Multi-resource allocation: analysis of a paid spectrum sharing approach based on fluid models

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Abstract—**Nowadays industrial and academic communities are focusing much of their efforts to define the main characteristics of 5G. Cognitive Radio, offering the possibility to significantly increase the spectrum efficiency, will play a key role in the whole range of IoT communications (one of the use cases of 5G networks). While Cognitive Radio is one of the most discussed topics in contemporary spectrum management, there are still many issues and challenges to be solved, even more when we think about large networks. In this context, one of the main question that motivates this work is: how to encourage the spectrum sharing behavior of primary users (PUs)? With this in mind, we study a paid-sharing approach where secondary users (SUs) pay for spectrum utilization. We assume a preemptive system where PUs have strict priority over SUs, and the affected SUs will then be reimbursed, implying some cost for the PUs service provider. This paper bears on the analysis of the behavior of the system where the number of users is arbitrary large and an admission control policy over SUs is applied. We develop a computationally efficient methodology to find an accurate estimation of the optimal admission control boundary based on fluid limits.**

Index Terms—**admission control, cognitive radio, dynamic spectrum allocation, fluid limit**

I. INTRODUCTION

IN the near future, the telecommunications industry will be faced with two big challenges: a need for more radio spectrum and an ever-increasing demand for data. Spectrum, however, is a finite resource. The concept of cognitive networks has emerged as one of the efficient means for utilizing the scarce spectrum by allowing spectrum sharing between a licensed primary network and a secondary network. We envision that soon this paradigm will become a reality, this is reflected by the number of publications and conferences about cognitive radio in 5G wireless communications [1], [2], [3], [4], [5], [6]. However, several challenges still need to be solved, new standards must be

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approved, new regulation is needed, and the wireless industry will have to develop the necessary equipment. Thinking about the evolution of wireless technologies, rapidly comes to our minds a world with more and more devices connected to the Internet. In this scenario, Internet of things (IoT) paradigm poses new challenges to the way in which spectrum management is approached and implemented. Experts estimate that the IoT will consist of about 30 billion connected objects by 2022[7], the vast majority through wireless networks. In this sense, there are several issues that need to be addressed before cognitive radio technology can be used for Internet of things. This work contributes in this direction developing methodologies to evaluate the spectrum allocation problem where the number of wireless users is "unlimited".

The concept of Cognitive Radio (CR) is not new, it was first introduced by Mitola [8] in his PhD thesis. CR represents a promising technology which, based on dynamic spectrum access, strives at solving two important problems: spectrum under-utilization and spectrum scarcity. In this paradigm we can identify two classes of users: primary and secondary. Primary users (PUs) are those for which a certain portion of the spectrum has been allocated to (often in the form of a paid contract). Secondary users (SUs) are devices capable of detecting unused licensed bands and adapt their transmission parameters for using them.

The fundamental concept behind CR networks is to allow SUs to use the licensed resource in the absence of PUs in order to improve the spectrum utilization. The key requirement in this context is that the PUs ought to be as little affected as possible by the presence of SUs. In the ideal case, PUs would use the network without being affected at all by SUs, which will in turn make use of whatever resources are left available. Although a rather large number of solutions already exist in the literature, this dynamic spectrum allocation (DSA) is still one

of the main challenges in the design of CR due to the requirement of "peaceful" coexistence of both types of users [9], [10].

There are roughly two different approaches for dynamic spectrum sharing: paid-sharing or free-sharing. An effective collaboration is achieved when the spectrum sharing behavior of primary service providers is stimulated. It is essential to have an incentive framework, then paid-sharing methods (i.e. when SUs pay for spectrum utilization) seem to be more suitable for this purpose.

Keeping this in mind, the focus of our analysis is a paid spectrum sharing method based on admission control decisions over SUs. In particular, we consider a scenario without spatial reuse of channels where if a PU does not find enough free channels in the system, at least one of the SUs will be deallocated immediately. In other words, a preemptive system is considered where some SUs communications will be aborted whenever a PU needs certain amount of bandwidth and the system has an insufficient number of free bands. In addition to that, the affected SUs (the ones that are deallocated before their services were finished) will be reimbursed, implying some cost for the PUs service provider (SP). Clearly there is a trade-off between the number of SU accepted (each one paying for the use of the spectrum) and the reimbursement in case they should be deallocated. Our problem is then to maximize the total expected discounted revenue of the SP over an infinite horizon. That is to say, the goal is to find the optimal admission control policy a SP must apply in order to obtain the maximum possible profit from SUs.

Before introducing our solution to the optimization problem, let us briefly describe how this general formulation can be applied to different real life scenarios. For instance, the system could be a cellular network that employs frequency division duplexing where the operator has C frequency bands (channels) to be assigned to its users (PUs). Moreover, in a LTE system (which is one of the predecessors of 5G), we can consider a channel as one resource block and then, the set of resources C represent the cell capacity. Finally, we have digital TV spectrum bands. In all of the described scenarios, if there are free resources, the SUs could use them with the constraint that their communications can be interrupted at any time (preemption). It is important to emphasize that our approach is useful when there is a large amount of concurrent transmissions which is a representative scenario in IoT applications.

To solve the optimization problem we model the optimal revenue problem as an infinite horizon Markov Decision Process (MDP) where the arrivals of each class are independent Poisson processes and the service durations are also independent and exponentially distributed. We further assume that each class of user demands b_i resources, with b_i integer (i.e. a PU requests b_1 channel bands and a SU requests b_2). The set of available actions of the MDP are reflected in the admission control policy: accept or reject a new SU. One of the challenges when we think about large random networks (e.g. V2V (vehicle to vehicle), M2M (machine to machine) and the whole range of IoT topologies) is how to manage a network with a large amount of connections with random traffic patterns. However, the Markovian structure allows us to analyze its asymptotic behavior, where the number of resources as well as the arrival rates are arbitrary large, by means of a simpler deterministic approximation usually named fluid limit or fluid approximation. In our case, the approximation can be determined as the unique solution of an ordinary differential equation system. This approximation will allow us to define and determine the optimal admission control policy that maximizes the revenue of the SP. See for example [11], [12] where there are examples of control queueing problems analyzed through a fluid approximation.

In terms of the preemption, this alternative of termination model is already defined by FCC[1] in a different scenario (Block D at 700 MHz: public safety services would act as the PUs and commercial services would act as the SUs, where public safety users are able to preempt commercial users during an emergency), representing a natural way to implement that situation. Our work differs from the preceding work [11], [12] by focusing on preemptive systems and multi-resource demand. In particular, in [13], [14] there is a complete literature review of the most representative studies about control policies for systems with preemption. In those works, due to the assumption "one user with one channel", the preemption mechanism is active only when all channels are busy. In our scenario the preemption

occurs when there are not enough free resources to satisfy primary demand.

Some preliminary results were published in our previous article [15]. In this paper, we incorporate aspects such as the multichannel demand in each class of user (b_1 and b_2) and different arrivals and departure rates between services. The main contribution of this work consists in the definition and analysis of a fluid model of the corresponding MDP. We also develop a methodology in order to obtain an approximation of the optimal admission control boundary. The proposal combines fast calculation with a robust performance. We also perform a comparison of the approximation presented here and the optimal one.

