.2: LPG production, blending and delivery. A problem diagram illustrating resources and operations.

products.

To maximize economic efficiency, a mixture of streams is typically required. The refinery configuration determines the daily production rates of these products. A mixture of qualities requires the recirculation of products. These processes, along with the time required for sampling and analysis, must be considered in the model.

Spheres at the terminal cannot simultaneously receive, deliver, or recirculate products. The final qualities, *Propane* and *Mixture*, must meet a set of specifications and demands.

Given a time horizon, the scheduler's objective is to determine the optimal sequence of qualities delivered through the pipeline and the final quality mixtures supplied to the market while satisfying a set of constraints.

An example of a sequence of events for this particular configuration is shown in Appendix 8.

4.3. Mathematical programming model

This section presents an LPG continuous-time model (LPG-CM) based on assigning operations to a set of resources for detailed planning of the daily operations at an LPG production facility, considering blending and distribution at a terminal, as described in the previous section. LPG-CM is a MINLP model that can be expressed as a MILP model combined with a set of bilinear constraints. The model further develops the research on crude oil operations by (Zimberg et al., 2023b, 2020), which was funded by ANII (Agencia Nacional de Investigación e Innovación, Uruguay). It follows a continuous-time formulation introduced by (Mouret et al., 2009), based on the MOS approach (Mouret et al., 2011), in which scheduled operations are grouped into slots according to their properties and interactions. Figure 4.3 presents a graph illustrating the model's operations (edges) and resources (nodes). The refinery, represented by resource $\mathcal{R}_U$, supplies intermediate products to resource $\mathcal{R}_R$ (refinery spheres). Resource $\mathcal{R}_G$ (Gas pipe) receives intermediate LPG products from $\mathcal{R}_R$ and $\mathcal{R}_V$ (vessels). These products are then stored in the terminal resource $\mathcal{R}_T$.

Each sphere at the terminal is designed for multiple operations, including reception from the Gas pipe, transfer to other spheres for blending and delivery to the *Mixture* and *Propane* markets. Resource $\mathcal{R}_B$ represents a manifold for blending operations. Final product qualities can be achieved by blending components received from the gas pipe or by combining these components with products from other spheres at the terminal.

As a result of blending intermediate streams at the terminal, final products are stored and delivered to the *Mixture* market, represented by $\mathcal{R}_{DS}$, and the *Propane* market, represented by $\mathcal{R}_{DP}$. Required deliveries are denoted by $\mathcal{R}_{OS}$ and $\mathcal{R}_{OP}$ for the *Mixture* and *Propane* markets, respectively.

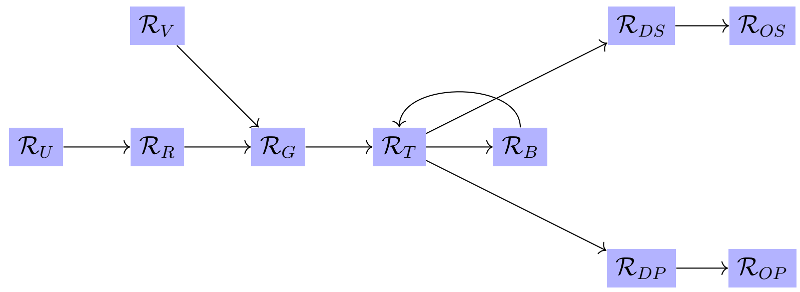

Figure 4.3: LPG production and blending. Graph of resources and operations. From left to right and top to bottom: refinery production units ($\mathcal{R}_U$), refinery spheres ($\mathcal{R}_R$) , vessel ($\mathcal{R}_V$) , Gas pipe ($\mathcal{R}_G$), terminal spheres ($\mathcal{R}_T$), Manifold ($\mathcal{R}_B$), *Mixture* market ($\mathcal{R}_{DS}$) and delivery ($\mathcal{R}_{OS}$), *Propane* market ($\mathcal{R}_{DP}$) and delivery ($\mathcal{R}_{OP}$).

Unless explicitly mentioned, constraints are valid for the following sets: slots $\mathcal{S}$, resources $\mathcal{R}$, operations $\mathcal{W}$, and qualities $\mathcal{Q}$. Please refer to Appendix 7 for a comprehensive description of sets, indexes, symbols, parameters, and variables.

4.3.1. Resource and operation constraints

Process operation requires that certain operations be executed, sharing partial or total times, while others cannot overlap. These can result from the physical design of the resources, pipeline network, and the time required for pumping, recirculation to ensure homogeneous quality, and laboratory analysis.

Figure 4.4 depicts the proposed model for operations on a terminal sphere. Each arrow represents an operation affecting the material balance of E15, including “from Gas pipe,” “to Mixture,” “to Propane,” “to Manifold,” and “to E15.” The Manifold represents a blending manifold that connects spheres, enabling multiple inlet and outlet operations. Blending within a sphere also occurs with products received from the Gas pipe. Recirculation and certification analysis are not modeled as operations but rather as time delays between blending and final delivery.

Based on the model presented in Figure 4.4, Figure 4.5 depicts the five allowed states with overlapping operations involving the terminal sphere E15 and the blending manifold. The sphere can receive products from the Gas pipe, while the

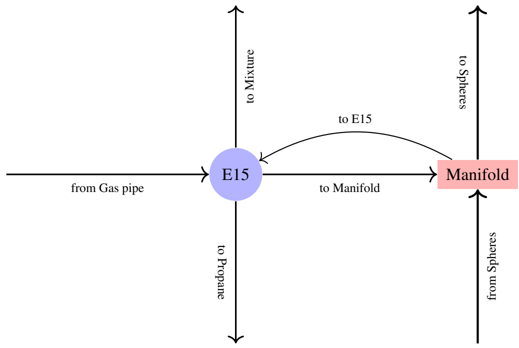

Figure 4.4: Proposed model for a terminal sphere

Manifold receives and delivers products from and to spheres different from E15. Similarly, the manifold operates while E15 delivers the final product, either *Mixture* or *Propane*. When E15 participates in the blending process, the manifold receives LPG from E15 and delivers it to another sphere, or E15 receives products from another source.

One common reason for the incompatibility of operations is the requirement for homogeneous quality when a resource delivers a product. Another reason may be the physical configuration of pipelines and pumps. Operations not represented within each of these states are forbidden, cannot be executed simultaneously, and consequently cannot be assigned to the same slot, in order to preserve material balance.

The problem assumes that production at the refinery is continuous. Figure 4.6 shows the four allowed states for the butane spheres E1 and E4.

Spheres E2 and E3 receive different products. Each of these resources can receive and deliver at the same time. The allowed states for E2 and E3 are presented in Figure 4.7.

Non-overlapping operations

The incompatibility between simultaneous operations can be modeled as a clique of operations. A clique $c \subseteq \mathcal{W}$ refers to a subset of operations that cannot be executed simultaneously in any slot i. The following constraints, Eq. (4.1), establish that, in a given slot i, at most one operation w from a clique c can be performed:

$$\sum_{w \in c} z_w^i \leq 1, \quad \forall i \in \mathcal{S}, c \in \mathcal{C} \tag{4.1}$$

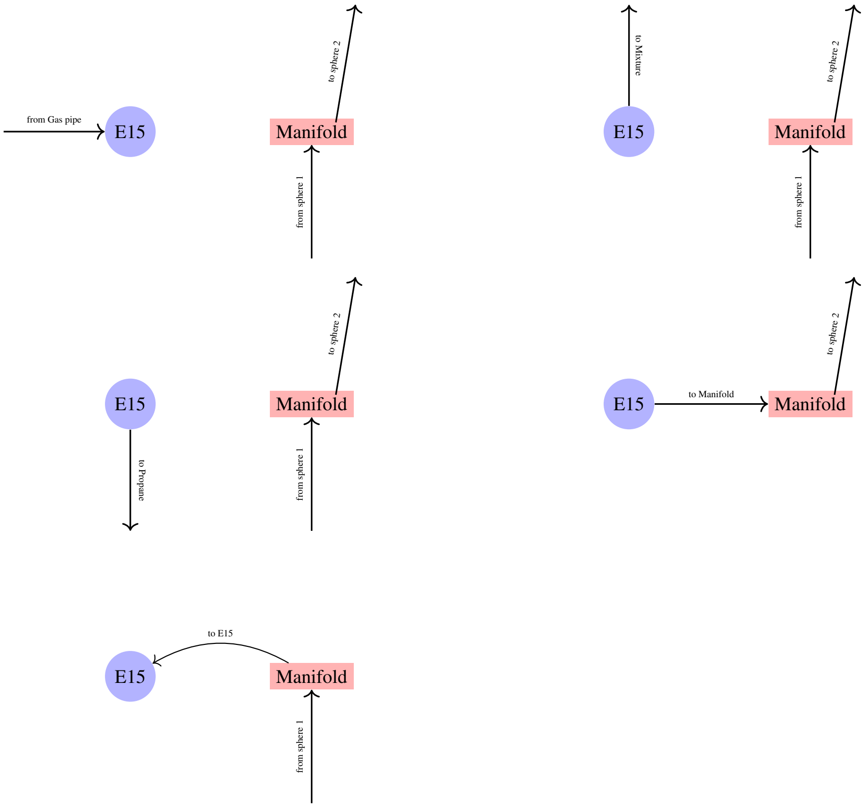

Figure 4.5: Five allowed states with overlapping operations for a terminal sphere and the blending manifold. From left to right and top to bottom: from Gas pipe, to Mixture, to Propane, to another sphere, from another sphere. Spheres 1 and 2 represent generic units.

where z_w^i is a binary variable that takes on value 1 if operation w is performed in slot i, otherwise it takes on value 0, and $\mathcal{C}$ is the set of cliques.

Blocking and grouping resources for problem size reduction

Given the set of spheres at the terminal, if certain resources cannot be used in advance during the scheduling time horizon, operations involving these resources are blocked to reduce the model's size, Eq. (4.2):

$$z_w^i = 0, \quad \forall i \in \mathcal{S}, r \in \mathcal{F}, w \in \mathcal{W}, w \in \mathcal{I}_r, w \in \mathcal{O}_r \tag{4.2}$$

For an industrial problem, it is common practice to assign a group of spheres at the terminal, $r \in \mathcal{R}_T$, to a specific product delivered through $r_1 \in \mathcal{R}_D$ (delivery manifold) to minimize the risk of undesired mixtures. This practice also involves

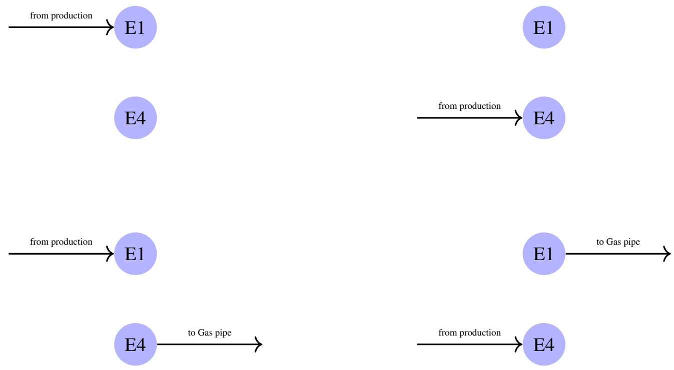

Figure 4.6: Four allowed states for refinery spheres E1 and E4. From left to right and top to bottom: production to E1, production to E4, production to E1 while E4 to Gas pipe, production to E4 while E1 to Gas pipe.

Figure 4.7: Two allowed states for refinery spheres E2 and E3 are: production to the sphere, or simultaneous production to the sphere and delivery to the gas pipe.

assigning specific resources to deliver a particular final product, taking into account both resource capacity and product demand. As a result, certain operations are voided on specific resources.

Equation (4.3) states that a resource r cannot deliver product r_1 if $r \notin \mathcal{B}_{r_1}$, where $\mathcal{B}_{r_1}$ denotes the set of allowed blending resources for the delivery of r_1. Each r_1 corresponds to a final product quality, either *Mixture* or *Propane*.

$$z_w^i = 0, \quad \forall i \in \mathcal{S}, r \in \mathcal{R}_T, r_1 \in \mathcal{R}_D, w \in \mathcal{I}_{r_1}, w \in \mathcal{O}_r, r \notin \mathcal{B}_{r_1} \tag{4.3}$$

Figure 4.8 modifies Figure 4.2 to illustrate these concepts. Note that the delivery of the product to the market is divided into two subsets of spheres, but all active spheres are connected through the manifold.

Volume consistency

For a given slot i, total volumes are obtained by accumulating partial volumes at resource r or transferring volumes through operation w for every quality q, Eq.

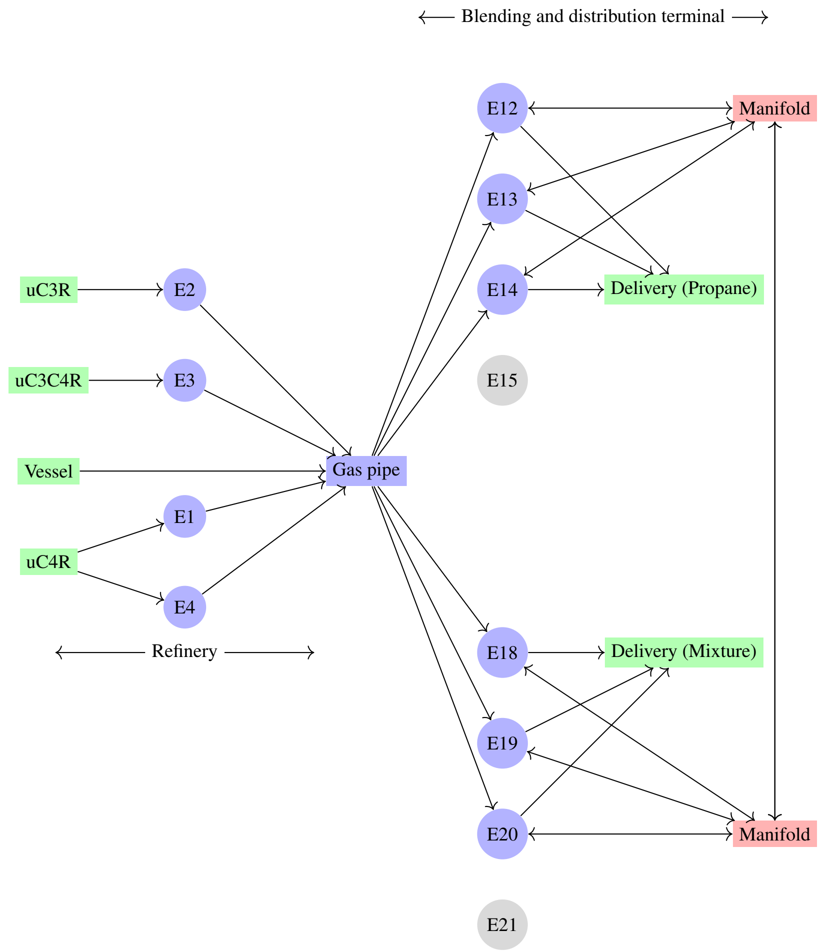

Figure 4.8: Resource assignments and voided operations in the scheduling process.

