Feed resource allocation optimization in dairy systems

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ABSTRACT

In this thesis, the problem of feed resources allocation to a heterogeneous dairy herd was studied. We focused on how to allocate available feed resources by grouping cows based in their energy requirements and distribute them to the available feed resources like pasture and/or supplements. This problem was modeled as a combinatorial optimization problem and solved with exact methods and Evolutionary Algorithms (EA). Considering that exact methods may have limitations due to a great computational demand (which causes extremely high executions times), initially, our approach used a single objective mathematical model and a Genetic Algorithm (GA). An experimental evaluation was performed in order to analyze the quality solution of the GA and to study how the resource allocation should be performed by interpreting the solutions' structure for both methods. The results showed that the values obtained by the GA were very close to the exact values (the maximum gap value was 1.09% and the average gap value was 0.50%), but generating different assignment structures, presenting a good diversity and a wider exploration of the solutions' space. In particular, we found many solutions with a very low gap value and large structural difference, reaching a maximum of 49.9%. Then, due to the complexity of dairy systems and the need to contemplate several objectives, a Pareto-based multi-objective optimization with the Differential Evolution (DE) algorithm was applied. To evaluate the DE algorithm, we performed experiments to compare the solutions quality of the DE with exact Linear Programming (LP) solutions. As part of this analysis, the influence of different stocking rates (number of cows/ha) on milk production, feed allocation and economic performance indicators was also evaluated as a source of variation. The DE solutions that minimize the feeding costs for different stocking rates closely approached the solutions derived with LP (with average values slightly higher, between 0.4% and 5.6%), confirming the quality of the DE algorithm. The multi-objective model scenarios demonstrated that increasing stocking density would enhance milk production and gross margin per unit of area at largely unchanged productivity per animal, by shifting the feed ration from roughage to a large proportion of supplementary concentrate feed. In particular, for stocking rates of 1.1, 1.6, 2.1 and 2.6 cows/ha, gross margins of 6.1, 8.9, 11.8 and 14.7 US dollars/ha/day were obtained, respectively. From the results, we concluded that the multi-objective optimization with a Pareto-based DE algorithm was highly effective to explore the interrelations among conflicting objectives and to find suitable solutions. Finally, and considering there are many variants of EA which have different performances depending on the problem being solved, we decided to evaluate some of the most successful algorithms presented in the literature to address the feed resource allocation problem. In particular, a performance evaluation of four methods (two GA: NSGA-II, SPEA-2; and two DE algorithms: GDE-3, and the Pareto-based DE) was done. The algorithms were evaluated taking into account execution times, objective functions values attained, Pareto front comparisons and performance metrics values. The results showed significant differences between the algorithms in their ability to approach solutions in the Pareto front and in their computational times. In particular, the SPEA-2 algorithm obtained optimal values for all objectives, its solutions represented a large part of the Pareto front approximation, and it presented the best results in terms of convergence, diversity and cardinality; but required higher execution times. Depending on the parametric settings of the algorithms, the execution times of NSGA-II and GDE-3 were between 5 and 23 seconds, while the times of SPEA-2 were between 105 and 28400 seconds.

Keywords: Resource allocation, Dairy production, Evolutionary algorithms, Multi-objective optimization

RESUMEN

En esta tesis, se estudió el problema de asignación de recursos alimenticios a un rodeo lechero heterogéneo. Nos enfocamos en cómo asignar los recursos alimenticios disponibles agrupando las vacas en función de sus necesidades energéticas y distribuyéndolas en los recursos alimenticios disponibles, como pasturas y/o suplementos. Este problema se modeló como un problema de optimización combinatoria y se resolvió con métodos exactos y Algoritmos Evolutivos (EA, por sus siglas en inglés). Dado que los métodos exactos pueden tener limitaciones debido a una gran demanda computacional (lo que causa tiempos de ejecución extremadamente largos), inicialmente, nuestro enfoque utilizó un modelo matemático y un Algoritmo Genético (GA, por sus siglas en inglés) contemplando un solo objetivo. Se realizó un experimento para analizar la calidad de las soluciones del GA y estudiar cómo se debe hacer la asignación de recursos mediante la interpretación de la estructura de las soluciones de ambos métodos. Los resultados mostraron que los valores obtenidos por el GA son cercanos a los valores exactos (el valor máximo de gap fue 1.09% y el valor promedio de gap fue 0.50%), pero generaron estructuras de asignación diferentes, presentando una buena diversidad y una exploración más amplia del espacio de soluciones. En particular, encontramos muchas soluciones con un valor de gap muy bajo y una gran diferencia estructural, alcanzando un máximo de 49.9%. Luego, debido a la complejidad de los sistemas lecheros y la necesidad de contemplar varios objetivos, se aplicó una optimización multiobjetivo basada en Pareto, para la cual se usó un algoritmo Diferencial Evolution (DE, por sus siglas en inglés). Para evaluar el algoritmo DE, se realizaron experimentos que compararon la calidad de las soluciones del DE con soluciones exactas de un modelo de Programación Lineal (LP, por sus siglas en inglés). Como parte de este análisis, también se evaluó, como fuente de variación, la influencia de diferentes cargas animales (número de vacas/ha) en la producción de leche, asignación de alimentos y resultados económicos. Las soluciones del DE que minimizan los costos de alimentación para diferentes cargas se acercaron mucho a las soluciones obtenidas con el modelo LP (con valores promedio ligeramente superiores, entre 0.4% y 5.6%), lo que confirma la calidad del algoritmo DE. Los escenarios del modelo multiobjetivo demostraron que aumentar la carga animal mejoraría la producción de leche y el margen bruto por unidad de área, manteniendo en gran medida la productividad por animal, al cambiar la ración de forraje por una gran proporción de suplementos. En particular, para cargas animales de 1.1, 1.6, 2.1 y 2.6 vacas/ha, se obtuvieron márgenes brutos de 6.1, 8.9, 11.8 y 14.7 dólares estadounidenses/ha/día, respectivamente. A partir de los resultados, se concluye que la optimización multiobjetivo basada en Pareto, para la cual se usó un DE, fue altamente efectiva para explorar las interrelaciones entre objetivos conflictivos y encontrar soluciones adecuadas. Finalmente, considerando que hay muchas variantes de EA que tienen diferentes rendimientos según el problema que se está resolviendo, decidimos evaluar algunos de los algoritmos más exitosos presentados en la literatura para abordar el problema de asignación de recursos alimenticios. En particular, se realizó una evaluación de desempeño de cuatro métodos (dos GA: NSGA-II, SPEA-2; y dos algoritmos DE: GDE-3 y el DE basado en Pareto). Los algoritmos se evaluaron teniendo en cuenta los tiempos de ejecución, valores de las funciones objetivo, comparaciones del frente de Pareto y valores de distintos indicadores de calidad. Los resultados mostraron diferencias significativas entre los algoritmos en su capacidad para acercarse a las soluciones del frente de Pareto y en sus tiempos computacionales. En particular, el algoritmo SPEA-2 obtuvo valores óptimos para todos los objetivos, sus soluciones representaron gran parte de la aproximación del frente de Pareto y presentó los mejores resultados en términos de convergencia, diversidad y cardinalidad; pero requirió tiempos de ejecución más altos. Dependiendo de la configuración paramétrica de los algoritmos, los tiempos de ejecución de NSGA-II y GDE-3 estuvieron entre 5 y 23 segundos, mientras que los tiempos de SPEA-2 estuvieron entre 105 y 28400 segundos.

Palabras clave: Asignación de recursos, Producción lechera, Algoritmos Evolutivos, Optimización multiobjetivo

Contents

Chapter 1

Introduction

The dairy industry plays a significant role in providing essential nutrients and economic benefits. Milk and dairy products provide important sources of high-quality protein and micronutrients in many low and middle-income countries where undernutrition is prevalent [55]. Furthermore, the dairy industry significantly contributes to the economies of many countries, generating income and employment opportunities for millions of people worldwide, especially in rural areas [57]. The world market for dairy products is constantly growing and evolving, highlighting a significant increase in production in recent years [87]. Milk is one of the most valuable agricultural products in the world, and together with its derived products represent 14% of world agricultural trade. It contributes 27% of the global value added of livestock and 10% to that of agriculture. In 2013, when its value approached the maximum of the last decade, it reached a total production of 770,000 million liters valued at 328,000 million dollars, ranking third by production tonnage. It was the main agricultural product in terms of value in all over the world [56].