The rest of the paper is structured as follows. In Section II we introduce the hypotheses and the main characteristics of the considered system. In Section III we present our main results based on a simplified scenario (the base-case), in particular we provide the results about the approximation by a deterministic process and we present the methodology in order to estimate the optimal admission control boundary. In Section IV we show how our results, obtained in Section III, may be adapted for the general model hypotheses. In Section V we explain how to deal with piecewise stationary prices. In Section VI we validate our results presenting numerical examples in several scenarios. Finally, we conclude and discuss future work in Section VII.

II. PROBLEM FORMULATION

Let us begin by describing our working scenario and introducing the notation, definitions and hypotheses. We assume that the spectrum is divided into C non-overlapping channels to be distributed between PUs and SUs.

Let $X_i(t)$ and $Y_j(t)$ be the number of PUs and SUs in the system at time t respectively (in order to simplify the notation, we indistinctly use X_i and $X_i(t)$, as well as Y_j and $Y_j(t)$). The indexes i and j represent the different classes of PUs and SUs. Let λ_1^i and μ_1^i be the arrival and departure rates for class i of PUs respectively (independent Poisson arrivals and exponentially distributed service times). In the same way, λ_2^j and μ_2^j represent the arrival and departure rates for class j of SUs. We assume that each PU of class i demands b_1^i resources,

and analogously each SU of class j requires b_2^j channel bands to its transmission. In this sense we identify different classes of PUs and SUs, each class corresponds to a set of parameters (λ, μ, b).

We consider a paid-sharing mechanism where SUs pay to the primary SP for its spectrum utilization. We assume static prices similar to the ones considered in previous articles [13], [16], [17]. Our analysis can be applied to dynamic prices if the dynamics preserves the stationarity of the process as is explained in Section V. Let $R > 0$ be the reward collected for each band when a SU is allowed to exploit the PU's resource (i.e. $b_2^j R$ is the collected reward when a SU of class j is accepted in the system). We also consider a preemptive system where PUs have strict priority over SUs. This means that SUs can be removed from the system if there is insufficient free capacity when a PU arrives. In this model, these affected SUs will be reimbursed with $b_2^j K$ ($K > 0$), implying a punishment for the SP. We take into account a discount rate $\alpha > 0$, that is, the rewards and costs at time t are scaled by a factor $e^{-\alpha t}$.

We thus have a Continuous Time MDP (CTMDP) with state space $S = \{(X_i, Y_j) \in \mathbb{N}^{i+j} : 0 \leq \sum_i b_1^i X_i + \sum_j b_2^j Y_j \leq C\}$ where the objective is to maximize the total expected discounted profit over an infinite time horizon applying admission control decisions over SUs. In other words, we want to find the optimal policy $\pi^* \in \Pi$ that defines the admission control action $a(s) \in A_s$ in each state $s \in S$ maximizing the SP's revenue. The set of possible policies is represented by $\Pi = \{\pi : S \rightarrow A_s\}$, where A_s is the action space. $A_s = \{0, 1\}$ or $A_s = \{0\}$, depending on $s \in S$, where action 0 corresponds to refusing a SU's arrival and 1 to admitting it. Note that for all $s = (X_i, Y_j) : \sum_i b_1^i X_i + b_2^j Y_j > C - b_2^j$ the action space is $A_s = \{0\}$ since the system has an insufficient number of free bands to satisfy the demand of a SU of class j.

In order to keep the analysis more understandable to the reader, in the next section we consider one class of PU (λ_1, μ_1, b_1) and one of SU (λ_2, μ_2, b_2). It is important to highlight that the analysis of this base-case is fully scalable for the general case considering more than two classes. Then, in Section IV we return to the general problem an we show the results for more than two classes of users.

III. TWO CLASSES ANALYSIS

In this illustrative scenario we have the CTMDP with state space $S = \{(X,Y) \in \mathbb{N}^2 : 0 \le b_1 X + b_2 Y \le C\}$. Making the context clear. in Fig. 1 we represent the state space and the different economic zones in the particular case where there is no admission control.
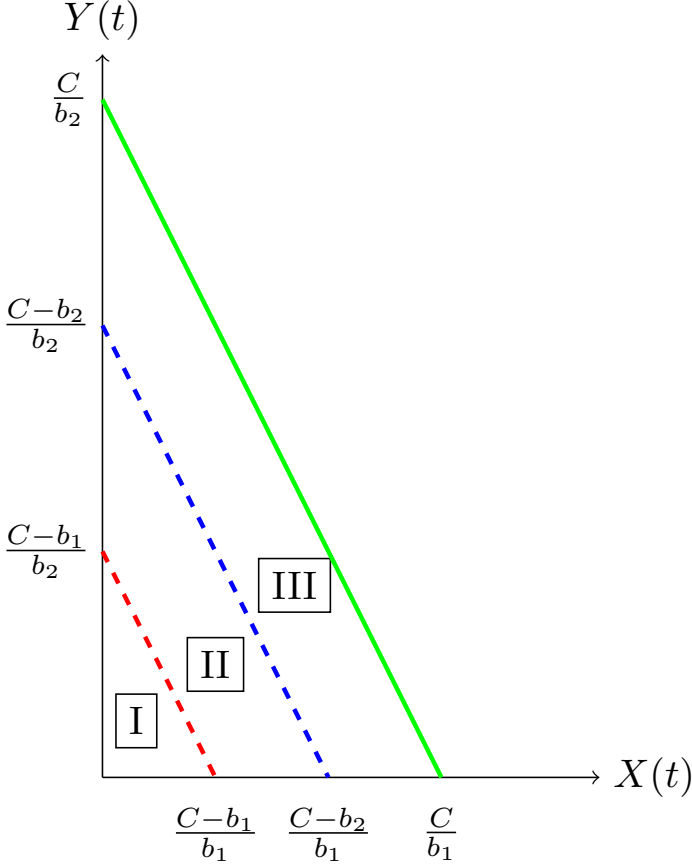

Fig. 1. In the two classes analysis, the state space is $S = \{(X,Y) : 0 \le b_1 X + b_2 Y \le C\}$, then $0 \le X \le C/b_1$ and $0 \le Y \le C/b_2$. If $a(X,Y) = 1 \forall (X,Y) : b_1 X + b_2 Y \le C - b_2$ and $b_1 > b_2$, we can divide the state space in three economic zones: I, II and III. In zone I for each SU that enters the system, the SP earns $b_2 R$. On the other hand, in zone II, the SP has to pay $Z b_2 K$ when a PU arrives (if $Y \ge Z$). In addition, it also earns $b_2 R$ for each SU accepted. Finally, in zone III the SP only pays $Z b_2 K$ (if $Y \ge Z$) for each PU arrived. Note: Z represents the number of deallocated SUs, see Eq. (1).

According to the previous definitions, the transition rates between states (X,Y) and (X',Y'), $q((X,Y),(X',Y'))$, of the CTMDP are:

- $q((X,Y),(X+1,Y)) = \lambda_1$ if $b_1 X + b_2 Y \le C - b_1$,
- $q((X,Y),(X-1,Y)) = \mu_1 X$,
- $q((X,Y),(X,Y+1)) = a(X,Y)\lambda_2$ if $b_1 X + b_2 Y \le C$,
- $q((X,Y),(X,Y-1)) = \mu_2 Y$,
- $q((X,Y),(X+1,Y-Z)) = \lambda_1$ if $C - b_1 < b_1 X + b_2 Y \le C$ and $Y \ge Z$ (preemption),

where $a(X,Y) \in A_s$ represents the admission control decision in each state and Z represents the number of preempted SUs:

$$Z = \left\lceil \frac{b_1 X + b_2 Y - C + b_1}{b_2} \right\rceil. \qquad (1)$$

Note that if the system is in state $s = (X,Y)$ and a SU arrives, it will access the system only if $a(X,Y) = 1$.