(4.4):

$$
\begin{aligned}
v_r^i &= \sum_{q\in\mathcal{Q}} vq_{r,q}^i, \qquad \forall i\in\mathcal{S}_1, r\in\mathcal{R}\\
dv_w^i &= \sum_{q\in\mathcal{Q}} dvq_{w,q}^i, \qquad \forall i\in\mathcal{S}, w\in\mathcal{W}
\end{aligned}
\tag{4.4}
$$

where v_r^i is the volume in resource r at slot i, $vq_{r,q}^i$ is the accumulated level of quality q in resource r at slot i, dv_w^i is the total volume transferred by operation w in slot i, and $dvq_{w,q}^i$ is the volume of quality type q transferred by operation w in slot i.

Resource limits

Equations (4.5) establish resource limits:

$$
\begin{aligned}
\underline{V_r} \le v_r^i \le \overline{V_r}, &\qquad \forall i\in\mathcal{S}_1, r\in\mathcal{R}\\
vq_{r,q}^i \le \overline{V_r}\, xvq_{r,q}^i, &\qquad \forall i\in\mathcal{S}_1, r\in\mathcal{R}_T\cup\mathcal{R}_V, q\in\mathcal{Q}
\end{aligned}
\tag{4.5}
$$

where $xvq_{r,q}^i$ is a binary variable that assumes value 1 if resource r has a nonzero volume of quality q in slot i, or else it assumes value 0, and $\underline{V_r}$ ($\overline{V_r}$) is the minimum (maximum) volume of products stored in resource r.

At the specified time horizon, minimum volumes of marketable products are required to be met, Eq. (4.6) :

$$
\sum_{r_1\in\mathcal{B}_r} v_{r_1}^{N+1} \ge \underline{Vfr_r}, \qquad \forall r\in\mathcal{R}_D \tag{4.6}
$$

where N is the number of priority slots and $\underline{Vfr_r}$ is the minimum final volume required in resource r.

Cargo unloading

Total volume of propane and butane in cargo $r\in\mathcal{R}_V$ must be fully unloaded, Eq. (4.7):

$$
\sum_{i\in\mathcal{S}, w\in\mathcal{O}_r} dv_w^i = V0_r, \qquad \forall r\in\mathcal{R}_V \tag{4.7}
$$

Cargo unloading starts with propane and ends with butane. Butane quality q_2 is unloaded when no propane q_1 remains in the vessel, as stated in Eq. (4.8).

$$dvq^i_{w,q_2} \leq \overline{V_r}(1 - xvq^i_{r,q_1}), \quad \forall i \in \mathcal{S}, r \in \mathcal{R}_V, w \in \mathcal{O}_r, (q_1, q_2) \in \mathcal{QQ} \tag{4.8}$$

Operation limits

Eqs. (4.9) describe the operation bounds. Given that the volume capacity of the Gas pipe is less than half of the daily demand for final products, it is a reasonable approximation to assume that each delivery from the refinery will flush the remaining volume in the pipeline to allocate pure qualities during the inlet operation. Thus, parameter $\underline{DV}_w$ in the data, the minimum volume transfer for an operation w in the pipeline, is set equal to the pipeline capacity.

The data defines the minimum and maximum flow rates for all inlet and outlet operations. Assuming that the daily refinery production and blending manifold flow rates are given for these operations, the parameters $\underline{VF}_w$ and $\overline{VF}_w$, respectively, the minimum and maximum flow transfer for operation w, are both set equal in the data for these operations.

$$\begin{aligned} \underline{DV}_w \, z^i_w \leq dv^i_w \leq \overline{DV}_w \, z^i_w, \qquad & \forall i \in \mathcal{S}, w \in \mathcal{W}_{GI} \\ dv^i_w \leq \overline{Vr} \, z^i_w, \qquad & \forall i \in \mathcal{S}, r \in \mathcal{R}, w \in \mathcal{O}_r \\ \underline{VF}_w \, td^i_w \leq dv^i_w \leq \overline{VF}_w \, td^i_w, \qquad & \forall i \in \mathcal{S}, w \in \mathcal{W} \end{aligned} \tag{4.9}$$

where td^i_w is the duration of operation w when performed during slot i. Notice that the latter constraint ensures that the total volume transferred by operation w is consistent with its flowrate and time duration.

Material balance

Equation (4.10) enforces the dynamics of the inventory of quality q in resource r at slot i. Volume additivity assumes similarity in the chemical composition of the components.

$$vq^i_{r,q} = Vq0_{r,q} + \sum_{\substack{j,w \in \mathcal{I}_r \\ j<i}} dvq^j_{w,q} - \sum_{\substack{j,w \in \mathcal{O}_r \\ j<i}} dvq^j_{w,q}, \quad \forall i \in \mathcal{S}_1, r \in \mathcal{R}, q \in \mathcal{Q} \tag{4.10}$$

where $Vq0_{r,q}$ is a parameter with the initial volume of quality q in resource r. This constraint states that the volume of q in r and slot i, $vq^i_{r,q}$, is calculated by adding to the initial volume the volume transferred by all inlet operations $w \in \mathcal{I}_r$ and subtracting the volume removed by all outlet operations $w \in \mathcal{O}_r$, which were performed in slots preceding i.

The volumes in pipeline, blending manifold, and delivery manifold resources $r \in \mathcal{R}_G \cup \mathcal{R}_B \cup \mathcal{R}_D$ are constant. This requirement is implemented in the data parameters by imposing $V0 = \underline{V_r} = \overline{V_r}$. Due to constraints in Eqs. (4.4), (4.5), and (4.10), the total input and output volumes are equal for these resources. Further, the data also establish that only one inlet and one outlet operation can be performed in any slot for these resources, namely $\mathcal{R}_G$, $\mathcal{R}_B$, and $\mathcal{R}_D$. In particular, the total volume is 0 for $r \in \mathcal{R}_B \cup \mathcal{R}_D$. As a consequence, the quality volumes of input and output streams are equal for $r \in \mathcal{R}_B \cup \mathcal{R}_D$.

Overlapping operations in spheres

In the problem under consideration, there is only one sphere that receives and delivers the propane production, and similarly for the mixture of propane and butane production. Refer to spheres E2 and E3 in Figures 4.7 and 4.8.

Inlet and outlet operations of these spheres can overlap. Modeling of resources with overlapping operations may result in the fulfillment of Eq. (4.10), even if the inlet and outlet quantities exceed the maximum capacity of the resources.

This undesirable behavior is prevented by introducing two virtual resources for each real one and imposing the necessary non-overlapping constraints between the inlet and outlet operations. The set $\mathcal{R}_Z$ includes the required real resource r_1, while $\mathcal{E}_{r_1}$ contains the pair of virtual spheres associated with r_1. Figure 4.9 shows the equivalent four allowed states for sphere E2.

The accumulated volumes of spheres $r \in \mathcal{E}_{r_1}$ correspond to one real sphere, Eq. (4.11).

$$2 \sum_{r \in \mathcal{E}_{r_1}} v_r^i \leq \sum_{r \in \mathcal{E}_{r_1}} \overline{V_r}, \qquad \forall i \in \mathcal{S}_1, r_1 \in \mathcal{R}_Z, \tag{4.11}$$

Gas pipeline

As previously mentioned, the capacity of the Gas pipe is small compared to the transferred volumes. The model assumes that all volumes transported through the pipeline have a minimum value determined by its capacity. This condition is specified in the data for all input operations in the Gas pipe, as expressed mathematically

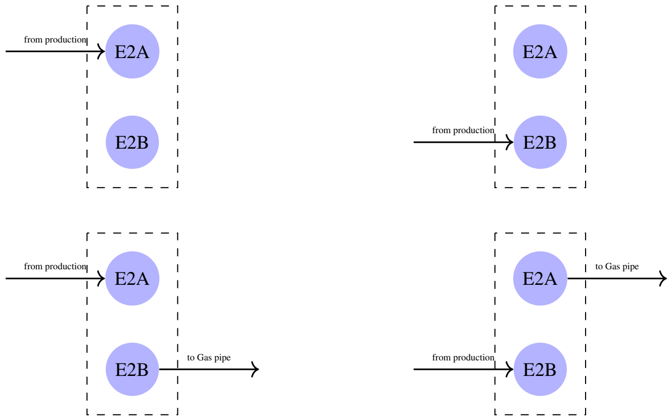

Figure 4.9: Four allowed states for virtual refinery spheres E2A and E2B. From left to right and top to bottom: production to E2A, production to E2B, production to E2A while E2B to Gas pipe, production to E2B while E2A to Gas pipe.

in Eq. (4.12).

$$\begin{cases} dvq^i_{w,q} \leq vq^i_{r,q} + dvpq^i_{w,q} + \overline{Vr}(1 - z^i_w), \\ dvq^i_{w,q} \geq vq^i_{r,q} + dvpq^i_{w,q} - \overline{Vr}(1 - z^i_w), \ \forall i \in \mathcal{S}, r \in \mathcal{R}_G, w \in \mathcal{O}_r \\ dvpq^i_{w,q} \leq \overline{Vr}\, z^i_w, \end{cases} \tag{4.12}$$

where $dvpq^i_{w,q}$ represents the additional volume of quality q transferred to and exiting pipeline r during operation w. Suppose operation w is performed in slot i. Then, the first two constraints state that the volume transferred out of resource r by the outlet operation w must be equal to the volume stored before the operation takes place in slot i, in addition to $dvpq^i_{w,q}$. The data confirms that the initial volume is the minimum quantity to be pumped out of the pipeline. Since the volume of the Gas pipe is fixed, a new volume of the same or different quality must remain in the pipeline, and an additional volume can be delivered to the final destination.

An example of the Gas pipe's operation and quality change is depicted in Table 4.2 for qualities C3R and C4R. Qualities C3R, C4R, C3C4R, C3I, and C4I are assumed to be pure.

Figure 4.10, shows the state of the Gas pipe across a sequence of slots, depending on the quality of the gas in both the Gas pipe and the inlet.

In the case of a long-distance pipeline, different qualities could be represented as a series of packets along the line as proposed in Zimberg et al. (2020, 2023b). The resource would be modeled using a continuous-time formulation and a multiple-operations-per-slot approach. Assuming that each LPG packet contains a pure pro-

Table 4.2: Gas pipe operations. Example for two qualities.

Initial $vq^i_{r,q}$	Additional $dvpq^i_{w,q}$	Final $vq^{i+1}_{r,q}$
C3R	C3R	C3R
C3R	C4R	C4R

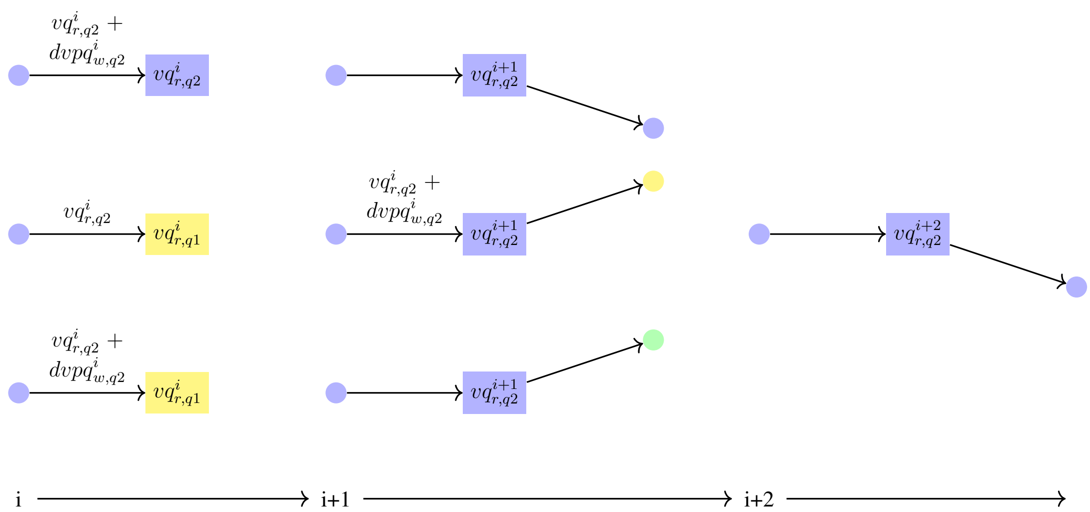

Figure 4.10: Operation of the pipeline proceeds from left to right and top to bottom as follows: A gas pipeline with quality q_2 (blue) in slot i, followed by a gas pipeline with quality q_2 in slot $i+1$ after delivery of quality q_2. Then, a gas pipeline with quality q_1 (yellow) in slot i, followed by a gas pipeline with quality q_2 in slot $i+1$ after delivery of quality q_1, and a gas pipeline with quality q_2 in slot $i+2$ after delivery. Finally, a gas pipeline with quality q_1 receives quality q_2, and the mixture of qualities (green) is delivered to a sphere. Please notice, that slot numbers can increase in a different way.

duct, bilinear constraints would not be required.

Manifold

Eq. (4.13) defines the assignment of operations in the blending manifold. Given a pair of input and output operations w_1 and w_2 in slot i, it assures that all qualities in the input stream are present in the output. Due to the volume balance of quality q in the Manifold, Eq. (4.10), the condition given in the data where the Manifold has zero volume, and the clique structure, there is only one pair of inlet operation w_1 and outlet operation w_2 that allows $dvq^i_{w1,q} = dvq^i_{w2,q}$. If a Manifold inlet operation w_1 is performed in slot i, then the presence of quality q in terminal sphere r (receiving outlet operation w_2) is determined by the presence of q in w_1:

$$\begin{cases} xvq^i_{r,q} \leq xdvq^i_{w,q} - z^i_w + 1, \\ xvq^i_{r,q} \geq xdvq^i_{w,q} + z^i_w - 1, \end{cases} \forall i \in \mathcal{S}, r \in \mathcal{R}_T, w \in \mathcal{W}_{BI}, q \in \mathcal{Q} \tag{4.13}$$

where $xdvq^i_{w,q}$ takes the value 1 if operation w transfers a nonzero volume of quality q, and 0 otherwise. $\mathcal{W}_{BI}$ represents the set of Manifold inlet operations. According to the inequalities above, if inlet operation w is performed and $z^i_w = 1$, then there exists a volume of quality q in resource r that delivers volume to the Manifold if and only if quality q is present in the inlet operation. Additionally, due to the volume balance of quality q in the Manifold, as mentioned earlier, quality q is present in the outlet operation for every quality q in resource r.