The livestock industry constitutes one of the most important productive sectors of the Uruguayan economy. In particular, milk production represents 9% of the gross value of agricultural production and occupies 5% of the Uruguay's agricultural area, being the sector with the highest export earnings per hectare (70% of the milk produced is exported to more than 60 countries as powdered milk, cheese and butter). There are currently 3900 dairy producers. Although there is a wide range of production systems and scales, an average dairy farm has 150 cows, where each one produces 18 liters per day in a feeding system that combines direct grazing of implanted pastures and supplementation with concentrate and conserved forage [89]. In recent decades there has been an unprecedented increase in national milk production, accompanied by a concentration process of production systems. Milk production increased by 52% during the last 15 years, going from 1.3 to 2 million liters per year, while the number of dairy farms decreased by 31%, going from 5.1 to 3.9 thousand dairy farms [44]. This process, in a global context of changes, led production systems to face new problems.

The primary factor driving global productivity growth is the combined effect of higher stocking rates and increased production per cow. A trend that is prevalent worldwide is the rise in farm and herd sizes coupled with a decrease in the number of farmers. In particular, in Uruguay, between 1985 and 2016, dairy production soared from 597 to 2083 million liters (L), despite a reduction in the land area allocated to dairy production from 1196 to 764 thousand hectares (ha) and a decline in the number of dairy farmers from 7102 to 3873 [58]. In grassland-based dairy systems, the stocking rate (number of cows per hectare), the stocking method and feed supplementation are important decision-making instruments that determine the effectiveness of the systems, directly impacting on feed intake, milk production and management efficiency in terms of labor productivity and economic viability. Variability in food availability and quality can be high, particularly if a large proportion of the *Dry Matter*[1] intake (DMI) is derived from pastures [75].

The dairy production system in Uruguay is defined as a pastoral system with supplementation [29], where the food supply structure is defined by the pastures directly grazed by cows and supplements. The places where the cows receive supplements are often a feed bunk where cows receive different mixes of concentrate and forages. These feed bunks are close to the milking parlour to minimize walking distance and machinery logistics. In Uruguay, the intensification of milk production has been based on a significant increase in the use of concentrates and preserved forages, while grazing remained unchanged [58]. However, the viability of these practices and their productive and economic sustainability is debatable. While some specialists promote the intensification of dairy systems, others question it. In particular, managing a larger number of cows/ha is considered to have a negative impact on the dairy system, since infrastructure constraints and animal welfare in this context are challenging. Also, in a context of economic uncertainty, where feed costs are high and the price of milk is volatile, many producers are trying to harvest as much grass as possible, positioning grass as the main feed for dairy cows, and using supplements only when grass is scarce. In this scenario, to achieve a successful system, it is necessary to have control over feeding, the dairy herd and also a high level of management that guarantees an efficient allocation of feed resources to the dairy herd [58].

Feed resource allocation is a central decision problem in dairy systems, and consists in determining how to asses the available feed resources into the dairy herd for feeding purposes. To perform the allocation, the characteristics of the cows and the feed must be considered. Animals differ in their feed requirements, in terms of both quantity and quality, based on their size, activity, growth and physiological status (lactation or pregnancy). Feed differs in energy content, availability, cost and their distance to the milking parlour. For management purposes, the herd is usually split into groups (typically based on parity and/or the level of production), and each group is distributed to the feeding options. The composition of each group remains unchanged for a certain period of time, typically one month. After that period new groups of cows are defined, so the distribution process into the feeding areas starts over. This procedure is repeated throughout the whole year. In Uruguay, this feed allocation process is usually carried out based on the experience and intuition, and even traditions of the producers, following management rules. This problem is difficult to solve when the problem size increases (since there are many possible combinations for grouping and distribution) and/or when resources are scarce. For this reason, addressing this problem by studying techniques to support in the decision making process can be of great help to the dairy sector. In this sense, the application of Operational Research (OR) in agriculture has been extensive and varied [168]. However the literature review performed has shown that this study is original, specially considering how the problem has been addressed, since there are no direct antecedents that solve the feed resources allocation to the dairy herd in a quantitative way.

The feed resource allocation to the dairy herd can be modeled as a combinatorial optimization problem. These models can be solved by applying exact methods (or exhaustive search techniques), although they may have limitations. In particular, when the size of the problem is large, they generate a great computational demand, which causes extremely high running times. In combinatorial optimization problems, obtaining solutions quickly is beneficial not only for computational efficiency in the use of resources, but also because it allows a more effective exploration of the search space. The problem of feed resource allocation in dairy systems is very

[1]The dry matter is a measurement of the mass of something when completely dried.

complex, and being able to analyze different strategies quickly is extremely important. The use of techniques that provide fast solutions makes it possible to explore a broader search space, and therefore simulate and analyze different scenarios in a given period of time. In turn, performing a quick scan could allow to make more agile decisions and react to changes in real time. Particularly for the problem addressed, where the dairy manager is involved in the decision making, obtaining fast solutions allows for a more effective interaction between the user and the algorithm. This type of interactions allows simulations to be carried out with different planning scenarios. Among other things, it is possible to quickly evaluate the convenience of modifying herd management, considering an increase or decrease in the stocking rate, or contemplating different needs that allow greater herbage intake, milk production or economic benefits (through a reformulation of the food supply).

When exact methods are not appropriate, an alternative to solve these problems are heuristic techniques. Heuristic techniques are simple procedures, generally based on common sense, which should provide a good solution in a simple and fast way. Unlike exact methods, these techniques do not guarantee obtaining an optimal solution to the problem, but they do aim to provide a good quality solution in reasonable execution times. Since it is not possible to guarantee an optimal solution, it is also not possible to know with certainty the error these methods have. Different heuristic techniques have been applied in different areas, presenting satisfactory results. With a higher-level problem-solving approach, metaheuristic techniques aim to efficiently explore the solution space based on different strategies, also incorporating mechanisms to avoid well known problems such as getting stuck in local optima. Evolutionary computing techniques constitute a set of metaheuristics generally used to solve combinatorial optimization problems, information searching, machine learning, among others. These techniques base their behavior on an emulation of natural evolution. They work on a population of solutions that are representations specifically coded according to the problem to be solved. Following Darwinian principles, these solutions interact with each other to generate better solutions to the problem. Following the biological process that occurs in nature, where living beings try to survive and thus maintain the species, evolutionary computing techniques apply the concepts of adaptation to the environment to solve problems in various areas. In this work, we worked with Evolutionary Algorithms since they are robust searching optimization techniques.

The main goal of this work is to study the problem of feed resources allocation to the dairy herd and its modeling as a combinatorial optimization problem. To address the problem, different evolutionary algorithms were proposed and experimental evaluations were performed using specially designed scenarios for this study. This approach enables a comparative analysis of the performance and quality of the obtained results, considering both computational and agronomic point of view.

The rest of this document is organized in 8 chapters:

In Chapter 2, we present general concepts about the dairy sector. We introduce the importance of dairy (at world and national level) and a brief historical review, where data to facilitate the understanding of the dairy evolution is presented. We also provide an explanation about pasture management in dairy systems and we present the concept of supplementation, including a justification for its possible use.

In Chapter 3, we introduce the concept of computational methods and present the most popular models in the scientific community. We introduce the exact methods and explain their limitations, justifying the need to address certain problems using alternative techniques. Then, we describe the evolutionary algorithms and their corresponding theoretical framework. Finally, we delve into evolutionary techniques, particularly genetic algorithms and differential evolutionary algorithms.

In Chapter 4, we describe the problem to be addressed, introducing the milk production model and explaining how to use it.

In Chapter 5, we mathematically formulate a first approximation of the resource allocation problem in order to maximize either dairy production or the economic benefit. We clarify the characteristics of its representation and the details of its implementation. We also present the resolution of the problem by using genetic algorithms. From the obtained results, we studied the resource allocation by interpreting the solutions' structure, and we analyzed the quality and diversity of these solutions. Part of this work was first presented in 2014 at the VIII ALIO/EURO Workshop on Applied Combinatorial Optimization [122]. In this version, the aim was to maximize milk production using a mathematical programming model, and also a Genetic Algorithm as an alternative method was used. Later, a subsequent version was presented at the Latin Ibero-American Conference on Operations Research (CLAIO 2014) and published in the proceedings of the conference [131]. In that version, the previously mentioned methods were used, and an experimental evaluation was performed to analyze the obtained results from an agronomic perspective. To do this, the structures of the obtained solutions were compared. Finally the complete work was published in the Agricultural Systems journal in 2016, extending the work from the previously mentioned conferences and incorporating the study of the quality and solutions diversity [130].