Since most of the MDPs resolution methods are for discrete time, in the next section we explain how to obtain an equivalent discrete time Markov decision process (DTMDP) of our CTMDP. For that purpose we use *uniformization* technique. Next, we briefly describe the most important aspects of this procedure. We suggest [18] and Chap. 11 of [19], as references for a more detailed analysis.

A. Uniformization

When the transition rates are identical for each state and action pair, one can convert a CTMDP into an equivalent discrete time Markov decision process (DTMDP). This means that the optimal action of both process coincide. In order to convert our process in a process with identical transition rates we resort to the technique known as *uniformization*. The idea behind *uniformization* is to use fictitious transitions from a state to itself to obtain a CTMDP equal in distribution to the original one and at the same time with identical transition rates. This new process can be then analyzed as a DTMDP.

Let (X,Y) be the CTMDP with transition probabilities $p(\cdot|s,a)$. Let $q(s,a)$ denote the transition rate out of state s when action a is taken, that is: $p(j|s,a) = \frac{q(j|s,a)}{q(s,a)}$. The infinitesimal generator of (X,Y) satisfies:

$$q(j|s,a) = p(j|s,a)q(s,a), \forall j \in S, j \ne s,$$
$$q(s|s,a) = -(1 - p(s|s,a))q(s,a) = -q(s,a).$$

Note that $p(s|s,a) = 0$, which means that, at the end of a sojourn in state s, the system will jump to a different state.

Let Γ be the uniformization constant; Γ is chosen such that $\Gamma > q(s,a), \forall s \in S, \forall a \in A_s$. Then, we can define the transition probabilities of the "uniformed process" $(\hat{X}, \hat{Y})$ as:

$$\hat{p}(j|s,a) = \frac{q(j|s,a)}{\Gamma} = \frac{q(s,a)}{\Gamma} p(j|s,a), \forall j \in S, j \ne s,$$

$$\hat{p}(s|s,a) = 1 - \frac{q(s,a)}{\Gamma}.$$

Therefore, the correspondent infinitesimal generator of $(\hat{X}, \hat{Y})$ is:

$$\hat{q}(j|s,a) = \hat{p}(j|s,a)\Gamma = q(j|s,a), \forall j \in S, j \neq s,$$
$$\hat{q}(s|s,a) = -(1 - \hat{p}(s|s,a))\Gamma = -q(s,a).$$

By creating fictitious transitions (from a state to itself), we are creating a stochastically equivalent process in which the transitions occur more often. We refer to $(\hat{X}, \hat{Y})$ as the uniformization of (X, Y) because it has an identical (or uniform) mean time between transitions (which corresponds to the decision epochs). Please note that the two processes are equal in distribution so they have the same probabilistic behavior.

Fig. 2 shows a generic state $s = (\hat{X}, \hat{Y})$ when the uniformization has already been done using $\Gamma = \lambda_1 + \lambda_2 + C(\mu_1/b_1 + \mu_2/b_2)$. In particular it is a state where the system has enough free capacity for primary demand $(b_1\hat{X} + b_2\hat{Y} \leq C - b_1)$.
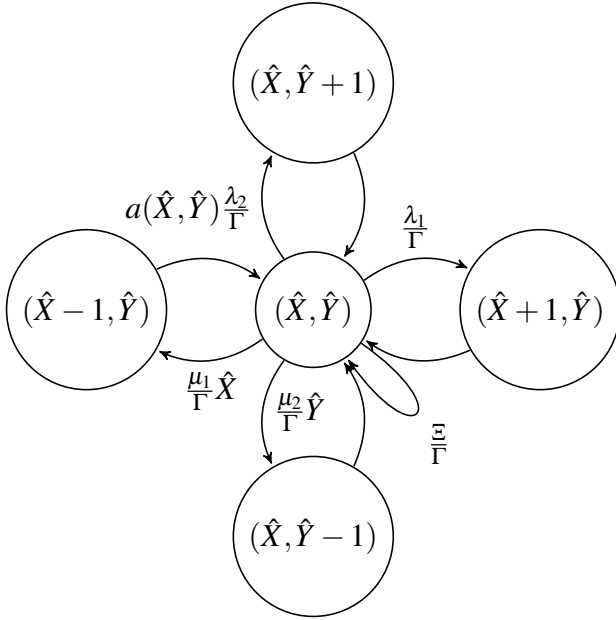

Fig. 2. Generic state $(\hat{X}, \hat{Y})$ obtained after the uniformization process (values correspond to outgoing transition probabilities). In this work $\Gamma = \lambda_1 + \lambda_2 + C(\frac{\mu_1}{b_1} + \frac{\mu_2}{b_2})$. Observe that this transformation implies "fictitious" transitions from a state to itself, $\Xi = C(\frac{\mu_1}{b_1} + \frac{\mu_2}{b_2}) - \mu_1\hat{X} - \mu_2\hat{Y} + \lambda_2(1 - a(\hat{X}, \hat{Y}))$.

Working with the uniformized process $(\hat{X}, \hat{Y})$, it is possible to demonstrate that it can be analyzed like a DTMDP with a discount factor $\beta = \frac{\Gamma}{\Gamma + \alpha}$ ($0 < \beta < 1$). The interested reader may consult [18] for

the details. In terms of the equivalent DTMDP, the problem can be written as:

$$V^*(s) = \max_{\pi \in \Pi^{MD}} E^\pi \left[\sum_{n=0}^\infty \beta^n r(s_n, a_n) | s_0 = s \right], \forall s \in S;$$

where:

- $V^*(s)$ is the maximal expected β-discounted reward for the system with initial state s,
- Π^{MD} represents all the possible Markov deterministic policies,
- t_n is the time of n-th transition (n-decision epoch),
- $a_n = a(t_n)$ and $s_n = s(t_n)$, and
- $r(s_n, a_n) = l(s_n, a_n, s_{n+1})$ (a lump reward at the moment of the transition) such that
  - if $a_n = 1, s_n = (\hat{X}_n, \hat{Y}_n)$ and $s_{n+1} = (\hat{X}_n, \hat{Y}_n + 1)$, $l(s_n, a_n, s_{n+1}) = b_2 R$,
  - if $s_n = (\hat{X}_n, \hat{Y}_n)$ and $s_{n+1} = (\hat{X}_n + 1, \hat{Y}_n - Z_n)$, $l(s_n, a_n, s_{n+1}) = -\hat{Z}_n b_2 K$,
  - otherwise, $l(s_n, a_n, s_{n+1}) = 0$.

In discrete time the best known practical algorithms for solving infinite-horizon MDPs based on dynamic programing are: Value Iteration (VI), Policy Iteration (PI) and Modified Policy Iteration (MPI). Numerical results reported in [20], [19] suggest that Modified Policy Iteration is more efficient than either VI o PI in practice. That is the main reason why in this work we have chosen MPI. In what follows we introduce the algorithm that we have used to validate our methodology.