It should be noted that Eqs. (4.10) and (4.13) do not ensure the transfer of quality q or the equality of the concentration of quality q in resource r with that in the inlet or outlet volume. The variables $vq^i_{r,q}$ and $dvq^i_{w,q}$ can be zero even when $xvq^i_{r,q}$ or $xdvq^i_{w,q}$ take the value 1 in Eqs. (4.5) and (4.14), respectively. Additional constraints addressing this issue are discussed in Section 4.3.3.

When the variable $xdvq^i_{w,q}$ is zero, quality q is not transferred, as enforced by Eq. (4.14):

$$dvq^i_{w,q} \leq \overline{Vr} \cdot xdvq^i_{w,q}, \quad \forall i \in \mathcal{S}, w \in \mathcal{W}_{BI}, q \in Q \tag{4.14}$$

Logistics

The goal is to fill a refinery sphere, denoted as $r \in \mathcal{R}_R$, before filling another sphere with the same product. The variable vr^i_r in Eq. (4.15) quantifies the remaining available volume in the sphere after filling it. In the context of slot i, variable vr^i_r represents the difference between the maximum capacity and the actual volume of the resource in slot $i + 1$. Its value is penalized in the objective function. The following constraints determine the calculation of the spare volume:

$$\begin{cases} vr^i_r \leq \overline{V_r} - v^{i+1}_r, \\ vr^i_r \geq \overline{V_r} - v^{i+1}_r - \overline{V_r} \cdot \left(1 - \sum\limits_{w \in \mathcal{I}_r} z^i_w\right), \end{cases} \forall i \in \mathcal{S}, i < N, r \in \mathcal{R}_R \tag{4.15}$$

Notice that vr^i_r equals the spare capacity $(\overline{V_r} - vr^i_r)$ only if an inlet operation is performed in resource r. Otherwise, vr^i_r is free and will be driven to zero due to the objective penalization—see term penalized by Cvr in Eq. 4.39. The penalization for spare capacity is incurred only once after an inlet operation.

The variable vm_r^i in Eq. (4.16) tracks the actual volume stored in tank $r \in \mathcal{R}_R$ when the scheduler performs an inlet operation, and its value is penalized in the objective.

$$\begin{cases} vm_r^i \leq v_r^i + \overline{V_r} \cdot \left(1 - \sum_{w \in \mathcal{I}_r} z_w^i\right), \\ vm_r^i \geq v_r^i - \overline{V_r} \cdot \left(1 - \sum_{w \in \mathcal{I}_r} z_w^i\right), \end{cases} \forall i \in \mathcal{S}, r \in \mathcal{R}_R. \tag{4.16}$$

Variable vm_r^i equals the stored volume v_r^i in the tank only when an inlet operation is performed. Thus, the volume stored in a tank is penalized only once for each time slot that the scheduler performs inlet operations. If inlet operations are not performed, then vm_r^i is forced to zero due to the objective penalization.

4.3.2. Quality specifications

Final products must meet a set of quality specifications that define the minimum or maximum values required by the market for specific quality attributes. Table 4.5 presents an example of specifications for *Mixture* and *Propane*, along with the properties of the production streams. The maximum concentration of butanes in *Propane* affects how the product vaporizes. Similarly, the composition of *Mixture* is related to burner design.

Eqs. (4.17), (4.18), and (4.19) guarantee that the products delivered to the market satisfy the required specifications:

$$\begin{cases} \sum_{w \in \mathcal{I}_{r_1} \cap \mathcal{O}_r, q \in \mathcal{Q}} dvq_{w,q}^i (K_{q,k} - \underline{K_{r_1,k}})/100 \geq -\overline{Vr}(1 - xvk_{r,r_1}^i) \\ \sum_{w \in \mathcal{I}_{r_1} \cap \mathcal{O}_r, q \in \mathcal{Q}} dvq_{w,q}^i (\overline{K_{r_1,k}} - K_{q,k})/100 \geq -\overline{Vr}(1 - xvk_{r,r_1}^i) \end{cases}$$

$$\forall i \in \mathcal{S}, r \in \mathcal{R}_T, r_1 \in \mathcal{R}_D, k \in \mathcal{KQ} \tag{4.17}$$

$$\begin{cases} \sum_{q \in \mathcal{Q}} vq_{r,q}^i (K_{q,k} - \underline{K_{r_1,k}})/100 \geq -\overline{Vr}(1 - xvk_{r,r_1}^i), \\ \sum_{q \in \mathcal{Q}} vq_{r,q}^i (\overline{K_{r_1,k}} - K_{q,k})/100 \geq -\overline{Vr}(1 - xvk_{r,r_1}^i), \end{cases}$$

$$\forall i \in \mathcal{S}, r \in \mathcal{R}_T, r_1 \in \mathcal{R}_D, k \in \mathcal{KQ} \tag{4.18}$$

$$z_w^i \leq xvk_{r,r_1}^i, \forall i \in \mathcal{S}, r \in \mathcal{R}_T, r_1 \in \mathcal{R}_D, w \in \mathcal{I}_{r_1} \cap \mathcal{O}_r, k \in \mathcal{KQ} \tag{4.19}$$

where the binary variable $xvk_{r,r_1}^i = 1$ if the quality in a terminal sphere r meets all specifications required for a delivery manifold r_1 in slot i; $\underline{K_{r_1,k}}$ ($\overline{K_{r_1,k}}$) is the minimum (maximum) value for specification k for product delivery from r_1, and $K_{q,k}$ is the contribution of quality q to specification k.

For each quality property k, the constraints in Eq. (4.17) ensure that the average content resulting from the inlet operation for delivery manifold r_1 meets the specification when xvk_{r,r_1}^i is flagged. Otherwise, if $xvk_{r,r_1}^i = 0$, the specifications are not guaranteed.

Similarly, the constraints in Eq. (4.18) determine that the average content in a terminal sphere r meets the specification for all quality properties required by a delivery manifold r_1, provided that $xvk_{r,r_1}^i = 1$. The quality properties are not guaranteed otherwise.

Finally, Eq. (4.19) determines that if an operation w is performed, transferring a product volume from a terminal sphere r (outlet operation) and to a delivery manifold r_1 (inlet operation) in slot i, which is flagged by $z_w^i = 1$, then the quality specifications must hold since xvk_{r,r_1}^i is also flagged.

For a final product denoted as $r_1 \in \mathcal{R}_D$, the combination of intermediate products in $\sum_{w \in \mathcal{I}_{r_1}} dv_w^i$ ensures compliance with the set of specifications for each quality attribute $k \in \mathcal{KQ}$. Similarly, the mixture of intermediate products in $v_r^i, r \in \mathcal{R}_T$, also guarantees compliance with the quality specifications.

The resulting quality attributes are not aligned between resources r and r_1. Similar to what was mentioned in the blending manifold model, Eqs. (4.17), (4.18), and (4.19) do not guarantee the equality of the concentration of quality q in resource r to that in the outlet volume from r (inlet volume to resource r_1). Additional constraints are required to enforce equal quality and will be presented in the next section.

Variables xvk_{r,r_1}^i are set to zero for void operations, Eq. (4.20):

$$xvk_{r,r_1}^i = 0, \quad \forall i \in \mathcal{S}, r \in \mathcal{R}_T, r_1 \in \mathcal{R}_D, w \in \mathcal{I}_{r_1}, w \in \mathcal{O}_r, r \notin \mathcal{B}_{r_1} \tag{4.20}$$

4.3.3. Bilinear terms

The quality of the volume delivered in slot i must be equal to the quality in the corresponding resource, which gives rise to the nonlinear relations defined in Eq. (4.21). These nonlinear constraints apply to resources containing mixtures of intermediate products for $r \in \mathcal{R}_T$, as follows:

$$\frac{vq^i_{r,q}}{v^i_r} = \frac{dvq^i_{w,q}}{dv^i_w}, \qquad \forall i \in \mathcal{S}, r \in \mathcal{R}_T, w \in \mathcal{O}_r, q \in \mathcal{Q} \tag{4.21}$$

which can be alternatively expressed as a bilinear equation, Eq. (4.22):

$$vq^i_{r,q} \cdot dv^i_w = v^i_r \cdot dvq^i_{w,q}, \qquad \forall i \in \mathcal{S}, r \in \mathcal{R}_T, w \in \mathcal{O}_r, q \in \mathcal{Q} \tag{4.22}$$

As mentioned and illustrated in Figure 4.5, E15 may either receive a specified volume matching the quality composition of another sphere when blending is planned within E15, or deliver a volume with its own quality composition to another sphere.

4.3.4. Time and sequencing constraints

Time constraints

Time relations for operations are given by Eq. (4.23):

$$\begin{cases} te^i_w = ts^i_w + td^i_w, \\ te^i_w \leq T z^i_w, \end{cases} \forall i \in \mathcal{S}, w \in \mathcal{W} \tag{4.23}$$

in which te^i_w is the end time, ts^i_w is the start time, and td^i_w is the time duration of operation w in slot i, and T is the scheduling time horizon. Notice that if the operation is not performed in the slot, then all these variables assume value zero.

Equality between the start and end times of input and output operations for the Gas pipe ($\mathcal{R}_G$), Manifold ($\mathcal{R}_B$), and demand and supply resources ($\mathcal{R}_D = \mathcal{R}_{DS} \cup \mathcal{R}_{DP}$) must be ensured. The clique structure dictates that, at most, one input (output) operation can be active per slot in these resources. The constraints are formulated in Eq. (4.24) as follows:

$$\begin{cases} \sum\limits_{w \in \mathcal{I}_r} ts^i_w = \sum\limits_{w \in \mathcal{O}_r} ts^i_w, \\ \sum\limits_{w \in \mathcal{I}_r} td^i_w = \sum\limits_{w \in \mathcal{O}_r} td^i_w, \end{cases} \forall i \in \mathcal{S}, r \in \mathcal{R}_G \cup \mathcal{R}_B \cup \mathcal{R}_D \tag{4.24}$$

which models the condition that if an inlet operation is performed in resource r, then an outlet operation is also performed, and they both start and end at the same time.

Continuous production

The continuous production imposes the following constraints, Eq. (4.25):

$$\begin{aligned}
\sum_{i \in \mathcal{S}, w \in \mathcal{O}_r} td_w^i &= T, \quad \forall r \in \mathcal{R}_{UB} \\
\sum_{i \in \mathcal{S}, w \in \mathcal{O}_r} td_w^i &= T, \quad \forall r \in \mathcal{R}_{UP} \\
\sum_{i \in \mathcal{S}, w \in \mathcal{O}_r} td_w^i &= T, \quad \forall r \in \mathcal{R}_{US}
\end{aligned} \tag{4.25}$$

enforce the continuous, non-stopping production of butane ($\mathcal{R}_{UB}$), propane ($\mathcal{R}_{UP}$), and mixed products ($\mathcal{R}_{US}$) at the refinery, respectively.

The continuous supply of gas products constraints, Eq. (4.26),

$$\begin{aligned}
\sum_{i \in \mathcal{S}, w \in \mathcal{I}_r} td_w^i &= T, \quad \forall r \in \mathcal{R}_{DS} \\
\sum_{i \in \mathcal{S}, w \in \mathcal{I}_r} td_w^i &= T, \quad \forall r \in \mathcal{R}_{DP}
\end{aligned} \tag{4.26}$$

keep the non-stopping delivery of *Mixture* ($\mathcal{R}_{DS}$) and *Propane* ($\mathcal{R}_{DP}$) to the market (see Figure 4.3).

Strict time constraints

Cargo reception ($\mathcal{R}_V$) and delivery operations ($\mathcal{R}_O = \mathcal{R}_{OP} \cup \mathcal{R}_{OS}$) are restricted between given start and end times, Eqs. (4.27) and (4.28), respectively,

$$\begin{cases} ts_w^i \geq Ts_r \cdot z_w^i, \\ te_w^i \leq Te_r \cdot z_w^i, \end{cases} \forall i \in \mathcal{S}, r \in \mathcal{R}_V, w \in \mathcal{O}_r \tag{4.27}$$

$$\begin{cases} ts_w^i \geq Ts_r \cdot z_w^i, \\ te_w^i \leq Te_r \cdot z_w^i, \end{cases} \forall i \in \mathcal{S}, r \in \mathcal{R}_O, w \in \mathcal{I}_r \tag{4.28}$$

Time and operations on virtual resources

Given a resource $r_3 \in \mathcal{R}_Z$ and the pair of spheres $r_1, r_2 \in \mathcal{E}_{r_3}$, the following constraint ensures material balance for a given slot i. To this end, the outlet operation of r_2 must start before the inlet operation of r_1, a condition imposed by Eq. (4.29).

$$ts^i_{w_2} \leq ts^i_{w_1}, \forall i \in \mathcal{S}, w_1 \in \mathcal{I}_{r_1}, w_2 \in \mathcal{O}_{r_2}, (r_1 \wedge r_2 \in \mathcal{E}_{r_3}), r_3 \in \mathcal{R}_Z, r_1 \neq r_2 \quad (4.29)$$

Sequencing constraints

The cargo of vessel r_1 must be completely unloaded before the unloading of the next cargo, r_2, begins. This condition is indicated by $Ts_{r_1} \leq Ts_{r_2}$. This requirement is expressed in terms of the relationship between the end time of an unloading operation in vessel r_1 and the corresponding start time for vessel r_2, which is implemented by the first constraint of Eq. (4.30).

$$\max_{j,w \in \mathcal{O}_{r_1}} te^j_w \leq \min_{j,w \in \mathcal{O}_{r_2}} \left(ts^j_w + T(1 - z^j_w)\right), \quad \forall r_1, r_2 \in \mathcal{R}_V, Ts_{r_1} \leq Ts_{r_2} \quad (4.30)$$

$$\sum_{\substack{j,w \in \mathcal{O}_{r_1} \\ j<i}} z^j_w \geq \sum_{\substack{j,w \in \mathcal{O}_{r_2} \\ j<i}} z^j_w, \quad \forall i \in \mathcal{S}, r_1, r_2 \in \mathcal{R}_V, Ts_{r_1} \leq Ts_{r_2} \quad (4.31)$$

Constraint 4.30 states that the end time of any output operation performed in r_1 (vessel that arrives earlier), in any slot, must precede the start time of any output operation performed in r_2 (vessel that arrives later). Jointly with the first constraint, constraint 4.31 mandates that any operation w_1 in r_1 must be performed before an operation w_2 is performed in r_2, *i.e.*, if $z^i_{w_1} = 1$ then $z^j_{w_2} = 0$ for all $j < i$.