In Chapter 6, we present an extension of the model presented in the previous chapter using a multi-objective and multi-period approach. To solve the model, we used a multi-objective differential evolutionary optimization algorithm to generate an approximation of the Pareto front. In addition to developing, testing and validating the usefulness of the methodology in the context of the dairy feeding systems problem, an additional aim was to apply it to typical Uruguayan dairy farming systems to identify and analyze the trade-offs between the different objectives. A first approximation of the extended model was presented in 2017 at the EFITA WCCA Congress, the European conference dedicated to the future use of ICT in the agri-food sector, bioresource and biomass sector [129]. Then, in 2018, results obtained from real data and considering experiments with different stocking rates were presented at the 29th European Conference On Operational Research [125]. Also, that same year, new experiments based on information from an ongoing dairy project were carried out, and the results were presented in the Big DSS Agro Conference [124]. These results were compared with the results obtained in the dairy project, which used simple objective exact methods (reaching optimal solutions), which allowed us to identify the quality of the solutions obtained by our model. Finally, a complete work, which extended part of the work presented in the previously mentioned conferences and included the analysis of the trade-offs between the different objectives, was published in the Agricultural Systems journal in 2020 [123].

In Chapter 7, we present a performance evaluation of four different evolutionary algorithms for the multi-objective and multi-period model. To evaluate the algorithms, computational experiments based on different sets of evolutionary parameter values were performed. Also, different quality metrics were used for this purpose. The performance evaluation proposal of different algorithms was presented at the Big DSS Agro Conference in 2019 [126]. Then, in the following edition of the Conference (2020), preliminary results of the conducted computational experiments were shown [127]. Finally, at the IEEE Latin American Conference on Computational Intelligence (LA-CCI) in 2021, more detailed results were published, and the advantages and disadvantages of each evaluated algorithm were discussed [128].

In Chapter 8, we present the conclusions of this work together with the proposal of possible lines of future work.

Chapter 2

Dairy Systems

Milk is produced and consumed throughout the world, and in most countries it ranks among the top agricultural products, in terms of quantity and value. Dairy production represents the economic and welfare support for millions of people. It is estimated a figure close to 120 million dairy farms in the world, which translates into approximately 70 million jobs [88]. In the last three decades, world milk production has increased by more than 60%, from 530 million tons in 1988 to 852 million tons in 2019 [56]. India is currently the world's largest producer of milk (196 million tons), followed by the United States (99 million tons), Pakistan (47 million tons), Brazil (35 million tons) and China (32 million tons) [88]. This increase is due, among other things, to the growing global demand for dairy products, which is driven by the increase in the world population and by higher per capita consumption. The global trend is also accompanied by a progressive increase in the size of dairy farms and greater professionalization, approaching models such as those in the United States or New Zealand, although the potential of family farms should not be underestimated. In turn, productive growth is accompanied by a greater number of animals (mainly in countries such as India and Pakistan), improvement in collection processes, efficiency improvements in integrated milk production systems (as in Turkey), increased production yields (as in the European Union and the United States), and an improvement in the utilization of installed capacity together with a high demand of imports for the dairy processing sector (as occurs in some countries in South America) [45]. The future of the dairy sector is promising, mainly due to markets in Asia and Africa, where significant growth is expected in the coming years.

Developing countries have driven this global productivity growth since the last decades. In particular, since the 1970s, the largest increase in milk production has been in South Asia. Some of the developing countries (located in the Mediterranean or the Near East, the Indian subcontinent, the savanna regions of West Africa, the highlands of East Africa, and parts of Latin and Central America) have a long tradition of dairy production, and milk or its derived products play an important role in their diet. On the other hand, other of these countries (located mainly in Southeast Asia and tropical regions) have only shown significant growth in production in recent years [6].

Regarding the price of milk, there is great volatility in the last 20 years. This volatility is due to the fact that production has a delay in responding and self-regulating to the signs of evolution and market adaptation of demand and prices. In particular, the price of milk presented an average of USD 0.25 per liter (with 4% fat and 3.3% protein) in the 2000-2006 period, increasing to an average of USD 0.40 per liter between 2007 and 2015. Then it decreased to USD 0.35 from 2017. Regarding production costs, the situation is uneven, since it depends on several factors, such as cost of food and labor, subsidies provided by the government, or climatic situation. Due to climatic differences, there are two well differentiated milk production systems in the world. The first system corresponds to regions with extreme temperatures, so the cows must be protected from cold or heat. For example, in countries in the northern hemisphere, cows are housed in barns during winter due to snow. On the other hand, in countries with very high temperatures, cows are housed to provide shade and artificial cooling, avoiding heat stress. In these production systems, the proportion of pasture allocated to livestock is minimal or zero. On the other hand, in the southern hemisphere, the climates are more temperate and the production systems have pastures as a feeding base [107]. The region in which Uruguay, Argentina, Brazil and Chile are located has some advantages compared to other regions. Among them, the possibility of achieving very good forage yields and using energy supplements due to the agroecological aptitude in the region stands out [24]. Grassland-based systems with supplementation have shown to be very competitive in conditions of high variability in both the market and the climate [27].

2.1 Grassland-based dairy systems

When herbage is the main food in a dairy farm, the production system is considered better for animal welfare. The greater the forage production, the greater the number of animals that the system can supply. This number determines the potential of the system to harvest grass, which defines the use of the forage produced. The amount of forage that each animal can consume depends on several factors: forage quality, pasture structure, and grazing management. The higher the DMI, the higher the milk production. Particularly, the higher the nutrient content per unit of biomass consumed, the potential for transformation of the consumed grass into animal product increases [146].

When maximizing milk production is the main goal and the pastures consumption is the main source of feed, DMI and hence individual milk production will be conditioned by nutritional and non-nutritional factors. Particularly, in countries where pastures are the most important source of food, it is usual to find differences in quantity and quality throughout the year, and they may not be able to satisfy the nutrient demand of the animals. Non-nutritional factors are determined by the structure of the grasses and their morphological characteristics (height, composition, resistance to cutting, among others), while the most important nutritional factor corresponds to the digestibility of the forage. In turn, the higher the digestibility, the higher the consumption. Due to this, the digestibility of the forage is conditioned by its chemical composition (mainly the fiber content) and by factors inherent to the animal [53].

In this type of production systems it is important to consider two key aspects: the use of the forage produced and the conversion of the grass consumed into milk. In the first place, the use of the forage produced is essential for the success of these systems, since unused grass is lost. On the other hand, the process of converting the grass consumed into milk is defined by the content of nutrients consumed. The higher the nutrient content per unit of biomass consumed, the higher the animal product. The key to harvesting and obtaining more milk from pasture depends on the capacity of the implemented system to take advantage of the accumulated biomass before its quality decreases and part of it get lost [146].

The production capacity of the dairy herd is directly related to the characteristics of the animal physiological state (genotype) and the environmental conditions (physical conditions of the environment, topography, climate, soil conditions, etc.). To design a production system, the characteristics mentioned above, as well as, some management decisions (number of animals, type of animals regarding its characteristics, amount and type of feeding activity, calving season, among others), have to be taken. These decisions determine the supply and demand structure of the production system, directly affecting its productive, environmental and economic capacity. Since dairy systems exhibit a structural imbalance in time between nutrient supply and demand, it is necessary to balance it them with supplementation (conserved forages and concentrates), which increases complexity from the point of view of operations, infrastructure and particularly in the management and control of food resources [29].

Pastoral systems rely more and more on balanced energy supplementation. In a context of intensive grazing, the main objectives of providing supplementation to dairy cattle are to obtain higher milk production, increase profits and avoid compromising the health of the animals. Even in systems where pastures provide a significant amount of nutrients, it is common to include supplementation in the diet. This supplementation allows to achieve milk productions that exceed the normal average values [113]. In some regions, grazing shows great seasonality throughout the year, with periods that present surpluses of pasture and forage, as well as periods of insufficiency that makes it impossible to maintain the herd under these systems. For this reason, supplementation is essential to achieve the objectives of balancing the supply and demand of nutrients, also having a positive impact on profitability. In turn, the need to use supplementation in autumn-winter is caused by a decrease in the growth rate of grasses [25]. Supplementation at that time of year seeks to increase energy values (limited in the case of exclusive grazing), but also to allow a high stocking rate that results in a high harvest efficiency in the spring-summer period. Precisely in spring, supplementation is less important, but it is still advisable to carry out a diet with a supplement to adjust the basic needs of the dairy herd and achieve a good productive performance. Finally, in summer, the need to use supplements arises because the pastures decrease their quality.