B. Modified policy iteration (MPI)

Algorithm 1 Pseudo-code of MPI algorithm

Require: C, λ_1, λ_2, μ_1, μ_2, b_1, b_2, β, R, K
1: select an arbitrary decision rule π'
2: **repeat**
3:     $\pi := \pi'$;
4:     Policy evaluation: compute the value function of the policy π: $V_\pi(s), \forall s \in S$;
5:     Policy improvement: obtain π' such that $\arg\max_{a \in A_s} \left\{ \sum_{j \in S} \beta \hat{p}(j|s,a)(l(s,a,j) + V_\pi(j)) \right\}$;
6: **until** $\pi = \pi'$
7: **return** π

MPI mainly applies to stationary infinite-horizon problems. When S is finite and also A_s is finite for each $s \in S$, then it is possible to demonstrate

that MPI algorithm terminates in a finite number of iterations with a discount optimal policy. Please note that when no improvements are possible, then the policy is guaranteed to be optimal.

TABLE I
PERFORMANCE OF MPI ALGORITHM (I7 3770/16GB). IN ALL EXAMPLES $\lambda_1 = 200, \lambda_2 = 500, \alpha = 2, R = 1$ AND $K = 3$.

Parameters	$|S|$	Running time
$C = 100, \mu_1 = 3, \mu_2 = 1,$ $b_1 = 5, b_2 = 1$	1071	46 sec
$C = 100, \mu_1 = 3, \mu_2 = 1,$ $b_1 = 1, b_2 = 1$	5151	9045 sec
$C = 500, \mu_1 = 0.1, \mu_2 = 1,$ $b_1 = 5, b_2 = 1$	25351	32267 sec

However, since MPI works over the policy-space, the size of which is exponential in the number of system states, when there is a large number of system states, a big negative impact on the algorithm convergence time occurs (see Table I). Motivated by this fact, in this work we will present a computationally efficient alternative methodology to find an accurate estimation of the optimal admission control boundary based on a fluid model approximation of the MDP.

C. Fluid Model

1) Preliminaries and Motivation: Applications of fluid models to telecommunications appeared in the literature and were widely developed in the last decade. Some recent examples include for instance: peer-to-peer systems and mobile networks (see for example [21], [22], [23], [24], [25] and references therein). Even if the general framework appears as common to all these problems, each one has its own particularities.

Before introducing the results for our model, we briefly describe the general framework of the methodology. In many scenarios where the number of channels C and the user arrival rates (λ_i) are large, a deterministic fluid model may offer a good approximation to the original control problem [26], [11]. In a nutshell, we can say that by choosing a convenient scaling of the process it is possible to obtain in the limit, a description of the asymptotic behavior of the process as the solution of an ordinary differential equation (ODE) system which is denominated "fluid limit".

Let $\tilde{X}^N(t)$ and $\tilde{Y}^N(t)$ be the number of PUs and SUs in the system considering a N-parametric version of the original stochastic model. That means that the parameters of this new process are: $\tilde{C} = CN$, $\tilde{\lambda}_i = \lambda_i N$ and $\tilde{\mu}_i = \mu_i, i = 1, 2$. Consider now the scaled process $(X^N(t), Y^N(t)) = 1/N(\tilde{X}^N(t), \tilde{Y}^N(t))$, then it can be decomposed in the following way:

$$(X^N(0)Y^N(0)) + \frac{1}{N}\int_0^t Q_N(\tilde{X}^N(s), \tilde{Y}^N(s))ds + \frac{M^N(t)}{N}.$$

where $Q_N(l)$ is the so-called drift of the process at state l which may be calculated as $\sum_m (l - m)q(l, m)$ ($q(l, m)$ is the transition rate from state l to m) and $M^N(t)$ is a Martingale.

If there exists a Lipschitz function Q such that $\lim_{N\to\infty} ||Q_N(\tilde{X}^N(t), \tilde{Y}^N(t))/N - Q(X^N(t), Y^N(t))|| = 0$ and $M^N(t)/N$ converges to zero in probability, then when $N \to \infty$ $(X^N(t), Y^N(t))$ converges in probability over compact time intervals to a deterministic process $(x(t), y(t))$, described by the ODE:

$$(x'(t), y'(t)) = Q(x(t), y(t)). \tag{2}$$

In the next subsection we calculate the drift for our process in order to determine the fluid limit. In this case, it is important to highlight that we can obtain an explicit expression of $(x(t), y(t))$. Without loss of generality, in the rest of the section we will consider $b_2 = 1$.

2) Fluid Limit: The classical results on convergence of Markov processes in [27] or [26] assume some regularity properties of the fluid ODE, i.e. the vector field defining the ODE must be a Lipschitz function in the domain of interest. As we will see, due to the existence of an admission control and preemption policies, in our system this regularity condition does not hold.

Using the methodology proposed by Bortolussi in [28], it is possible to consider a piecewise-smooth (PWS) system (i.e. a dynamical system in which the vector field is discontinuous in the domain of interest, but with a controlled form of discontinuity). That is to say, considering $\frac{d}{dt}\mathbf{x} = f(\mathbf{x}), f : E \to \mathbb{R}^n, E \subseteq \mathbb{R}^n, \bigcup R_i \supseteq E$ (R_i $i = 1 \dots s$ is a finite set of disjoint regions), a PWS system is when f is smooth on R_i and can be discontinuous only on the boundaries of R_i. The author of [28] also proved that the sequence of CTMC converges to the trajectories of this hybrid dynamical system when the size of the system N goes to infinity. Let us then consider this result in order to obtain our fluid approximation.

Before we start, a question that arises is: do optimal admission control boundaries have a particular structure?. In Fig. 3 we show two generic AC (admission control) boundaries[2] obtained by MPI algorithm for two sets of system parameters. We can see the linear correlation between X and Y in the AC boundaries.
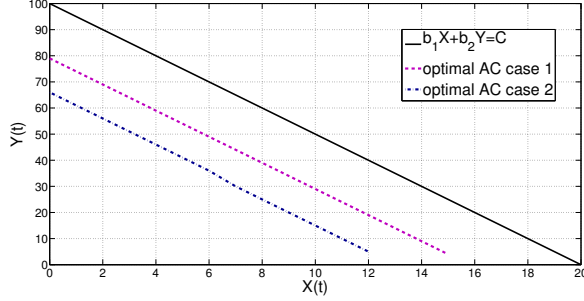

Fig. 3. Optimal AC boundaries. Parameters of case 1: $\lambda_1 = 200$, $\lambda_2 = 500$, $\mu_1 = 0.1$, $\mu_2 = 1$, $R = 1$, $K = 3$, $C = 100$, $b_1 = 5$, $b_2 = 1$ and $\alpha = 50$. Parameters of case 2: $\lambda_1 = 300$, $\lambda_2 = 500$, $\mu_1 = 0.1$, $\mu_2 = 0.5$, $R = 1$, $K = 3$, $C = 100$, $b_1 = 5$, $b_2 = 1$ and $\alpha = 50$. The state space is the same in both cases ($\{(X,Y) : 0 \le b_1 X + b_2 Y \le C\}$).
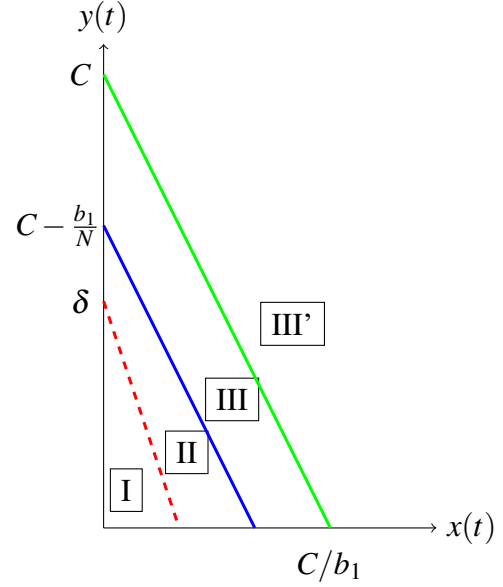

Fig. 4. Considering $b_2 = 1$, we can divide the state space in three different economic zones. SUs are accepted in zone I and rejected in zones II and III. Zone III includes the states where SUs are deallocated and Zone II represents the "neutral" one (neutral in the sense that the SP does not earn nor lose). Zone III' is an "artificial zone" used to the definition of the fluid approximation.