The first equation is linearized using $\overline{ter_1}$ and $\underline{tsr_2}$, which bound the $\max$ and $\min$ arguments, respectively, Eq. (4.32):

$$\begin{cases} te^i_w \leq \overline{te_{r_1}}, & \forall i \in \mathcal{S}, r_1 \in \mathcal{R}_V, w \in \mathcal{O}_{r_1}, \\ \underline{ts_{r_2}} \leq \left(ts^i_w + T \cdot (1 - z^i_w)\right), & \forall i \in \mathcal{S}, r_2 \in \mathcal{R}_V, w \in \mathcal{O}_{r_2}, \\ \overline{te_{r_1}} \leq \underline{ts_{r_2}}, & \forall r_1, r_2 \in \mathcal{R}_V, Ts_{r_1} < Ts_{r_2} \end{cases} \quad (4.32)$$

Similar constraints are considered for the market supply, Equations (4.33) and (4.34):

$$\sum_{\substack{j,w \in \mathcal{I}_{r_1} \\ j<i}} z_w^j \geq \sum_{\substack{j,w \in \mathcal{I}_{r_2} \\ j<i}} z_w^j, \quad \forall i \in \mathcal{S}, r_1, r_2 \in \mathcal{R}_{OS}, Ts_{r_1} \leq Ts_{r_2} \tag{4.33}$$

$$\sum_{\substack{j,w \in \mathcal{I}_{r_1} \\ j<i}} z_w^j \geq \sum_{\substack{j,w \in \mathcal{I}_{r_2} \\ j<i}} z_w^j, \quad \forall i \in \mathcal{S}, r_1, r_2 \in \mathcal{R}_{OP}, Ts_{r_1} \leq Ts_{r_2} \tag{4.34}$$

Operations in a clique $c \in \mathcal{C}$ must not overlap and are executed in different time slots i_1 and i_2, as enforced by Eq. (4.35). The time delay Tw between specific pairs of inlet and outlet operations is also considered in Eq. (4.36).

$$\sum_{w \in c} te_w^{i_1} + \sum_{\substack{j,w \in c \\ i_1<j<i_2}} td_w^j \leq \sum_{w \in c} ts_w^{i_2} + T\left(1 - \sum_{w \in c} z_w^{i_2}\right), \quad \forall i_1, i_2 \in \mathcal{S}, i_1 < i_2, c \in \mathcal{C} \tag{4.35}$$

$$te_{w_1}^{i_1} + Tw \cdot z_{w_1}^{i_1} \leq ts_{w_2}^{i_2} + (T + Tw)(1 - z_{w_2}^{i_2}), \quad \forall i_1, i_2 \in \mathcal{S}, i_1 < i_2, (w_1, w_2) \in \mathcal{D} \tag{4.36}$$

4.3.5. Symmetry breaking

In order to accelerate the optimization algorithm, symmetry-breaking relations between the assignment variables are introduced to guide the process toward specific types of solutions, as stated in Eq. (4.37).

The set $\mathcal{A}$ includes vessel and refinery operations. Refinery operations are considered continuous, and this requirement guarantees the feasibility of the constraints for every pair of operations in $\mathcal{A}$. The set of constraints enforces an assignment of operations to the lowest available slot.

$$\begin{cases} z_w^i \leq z_w^{i-1} + \displaystyle\sum_{\substack{w_1 \in \mathcal{W} \\ (w,w_1) \in \mathcal{A} \vee (w_1,w) \in \mathcal{A}}} (z_{w_1}^i + z_{w_1}^{i-1}), \forall i \in S, i > 1, \\ z_w^i \leq z_w^{i+1} + \displaystyle\sum_{\substack{w_1 \in \mathcal{W} \\ (w,w_1) \in \mathcal{A} \vee (w_1,w) \in \mathcal{A}}} (z_{w_1}^i + z_{w_1}^{i+1}), \forall i \in S, i < N, \end{cases} \forall w \in \mathcal{W}_A \tag{4.37}$$

As an example, let us define $\mathcal{A} = (201, 202)$ where operations 201 and 202 represent the inlet operations to spheres E2A and E2B respectively. The formulation of Eq. (4.37) yields Eq. (4.38).

$$\begin{cases} z_{201}^{i} \leq z_{201}^{i-1} + z_{202}^{i} + z_{202}^{i-1} \\ z_{201}^{i} \leq z_{201}^{i+1} + z_{202}^{i} + z_{202}^{i+1} \end{cases} \tag{4.38}$$

Operations 201 and 202 cannot overlap. If $z_{202}^{i-1} = 0$, $z_{202}^{i} = 0$, and z_{202}^{i+1} (keeps doing the same operation), then $z_{201}^{i} \leq z_{201}^{i-1}$ and $z_{201}^{i} \leq z_{201}^{i+1}$. Consequently, operation 201 will be assigned to the lowest available slot.

4.3.6. Objective function

The objective is to minimize the total cost by reducing the deviation in the delivery of final products from the required quantities and minimizing the total number of operational changes. Additionally, it prioritizes a group of operations to minimize symmetry, while driving reception to a minimum and filling refinery spheres to maximum capacity. The formulation also includes a reduction in operation end times.

The second and last terms of the objective cost relate to the efficiency of the schedule, aiming to minimize operational changes and reduce process times. A schedule with fewer operational changes, which continues the same operation unless necessary, results in reduced maintenance costs and fewer operational errors. Additionally, minimizing process times helps avoid demurrage costs.

The main aim of the other terms is to reduce symmetrical solutions and simplify operations, thereby enhancing the model's performance and aligning it more closely with actual practice. This includes prioritizing specific operations to certain spheres and maintaining reception at minimum capacity until filled, if feasible. The set $\mathcal{W}_{RB}$ includes refinery, pipeline and manifold operations.

Cost terms are adimensional and a normalization factor is added to all terms to scale them between 0 and 1 before applying the weightings to avoid a bias in the optimization towards larger quantities.

Eq. (4.39) expresses the objective considering the costs mentioned above along with Eqs. (4.40) and (4.41):

$$
\begin{aligned}
\text{mín } f = & \; Cv \sum_{i \in \mathcal{S}, r \in \mathcal{R}_O} \frac{dvol_r^i}{Vr_r} \\
& + Czz \sum_{\substack{i \in 1 \ldots N-1 \\ w \in \mathcal{W}_{RB}}} \frac{dzz_w^i}{(N-1)\, |\mathcal{W}_{RB}|} \\
& + \sum_{\substack{i \in \mathcal{S}, \\ w \in \mathcal{W}_{RB}}} Czb_w \frac{z_w^i}{N\, |\mathcal{W}_{RB}|} \\
& + Cvr \sum_{\substack{i \in \mathcal{S}, \\ r \in \mathcal{R}_R}} \frac{vr_r^i}{\overline{V}_r} \\
& + Cvm \sum_{\substack{i \in \mathcal{S}, \\ r \in \mathcal{R}_R}} \frac{vm_r^i}{\overline{V}_r} \\
& + Cte \sum_{\substack{i \in S, \\ w \in \mathcal{W}}} \frac{te_w^i}{T\, |\mathcal{W}|}
\end{aligned}
\tag{4.39}
$$

where

$$
\begin{cases}
dzz_w^i \geq z_w^{i+1} - z_w^i, \\
dzz_w^i \geq z_w^i - z_w^{i+1},
\end{cases}
\quad \forall i \in S, i < n, w \in \mathcal{W}_{RB}
\tag{4.40}
$$

$$
\begin{cases}
\sum_{w \in \mathcal{I}_r} (dv_w^i - \overline{Vf}_w \; td_w^i) \leq dvol_r^i, \\
\sum_{w \in \mathcal{I}_r} (\overline{Vf}_w \; td_w^i - dv_w^i) \leq dvol_r^i,
\end{cases}
\quad \forall \in S, r \in \mathcal{R}_O
\tag{4.41}
$$

4.3.7. Compact formulation

With the notation, objective, and constraints defined, the problem can be concisely formulated as follows:

$$
\begin{aligned}
& \text{mín } f \\
& \text{s.t. Eqs. (4.1)-(4.20),} \\
& \quad \text{Eqs. (4.22)-(4.31),} \\
& \quad \text{Eqs. (4.33)-(4.37),} \\
& \quad \text{Eqs. (4.39)-(4.41).}
\end{aligned}
\tag{4.42}
$$

A MINLP problem is formulated by combining a MILP model with the nonlinear constraint (4.22).

4.4. Heuristics

The model is solved in a two-step procedure: a linear step followed by a nonlinear, that has proved to be more efficient than a global optimization approach(Mouret et al., 2009). A Relax & Fix strategy is applied when an exact branch-and-cut solution can not be achieved in a feasible time. The strategy has been described in previous works(Zimberg et al., 2019, 2023b). The algorithm is depicted in Algorithm 2.

Initially, a variant of the model is solved, where a subset of the binary variables is relaxed for slots $i = \Delta + 1, \ldots, N$, where the step parameter Δ is provided by the user. Follows an iteration for slots $i = 1, \ldots, N - \Delta$.

Data: Δ : number of slots where integrality is kept
Data: $Model$: model instance
Result: Solution of a restricted $Model$
Relax a subset of binary variables for $i = \Delta + 1, \ldots, N$;
Solve;
for $i = 1, \ldots, N - \Delta$ **do**
    Fix binary variables in slot i;
    Restore integrality of relaxed binary variables in slot $i + \Delta$;
    Solve;
end

Algorithm 2: Relax-and-fix strategy

For a case where N=8 and Δ=5, the strategy works as follows:

- Relax specific binaries in slots 6 to 8 and solve.
- Fix all the binaries in slot 1 based on the previous solution. Restore the integrality of binaries in slot 6 and solve.
- Fix all the binaries in slot 2. Restore the integrality of binaries in slot 7 and solve.
- Fix all the binaries in slot 3. Restore the integrality of binaries in slot 8 and solve.

In this work, only a subset of the z variables is relaxed. For all slots, this subset is determined by the Gas pipe inlet and outlet operations (except for cargo unloading), the inlet and outlet operations of the blending manifold, and the inlet operations for *Mixture* and *Propane* delivery. Operations related to cargo unloading, refinery inlet processes, and final product deliveries are not relaxed. This configuration helps reduce the number of infeasible solutions for a given Δ parameter and the corresponding relaxation of binary variables.

4.5. Experiments

This section analyzes fourteen cases based on a single problem instance. The instances are formulated in the algebraic modeling language AMPL (Fourer et al., 2002). In the first step, where Eqs. (4.22) (nonlinear terms) are excluded, the resulting MILP model is solved using CPLEX 20.1.0.0 with a branch-and-cut algorithm or a Relax-and-Fix approach, Algorithm 2, to obtain a lower bound for the solution. Subsequently, the binary variables are fixed, the bilinear terms, Eq. (4.22), are added to the model, and the problem is solved with the IPOPT solver Wächter and Biegler (2006) to determine an upper bound for the solution. This strategy was previously applied in works such as (Mouret et al., 2009; Zimberg et al., 2023b).

4.5.1. Problem setup

The problem considers three LPG refinery qualities: propane (C3R), butane (C4R), and a propane-butane mixture (C3C4R). It also includes the reception of an LPG cargo (C3I and C4I). A feasible solution requires eight slots, and the main model results are reported and discussed.

The diagram presented in Figure 4.8 has been modified to provide a more detailed representation of the model's features. In Figure 4.11, refinery spheres E1 to E4 receive and deliver production. Virtual spheres replace real ones. Terminal spheres E12, E13, and E14 are assigned to the delivery of *Propane*, while spheres E18, E19, and E20 are designated for *Mixture*. Transfers between terminal spheres are modeled with the Manifold. It is represented in the figure by four interconnected blocks that receive and deliver products to and from every sphere in the terminal. The demands for *Mixture* and *Propane* are delivered to resources Z901 and Z001, respectively.

The initial inventories and maximum capacities are presented in Table 4.3, while the volumes of cargo reception and deliveries are shown in Table 4.4. The time horizon covered by this table is 10 days, with one cargo scheduled for unloading

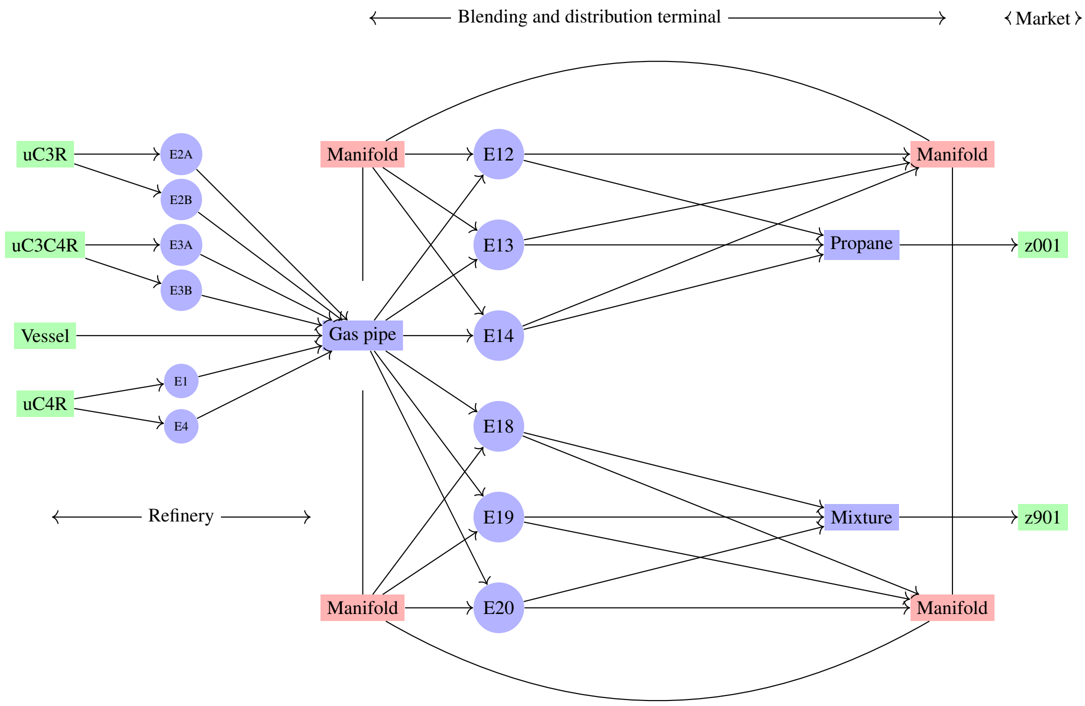

Figure 4.11: Diagram of resources and operations for the proposed problem.

between days 7 and 10. The cargo quality is a mixture of equal parts propane and butane, and the required delivery volumes are also included.

The combined information from both tables shows that there is not enough *Mixture* at the terminal to meet the demand within the given time horizon. Additionally, a shortage of C3R at the refinery suggests that blending between spheres may be necessary to make *Mixture* available within the required timeframe. This problem represents a more constrained scenario than the one presented in Appendix 8, aimed at showcasing the model's capabilities.

The quality and specifications of the products are defined by their butane content, as shown in Table 4.5.