The economic changes in recent years have revealed the need to free up the grazing area for agriculture, so achieving sustainable mixed systems requires a certain degree of dairy intensification. For this reason, housing dairy cow is taking more and more prominence. Paradoxically, in the United States, where housing systems based on the use of Totally Mixed Rations (TMRs) are very popular, the use of grazing in dairy cattle has been strongly considered in recent years. This occurs due to the need to reduce production costs (mainly food and labor) of US producers and because public perception of animal welfare and product health. Extreme intensification through the use of TMRs would directly impact competitiveness, since the advantages offered by grass to produce milk at low cost would be lost. In turn, it would lead to distancing from potential differentiated markets that seek milk and/or dairy products from cows fed on pastures. We must emphasize that high genetic merit dairy cows cannot reach their maximum production potential solely on the basis of grass consumption. Therefore, they require a daily supply of energy to meet the nutritional demand due to milk production. The different levels of intensification that producers apply to increase milk production depend on the price they receive for each liter of milk and the costs associated with the components to produce that liter of milk [26]. To determine how much supplementation to introduce into the system in order to achieve productive and economical objectives, producers must consider both the nutritional needs of the cows and the conditions of the feeding areas [144].

The management of grassland-based dairy systems with supplementation involves the periodic decision on how to combine the available feed resources considering the available animal potential. In this work, this problem is analyzed and modeled as a combinatorial optimization problem.

2.2 Dairy sector in Uruguay

In Uruguay there are commercial dairy production systems in all nineteen departments, although two areas stand out, the South Basin and the West region. The South Basin is made up of the departments of Montevideo, Canelones, San Jose and Florida, while the West Coast is made up of the departments of Colonia, Soriano, Rio Negro and Paysandu. The highest degree of concentration occurs in San Jose, Colonia and Florida. Together they accumulate the largest number of dairy farms (2353, representing 63.8% of the total), with an area of 454,891 hectares, which represents 60.3% of the total area [45].

In the last five decades, Uruguayan milk production has grown significantly, going from 400 to 2237 million liters per year in 2018. However, this was not due to an increase in the number of producers, since the number of them has declined significantly during the last 20 years, where more than 2000 producers have disappeared [24, 45]. This increase in milk production was supported by an improvement in productivity rates. National dairy farms greatly increased both production per hectare and per milking cow. In recent years, the use of conserved forages, feeds byproducts and grains has also increased, but the basis of the production systems in Uruguay continues to be essentially pastoral. Between 1977 and 1999, dairy production grew approximately 3% per year. This growth corresponds almost entirely to an increase in milk sent to processing plants, and destined for industrialization and manufacture of milk by-products, with very low growth in the market for milk destined to domestic consumption. In those years, establishments that did not reach 1000 liters per hectare per year (lt/ha/year) with others that were around 3000 lt/ha/year in a sustained and profitable way coexisted, based on a system with a pastoral base and using conserved forages and concentrates in a strategic but limited way. One of the main reasons that caused the change in Uruguayan dairy from the 70s to the present is due to the greater use of cultivated pastures. The increase in quantity and quality of feed impacted on productivity and productive performance, as well as, on the recovery of soil fertility [34]. Currently, the main destination of the produced milk continues to be the processing industry, reaching 83.6% of the total production (1870 million liters). The Uruguayan export coefficient (exported/available) is approximately 61.2% of the total production, while the rest is destined for domestic consumption. Uruguay is the largest milk producer per capita in Latin America, and it also has a high consumption, one of the highest in the world [45].

In the last decades, the sector has grown at rates of the order of 5% per year, accelerating in the last 6 years with growth rates of 7% per year. This growth process has been mainly sustained by increases in productivity, since the dairy area decreased by 10% [43, 90]. Productivity per cow is the factor that individually explains a greater proportion of the total productive growth (greater than 60%), while the increase in stocking rates explains the remaining percentage. As has happened in various pastoral systems, the strategy of intensification of milk production in Uruguay was based on an increase in the use of conserved forages and concentrates, while the direct harvest of forage by the cows remained unchanged [28]. This model of intensification of the Uruguayan dairy has been demanding investment, specially in the feeding process (incorporation of Mixers, feed bunks, etc.), milking capacity, structure, livestock management, etc. The requirements for qualified human resources and the organization of work have also increased. Regarding the evolution of employment in the sector, at the end of the 80s there were approximately 5000 employees, a number that dropped to 3600 by the end of the 90s. Then, in the first decade of the XXI century, dairy production grew significantly, placing dairy as one of the sectors with the highest participation in the industrial added value and with the highest level of exports, even facing changes in the conditions of competition with other agricultural activities and a significant increase in the price of land [61].

The efficient use of food and animal resources is essential for pastoral systems, so it is interesting to have a model for the resources allocation that supports decision-making, particularly facilitating a balanced management between supply and demand throughout the year.

Chapter 3

Computational Methods

Operations Research can be defined as a discipline that consists on the application of advanced analytical methods in order to support the decision-making process, identifying the best possible courses of action [140].

In this context, Operations Research uses techniques of mathematical modeling, statistical analysis and mathematical optimization, with the aim of reaching optimal solutions or close to them when faced with complex decision problems. Decisions reached through the use of an operation research model are expected to be significantly better compared to those decisions that could be made using the simple intuition or experience of the decision maker. This is particularly true when dealing with complex real-world problems, which consider hundreds, even thousands of decision variables and constraints. In formal terms, an optimization model considers one or more objective functions, made up of variables, that you want to maximize or minimize. The values that the decision variables can adopt are usually restricted by constraints that take the form of equations and/or inequalities that seek to represent the limitations associated to the addressed problem [159].

The focus of Operations Research is modeling. A model is an analytical tool that helps us to achieve a structured vision of reality. In this way, the purpose of the model is to provide a way to analyze the behavior of the components of a system in order to optimize their performance (identify the best possible course of action). In this sense, we can identify five stages: definition of the problem, construction of the model, solution of the model, validation, and implementation and control of the solution. Subsequently, and if necessary, it is possible to carry out an iterative and incremental analysis, in which the different stages are addressed again in order to correct, complement or enhance the model. A graphic representation of an iterative and incremental analysis that addresses the mentioned stages is presented in Figure 1. The first stage is to define the problem. In this instance it is important to identify the relevant factors in the problem and if it is possible to make a good decision without having to address the problem through an operations research approach. The quality of the optimization model will largely depend on the assertiveness in the definition of the decision problem. Then we proceed to build the model, which will seek to represent the problem in a simplified way. On the one hand, the model is intended to be representative of the addressed reality, but at the same time it must be simple to favor its resolution and understanding. Reaching this balance is not easy. For this reason, it is possible to define more than one optimization model that represents the problem with different levels of detail and abstraction. Once the optimization model has been built, the alternative solutions for it must be identified. Computer programs that use specific resolution algorithms depending on the characteristics of the model can be used. Finally, it is verified that the reached solution complies with the imposed constraints and it is implemented.

Modern optimization methods have their origins in World War II, when the US and British military administrations called in scientists to collaborate in solving strategic and tactical problems. In particular, they were asked to carry out military operational research. Scientists contributed to the triumph of Britain's air warfare by developing effective methods for using the new radar tool. Currently, optimization is a process that has great applications in almost all areas, where an improvement on the systems is achieved and therefore producing a reduction in costs, time, resources, etc. [84].

Figure 1: Stages of the Operations Research model.

3.1 Mathematical programming

Optimization problems are mathematically modeled using Mathematical Programming, which is divided into several branches according to the characteristics of the variables and the equations or inequalities that describe the model. If the equations are linear, then we are faced with a Linear Programming problem, otherwise we are facing a Non-Linear Programming problem. At the same time, it is possible to differentiate between Integer Programming and Continuous Programming problems, which use integer and continuous variables, respectively. Depending on the addressed problem, it may also be interesting to work with more than one objective to optimize, for these cases we refer to Multi-Objective Programming. It is also possible to differentiate the problems according to the characteristics of the values to be used. When the parameters that determine the problem have fixed values, we face a Deterministic Programming problem, while if the values came from a random distribution we face a Stochastic Programming problem [84].