Fig. 3 suggests that the admission control boundary in the limit can be assumed as a line with equation $y = Ax + \delta$ with unknowns A and δ ($A < 0$ and $0 \le \delta \le C$). This hypothesis will be used in the following analysis but it is important to highlight that our results and methodology can be extended to other characteristics of the admission control boundary.

In this context we have the three economic zones illustrated in Fig. 4 (I, II and III). Please note that in the limit ($N \to \infty$) zone III converges to the line $b_1 x + y - C = 0$ (i.e. in the limit zone III disappears), then $z(t) = b_1 x(t) + y(t) - C + b_1 = b_1$. As a consequence, in the limit we can think the system with only two economic zones (I and II).

If we turn our attention to these two regions (I and II), we have $f_1(x,y) = (\lambda_1 - \mu_1 x, \lambda_2 - \mu_2 y)$ and $f_2(x,y) = (\lambda_1 - \mu_1 x, -\mu_2 y)$ the velocity vectors, both continuous in I and II respectively. In order to be in the context of [28], as a way to see it, it is useful to artificially extend our processes beyond the region $\{b_1 x + y \le C\}$, assuming that in the region $\{b_1 x + y > C\}$ (zone III') the vector field is $f_3 = (\lambda_1 - \mu_1 x, -\lambda_1 b_1 - \mu_2 y)$ (representing the

preempted scenario). Then, in our PWS system we identify three zones (*I*, *II* and *III'*) and two surfaces ($\gamma : -Ax + y - \delta = 0$ and $\gamma' : b_1 x + y - C = 0$). According to that and excluding the behavior on the surfaces, the deterministic system is driven by the equations:

If $-Ax + y - \delta < 0$ (*I*):

$$\begin{cases} x' = \lambda_1 - \mu_1 x, \\ y' = \lambda_2 - \mu_2 y; \end{cases}$$

else, if $-Ax + y - \delta > 0$ and $b_1 x + y - C < 0$ (*II*):

$$\begin{cases} x' = \lambda_1 - \mu_1 x, \\ y' = -\mu_2 y. \end{cases}$$

Some remarks are in order concerning these results. Firstly, the equations in zones I and II are obtained directly using classical results on convergence of Markov processes. The difficult task consists in analyzing the system on γ and γ'.

Secondly, let $n(\mathbf{x})$ be the normal vector to γ at $\mathbf{x}$ ($\mathbf{x} \in \gamma$), we find the following possible behaviors of a solution starting in $\mathbf{x}$ depending on the value of $n^T(\mathbf{x}) f_1(\mathbf{x})$ and $n^T(\mathbf{x}) f_2(\mathbf{x})$:

- transversal crossing: if $n^T(\mathbf{x})f_1(\mathbf{x})$ and $n^T(\mathbf{x})f_2(\mathbf{x})$ are non zero and have the same sign,
- sliding motion: if $n^T(\mathbf{x})f_1(\mathbf{x}) > 0$ and $n^T(\mathbf{x})f_2(\mathbf{x}) < 0$, the system cannot escape from γ, then the solution follows a vector field obtained as convex combination of f_1 and f_2,
- tangential crossing: if $n^T(\mathbf{x})f_1(\mathbf{x}) = 0$ or $n^T(\mathbf{x})f_2(\mathbf{x}) = 0$.

Please note that it depends on the values of λ_1, λ_2, μ_1, μ_2, b_1, A and δ. We have an analogous situation considering $n'(\mathbf{x})$ (the normal vector to γ') and the values of $n'^T(\mathbf{x})f_2(\mathbf{x})$ and $n'^T(\mathbf{x})f_3(\mathbf{x})$, $\forall \mathbf{x} \in \gamma'$.

Finally, in the presence of fluid limits it is usual to infer from the fixed point analysis of the deterministic system the behavior of the stochastic process in the stationary regime. In particular, if there is a unique fixed point that is a global attractor, the stochastic invariant distribution converges in probability to this fixed point [29]. According to that, one of our main results is that the position of the ODE (or PWS) fixed points is decisive in defining an effective operating point of the system. In this situation, we can identify two scenarios depending on the value of λ_1/μ_1: (1) if $\lambda_1/\mu_1 \geq C/b_1$ or (2) if $\lambda_1/\mu_1 < C/b_1$.

System saturated by PUs: If we consider the primary system in its rush hour traffic, i.e. $\lambda_1/\mu_1 \geq C/b_1$, we have that $n'^T(\mathbf{x})f_3(\mathbf{x}) = -\mu_1 b_1 x - \mu_2 y < 0, \forall \mathbf{x} \in \gamma'$ and we can deduce that there is an interval of γ' (including C/b_1) where $n'^T(\mathbf{x})f_2(\mathbf{x}) = \lambda_1 b_1 - \mu_1 b_1 x - \mu_2 y > 0$. Then, the sliding motion condition is verified, at least, on an interval of γ'. In this case it is possible to demonstrate that fixed point of the PWS system is $(x^*, y^*) = (C/b_1, 0)$. The PWS is completely defined including the dynamic on γ' as: if $b_1 x + y - C = 0$ (γ'):

$$\begin{cases} x' = \lambda_1 - \mu_1 x, \\ y' = -\lambda_1 b_1 + \mu_1 b_1 x. \end{cases}$$

System unsaturated by PUs: On the other hand, if $\lambda_1/\mu_1 < C/b_1$ we have three possible cases depending on the unknowns values of A and δ. Let us give an intuitive explanation:

1) if $(\lambda_1/\mu_1, \lambda_2/\mu_2)$ is located in zone I $(-A\lambda_1/\mu_1 + \lambda_2/\mu_2 < \delta)$, the fixed point (x^*, y^*) will be $(\lambda_1/\mu_1, \lambda_2/\mu_2)$. Then, a solution starting in zone I will continue in zone

I. In this case the admission control does not apply (i.e. $A = -b_1$ and $\delta = C$).

2) if $-A\lambda_1/\mu_1 + \lambda_2/\mu_2 > \delta$ and $\lambda_1/\mu_1 > -\delta/A$, the fixed point (x^*, y^*) will be $(\lambda_1/\mu_1, 0)$ located in zone II. Then, a solution starting in zone I will continue in zone II and will die in II (transversal motion occurs on γ). In the same way as in the previous case, the admission control does not apply.