Table 4.6 shows general cost penalties for the objective terms. It highlights the priority of minimizing operation changes and ensuring demand fulfillment.Additional cost parameters are operation-dependent and are implemented to prioritize specific actions and reduce symmetry. For the terminal spheres, costs increase progressively with the sphere number. Reception from the gas pipeline and blending are assigned higher priority than reception from other spheres, reflected by correspondingly lower cost coefficients.

The clique structure of the problem establishes non-overlapping requirements

Table 4.3: Cases 1 to 14. Initial inventories and maximum capacities (10^3 m^3). Resources with zero values are omitted.

Initial Inventory	C3R	C4R	C3C4R	C3I	C4I	Max.Capacity
Vessel				2	2	4
uC3R	100					100
uC3C4R			100			100
uC4R		100				100
rE1		0.25				0.7
Gas pipe		0.25				0.25
tE12	0.7					0.7
tE13	0.7					0.7
tE14	0.4					0.7
tE19	0.5	0.5				2.5
tE20		1.2				2.5

Table 4.4: Cases 1 to 14. . Reception and delivery. Volume (10^3 m^3) and Time (day).

Reception	Mixture	Propane	C3I	C4I	T start	T end
Vessel			2	2	7	10
Delivery						
z901	5				0	10
z001		1.5			0	10

Table 4.5: Cases 1 to 14. Specifications of intermediate and final products.

Product/Spec	C4 %
Propane	0 to 20
Mixture	50 to 80
C3R	0
C4R	100
C3C4R	60
C3I	0
C4I	100

between operations:

- Only one inlet or one outlet operation is allowed per slot for all resources.
- For a given sphere in a specific slot, only one inlet or outlet operation is allowed. Spheres can either receive or deliver product during a specific time period. For other resources, such as the Gas pipe, Manifold, Mixture, and Propane, both inlet and outlet operations can coexist within the same slot.

Table 4.6: Cases 1 to 14. Objective penalties (adimensional).

Operation Change	Volume Delivered	Volume Max.	Volume Min.	End Time
0.9	0.9	0.5	0.5	0.5

4.5.2. Problem results and general discussion

Table 4.7 (Cases 1 to 9) and Table 4.8 (Cases 10 to 14) present the case-specific parameters, along with the main parameters of the model structure, solution times and the objective values for the first (linear) and second steps (objective function) of the solution algorithm.

To analyze how the quality of the solution changes with solution time, different Δ sizes and mipgap tolerances are selected in Cases 1 to 9. Cases 1 to 4 compare the results when the heuristic Δ parameter increases from 4 to 7. The optimization mipgap is set to 0.5 % for all steps, except for the last step, where it is set to 1 %. An exact solution is evaluated in Case 5 but the time limit is exceeded. In cases 6 to 9, the last step is run with a mipgap of 0.5 %.

A comparison between Cases 2 and 7, where the mipgap improves from 1 % to 0.5 % in the final step for $\Delta = 5$, shows a 0.1 % reduction in the objective function, while the solution time increases significantly from 951 seconds (Case 2) to 2410 seconds (Case 7). Case 2 is acceptable for practical applications, given the small differences in the objective values. The difference in the objective values between Case 2 and the best estimate of the exact solution, represented by Case 5, is 0.3 %.

Table 4.9 presents the objective costs for cases 2 and 7. The main differences are determined by the operation end times and the assignment of resources.

A comparison of the objective values from the linear and nonlinear stages of the algorithm shows a small gap between the solutions obtained in the two stages. In the context of the problem under study, the bilinear terms are addressed in the second step of the algorithm. In the first step, the manifold and specification constraints help the model preserve the quality components on both sides of the product transfer; however, feasibility is only guaranteed in the second step.

For cases 10 to 14 the last step considers a mipgap of 1 % but symmetry-breaking constraints defined by Eq. (4.37) are disabled, as is also the case for the exact solution in Case 14. Table 4.8 shows that solution times exceed the time limit compared to Cases 1 to 5.

Table 4.7: Cases 1 to 9. Case-specific parameters, model size, solution time (s) and objectives (adimensional). An asterisk (*) indicates that the maximum solution time was exceeded.

Case	Slots	Δ	Mipgap (%)	Last Mipgap (%)	Rows	Columns	Non zeros	Binaries	Time	Obj. Linear	Obj.
1	8	4	0.5	1	9742	3551	42780	918	2292	8.08453	8.08453
2	8	5	0.5	1	9730	3549	42399	966	951	7.84015	7.84015
3	8	6	0.5	1	9846	3547	42161	1030	3431	7.8375	7.8375
4	8	7	0.5	1	9276	3543	40762	1076	15183	7.83079	7.83787
5	8	8	0.5		9668	3525	40216	1123	36076*	7.81778	7.81928
6	8	4	0.5	0.5	9742	3551	42780	918	2299	8.0761	8.07610
7	8	5	0.5	0.5	9730	3549	42399	966	2410	7.83066	7.83066
8	8	6	0.5	0.5	9846	3547	42161	1030	7615	7.8375	7.83750
9	8	7	0.5	0.5	9276	3543	40762	1076	38112*	7.83017	7.88891

Table 4.8: Cases 10 to 14. Case-specific parameters, model size, solution time (s) and objectives (adimensional). Symmetry-breaking constraints are disabled. An asterisk (*) indicates that the maximum solution time was exceeded.

Case	Slots	Δ	Mipgap (%)	Last Mipgap (%)	Rows	Columns	Non zeros	Binaries	Time	Obj. Linear	Obj.
10	8	4	0.5	1	9672	3578	42080	942	386		
11	8	5	0.5	1	9661	3571	41703	990	28258	0.92927	1.19263
12	8	6	0.5	1	9642	3552	41239	1038	58774*	0.8413	0.84130
13	8	7	0.5	1	9742	3565	41171	1101	72007*	0.85944	0.85944
14	8	8	0.5		9691	3562	40796	1147	36011*	0.83358	0.83358

Table 4.9: Cases 2, and 7. Solution. Objective costs (adimensional)

Case	Volume Delivered	Operation Change	Operation Specifics	Volume Max.	Volume Min.	End Time
2	0	0.24596	0.00984	7.35713	0	0.22721
7	0	0.24596	0.01011	7.35713	0	0.21745

Table 4.10 presents the composition of the final products for Cases 2 and 7. The values are similar, and all products meet the required specifications.

Table 4.10: Cases 2 and 7. Solution. Quality of final products (%)

Case	Product	Delivery	C34R	C3I	C3R	C4I	C4R	*C4 %*
2	Mixture	z901	9.6		32.4		58	63.8
7	Mixture	z901	9.6		32.4		58	63.8
2	Propane	z001	8.9		91.1		0	5.2
7	Propane	z001	8.9		91.1		0	5.2

The Gantt charts for the solutions of Cases 2 and 7 are shown in Figure 4.12. The diagrams are generally similar, with differences in certain slot assignments, sphere allocations, and operation timings.

Cargo unloading sequences and refinery operations are identical in both cases, except for the operation Refinery Mixture to E1, which is assigned to slot 4 in Case 2 and to slot 5 in Case 7.

As expected from an MOS model, slot numbers do not necessarily increase with time. For example, the transfer of C3C4R from production to E3B is assigned to slot 4, while the operation from C4R production to E1 is scheduled for slot 7, with both occurring within a similar time window. Figure 4.12 also illustrates that different operations at different times can be assigned to the same slot, as shown by the assignment of certain operations to slot 3.

Figure 4.13 illustrates the time evolution of the inventory of refinery sphere E2, including its virtual spheres, for the solution of Case 2. The inventory of E2 is the sum of the inventories of its virtual spheres.

For Case 2, the sequence of operations over time (excluding final deliveries) is detailed below. The sequence of operations and blendings proposed by the solution is well-suited to the characteristics of the problem.

- E1 to Gas Pipeline to E18: C4R is transferred to E18. The pipeline initially contains C4R (see Table 4.3). Slot 1.
- E13 to Manifold to E20: C3R is blended with C4R in E20. The resulting mixture meets the *Mixture* specification (Table 4.5). Slot 3.
- E2B to Gas Pipeline to E18: The remaining C4R and C3R are transferred to the empty E18. Slot 2.
- E3A to Gas Pipeline to E19: C3R and C3C4R are transferred to E19, which initially contains a volume of *Mixture* (see Figure 4.14). Slot 3.
- E4 to Gas Pipeline to E19: C3C4R and C4R are transferred, along with the previous reception (*Mixture* quality). Slot 4.

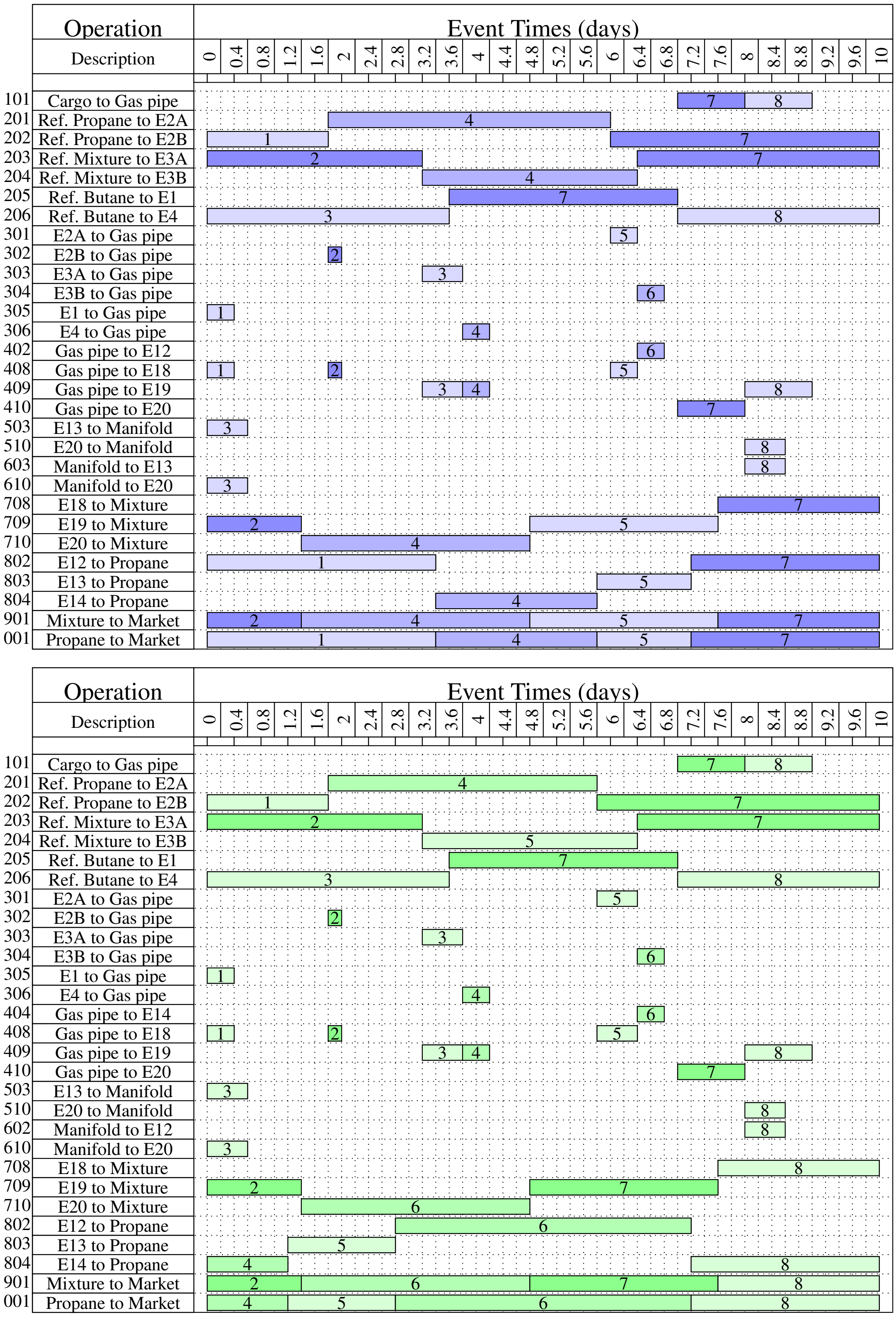

Figure 4.12: Cases 2 (above) and 7. Solution. Diagram of operations and times.

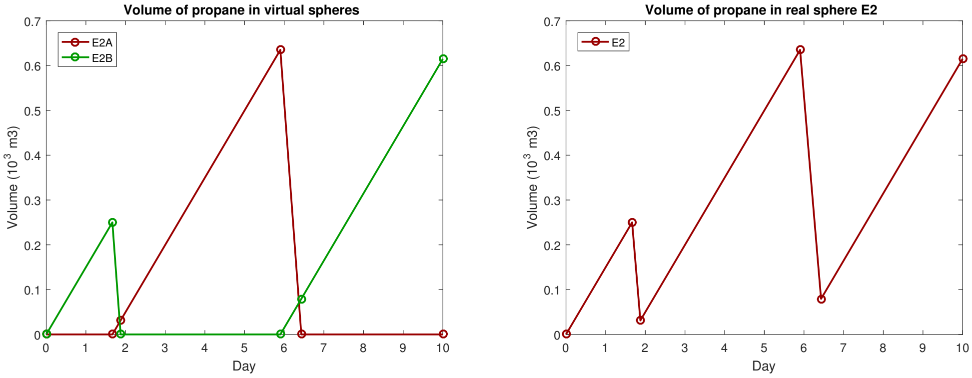

Figure 4.13: Case 2. Solution. Volume of propane in sphere E2, virtual and real spheres (10^3 m^3).

- E2A to Gas Pipeline to E18: C4R and C3R are transferred to E18. The resulting quality satisfies the *Mixture* specification. Slot 5.
- E3B to Gas Pipeline to E12: C3R and C3C4R are transferred to fill E12, which initially contains *Propane* (see Figure 4.14). Slot 6.
- Cargo to Gas Pipeline to E20: E20 is empty (see Figure 4.14) and is filled with C3C4R and C3I. Slot 7.
- Cargo to Gas Pipeline to E19: E19 is empty (see Figure 4.14). It is filled with C3I and C4I. Slot 8.
- E20 to Manifold to E13: E13 is initially empty (see Figure 4.14) and is filled with product from E20. The resulting blend satisfies the *Propane* specification. E20, having a larger capacity than E13, (see Table 4.3), initially receives most of the C3I product. Slot 8.

As the demands and timelines are presented in Table 4.4, the *Mixture* demand is fulfilled through sequential contributions from sphere E19 (initial inventory), along with blendings in spheres E20, E19, and E18. Meanwhile, the *Propane* demand is met by the initial quality in spheres E12, E14, and E13, supplemented by blending in E12. The sequence of deliveries is also illustrated in Figure 4.14.

A sensitivity analysis of the objective cost parameters is presented in Appendix 9. The results show that costs associated with maximum and minimum unfulfilled volumes have the greatest impact on the objective.