Another branch of Mathematical Programming is Combinatorial Optimization, which attempts to solve problems from an exploration of the solution space. These problems consist of obtaining an element of a finite or infinite countable set, which is optimal for the maximization or minimization proposed. The resolution mechanisms or methods for these problems can be divided into two categories, the exact methods and the heuristic methods (or approximate methods). Exact methods are those that provide an optimal solution to the problem, while heuristic methods provide a good but not necessarily optimal solution to the problem [135].

Exact methods guarantee to obtain a solution whose value is the global optimum (there can be several solutions with the same value). In cases of combinatorial optimization, a trivial exact method is to perform an exhaustive search within the set of all solutions to the problem. In some cases, based on the study and analysis of the structure and characteristics of the instances of a combinatorial optimization problem, it is possible to develop algorithms that obtain optimal solutions whose computational complexity has a lower order than the exhaustive search. However, in other cases, these same algorithms fail to reduce the order of computational complexity of the exhaustive search. Some of the most commonly used methods for solving linear programming problems and integer linear programming are the simplex method [105], interior point methods [106], and branch and bound [102]. At first glance, the application of exact methods seems a logical and simple solution, since it guarantees to obtain the global optimum. However, in many cases, and particularly in large and difficult instances of problems, this proposal becomes impractical. In these scenarios, the possible number of solutions grows exponentially according to the size of the instance, which makes the computational complexity of the exact methods grow similarly. The intuitive idea of "hard problems" is reflected in the NP-Hard problem family. For this type of problems, no exact algorithms are known for their resolution in polynomial time. In other words, these problems are those for which an optimal solution cannot be guaranteed in a reasonable time [135]. Due to the interest in solving NP-Hard problems and the use of larger instances, the scientific community has developed efficient procedures that allow determining good quality solutions, even if they are not optimal, in reasonable response times.

3.2 Metaheuristics

A heuristic (word derived from the Greek heuriskein, which means "to find" or "to discover"), or heuristic method, is a procedure that tries to find a good feasible solution to the addressed problem, but not necessarily an optimal solution. These types of methods, in general, do not provide any guarantee about the quality of the solution found, but a well-designed heuristic can provide a solution close to the optimal (or conclude that such solutions do not exist). These procedures are usually an iterative algorithm, where in each iteration it seeks to find solutions that present better characteristics than those found in previous steps. After a reasonable time has elapsed, the algorithm provides the best solution found. Heuristics are often based on simple ideas about how to find a good solution. These ideas must fit the problem, so, in general, each method is designed to address a specific type of problem rather than a variety of applications. Due to this, to develop a heuristic method, it is necessary to design a new algorithm from the scratch in order to adjust to a specific problem, as long as there is no algorithm that finds the optimal solution. This scenario has changed due to the development of powerful metaheuristics. A metaheuristic is a general solution method that provides both a general structure and strategic criteria for developing a specific heuristic method that fits a particular type of problem [84]. Metaheuristics emerged in 1986 when Glover used the term "meta-heuristics" to refer to heuristics with a higher level of abstraction [70]. Among other things, they arise from the need to define heuristic methods that can be used to solve different optimization problems. Metaheuristics seek to improve heuristic techniques, precisely from the integration of different search strategies to explore the solutions space more efficiently and effectively, from the incorporation of small modifications to adapt them to the particularities of each problem [9]. In other words, metaheuristics are high-level strategies that use different methods to explore the solution space. In these techniques, the balance between diversification and intensification is essential. On the one hand, diversification refers to the exploration of the solution space, while intensification emphasizes exploiting the experience generated through the search process [23, 135].

3.2.1 Classification of metaheuristics

Metaheuristics can be classified in different ways. Some classification criteria consists of analyzing whether they are inspired by nature or not, whether they have a static or dynamic objective function, whether they use memory or not, whether they use one or more neighborhood structures, and perhaps the most widely used classification is the one that determines whether the technique uses a single search point (based on trajectory) or works on a set of solutions (based on population) [9]. Trajectory-based methods consist of starting from an initial solution and updating it by exploring its neighborhood, thus generating a trajectory in the solutions space. The search ends when a maximum number of iterations is reached, a solution of acceptable quality is found, or a process stall is detected. Some examples of these techniques are Simulated Annealing [96], Tabu Search [70], GRASP [59], Variable Neighborhood Search [83], and Iterated Local Search [104]. On the other hand, population-based techniques consist of using a set of solutions (population of solutions) in each iteration, so they naturally provide an intrinsic path for exploring the solution space. The way in which the population is manipulated in each iteration determines the efficiency and effectiveness of the technique. Some examples of these techniques are Evolutionary Algorithms [5], Estimation of Distribution Algorithms [101], Scatter Search [71], Ant Colony Optimization [48], Parallel Ant Colony Optimization [136] and Particle Swarm Optimization [95].

3.3 Evolutionary algorithms

Evolutionary algorithms (EA) are heuristic and optimization techniques inspired by Darwin's principles on the evolution of species in the biological world [35]. In nature, during the evolutionary process, living beings try to survive so that their species prevails. Through this mechanism, evolutionary algorithms emulate the biological process of adaptation of living organisms to the environment, applying it to the resolution of problems in different areas. They are computationally modeled by simulating natural selection and genetic recombination through different evolutionary operators. Particularly, they work on a population of individuals that represent possible solutions. These solutions interact with each other, following Darwinian principles of natural evolution with the idea of iteratively producing better solutions to the problem. Particularly, these solutions are mixed and compete with each other, where the most suitable ones prevail over time, evolving towards better solutions. Potential solutions are evaluated using a fitness function, which takes into account the problem to be solved.

The pseudocode of the evolutionary algorithm is presented in Algorithm 1.

Evolutionary algorithms begin with a stage ($t = 0$) in which an initial population P of size N is randomly generated (although it is also possible to apply optimization techniques to create more promising individuals than those randomly generated). These solutions are

Algorithm 1 Evolutionary Algorithm

Create a random initial population $(P^{t=0})$
while NOT StopCriteria **do** *//Evolutionary process*
  $P^t = fitnessEvaluation(P^t)$
  $Pp^t = selection(P^t)$
  $Pc^t = evolutionaryOperators(Pp^t)$
  $P^{t+1} = replacement(Pc^t, P^t)$
end while
return the best solution

encoded in genotypes (information contained in chromosomes), which are represented by decision variables as a multi-dimensional vector p of z alleles. After generating the initial population $P^{t=0}$, the algorithm enters a cycle (that ends when a stop criteria is reached), which represents the mentioned evolutionary process. In this cycle four stages are distinguished: evaluation, selection, application of stochastic evolutionary operators and replacement; where a new generation $t+1$ of genotypes is created from solutions that evolve from their parents (which are in the population P of the current generation t). Initially, the solutions are evaluated by assigning a suitability value (fitness) to each individual in the population P^t. This value evaluates how well each individual solves the addressed problem, and is used to guide the evolutionary mechanism. The parents (appropriate candidates according to their fitness values) are then selected Pp^t, to which the evolutionary operators are applied in order to produce the next generation of individuals Pc^t. Finally, the replacement is carried out, a mechanism that performs generational change P^{t+1}, replacing individuals from the previous generation with new descendants. At the end, the algorithm returns the best solution found in the entire execution.

One of the most used criteria to determine the algorithm's stop condition lies in controlling the generations carried out, stopping the evolutionary cycle when reaching a previously established number of generations G. Other alternatives consider the variation of fitness values, stopping the evolutionary cycle when the process stagnates and does not obtain considerable improvements in fitness values. On the other hand, when the optimal value of the problem is known, it is possible to determine the stopping criterion taking into account the error made in each generation. Regarding the evolutionary operators, they determine the way in which the algorithm explores the solution space of the problem. There are different operators and particularities in their mode of application, providing different alternatives to evolutionary algorithms. The main operators are the recombination and the mutation. The recombination or crossover operator allows combining characteristics of two or more individuals with the idea of obtaining better adapted offspring, while the mutation introduce diversity through random modifications.