3) if $-A\lambda_1/\mu_1 + \lambda_2/\mu_2 > \delta$ and $\lambda_1/\mu_1 < -\delta/A$, the fixed point (x^*, y^*) will be $(\lambda_1/\mu_1, A\lambda_1/\mu_1 + \delta)$ located on γ (sliding motion). This condition is verified when $(\lambda_1/\mu_1, \lambda_2/\mu_2)$ is located in zone III' and $\lambda_1/\mu_1 < C/b_1$. In this case, the corresponding PWS is completely defined including the dynamic on γ as:

$$\begin{cases} x' = \lambda_1 - \mu_1 x, \\ y' = A(\lambda_1 - \mu_1 x). \end{cases}$$

3) Optimization problem formulation: We have characterized the behavior of the dynamical system. Now we will formulate the economic problem using the deterministic approximation. Let $R_{t_1}(x_0, y_0)$ be the SP profit function,

$$R_{t_1}(x_0, y_0) = \int_0^{t_1} \lambda_2 R e^{-\alpha t} dt - \int_{t_2}^{t_C} \lambda_1 K b_1 e^{-\alpha t} dt,$$

where t_1 verifies $-Ax(t_1) + y(t_1) = \delta$ (the system reaches the admission border), t_2 verifies $b_1 x(t_2) + y(t_2) = C$ (there is no free capacity) and t_C is such that $b_1 x(t_C) = C$ (all resources are occupied by PUs).

Working with the fluid approximations of $x(t)$ and $y(t)$, we can re-write the optimization problem as:

$$\begin{aligned} \underset{t_1}{\text{maximize}} \quad & R_{t_1}(x_0, y_0) \\ \text{subject to} \quad & 0 \leq t_1 \leq t_C \\ & b_1 x(t_2) + y(t_2) = C \\ & b_1 x(t_C) = C. \end{aligned} \qquad (3)$$

We are interested in obtain the admission control boundary independently of the initial condition of the system, so we propose the following methodology:

- Step 1: Choose a set of possible initial conditions $\{(x_0^i, y_0^i)\}$ (when the AC is considered as a line, only two tuples are necessary).
- Step 2: Run the optimization problem to obtain t_1^i, $x(t_1^i)$ and $y(t_1^i)$,

- Step 3: Apply least squares fitting to obtain the parameters of the AC border as those that minimize the mean square error (when the AC is considered as a line, the estimation of A and δ is direct).

Please note that t_C is not defined when the system is unsaturated by PUs, in particular in the third case (sliding motion) when the admission control decision makes sense, the solution of the optimal problem is $t_1 = t_2$ such that $b_1 x(t_2) + y(t_2) = C$.

IV. GENERAL CASE ANALYSIS

In the previous sections we have analyzed what we termed the base-case. In the following two subsections we briefly illustrate how our results and methodology are extended to study more complex scenarios. In particular, we will consider different primary arrival and departure rates, and also different primary resource demands.

A. Different primary arrival rates

In the same way as in the base-case, the arrival processes for primary and secondary users are independent Poisson processes and the service durations are independent and exponentially distributed. We consider now different primary arrival rates. In order to simplify the analysis we consider first the same departure rate for all classes of users. The case of different departure rates will be considered in the next section. In particular, we present the analysis where two primary arrival rates are involved.

The model assumptions for this case are the following:

- $X_1(t)$, $X_2(t)$: number of PUs at time t with arrival rates λ_1 and λ_1' respectively,
- $Y(t)$: number of SUs at time t
- C: total number of identical channels,
- λ_2: arrival rate for SUs,
- μ: departure rate for all classes of users, and
- $b_1 = b_1' = b_2 = 1$: we associate one user with one channel (in order to simplify the notation).

The state space is then:

$$S = \{(X_1, X_2, Y) \in \mathbb{N}^3 : 0 \leq X_1 + X_2 + Y \leq C\},$$

According to these hypotheses, it is possible to demonstrate that the admission control border will be a plane with equation

$$X_1(t) + X_2(t) + Y(t) - \Delta = 0.$$

Considering the same scaling for the process as for the base-case, we can obtain the fluid approximation of the described stochastic system:
If $x_1 + x_2 + y - \Delta < 0$:

$$\begin{cases} x_1' = \lambda_1 - \mu x_1, \\ x_2' = \lambda_1' - \mu x_2, \\ y' = \lambda_2 - \mu y; \end{cases}$$

else, if $x_1 + x_2 + y - \Delta > 0$ and $x_1 + x_2 + y - C < 0$:

$$\begin{cases} x_1' = \lambda_1 - \mu x_1, \\ x_2' = \lambda_1' - \mu x_2, \\ y' = -\mu y. \end{cases}$$

When the sliding motion condition is verified, the system of differential equations is:

$$\begin{cases} x_1' = \lambda_1 - \mu x_1, \\ x_2' = \lambda_1' - \mu x_2, \\ y' = -(\lambda_1 + \lambda_1') + \mu(x_1 + x_2). \end{cases}$$

Please note that the same zones as for the base-case can be identified, but with an admission control border defined by a plane instead of a line.

Finally, the economic model is as before:

$$\begin{aligned} \text{maximize} \quad & R_{t_1}(x_{10}, x_{20}, y_0) \\ \text{subject to} \quad & 0 \leq t_1 \leq t_C \\ & x_1(t_2) + x_2(t_2) + y(t_2) = C \\ & x_1(t_C) + x_2(t_C) = C, \end{aligned} \tag{4}$$

where

$$R_{t_1}(x_{10}, x_{20}, y_0) = \int_0^{t_1} \lambda_2 R e^{-\alpha t} dt - \int_{t_2}^{t_C} (\lambda_1 + \lambda_1') K e^{-\alpha t} dt.$$

Please note that we can extend this result for more than two different primary arrival rates, considering more than two classes of PUs. In that context the analytical expressions are totally analogous to the presented ones. The difference is that the admission control border will be a hyperplane instead of a plane, but the analytical result will be totally analogous.

B. Different primary arrival and departure rates, and different primary demands

Let us study the general case considering different primary arrival and departure rates and different demands. In this context, we identify each set $(\lambda_1^i, \mu_1^i, b_1^i)$ with one class of user. The main difference with the previous case is that the admission control border will be approximated as an hyperplane.

Working with m classes of PUs, $b_2 = 1$ and assuming that the AC hyperplane has the equation $\sum_i A_i x_i + By - \Delta = 0$ (with unknown $A_i > 0$, $B > 0$ and $0 < \Delta \leq C$), we have that the fluid limit verifies:
If $\sum\limits_{i=1}^{m} A_i x_i + By - \Delta < 0$:

$$\begin{cases} x_i' = \lambda_1^i - \mu_1^i x_i, i = 1 \dots m, \\ y' = \lambda_2 - \mu_2 y; \end{cases}$$

else, if $\sum\limits_{i=1}^{m} A_i x_i + By - \Delta > 0$ and $\sum\limits_{i=1}^{m} b_1^i x_i + y - C < 0$:

$$\begin{cases} x_i' = \lambda_1^i - \mu_1^i x_i, i = 1 \dots m, \\ y' = -\mu_2 y. \end{cases}$$

Depending on the parameters λ_1^i, μ_1^i, λ_2, μ_2 and b_1^i we can identify sliding motion on one of the following hyperplanes $\sum\limits_{i=1}^{m} A_i x_i + By - \Delta = 0$ (AC border) or on $\sum\limits_{i=1}^{m} b_1^i x_i + y - C = 0$ (State Space border). Then the fluid limit is completely defined by the next equations.
If $\sum\limits_{i=1}^{m} A_i x_i + By - \Delta = 0$ and the sliding motion occurs on the AC border:

$$\begin{cases} x_i' = \lambda_1^i - \mu_1^i x_i, i = 1 \dots m, \\ y' = -(\sum_i A_i \lambda_1^i) + \sum_i A_i \mu_1^i x_i; \end{cases}$$

else, if $\sum\limits_{i=1}^{m} b_1^i x_i + y - C = 0$ and the sliding motion occurs on the state space border:

$$\begin{cases} x_i' = \lambda_1^i - \mu_1^i x_i, i = 1 \dots m, \\ y' = -(\sum_i b_1^i \lambda_1^i) + \sum_i b_1^i \mu_1^i x_i. \end{cases}$$

Again for this case, the optimization problem is analogous to the previous ones.