A sensitivity analysis of the number of spheres and the time horizon is presented in Appendix 10. Objective values appear to be more affected by changes in the time horizon, while solution time increases significantly when the number of spheres is

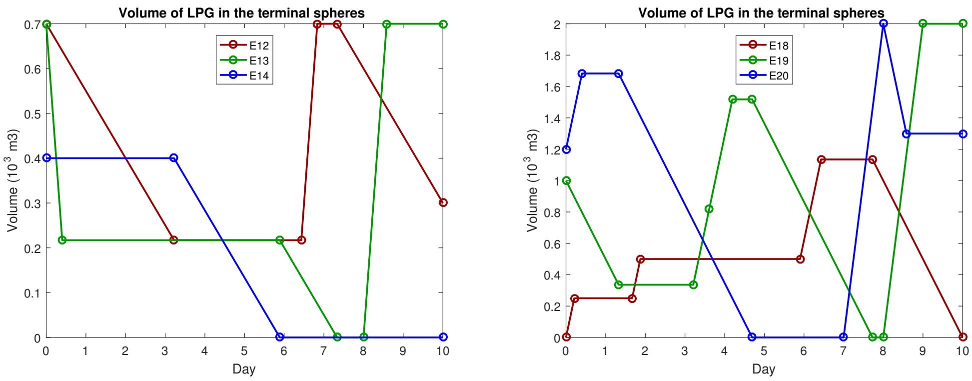

Figure 4.14: Case 2. Solution. LPG inventory in the terminal spheres (10^3 m^3).

increased.

4.6. Conclusions

This work introduces a new model to address the problem of LPG production, blending, and delivery. The available literature on this subject is limited and does not consider quality specifications of mixtures, pipeline transfer of products, or a terminal where each resource can store intermediate or final products based on the schedule, without a predefined allocation.

The model provides a flexible and representative framework for typical LPG facility operations.

In terms of modeling contributions, we extend the application of a continuous-time model with multi-operation sequencing to address a scheduling problem involving the production, blending, and delivery of LPG. Specifically, resources with overlapping operations are modeled as two virtual spheres. The pipeline's size, which connects the refinery to the terminal, allows us to assume a single quality remains in the gas pipe, thereby limiting bilinear constraints to mixtures and deliveries at the terminal. The need for binary variables is reduced in the blending model at the terminal by introducing the manifold model. Quality specifications are tracked through dedicated constraints. The objective aligns with best practices, focusing on minimizing deviations between deliveries and required demands. It also aims to reduce the number of operation changes, minimize operation end times, prioritize certain operation sequences to reduce symmetry, and adhere to resource best practices.

With reference to the solution methodology, we identify contributions that enhance feasibility. The constraints defined for the manifold and quality specifications establish relationships between certain binary variables during the linear step of the solution algorithm, thereby ensuring feasibility. Binary variables are fixed, and bilinear terms are incorporated into the model during the second step of the solving procedure. Solution time is improved through multiple strategies. First, symmetry-breaking constraints reduce processing time. Additionally, reducing the problem size enhances solution time by adopting the common practice of using subsets of resources to deliver specific final products. Furthermore, for a given time horizon, certain spheres may be excluded from consideration in specific studies. Finally, the Relax & Fix heuristics is applied to further improve solution times.

Examples are presented where exact solutions cannot always be achieved within practical time limits. Increasing the mipgap in the final step of the Relax & Fix heuristics has proven effective in reducing solution time while ensuring a good solution for practical applications.

The results show a small gap between the objective values from the linear and nonlinear stages of the algorithm.

Fixed blending specifications could improve solution time depending on other parameters such as initial inventories and flow rates. Increasing pipeline capacity may require modeling the sequence of packets in the pipe as in Zimberg et al. (2020, 2023b). The present work considers a configuration where the pipeline is small enough to assume a single packet occupies the entire length. Increasing the number of non-overlapping constraints may also raise the minimum number of time slots required to obtain a feasible solution. The entire analysis represents an extension of the current work.

Future work will focus on exploring additional heuristics and problem decomposition techniques to further improve solution quality and computational efficiency.

Capítulo 5

Conclusions and future work

5.1. Conclusions

The works presented in the previous chapters extend the concept of multi-operation sequencing to model two distinct problems. Chapter 3 addresses the reception, blending, pipeline delivery, and processing of crude oil at a refinery. Chapter 4, by contrast, focuses on the production, blending, and distribution of LPG. Both models introduce structural features that eliminate bilinear terms in the pipeline equations. Furthermore, they address real-world operational challenges, with representative examples provided, analyzed, and discussed. Chapter 4 also introduces the modeling of overlapping operations and dynamic storage assignment. New symmetry-breaking constraints are introduced to reduce the size of the search space.

The manifold model in the crude oil system suppresses pipeline nonlinearities by selecting a single quality from a predefined set of admissible qualities. Each quality is characterized by a set of specifications, and the total number of qualities is determined based on previous planning solutions and operational experience with blending. This approach linearizes the pipeline representation and simplifies computation.

Pipeline size can significantly affect the solution procedure, as computational complexity tends to escalate. The maximum parcel that can enter the pipeline within a given time slot is limited by pipeline capacity. As pipeline capacity increases, more time slots are required to adequately represent the problem, which in turn increases computational complexity. In addition, depending on the residence time of a given packet in the pipeline, the likelihood of mixtures forming at interfaces increases.

As noted in Chapter 1, planning requirements are incorporated into the objective function through quality and demand constraints. Additional objective terms account for minimum processing times and operational uniformity. These objectives align with best practices in the refining industry.

The influence of the objective terms on the final schedule depends strongly on the relative cost coefficients assigned to them. Poor calibration of these terms may distort the solution, whereas appropriate weighting can improve realism and operational robustness.

Regarding the solution methodology, a two-step solving approach is applied. In the first step, a Branch & Bound algorithm is used, and Relax & Fix heuristics are applied to improve solution times. In the second step, the binary variables are fixed, and the resulting nonlinear problem is solved.

The main computational challenges in the LPG model arise from the presence of bilinear terms, overlapping operations, and dynamic storage assignment. In the first solution step, bilinearities are neglected, and the manifold model and the specification constraints are applied to ensure that all qualities are transferable. In the second stage, binary variables are fixed and bilinear terms are introduced, which significantly increases the likelihood of obtaining a feasible solution.

The formulations and implementations developed in this work provide a flexible framework suitable for practical application. Schedule optimization can become part of the refinery's daily operations if it delivers high-quality solutions within a reasonable processing time. In this context, "quality" refers to adherence to best practices. It is also essential to construct a scheduling case within an acceptable timeframe; therefore, in practice, the software application should integrate with mass balance systems, SCADA, and laboratory data.

5.2. Future Work

Improving solution speed is essential for enhancing model performance. Future work should explore alternative heuristics and decomposition techniques to further reduce problem size and computation time. Additional areas of investigation include modeling the final system state in terms of product qualities and volumes, as well as improving the manifold model presented in Chapter 4.

In order to reduce model complexity and improve runtime a rolling-horizon approach is considered particularly attractive because it preserves integrality. However, when using a slot-based representation, careful analysis is required to understand how solutions evolve as the horizon is extended day by day. Lagrangian decomposition has also been successfully applied in related works and represents another promising methodology for reducing complexity and improving computational performance.

Referencias bibliográficas

Cafaro, V. G., Pautasso, P. C., Cerda, J., and Cafaro, D. C. (2019). Efficient planning of crude oil supplies through long-distance pipelines. *Computers & Chemical Engineering*, 122:203–217.

Castillo-Castillo, P. A. and Mahalec, V. (2016). Improved continuous-time model for gasoline blend scheduling. *Computers & Chemical Engineering*, 84:627–646.

Castro, P. (2015). Tightening piecewise mccormick relaxations for bilinear problems. *Computers & Chemical Engineering*, 72:300–311.

Castro, P. and Grossmann, I. (2014). Global optimal scheduling of crude oil blending operations with RTN continuous-time and multiparametric disaggregation. *Industrial & Engineering Chemistry Research*, 53(39):15127–15145.

Chen, X., Grossmann, I., and Zheng, L. (2012). A comparative study of continuous-time models for scheduling of crude oil operations in inland refineries. *Computers & Chemical Engineering*, 44:141–167.

Chen, Y., Ding, J., and Chai, T. (2022). A knowledge transfer based scheduling algorithm for large-scale refinery production. *IEEE Transactions on Industrial Informatics*, 18(2):869 – 879.

Chen, Y., Ding, J., and Chen, Q. (2024). A reinforcement learning based large-scale refinery production scheduling algorithm. *IEEE Transactions on Automation Science and Engineering*, 21(4):6041–6055.

Chen, Y. and Jiang, Z. (2024). A refinery production scheduling model with operation mode transition based on variable driven modelling approach. *Journal of Chemical Engineering of Japan*, 57(1):2379339.

de Assis, L. S., Camponogara, E., and Grossmann, I. E. (2021). A MILP-based clustering strategy for integrating the operational management of crude oil supply. *Computers & Chemical Engineering*, 145:10716.

de Assis, L. S., Camponogara, E., Menezes, B. C., and Grossmann, I. (2019). An MINLP formulation for integrating the operational management of crude oil supply. *Computers & Chemical Engineering*, 123:110–125.

de Assis, L. S., Camponogara, E., Zimberg, B., Ferreira, E., and Grossmann, I. E. (2017). A piecewise McCormick relaxation-based strategy for scheduling operations in a crude oil terminal. *Computers & Chemical Engineering*, 106:309–321.

Floudas, C. A. and Lin, X. (2004). Continuous-time versus discrete-time approaches for scheduling of chemical processes: a review. *Computers & Chemical Engineering*, 28(11):2109–2129.

Fourer, R., Gay, D. M., and Kernighan, B. W. (2002). *AMPL: A Modeling Language for Mathematical Programming*. Duxbury Press.

Franzoi, R. E., Menezes, B. C., Kelly, J. D., Gut, J. A., and Grossmann, I. E. (2024). Large-scale optimization of nonconvex minlp refinery scheduling. *Computers & Chemical Engineering*, 186:108678.

Harjunkoski, I., Maravelias, C. T., Bongers, P., Castro, P. M., Engell, S., Grossmann, I. E., Hooker, J., Méndez, C., Sand, G., and Wassick, J. (2014). Scope for industrial applications of production scheduling models and solution methods. *Computers & Chemical Engineering*, 62:161–193.

He, R., Xie, Y., Zhang, S., Xu, F., and Long, J. (2025). Knowledge-assisted hybrid optimization strategy of large-scale crude oil scheduling integrated production planning. *Computers & Chemical Engineering*, 192:108904.

Ikonen, T. J., Kim, B., Maravelias, C. T., and Harjunkoski, I. (2025). Supervised learning of the optimal objective function value in chemical production scheduling. *Industrial & Engineering Chemistry Research*, 64(8):4425–4438.

Jia, Z. and Ierapetritou, M. (2004). Efficient short-term scheduling of refinery operations based on a continuous time formulation. *Computers & Chemical Engineering*, 28:1001 – 1019.

Karuppiah, R. and Grossmann, I. (2006). Global optimization for the synthesis of integrated water systems in chemical processes. *Computers & Chemical Engineering*, 30:650–673.

Khor, C. S. and Varvarezos, D. (2017). Petroleum refinery optimization. *Optimization and Engineering*, 18:943–989.

Kolodziej, S., Castro, P., and Grossmann, I. (2013). Global optimization of bilinear programs with a multiparametric disaggregation technique. *Journal of Global Optimization*, 57:1039–1063.

Kondili, E., Pantelides, C., and Sargent, R. (1993). A general algorithm for short-term scheduling of batch operations—i. milp formulation. *Computers & Chemical Engineering*, 17(2):211–227. An International Journal of Computer Applications in Chemical Enginnering.

Lee, H., Pinto, J. M., Grossmann, I. E., and Park, S. (1996). Mixed-integer linear programming model for refinery short-term scheduling of crude oil unloading with inventory management. *Industrial & Engineering Chemistry Research*, 35(5):1630–1641.

Li, F., Yang, M., Du, W., and Dai, X. (2020). Development and challenges of planning and scheduling for petroleum and petrochemical production. *Frontiers of Engineering Management*, 7(3):373.

Li, J. and Karimi, I. A. (2011). Scheduling gasoline blending operations from recipe determination to shipping using unit slots. *Industrial & Engineering Chemistry Research*, 50(15):9156–9174.

Li, J., Karimi, I. A., and Srinivasan, R. (2010). Recipe determination and scheduling of gasoline blending operations. *AIChE Journal*, 56(2):441–465.

Li, J., Misener, R., and Floudas, C. (2011). Continuous-time modeling and global optimization approach for scheduling of crude oil operations. *American Institute of Chemical Engineering*, 58(1):205–226.

Maravelias, C. T. and Sung, C. (2009). Integration of production planning and scheduling: Overview, challenges and opportunities. *Computers & Chemical Engineering*, 33(12):1919–1930. FOCAPO 2008 – Selected Papers from the Fifth International Conference on Foundations of Computer-Aided Process Operations.

McCormick, G. P. (1976). Computability of global solutions to factorable non-convex programs: Part I — Convex underestimating problems. *Mathematical Programming*, 10:147–175.

Moro, L. F. L. and Pinto, J. M. (2004). Mixed-integer programming approach for short-term crude oil scheduling. *Industrial and Engineering Chemistry Research*, 43(1):85–94.

Mouret, S., Grossmann, I. E., and Pestiaux, P. (2009). A novel priority-slot based continuous-time formulation for crude-oil scheduling problems. *Industrial and Engineering Chemistry Research*, 48:8515–8528.

Mouret, S., Grossmann, I. E., and Pestiaux, P. (2011). Time representations and mathematical models for process scheduling problems. *Computers & Chemical Engineering*, 35:1038–1063.

Oddsdottir, T. A., Grunow, M., and Akkerman, R. (2013). Procurement planning in oil refining industries considering blending operations. *Computers & Chemical Engineering*, 58:1–13.

Oliveira, F., Hamacher, S., and Almeida, M. (2011). Process industry scheduling optimization using genetic algorithm and mathematical programming. *Journal of Intelligent Manufacturing*, 22:801–813.

Pantelides, C. (1994). Unified frameworks for optimal process planning and scheduling. In *Second Int. Conf. Found. Comput.-Aided Process Operat.*, Crested Butte, Colorado, USA.

Pinto, J. M. and Grossmann, I. E. (1998). Assignment and sequencing models for the scheduling of process systems. *Annals of Operations Research*, 81:433–466.

Pinto, J. M. and Moro, L. F. L. (2000). A mixed integer model for LPG scheduling. *Computer Aided Chemical Engineering*, 8:1141–1146.

Reklaitis, G. V. (1996). Overview of scheduling and planning of batch process operations. In Reklaitis, G. V., Sunol, A. K., Rippin, D. W. T., and Hortaçsu, Ö., editors, *Batch Processing Systems Engineering*, pages 660–705, Berlin, Heidelberg. Springer Berlin Heidelberg.