Part of a EA's performance is determined by the diversity of the population throughout evolution. In generic terms, diversity is understood as the variety of individuals existing in the population, and in particular the variety of fitness. The need for diversity of individuals is due to the fact that with similar individuals the recombination operator almost completely loses the ability to exchange useful information between individuals, and consequently the search stagnates. The need to keep fitness diversity lies in the practical impossibility of working with an infinite population. With little diversity in fitness, all individuals will have more or less the same chance of survival, and selection will not increase diversity. From this, the importance of the search will fall on the evolutionary operators, which will result in a random search. For the selection to be effective, the population must contain (in all generations) a certain variety of fitness. When the population is finite there should not be a great disparity in fitness either, as it can negatively affect the diversity of the population. A good control of diversity can avoid premature convergence, either due to too much diversity ("superindividuals" at a certain moment but not globally optimal) or even due to too little diversity (genetic drift).

Regarding the instances of selection and replacement, there are different policies that allow modifying the characteristics of the evolutionary algorithm. By applying suitable policies it is possible to privilege the most adapted individuals in each generation, increase the selective pressure over better adapted individuals, generate a reduced number of descendants in each generation, among others.

When using metaheuristics to solve optimization problems, it is also very important to determine the best possible configuration of the parameter values that control the algorithms. The parametric configuration directly impacts the performance of the metaheuristic. Although there are guides that are intended to help the researcher in making decisions to determine the best values, the calibration of parameters is often based on the experience and intuition of the researcher, and above all to through the observation and application of a trial and error methodology. In EA, parameter adjustment is one of the main weaknesses of this type of algorithm, and therefore can be critical for its success. In some cases a very precise calibration is required, to such an extent that a wrong adjustment can lead to an inappropriate behavior of the technique.

3.3.1 History of evolutionary algorithms

Natural evolution was seen as a learning process since 1930s, when Walter D. Cannon suggested that the evolutionary process is similar to the trial and error learning that often occurs in humans [21]. However, the first ideas about evolutionary algorithms emerged in 1950, when relationships between learning processes and natural evolution were detected. In particular, Alan Turing recognizes a connection between both aspects, proposing to use evolutionary techniques capable of simulating intelligent activities developed by human beings [162].

In the late 1950s and early 1960s, biologist Alexander S. Fraser published works on the evolution of biological systems on a digital computer [62, 63], giving the inspiration for the genetic algorithm proposed by Holland in 1975 [86]. Fraser's work included the use of a binary representation, a probabilistic crossover operator, a population of parents that generated a new population of individuals after recombining, and the use of a selection mechanism. His work on this topic is summarized in a book entitled "Computer Models in Genetics" [65]. Fraser also used the term "learning" to refer to the evolutionary process carried out in his simulations, and anticipated the inversion operator, the definition of a fitness function, and the statistical analysis of the convergence of the selection process [64]. Around the same time as Fraser, the English statistician George Box suggested an evolutionary approach to optimizing industrial production. Following an analogy with the development of chemical processes in nature, he proposed modifying the traditional static operating systems by dynamic mechanisms, evaluating their effects and modifying the process to improve the obtained results. His technique, called EVOP (Evolutionary Operation) is still in use in the chemical industry [10, 11]. In 1958, R. M. Friedberg proposed to evolve computer programs, being one of the first scientists to try it [66]. His experiments were not very successful, and they caused a lot of criticism on the part of researchers of classical Artificial Intelligence. In turn, between 1953 and 1956, Nils Aall Barricelli developed the first simulation of an evolutionary system on a digital computer. His experiments followed the guidelines of a discipline baptized in the early 1980s as "artificial life" [149]. Hans Joachim Bremermann was perhaps the first to visualize evolution as an optimization process, in addition to performing one of the first simulations of evolution using binary strings that were processed through reproduction (sexual or asexual), selection and mutation (a clear predecessor of the genetic algorithm) [15]. Bremermann also used an evolutionary technique for optimization problems with linear constraints [16, 18]. The main idea consisted in using an individual that was modified through a mutation operator. In extending this technique to more complex problems, he also used specialized recombination operators [19]. Bremermann was a pioneer in using the concept of "population" in the simulation of evolutionary processes. He also intuited the importance of co-evolution [15] (use of two populations that evolve in parallel and whose aptitudes are related to each other) and visualized the potential of evolutionary techniques to train neural networks [17]. John von Neumann was also interested in combining computation with evolutionary techniques, particularly in the case of cellular automata. In the final stage of his life he was working with cellular automata, as evidenced by his unfinished text "Theory of AutoReplicable Automata" [165], which would be edited after his death by his colleague A. Burke. Von Neumann proposed evolutionary mechanisms based on programming to implement automata with computational power equivalent to a universal Turing machine.

Other researchers also made important contributions to forge what is now known as "Evolutionary Computing". In the mid-1960s, Lawrence J. Fogel conceived the use of simulated evolution to solve prediction problems. His technique was called "Evolutionary Programming". Also in that decade, Peter Bienert, Ingo Rechenberg and Hans-Paul Schwefel developed a method of random discrete adjustments inspired by the mutation mechanism that occurs in nature. His technique was called "Evolutionary Strategies". John H. Holland developed "reproductive" and "adaptive" plans in the early 1960s in an attempt to make computers learn by mimicking the process of evolution. This technique would later be known worldwide as the "Genetic Algorithm". John R. Koza (1989) proposed the use of a tree representation in which a crossover operator was implemented to swap sub-trees between the different programs of a randomly generated population. In doing this work, Koza used an automated fitness function, setting it apart from other similar studies that required manual fitness assignment. Finally, this proposal was called "Genetic Programming', and today it is very popular and has a wide range of applications. In recent years, a technique called "Differential Evolution" has gained increasing interest for solving optimization problems in many fields. The method was originally proposed by Storn and Price in 1995 [157], and today it is considered one of the most popular optimization algorithms, presenting very good performance for different problems.

3.4 Genetic Algorithms

Genetic Algorithms (GA) are a type of evolutionary algorithms that constitutes one of the most widely used techniques today as a consequence of their adaptability to a wide range of problems. By constituting a case of evolutionary technique, genetic algorithms are based on the genetic processes of biological organisms, emulating the natural evolution of living beings. These algorithms combine the survival of the fittest individuals with the exchange of information or genes (randomly) between individuals of a population, thus seeking the best solution to the problem, which is evaluated by the fitness function (definition of a function of adaptation to the problem). For this, the individuals of the population are represented by chromosomes, which evolve according to the principles presented by Charles Darwin on natural selection and the survival of the fittest. In particular, it is expected that, through the application of evolutionary operators, and after a certain number of generations, the population will converge towards a solution close to the global optimum of the problem.

Bremermann's research works are recognized as pioneers, but in practice Holland is considered to be the founder of GA, as he was the one who formalized its operating mechanism in 1975 [86]. Subsequently, many researchers worked with Holland to continue developing and enhancing the GA technique. In particular, De Jong presented a series of computational experiments applied to the optimization of functions, which were consolidated as the set of standard test functions to determine the usefulness of evolutionary techniques [120]. In the 1980s, GAs gained popularity as a combinatorial and design problem-solving strategy. The growing interest of a community of researchers in the area led to the holding of the first International Conference on Genetic Algorithms in 1985. The publication of Goldberg [72] made it possible to further disseminate this evolutionary technique, presenting the theoretical foundations and a varied range of applications in the areas of optimization, search and learning. Since then, the interests of the scientific community working in the area of genetic algorithms have diversified, extending theoretical analyzes, proposing new models and addressing a wide range of applications.

Genetic algorithms are based on the generic scheme of an evolutionary algorithm, introducing the crossover as the main evolutionary operator and the mutation as secondary evolutionary operator. The crossover operator allows to recombine the genetic information of the solutions, while the mutation operator allows to modify the genetic information of the solutions (generally used to generate greater diversity among the solutions). The pseudocode of the GA is presented in Algorithm 2.

Algorithm 2 Genetic Algorithm

Create a random initial population $(P^{t=0})$
while NOT StopCriteria **do** *//Evolutionary process*
  $P^t = fitnessEvaluation(P^t)$
  $Pp^t = selection(P^t)$
  $Pc^t = crossover(Pp^t)$
  $Pc^t = mutation(Pc^t)$
  $P^{t+1} = replacement(Pc^t, P^t)$
end while
return the best solution

In order to apply a GA it is necessary to consider five basic components: a representation of the potential solutions of the problem, a way to create an initial population of possible solutions (usually a random process), an evaluation function that plays the role of the environment (classifying the solutions in terms of their "aptitude"), genetic operators to alter the composition of individuals of different generations, and values for the different parameters used by the GA.