V. PIECEWISE STATIONARY PRICES

The introduction of dynamic prices in our framework is indeed possible, at least if the dynamics preserves the stationarity of the process. A possible scenario is where the prices depend on the arrival rate of the PUs. If we consider that these rates remain constant during some time periods (consider workdays or weekends or day and night) then the system can be divided in stationary periods and for each one of these periods our proposal and its results are still valid. To illustrate this scenario we can define $w : \{1 \dots W\}$ time periods. In each period w we have a set of constant system parameters $(\lambda_{1w}^i, \lambda_{2w}^j, \mu_{1w}^i, \mu_{2w}^j, b_{1w}^i, b_{2w}^j, R_w, K_w)$. In this context, the optimization problem has to be solve for each w obtaining different admission control policies which the SP must apply in each period w.

It is important to remark that to cope with instant variations of the prices, it is necessary to introduce a different methodology. In the next section we shall present some simulations that will help us to gain insight into the framework and analyze the accuracy of our results.

VI. SIMULATED EXPERIMENTS AND RESULTS

In order to validate our proposal we will present some examples considering systems with a large but finite number of channel bands. In each scenario, we calculate the admission control boundary using the proposed methodology and we compare it with the results obtained by MPI algorithm. We divide the validation tests into four groups: three from the base-case and another considering that PUs arrive with different arrival rates and same departure rate.

In particular, in the base-case, we first study scenarios where system parameters verify $\lambda_1/\mu_1 < C/b_1$. Then, for $\lambda_1/\mu_1 \geq C/b_1$ (rush hour) we analyze the impact of the parameter b_1 considering the particular case when $\mu_1 = \mu_2$. In this situation, as can be demonstrated (see [13]), the optimal admission control boundary is a line with equation $b_1 x + b_2 y = \delta$ (where δ is the only parameter that has to be determined). Finally, also for the system in its rush hour, we show the performance of the approximation in general cases (e.g. different arrival and departure rates, and different demands).

A. Group 1: System unsaturated by PUs

According to the analysis of the fluid limit we concluded that $A = -b_1$ and $\delta < C$ but very closely to C. In terms of the stochastic system, this thresh-

old will be highly dependent on N, then we can approximate the AC as the line $b_1x + y = C$.

In table II we present a set of cases which verify $\lambda_1/\mu_1 < C/b_1$ and in Fig. 5 the correspondent AC boundaries obtained by MPI. We can conclude that the fluid approximation is accurate.

TABLE II
AC BOUNDARIES OBTAINED BY MPI. PARAMETERS COMMON TO ALL CASES: $N = 100$, $\mu_1 = 0.1$, $\mu_2 = 1$, $R = 1$, $C = 1$, $b_1 = 5$, $b_2 = 1$ AND $\alpha = 5$. THE LINE $b_1X^N + Y^N(t) = C$ REPRESENTS AN ESTIMATION OF THE AC BOUNDARY OBTAINED BY OUR APPROXIMATION.

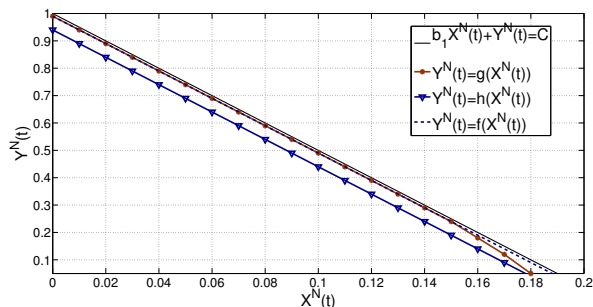| K	λ_1	λ_2	AC boundary
3	0.01	2	$y = f(x)$
10	0.01	2	$y = g(x)$
10	0.01	5	$y = h(x)$

Fig. 5. Optimal AC boundaries $f(x)$, $g(x)$ and $h(x)$ obtained by MPI (see Table II).

For the third case, where the optimal AC border is given by $y = h(x)$, we run the MPI algorithm considering different N values. In Fig. 6 we can see the results, and as expected that the fluid approximation improves (and so the distance between the optimal AC border and its approximation decreases) when N
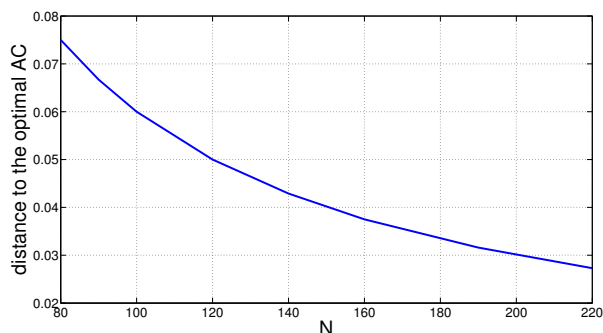

Fig. 6. Distance between Optimal AC boundary $y = h(x)$ (see Table II) and the fluid AC estimation $b_1x + b_2y = C$.

increases. Although our approximation is valid for infinity networks ($N \to \infty$), we want to emphasize that we obtain an excellent performance considering large but finite number of users.

B. Group 2: Different primary bandwidth requirements considering $\frac{\lambda_1}{\mu_1} \geq \frac{C}{b_1}$.

As a first illustration of the accuracy of our proposal when the system is operating in saturated traffic conditions of PUs, we consider scenarios where $\mu_1 = \mu_2$ (the same departure rate in both classes of users). In this particular case, where some characteristics of the admission control boundary are already known, the methodology requires only one initial condition on Step 1. This represents an immediate calculation of policy parameters. We test the approximation with different values of b_1 ($b_1 = 1, 2, 5, 10$). The rest of the network parameters (e.g. rates, prices, number of total channel bands) are the same in all the examples.

TABLE III
AC BOUNDARIES AND REWARD CONFIDENCE INTERVALS (0.95 LEVEL OF CONFIDENCE) OBTAINED BY BOTH METHODS. SYSTEM PARAMETERS: $N = 100$, $\lambda_1 = 3$, $\lambda_2 = 5$, $\mu_1 = \mu_2 = 0.1$, $R = 1$, $K = 3$, $C = 1$, $b_2 = 1$ AND $\alpha = 50$.

b_1	AC boundary	MPI		Fluid Model	
		δ	Reward	δ	Reward
1	$x + y = \delta$	0.93	10.6 ± 0.6	0.92	10.7 ± 1.0
2	$2x + y = \delta$	0.86	9.76 ± 0.55	0.85	9.76 ± 0.45
5	$5x + y = \delta$	0.65	5.86 ± 0.74	0.62	5.95 ± 0.65
10	$10x + y = \delta$	0.29	0.66 ± 0.40	0.24	0.81 ± 0.47

In table III we present the ACs boundaries for all the cases. In order to test how close to the optimal is the approximation, we make several experiments ($n = 30$) with both boundaries (the optimal and its approximation) and compute the profit of the SP. Each experiment consists in one realization of the continuous time Markov chain using the appropriate AC boundary. In each transition the discount profit of the SP is computed. We build the reward confidence intervals and they are included in table III.