Schilling, G. and Pantelides, C. (1996). A simple continuous-time process scheduling formulation and a novel solution algorithm. *Computers & Chemical Engineering*, 20:S1221–S1226. European Symposium on Computer Aided Process Engineering-6.

Shah, N. (1996). Mathematical programming techniques for crude oil scheduling. *Computers & Chemical Engineering*, 20:S1227–S1232. European Symposium on Computer Aided Process Engineering-6.

Shah, N. K. and Ierapetritou, M. G. (2015). Lagrangian decomposition approach to scheduling large-scale refinery operations. *Computers & Chemical Engineering*, 79:1–29.

Velez, S. and Maravelias, C. (2013). Multiple and nonuniform time grids in discrete-time MIP models for chemical production scheduling. *Computers & Chemical Engineering*, 53:70–85.

Wang, C., Yang, M., Dai, X., Fan, C., and Du, W. (2024). A novel multi-operation sequencing formulation for the crude oil scheduling problem with restricted overlapping constraints. *Computers & Chemical Engineering*, 187:108714.

Wächter, A. and Biegler, L. T. (2006). On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming. *Mathematical Programming*, 106:25–57.

Yadav, S. and Shaik, M. (2012). Short-term scheduling of refinery crude oil operations. *Industrial & Engineering Chemistry Research*, 51:9287–9299.

Zhang, S. and Xu, Q. (2015). Refinery continuous-time crude scheduling with consideration of long-distance pipeline transportation. *Computers & Chemical Engineering*, 75:74 – 94.

Zhang, X. and Sargent, R. (1996). The optimal operation of mixed production facilities—a general formulation and some approaches for the solution. *Computers & Chemical Engineering*, 20(6):897–904. Fifth International Symposium on Process Systems Engineering.

Zhao, Y., Wu, N., and Li, Z. (2017). A novel solution approach to scheduling of crude-oil operations based on an improved continuous-time formulation. In *Proceedings of the IEEE 14th International Conference on Networking, Sensing and Control (ICNSC)*, pages 482–487.

Zimberg, B., Camponogara, E., and Ferreira, E. (2015). Reception, mixture, and transfer in a crude oil terminal. *Computers & Chemical Engineering*, 82:293 –302.

Zimberg, B., Camponogara, E., and Monzón, P. (2023a). Liquefied petroleum gas scheduling, blending, and optimization model. In *Proceedings of the IFORS 2023 Conference*, pages 145–146, Santiago de Chile, Chile. IFORS.

Zimberg, B., Camponogara, E., and Monzón, P. (2025). A continuous-time scheduling model for liquefied petroleum gas production, blending and delivery. *Industrial & Engineering Chemistry Research*, 64(40):19580–19599.

Zimberg, B., Ferreira, E., and Camponogara, E. (2023b). A continuous-time formulation for scheduling crude oil operations in a terminal with a refinery pipeline. *Computers & Chemical Engineering*, 178:108354.

Zimberg, B., Ferreira, E., Camponogara, E., and Fonseca, M. (2020). Modelo planificación de la logística de recepción, mezcla y entrega de crudo en la terminal de Uruguay. Informe. *ANII FSE 1 2016 1 131614*.

Zimberg, B., Testuri, C. E., and Ferrari, G. (2019). Stochastic modeling of fuel procurement for electricity generation with contractual terms and logistics constraints. *Computers & Chemical Engineering*, 123:49 – 63.

Capítulo 6

Notation for the Crude Oil Model

6.1. Indexes and Sets

The following tables define the sets and indexes that appear in the model. Several dimensional units may be considered for parameters and variables, provided the required consistency in the constraints formulation. In particular, the objective function cost terms are adimensional.For instance, in the cases provided, volume data is given in (10^3 m^3), Time (day), flowrate (10^3 m^3 /day) and quality composition is expressed as (%).

Table 6.1: Indexes and Sets

Element	Description
Slot sets:	
N	Number of priority slots
$\mathcal{S}$	Slot set $\mathcal{S} = \{1, \ldots, N\}$
$\mathcal{S}_0$	Slot set corresponding to the initial distribution of qualities along the pipeline
$\mathcal{S}_1$	Extended slot set $\mathcal{S}_1 = \mathcal{S} \cup \mathcal{S}_0$
$\mathcal{S}_2$	Extended slot set $\mathcal{S}_2 = \mathcal{S} \cup \{N+1\}$
Operation sets:	
$\mathcal{W}$	All operations except maintenance
$\mathcal{W}_1$	All operations including maintenance

$\mathcal{W}_V$	Vessel unloading operations
$\mathcal{W}_B$	Subsea pipeline unloading operations
$\mathcal{W}_T$	Tank-to-manifold transfer operations
$\mathcal{W}_{PI}$	Pipeline inlet operations
$\mathcal{W}_{PO}$	Pipeline output operations
$\mathcal{W}_D$	Distillation operations
$\mathcal{W}_O$	Output operations
$\mathcal{W}_M$	Maintenance operations set

Resource sets:

$\mathcal{R}$	All resources
$\mathcal{R}_V$	Vessels
$\mathcal{R}_B$	Buoy and sub sea pipeline
$\mathcal{R}_T$	Storage tanks in terminal
$\mathcal{R}_I$	Manifold to pipeline input
$\mathcal{R}_P$	Pipelines
$\mathcal{R}_C$	Charging tanks in refinery
$\mathcal{R}_D$	Distillation units
$\mathcal{R}_O$	Output resources

Quality sets:

$\mathcal{Q}$	All qualities
$\mathcal{Q}_T$	Terminal crude oil qualities
$\mathcal{Q}_C$	Refinery crude oil qualities
$\mathcal{Q}\mathcal{Q}_T \subseteq \mathcal{Q}_T \times \mathcal{Q}_T$	Pairs (q_1, q_2) of incompatible crudes in terminal tanks
$\mathcal{Q}\mathcal{Q}_C \subseteq \mathcal{Q}_C \times \mathcal{Q}_C$	Pairs (q_1, q_2) of incompatible crudes in refinery tanks

Compound sets:

$\mathcal{C} \subseteq 2^{\mathcal{W}}$	Cliques of non-overlapping operations
$\mathcal{A}(\mathcal{W}_1 \times \mathcal{W}_1)$	Set of adjacent operations
$\mathcal{M}(\mathcal{R}_T \cup \mathcal{R}_C)$	Maintenance operation resource assignment
$\mathcal{I}_r$	Inlet transfer operations on resource $r \in \mathcal{R}$

$\mathcal{O}_r$	Outlet transfer operations on resource $r \in \mathcal{R}$
$\mathcal{Q}_r$	Allowed crude qualities in resource $r \in \mathcal{R}$

6.2. Parameters

Table 6.2: Detailed description of model parameters.

Parameter	Description
Time parameters:	
T	Scheduling end time
Ts_r	Allowed start time to fill output tank r
Te_r	Allowed end time to fill output tank r
Dt_r	Maximum demurrage time to fill output tank r
Tt_C	Wait time between operations in slot $C \in \mathcal{C}$
TMs_w	Allowed start time to perform maintenance operation w
TMe_w	Allowed end time to perform maintenance operation w
Operation parameters:	
$\underline{DV_w}$	Minimum volume transfer in operation w
$\overline{DV_w}$	Maximum volume transfer in operation w
$\underline{VF_r}$	Minimum volume flow transfer in operation w
$\overline{VF_r}$	Maximum volume flow transfer in operation w
$\underline{DVp}$	Minimum pipeline delivery to charging tanks
Resource parameters:	
$\underline{V_r}$	Minimum volume in resource r
$\overline{V_r}$	Maximum volume in resource r
$\overline{Vs}$	Terminal volume capacity, e.g. $\overline{Vs} = \sum_{r \in \mathcal{R}_T} \overline{V_r}$

$\overline{R}$	Maximum number of tanks allowed for simultaneous delivery to the pipeline
$Vqr_{r,q}$	Required volume of crude type $q \in \mathcal{Q}_C$ in output resource $r \in \mathcal{R}_O$
Vr_r	Required volume in output resource r , $Vr_r = \sum_{q \in \mathcal{Q}_C} Vqr_{r,q}$
$\overline{Vq_q}$	Maximum volume of quality $q \in \mathcal{Q}_C$ delivered by the terminal
$Vq0_{r,q}$	Initial inventory of crude type $q \in \mathcal{Q}$ in resource $r \in \mathcal{R}$
$V0_r$	Initial inventory in resource r , $Vr_r = \sum_{q \in \mathcal{Q}} Vq0_{r,q}$
$Vpq0^j_{r,q}$	Initial inventory of crude type $q \in \mathcal{Q}_C$ in the pipeline $r \in \mathcal{R}_P$ that enters in slot $j \in \mathcal{S}_0$
$Vqf_{r,q}$	Final inventory of crude type $q \in \mathcal{Q}_C$ in resource $r \in \mathcal{R}_C \cup \mathcal{R}_P$
Vf_r	Final inventory in resource r , $Vf_r = \sum_{q \in \mathcal{Q}_C} Vqf_{r,q}$
$\underline{Vfc}$	Minimum final volume in charging tanks

Quality parameters:

$\overline{Q_T}$	Maximum number of allowed qualities in terminal tanks
$\overline{Q_C}$	Maximum number of allowed qualities in refinery tanks
$\overline{Q_{qr,qt}}$	Maximum content of quality $qt \in \mathcal{Q}_T$ in quality mixture $qr \in \mathcal{Q}_C$
$\underline{Q_{qr,qt}}$	Minimum content of quality $qt \in \mathcal{Q}_T$ in quality mixture $qr \in \mathcal{Q}_C$

Maintenance:

Mo_w	Type of maintenance operation w

Objective function costs:

Cd_r	Demurrage time gap in output resource r
Cm_w	Non-fulfillment of maintenance operation w
Cv_r	Volume gap in output resource r
$Cvq_{r,q}$	Volume gap of crude quality $q \in \mathcal{QR}$ in output resource r
Cvm	Reception of crude oil in non empty tanks
Cvf	Final volume gap
$Cvqf$	Final volume gap by crude quality
Cvr	Cost due to not filling tanks during crude reception

Cdt	Demurrage time during crude reception and delivery
Cts	Final time cost
$Cxvp$	Number of pipeline batches
$Cxvq$	Number of qualities in the terminal tanks
Cz	Number of performed operations

6.3. Variables

Table 6.3 depicts the variables, which are all real and nonzero, unless otherwise stated. Binary variables are 0 assigned unless specified.

Table 6.3: Model Variables

Variable	Description
Time variables. Operation $w \in \mathcal{W}_1$ and slot $i \in \mathcal{S}$	
ts_w^i	start time of operation w if assigned to slot i, 0 otherwise
te_w^i	end time of operation w if assigned to slot i, 0 otherwise
$\underline{ts_r}$	minimum start time of unloading operations for vessel $r \in \mathcal{R}_V$
$\overline{te_r}$	maximum end time of unloading operations for vessel $r \in \mathcal{R}_V$
td_w^i	time duration of operation w if assigned to slot i, 0 otherwise
dt_r	delay time of unloading or output operation, $r \in \mathcal{R}_T \cup \mathcal{R}_O$
Operation variables. Unless mentioned, $w \in \mathcal{W}$ and slot $i \in \mathcal{S}$	
z_w^i	binary, 1 if operation $w \in \mathcal{W}_1$ is performed in slot i.
dv_w^i	total volume transferred by operation w in slot i
$dvq_{w,q}^i$	volume of crude type $q \in \mathcal{Q}$ transferred by operation w in slot i
$dvbq_{w,q}^i$	volume of crude type $q \in \mathcal{Q}_T$ transferred to the buoy by an unloading operation $w \in \mathcal{W}_V$ in slot i.
Resource variables. Volume in slot i refers to volume before operations	

v_r^i Volume in resource $r \in \mathcal{R}$, slot $i \in S_2$

$vf_{r,q}$ final crude volume in resource $r \in \mathcal{R}_T \cup \mathcal{R}_C \cup \mathcal{R}_P$.

$vfq_{r,q}$ final volume of crude type $q \in \mathcal{Q}$ in resource $r \in \mathcal{R}_T \cup \mathcal{R}_C \cup \mathcal{R}_P$.

$vq_{r,q}^i$ accumulated level of crude $q \in Q$ in tank $r \in \mathcal{R}$, slot $i \in \mathcal{S}_2$

vm_r^i crude volume in tank $r \in \mathcal{R}_T \cup \mathcal{R}_C$ and slot $i \in \mathcal{S}$ when an operation is activated, 0 otherwise.

vr_r^i remaining volume to fill tank $r \in \mathcal{R}_T \cup \mathcal{R}_C$ and slot $i \in \mathcal{S}$.

$xvq_{r,q}^i$ binary, 1 if resource $r \in \mathcal{R}_T \cup \mathcal{R}_C$ has non-zero volume of crude quality $q \in \mathcal{Q}$ in slot $i \in \mathcal{S}_2$.

$yvq_{r,q}^j$ binary, 1 if volume that enters the pipeline $r \in \mathcal{R}_P$ in slot $j \in \mathcal{S}$, has quality $q \in \mathcal{Q}_C$.

Pipeline variables:

$dvp_r^{j,i}$ Volume of packet that entered the pipeline $r \in \mathcal{R}_P$ in slot $j \in \mathcal{S}_1$, being transferred out of the pipeline, partially or fully, in slot $i \in \mathcal{S}$.

$dvpq_{r,q}^{j,i}$ Volume of crude type q of packet that entered the pipeline $r \in \mathcal{R}_P$, in slot $j \in \mathcal{S}_1$ being transferred out of the pipeline in slot $i \in \mathcal{S}$.

$vp_r^{j,i}$ Volume of packet that enters the pipeline $r \in \mathcal{R}_P$, at slot $j \in \mathcal{S}_1$ and remains in the pipeline, partially or fully, in slot $i > j$, $i \in \mathcal{S}_2$.

$vpq_{r,q}^{j,i}$ Volume of crude type $q \in \mathcal{Q}_C$ in packet that enters the pipeline $r \in \mathcal{R}_P$, in slot $j \in \mathcal{S}_1$ and remains in the pipeline until slot $i > j$, $i \in \mathcal{S}_2$.

$xvp_r^{j,i}$ binary, 1 if a volume that enters the pipeline $r \in \mathcal{R}_P$, in slot $j \in \mathcal{S}_1$, leaves the pipeline in slot $i > j$, 0 otherwise, $j < i$, $i \in S$.

Objective cost variables

cdt Demurrage time.

cm Compliance with maintenance operations.