3.4.1 Genetic representation

When approaching a specific problem using GA, it is necessary to define a suitable genetic encoding that allows to represent the possible solutions of the problem. In particular, the representation scheme defines how the chromosomes correspond to the solutions of the problem. For its design, the preponderant parameters of the solutions are identified, and then are encoded within the chromosome. The structure of the solutions can be visualized as a set of parameters (genes), which grouped together form a vector of values, known as a chromosome. Each value in the vector is called an allele. In other words, a chromosome is a vector in which all the genetic information of an individual is stored. The illustration of a population with four chromosomes (individuals), in which a gene and an allele are exemplified can be seen in Figure 2.

In biology, genotype refers to the set of chromosomes that define the characteristics of an

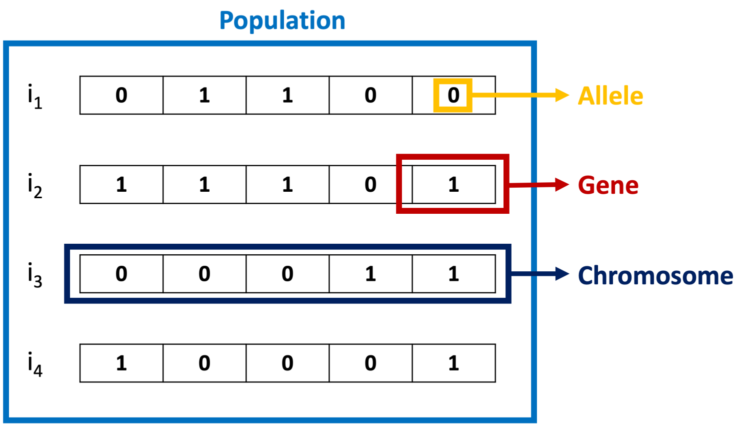

Figure 2: Illustration of a population with four chromosomes.

individual. In GA, the genotype is also made up of chromosomes, although in general a single chromosome per individual or solution to the problem is considered. On the other hand, the phenotype describes the physical appearance of an individual, representing in terms of GA a point in the solution space of the problem [147]. In order to represent any point in the solution space, it is necessary to identify a suitable coding of individuals. These algorithms were originally intended to work with solutions represented by binary strings, and generally use fixed-length binary encodings, so solutions are encoded as a string of bits of a given length. Despite this, other representations have been used over time, such as permutations, vectors of integers or reals, and even complex data structures [120]. The GA are characterized by having a linear representation of the solutions and generally do not include self-adaptation parameters in the individuals. Historically, GAs have tried to be universal, so binary encoding was seen as a standard representation and adjustable to a wide range of problems. In fact, from a string of bits it is possible to encode integers, reals, sets or even more complex representations. Binary chromosomes are also easy to implement and manipulate, allowing mutation and crossover operators to be done easily. In some circumstances, bit strings may not be the most appropriate encodings for some types of problems [72], so it is possible to find representations that give the algorithm more sense and coherence. It should also be noted that a more complex representation requires a set of specifically adapted evolutionary operators.

3.4.2 Initial population

The initial population is made up of a set of chromosomes, which is usually defined randomly due to lack of problem information. Although in practice is chosen randomly, specific knowledge can help to create a feasible initial population with some individual close to the optimum. If the definition is not random, it is important to guarantee that there is structural diversity within the initial population, otherwise it is possible that a large part of the solution space is not reached and premature convergence is obtained. In general terms, the initial population should be as diverse as possible to achieve a uniform fitness distribution.

Regarding the size of the population to use, it is important to make a correct choice. Small populations run the risk of not adequately covering the solution space, while working with large populations can lead to problems related to excessive computational cost. For binary representations, Goldberg carried out a theoretical study where he concluded that the optimal size of the population grows exponentially with the chromosome size, while Koljonen and Alander (based on empirical evidence) suggest that the size of the population should be between one and two times the size of the chromosome [97].

3.4.3 Fitness function

The fitness function quantifies the aptitude of each chromosome as a solution to the problem. It also determines the probability with which a chromosome is selected for the reproduction phase, thus being able to transfer part of its genetic material to the next generation. This function is very important for the algorithm to work correctly, as it puts pressure on the population to evolve towards more suitable chromosomes. Since the evolutionary process tends to retain the genetic material of chromosomes with high fitness values, an appropriate choice of this function will give a greater probability of retaining characteristics of solutions close to the optimum.

The fitness function must reflect the value of the individual in a "real" way according to the context of the problem addressed. In many combinatorial optimization problems, where there are a large number of constraints, part of the points in the solution space represent invalid individuals, or not feasible individuals. Several alternative solutions have been proposed for approaches where individuals are subject to constraints. Perhaps the most widely used approach consists in discarding the individuals that do not verify the constraints of the problem. For these cases, crossover and mutation operations are applied until feasible individuals are obtained, or a correction function is used to modify the resulting individuals so that they become feasible. Another approach is based on the fitness function penalty. The general idea is to penalize the individual's fitness value by a certain number (the penalty), which is related to the constraints that the individual violates.

A common problem in GA execution occurs due to the convergence speed of the algorithm. In some cases the convergence is very fast, which is called premature convergence (the algorithm usually converges towards local optimum), while in other cases the problem is just the opposite, where a slow convergence occurs. The problem of premature convergence often occurs when the selection of individuals is made in proportion to their fitness function. In this context, there may be individuals with an adaptation to the problem far superior to the rest, and as the algorithm advances, they "dominate" the population. For these cases it is intended that, from a transformation of the fitness function, these "superindividuals" do not dominate the population.

3.4.4 Genetic operators

Most GA variants use selection, crossover, and mutation as their main operators.

Different techniques can be used to select individuals from the population. The selection process must allow to choose the most adapted chromosomes according to the fitness function. The probability of an individual to be selected is proportional to his fitness. Selective pressure is defined as the degree to which the best individuals are favored. This selective pressure leads to improving the fitness of the population over successive generations [145]. The most common selection mechanisms are the proportional selection, ranking-based selection, roulette selection, and tournament selection. The proportional selection chooses individuals based on their fitness values relative to the aptitude of the other individuals in the population. The ranking-based selection chooses individuals considering their relative position in a population fitness ranking. This mechanism introduces a high degree of elitism, since it maintains a high percentage of the best individuals in the population between generations. In the roulette selection, each individual is assigned a segment of the roulette whose size is proportional to its fitness value. The sum of the lengths of the segments must be equal to 1. The process consists of generating a random number between 0 and 1, and the individual whose segment comprises the generated number is selected for reproduction. The procedure is repeated until the desired number of individuals is obtained. In the tournament selection, a certain number of individuals are randomly chosen from the population, which compete among themselves (considering their fitness value) to determine which ones will be selected to reproduce. Choosing the best members of a tournament produces a relatively strong selective pressure, therefore the best is generally chosen according to a certain probability.

Once the individuals are selected, they are recombined to produce the offspring that will be inserted in the next generation. The different crossover methods may operate in two different ways. If a non-elitist strategy is chosen, the descendants are inserted into the temporary population even if their parents have a better fitness value. On the contrary, using an elitist strategy, the offspring pass to the next generation only if they exceed the aptitude of the parents (or of the individuals to be replaced). The main idea of the crossover is based on the fact that, if two parents are well adapted to the environment, and their children share genes from both parents, there is a possibility that the inherited genes are those that provide good quality fitness to the parents. The resulting offspring should have a greater fitness value than each of the parents separately. If the crossover does not group the best features in one of the children and the offspring have a worse aptitude than the parents, it does not mean taking a step back in the evolutionary process. Using a non-elitist crossover strategy ensures that genes from the best individuals are passed on to the next generation. If, even with a worse fitness value, the offspring is chosen to be inserted, since the genes of the parents will continue in the population, these parents could be obtained again. In this case, the previously decrease in the fitness value would be recovered.

The crossover is a process that consists of taking two parent solutions and producing children from them. One of the most commonly used crossover operators is the N-points crossover. It takes two parents and cuts them into N + 1 segments using N randomly taken crossover points. Then the segments are exchanged, thus generating two children. In Figure 3 the application of the one-point crossover for a binary representation is exemplified. GA originally used one-point crossover, which cuts two chromosomes at a random point and combines the segments to generate new individuals. From the random point, one parent contributes its leading segment and the other parent contributes its trailing segment to produce one of the children. This operator was used by Goldberg in the Simple Genetic Algorithm [72].