Some remarks regarding the obtained results are in order. Firstly, these results lead us to conclude that our fluid limit provides an excellent approximation of the optimal one. Secondly, we observe that the deterministic approximation is more conservative than the stochastic one. This is a direct consequence of the limit of the penalty zone. Lastly,
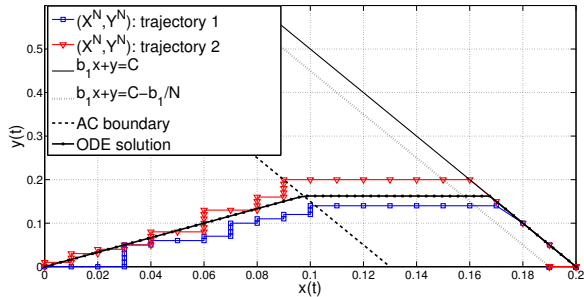
Fig. 7. Evolution on the plane of the Markov Chain (two realizations) and the ODE. Parameters: $N = 100$, $\lambda_1 = 3$, $\lambda_2 = 5$, $\mu_1 = \mu_2 = 0.1$, $R = 1$, $K = 3$, $C = 1$, $b_1 = 5$, $b_2 = 1$ and $\alpha = 50$. AC boundary: $b_1 x + y = 0.63$.
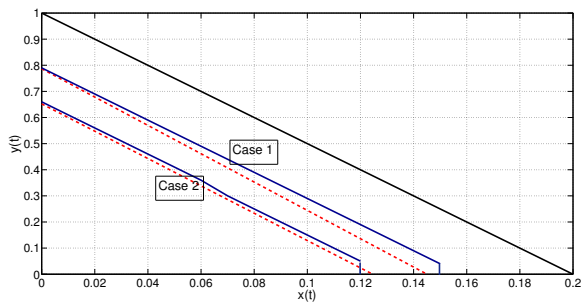

Fig. 8. Solid and dashed lines represent optimal and estimated AC boundaries respectively. Parameters of case 1: $N = 100$, $\lambda_1 = 2$, $\lambda_2 = 5$, $\mu_1 = 0.1$, $\mu_2 = 1$, $R = 1$, $K = 3$, $C = 1$, $b_1 = 5$, $b_2 = 1$ and $\alpha = 50$. Parameters of case 2: $N = 100$, $\lambda_1 = 3$, $\lambda_2 = 5$, $\mu_1 = 0.1$, $\mu_2 = 0.5$, $R = 1$, $K = 3$, $C = 1$, $b_1 = 5$, $b_2 = 1$ and $\alpha = 50$.

we can observe the accuracy of our model in the curves of Fig. 7 where we show the evolution on the plane of two realizations of the MDP together with the PWS system solution.

C. Group 3: Different arrivals and departure rates considering $\frac{\lambda_1}{\mu_1} \geq \frac{C}{b_1}$.

When we think about cognitive radio networks and even more when we think about IoT applications using licensed spectrum as secondaries, we commonly imagine different primary and secondary services (e.g. Internet access through White Space frequency bands [30]), then the natural situation is to model them with different arrival and service rates. We test now the methodology in generic cases.

In Fig. 8 and in Table IV we present the results of two examples. The differences between them are the arrival and departure rates of PUs and SUs. The estimation of the AC boundary is obtained using the methodology introduced before for different values

of initial conditions. Please note that the optimal AC boundaries are not necessarily lines when $\mu_1 \neq \mu_2$, in particular see case 2 where the boundary is a piecewise linear function with three line segments (one when $x(t) \in [0, 0.06]$, the second when $x(t) \in [0.06, 0.065]$ and the third when $x(t) \in [0.065, 0.12]$. Also in case 2, we can say that for $x(t) > 0.12$ all SU arrivals will be rejected.

TABLE IV
REWARD CONFIDENCE INTERVALS (0.95 LEVEL OF CONFIDENCE) OBTAINED BY BOTH METHODS. PARAMETERS OF CASE 1: $N = 100$, $\lambda_1 = 2$, $\lambda_2 = 5$, $\mu_1 = 0.1$, $\mu_2 = 1$, $R = 1$, $K = 3$, $C = 1$, $b_1 = 5$, $b_2 = 1$ AND $\alpha = 50$. PARAMETERS OF CASE 2: $N = 100$, $\lambda_1 = 3$, $\lambda_2 = 5$, $\mu_1 = 0.1$, $\mu_2 = 0.5$, $R = 1$, $K = 3$, $C = 1$, $b_1 = 5$, $b_2 = 1$ AND $\alpha = 50$.

Case	MPI	Fluid Model
1	8.16 ± 0.82	7.60 ± 0.70
2	4.87 ± 0.51	4.30 ± 0.70

Again, our estimation shows an excellent performance in both cases. This demonstrates the versatility of our technique.

D. Group 4: Different primary arrival rates

In the same way as in Group 2, in this case we consider scenarios operating in saturated traffic conditions of PUs and we consider the same departure rate in all classes of users. We recall that in case the assumptions are the following:

- λ_1 and λ_1': primary arrivals rates,
- λ_2: secondary arrival rate,
- μ: departure rate of all classes,
- $b_1 = b_1' = b_2 = 1$: one user one channel.

Using the results of Subsection IV-A we can obtain the parameter Δ which completely determines the AC border. The obtained AC border is a plane with normal vector $(1, 1, 1)$. In Table V we show how the methodology performs for two different arrival rates sets.

TABLE V
AC BOUNDARIES AND REWARD CONFIDENCE INTERVALS (0.95 LEVEL OF CONFIDENCE) OBTAINED BY BOTH METHODS. SYSTEM PARAMETERS: $N = 10$, $\lambda_2 = 10$, $\mu = 1$, $R = 1$, $K = 3$, $C = 1$, $b_2 = 1$ AND $\alpha = 5$.

λ_1/λ_1'	MPI		Fluid Model	
	Δ	Reward	Δ	Reward
20/10	0.4	0.50 ± 0.20	0.43	0.60 ± 0.20
15/8	0.6	0.93 ± 0.28	0.65	0.82 ± 0.24

VII. Conclusions

The main contribution of this work is the analysis and characterization of a possible model of spectrum sharing in cognitive radio networks. First, we characterize the behavior of the system when an admission control is applied using a fluid approximation of the stochastic model. Then, we model the economical problem and we develop a computationally efficient way to find an estimation of the admission control boundary in order to optimize the profit of primary SP. The proposal is flexible enough to allow different arrival rates for the primary users as well as different departures rates. Through extensive simulations we have verified that the proposed approximation is accurate.

We believe that the results in this work are inspiring and applicable to important emerging classes of wireless networks. However, it is important to remark that the studied problem is much more general than a cognitive radio network analysis, since it can also model many other economic scenarios of dynamic control of queueing systems which consider preemptive situations with reimbursement, admission control decisions and multi-resource allocation.

Finally, the next stage in our research line would be to incorporate to our problem dynamic pricing features like for example the ones defined in [14].

Acknowledgments

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