$cost$ Total cost.

cts Unloading start time.

cv Volume gap.

cvf	Final volume gap.
cvp	Idle pipeline.
cvq	Quality gap.
$cvqf$	Final quality gap.
cvm	Tank at minimum capacity before reception.
cvr	Tank at maximum capacity after reception.
$cxvp$	Number of packages delivered through the pipeline.
$cxvq$	Mixture of qualities in terminal tanks.
cz	Number of operations.
$dvol_r$	Absolute volume gap error for resource $r \in \mathcal{R}_O$.
$dqual_{r,q}$	Absolute quality gap error for quality $q \in \mathcal{Q}_C$ and resource $r \in R_O$.
$dvolf_r$	Absolute volume gap error for tank $r \in \mathcal{R}_C \cup \mathcal{R}_P$.
$dqualf_{r,q}$	Absolute quality gap error for quality $q \in \mathcal{Q}_C$ and tank $r \in \mathcal{R}_C \cup \mathcal{R}_P$.

Capítulo 7

Notation for the LPG Model

7.1. Indexes and Sets

Table 7.1 defines the sets and indexes of the model.

Table 7.1: Indexes and Sets

Element	Description
Slot sets:	
N	Number of priority slots
$\mathcal{S}$	Slot set $\mathcal{S} = \{1, \ldots, N\}$
$\mathcal{S}_1$	Extended slot set $\mathcal{S}_1 = \mathcal{S} \cup \{N+1\}$
Operation sets:	
$\mathcal{W}$	All operations
$\mathcal{W}_R$	Refinery operations
$\mathcal{W}_{BI}$	Manifold inlet operations
$\mathcal{W}_{BO}$	Manifold outlet operations
$\mathcal{W}_{GI}$	Pipeline inlet operations
$\mathcal{W}_{GO}$	Pipeline outlet operations
$\mathcal{W}_{SI}$	Terminal spheres to *Mixture* delivery
$\mathcal{W}_{SO}$	*Mixture* delivery
$\mathcal{W}_{PI}$	Terminal spheres to *Propane* delivery
$\mathcal{W}_{PO}$	*Propane* delivery

$\mathcal{W}_{RB} \subseteq \mathcal{W}$	Priority resource set, $\mathcal{W}_{RB} = \mathcal{W}_R \cup \mathcal{W}_{GI} \cup \mathcal{W}_{GO} \cup \mathcal{W}_{BI} \cup \mathcal{W}_{BO}$
$\mathcal{W}_A \subseteq \mathcal{W}$	Set of operations to model symmetry-breaking constraints

Resource sets:

$\mathcal{F}$	Set of blocked resources
$\mathcal{R}$	All resources
$\mathcal{R}_V$	LPG cargo vessel
$\mathcal{R}_U$	Refinery production
$\mathcal{R}_{UB}$	Refinery butane production
$\mathcal{R}_{UP}$	Refinery propane production
$\mathcal{R}_{US}$	Refinery mixture production
$\mathcal{R}_R$	Refinery spheres
$\mathcal{R}_{RB}$	Refinery butane spheres
$\mathcal{R}_{RP}$	Refinery propane spheres
$\mathcal{R}_{RS}$	Refinery mixture spheres
$\mathcal{R}_G$	Gas pipe
$\mathcal{R}_B$	Blending manifold (Manifold)
$\mathcal{R}_T$	Terminal spheres
$\mathcal{R}_D$	*Mixture* and *Propane* delivery manifold
$\mathcal{R}_{DS}$	*Mixture* delivery manifold
$\mathcal{R}_{DP}$	*Propane* delivery manifold
$\mathcal{R}_O$	*Mixture* and *Propane* deliveries
$\mathcal{R}_{OS}$	*Mixture* deliveries
$\mathcal{R}_{OP}$	*Propane* deliveries
$\mathcal{R}_Z$	Set of resources with overlapping inlet and outlet operations

Quality sets:

$\mathcal{Q}$	Set of qualities
$\mathcal{KQ}$	Quality specifications

Sets over other sets:

$\mathcal{A} \subseteq 2^{\mathcal{W}_A}$	Pairs of operations $w \in \mathcal{W}_A$ for symmetry breaking constraints

$\mathcal{B}(\mathcal{R}_D)$	Resources allowed for delivery to $r \in \mathcal{R}_D$
$\mathcal{C} \subseteq 2^{\mathcal{W}}$	Cliques of non-overlapping operations
$\mathcal{D} \subseteq 2^{\mathcal{W}}$	Pairs of input and output operations for time delay modeling
$\mathcal{E}(\mathcal{R}_Z)$	Virtual resources for $r \in \mathcal{R}_Z$
$\mathcal{I}(\mathcal{R})$	Input transfer operations of resource $r \in \mathcal{R}$
$\mathcal{O}(\mathcal{R})$	Output transfer operations of resource $r \in \mathcal{R}$
$\mathcal{QQ}$	Pairs of non compatible qualities

7.2. Parameters

Table 7.2 describes the parameters of the model. Notation $(qual)$ refers to property units.

Table 7.2: Parameters

Parameter	Description
Time parameters:	
T	Scheduling time horizon (day)
Te_r	Allowed end time to fill output sphere $r \in \mathcal{R}_V \cup \mathcal{R}_O$ (day)
Ts_r	Allowed start time to fill output sphere $r \in \mathcal{R}_V \cup \mathcal{R}_O$ (day)
Tw	Waiting time between certain input and output operations (day)
Operation parameters:	
$\overline{DV}_w$	Maximum volume transfer in operation $w \in \mathcal{W}_{GI}$ (10^3m^3)
$\underline{DV}_w$	Minimum volume transfer in operation $w \in \mathcal{W}_{GI}$ (10^3m^3)
$\overline{VF}_w$	Maximum volume flow transfer in operation $w \in W$ $(10^3m^3/day)$
$\underline{VF}_w$	Minimum volume flow transfer in operation $w \in W$ $(10^3m^3/day)$
Resource parameters:	
$\overline{V_r}$	Maximum volume of resource $r \in \mathcal{R}$ (10^3m^3)
$\underline{V_r}$	Minimum volume of resource $r \in \mathcal{R}$ (10^3m^3)
$\underline{Vfr_r}$	Minimum total final volume, $r \in R_D$ (10^3m^3)

$\overline{Vr}$	Terminal volume capacity, e.g. $\overline{Vr} = \sum_{r \in \mathcal{R}_S} \overline{V_r}$
Vr_r	Required volume for delivery from resource $r \in \mathcal{R}_O$, $Vr_r = \sum_{w \in \mathcal{I}_r} \overline{VF_w}(Te_r - Ts_r)$

Initial conditions:

$V0_r$	Initial volume in resource $r \in \mathcal{R}_V$ $(10^3 m^3)$
$Vq0_{r,q}$	Initial volume of quality $q \in Q$ in resource $r \in R$ $(10^3 m^3)$

Objective function costs:

Cte	End time operation cost
Cv	Unfulfilled delivery
Cvm	Reception at minimum volume
Cvr	Reception and spare capacity cost
Czb_w	Priority operation cost for z_w^i , $w \in \mathcal{W}_{RB}$
Czz	Cost of changing operations

Quality parameters:

$K_{q,k}$	contribution of quality $q \in Q$ to specification $k \in \mathcal{KQ}$ $(qual)$
$\overline{K_{r,k}}$	Maximum value of specification $k \in \mathcal{KQ}$ for resource $r \in R_D$ $(qual)$
$\underline{K_{r,k}}$	Minimum value of specification $k \in \mathcal{KQ}$ for resource $r \in R_D$ $(qual)$

7.3. Variables

Table 7.3 describes the variables which are all real and nonzero, unless otherwise stated. All variables will be assigned to the value 0 if not active unless specified.

Table 7.3: Model Variables

Variable	Description
	Binary variables, assignments (0 unless specified).
$xdvq_{w,q}^i$	1 if volume transferred in operation $w \in \mathcal{I}_r$, $r \in \mathcal{R}_B$ has a non-zero volume of quality $q \in Q$ at slot $i \in S$
$xvq_{r,q}^i$	1 if resource $r \in \mathcal{R}_T \cup \mathcal{R}_V$ has a non-zero volume of quality $q \in Q$ at slot $i \in S_1$

xvk_{r,r_1}^i 1 if quality in resource $r \in \mathcal{R}_T$ satisfies all the specifications $k \in \mathcal{KQ}$ for $r_1 \in \mathcal{R}_D$ at slot $i \in S_1$

z_w^i 1 if operation $w \in \mathcal{W}$ is assigned to slot $i \in S$

Time variables:

td_w^i Time duration of operation $w \in \mathcal{W}$ at slot $i \in S$, 0 otherwise (day)

te_w^i End time of operation $w \in \mathcal{W}$ at slot $i \in S$, 0 otherwise (day)

ts_w^i Start time of operation $w \in \mathcal{W}$ at slot $i \in S$, 0 otherwise (day)

$\overline{te_r}$ maximum end time of unloading operations for vessel $r \in \mathcal{R}_V$

$\underline{ts_r}$ minimum start time of unloading operations for vessel $r \in \mathcal{R}_V$

Operation variables:

dv_w^i Total volume transferred in operation $w \in \mathcal{W}$ and slot $i \in S$ $(10^3 m^3)$

$dvq_{w,q}^i$ Volume of quality type $q \in \mathcal{Q}$ transferred in operation $w \in \mathcal{W}$ and slot $i \in S$ $(10^3 m^3)$

dzz_w^i Absolute error between consecutive operations, $w \in \mathcal{W}_{RB}$, $i \in S$

$dvpq_{w,q}^i$ Volume of quality $q \in \mathcal{Q}$ exiting the pipeline during operation $w \in \mathcal{W}_{GO}$ and slot $i \in S$

$dvol_r^i$ Unfulfilled delivery volume for $r \in \mathcal{R}_O$ in slot $i \in S$ $(10^3 m^3)$

Resource variables:

v_r^i Volume in resource $r \in \mathcal{R}$ and slot $i \in S_1$ $(10^3 m^3)$

$vq_{r,q}^i$ Volume of quality $q \in Q$ in sphere $r \in \mathcal{R}$ and slot $i \in S_1$ $(10^3 m^3)$

vr_r^i Spare capacity in sphere $r \in \mathcal{R}_R$ at slot $i \in S$ if an inlet operation is performed, 0 otherwise $(10^3 m^3)$

vm_r^i Remaining volume in sphere $r \in \mathcal{R}_R$ at slot $i \in S$ if an inlet operation is performed, 0 otherwise $(10^3 m^3)$

Capítulo 8

A real LPG operation

Table 8.1 presents an example of operations over two consecutive days. The refinery product C3R is continuously sent to sphere E2, while C4R is received into sphere E1, and C3C4R is delivered to E3. Sphere E4 does not receive any product during this period.

The operations proceed as follows:

- Between hours 7 and 11, E2 delivers C3R to the Gas pipe. At the same time, E20 receives C3C4R, which was previously stored in the pipeline.
- Next, until hour 15, C3R exits the Gas pipe and is transferred to sphere E13.
- Between hours 20 and 28, E3 delivers C3C4R to the pipeline. Meanwhile, the initial content of C3R is sent to E20 between hours 20 and 24. Subsequently, C3C4R is received in E21. Between hours 28 and 36, E21 continues to receive C3C4R, while E4 delivers C4R to the Gas pipe.
- Regarding final product deliveries, on the first day at the terminal facility, *Propane* and *Mixture* are sold from spheres E15 and E21, respectively. On the following day, spheres E14 and E19 deliver these products to the market. The process will continue with a terminal sphere receiving C4R. Once sphere E21 finishes selling *Mixture* to the market, it begins receiving fresh product at night.

This example illustrates that final qualities can be achieved in a given sphere by mixing streams received from the gas pipeline. E13 receives C3R, E20 receives C3C4R and C3, while E21 receives C3C4R. These mixtures meet the quality requirements of the marketable products, as shown in Section Experiments, Table 4.5.

Table 8.1: LPG operations on two consecutive days.

Quality	Operation	Period (h)
C3R	Ref-E2	0-48
C4R	Ref-E1	0-48
C3C4R	Ref-E3	0-48
Mixture	E21-Mkt	6-16
Propane	E15-Mkt	6-16
C3R	E2-Pipe	7-11
C3C4R	Pipe-E20	7-11
C3R	E2-Pipe	11-15
C3R	Pipe-E13	11-15
C3C4R	E3-Pipe	20-24
C3R	Pipe-E20	20-24
C3C4R	E3-Pipe	24-28
C3C4R	Pipe-E21	24-28
C4R	E4-Pipe	28-36
C3C4R	Pipe-E21	28-36
Mixture	E19-Mkt	30-40
Propane	E14-Mkt	30-40

Capítulo 9

Sensitivity Analysis of LPG Costs

Table 9.1 presents the sensitivity analysis of the objective cost parameters. Each parameter is increased or decreased by 10 % relative to the values presented in Section Experiments. The reference case is shown in italics. All cases were solved using the Relax & Fix strategy with a step size of 6. The resulting change in the objective is reported in the last column of the table. Costs associated with maximum and minimum unfulfilled volume have the most significant impact on the objective.

Table 9.1: Sensitivity on the objective cost parameters (adimensional)

Operation Change	Volume Delivered	Volume Max.	Volume Min.	End Time	Operation Dependent	Objective	Objective change (%)
0.9	*0.9*	*0.5*	*0.5*	*0.5*	*Base*	*7.83750*	
0.81						7.81557	-0.3
0.99						7.86596	0.4
	0.81					7.83775	0
	0.99					7.84296	0.1
		0.45				7.12883	-9
		0.55				7.98099	1.8
			0.45			7.26797	-7.3
			0.55			7.85567	0.2
				0.45		7.82502	-0.2
				0.55		7.87596	0.5
					0.9 Base	7.83749	0
					1.1 Base	7.84307	0.1

Capítulo 10

LPG Model Sensitivity: Spheres and Time Horizon

Based on the problem presented in the Section Experiments, Table 10.1 initially compares cases in which the time horizon is extended from 10 to 12. The reference case is shown in italics. All cases were solved using the Relax & Fix strategy with a step size of 5. Each case considers 6 refinery and 6 terminal spheres, except for the last two, in which the number of terminal spheres is increased to 7 and 8, respectively.

Objective values are more affected by changes in the time horizon, while solution time increases significantly when the number of spheres is increased.

Table 10.1: Sensitivity to time horizon and additional spheres (last two rows). An asterisk (*) indicates that the maximum solution time was exceeded.

T Day	# Spheres	Mipgap %	Last mipgap %	Rows	Columns	Non zeros	Binaries	Time	Objective	Objective change (%)
10	*12*	*0.5*	*1*	*9730*	*3549*	*42399*	*966*	*951*	*7.84015*	
11	12	0.5	1	9730	3549	42399	966	2733	7.50079	-4.3
12	12	0.5	1	9730	3549	42399	966	2027	8.37495	7.1
10	13	0.5	1	11180	3958	48368	1052	31426	7.87937	0.5
10	14	0.5	1	12635	4378	54481	1129	42938*	7.90784	0.9