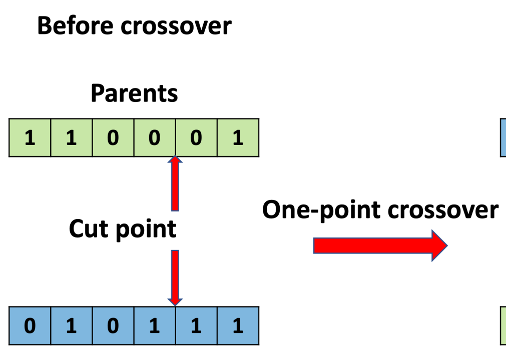

Figure 3: One-point crossover scheme for a binary representation.

In Figure 4 the application of the two-point crossover for a binary representation is exemplified. Although it is recognized that the two-point crossover provides a substantial improvement over the one-point crossover, adding a greater number of crossover points reduces the performance of the GA [37]. It could also lead to infeasible solutions due to violations of the constraints of the problem. However, adding more crossover points makes the solution space of the problem better explored.

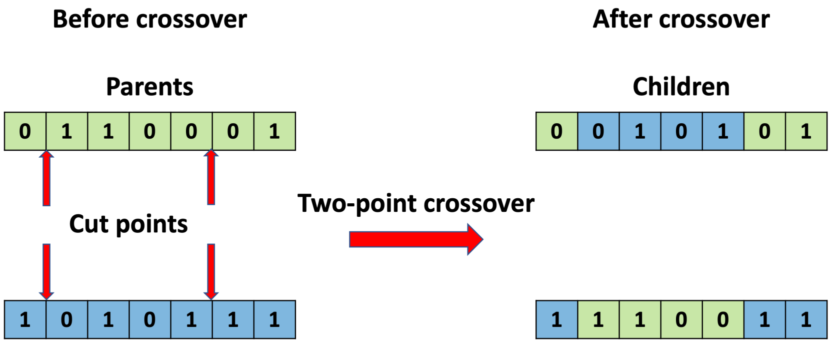

Figure 4: Two-point crossover scheme for a binary representation.

Another widely used technique is the uniform crossover [158], where each gene in the offspring has the same probability of belonging to one of the parents. Although it can be implemented in many different ways, the technique involves generating a crossover mask with binary values. If there is a 0 in one of the positions of the mask, the gene located in that position in one of the children is copied from the first parent. On the other hand, if there is a 1, the gene is copied from the second parent. To produce the second child, the roles of the parents are exchanged, i.e., the interpretation of the ones and the zeros of the crossover mask is exchanged. In Figure 5, the application of the uniform crossover for a binary representation is exemplified.

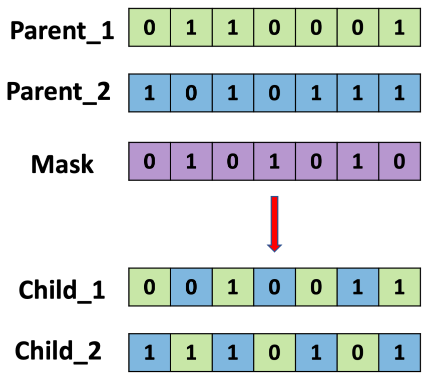

Figure 5: Uniform crossover scheme for a binary representation.

Another of the operators used by GAs is the mutation. This operator makes random changes to some values of the genes of a chromosome. While the crossover operator allows to take advantage of the information present in the current solutions to find better solutions, the mutation operator helps to explore the whole search space since it introduces diversity in the evolutionary process. Although individuals can be selected directly from the current population and mutated before introducing them into the new population, the mutation is often used together with the crossover operator. First, two individuals are selected from the population to apply the crossover. If the crossover is successful then one of the children, or both, are mutated with some probability. In this way, this behavior imitates nature, producing some type of error (usually without major importance) in the passage of the genetic component from parents to children. There are different ways to apply the mutation. For example, in the case of a binary representation, the commonly used mutation operator inverts the value of some genes with a small probability. An application example of the Bit Flip mutation for a binary representation can be seen in Figure 6.

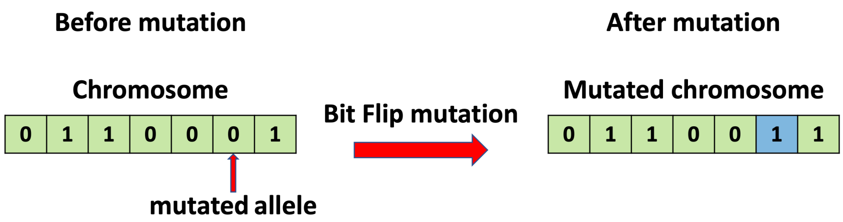

Figure 6: Bit Flip mutation scheme for a binary representation.

3.4.5 Parameters of Genetic Algorithms

The GA requires four parameters to generate the evolutionary process: CR (crossover probability), M (mutation probability), N (population size) and G (number of generations of the population). Both mutation and crossover operators are applied probabilistically, depending on the operator. Generally the application rate of the crossover operator is high (between 0.5 and 0.9), while the application rate of the mutation operator is very low (typically the inverse of the chromosome size) [120]. Regarding the population size, for binary representations, Koljonen and Alander suggested a population size between one and two times the size of the chromosome [97]. However, there are different suggestions regarding the definition of the population size. Some researchers suggest large sizes, but warn that the computational cost can be high [99, 137]. On the other hand, the number of generations is directly related to the addressed problem. Due to this, it is suggested to study, for each problem, the number that allows a correct evolution of individuals without compromising the aptitude of the solutions.

3.5 Differential Evolution Algorithm

The Differential Evolution (DE) algorithm emerged as a very competitive form of evolutionary computing in 1995, when Storn and Price wrote a technical report about it [157]. The success of DE was demonstrated at the First International Contest on Evolutionary Optimization in 1996. A year later, it turned out to be one of the best algorithms in the second international competition on evolutionary optimization. Thereafter, several journal articles were published describing and highlighting the potential of the algorithm [36].

DE operates computationally using the same criteria used by a standard EA. However, unlike traditional EA, it encodes solutions as vectors and uses different operations to construct new solutions from the existing ones. When a new solution, also called candidate, is constructed, it is compared to its parent. If the candidate is better than its parent, it replaces the parent in the population, otherwise, the candidate solution is discarded. In particular, DE differs significantly in the sense that distance and direction information from the current population is used to guide the search process. The mutation stage is applied first to generate a trial vector, which is then used within the crossover operator to produce one offspring. The sizes of the mutation steps are not sampled from a prior known probability distribution function, but are influenced by differences between individuals in the current population. The pseudocode of general DE algorithm is presented in Algorithm 3.

Algorithm 3 Differential Evolution Algorithm

Create a random initial population $(P^{t=0})$
while NOT StopCriteria **do** *//Evolutionary process*
    for each individual i in P^t **do**
        $fitnessEvaluation(p_i^t)$
        selectionOperation (to participate in mutation)
        Create the trial vector tv_i^t *by applying the mutation operator to* p_i^t
        Create an offspring pc_i^t *by applying the crossover operator to* tv_i^t
        if pc_i^t *is better than* p_i^t **then**
            Add pc_i^t *to* P^{t+1}
        else
            Add p_i^t *to* P^{t+1}
        end if
    end for
end while
return the best solution

3.5.1 Initial population

Individual positions provide significant information about their fitness. Considering a good random initialization method is used to construct the initial population, the initial individuals will provide a good representation of the entire solution space, with relatively large distances between the individuals. As the search process progresses, the distances between the individuals become smaller, and all the individuals converge towards the same solution. The initial distances between individuals is determined by the size of the population. The greater the number of individuals in a population, the smaller the magnitude of the distances between them.

3.5.2 Evolutionary operators

The mutation operator generates a trial vector tv_i^t for each individual of the current population $P^{t=0}$ by mutating a target vector with a weighted differential. This trial vector is then used by the crossover operator to produce the offspring. To generate the trial vector three individuals from the population are selected: a target vector p_j^t and two other randomly selected individuals (p_m^t and p_n^t), where $i \neq j$ and $m \neq n$. Generally the target vector is randomly selected or the best individual is selected [161]. Then, the trial vector is calculated by modifying the target vector from a scale factor F, which controls the amplification of the differential variation between the other two selected individuals. This calculation is indicated in Equation 3.1.

$$tv_i^t = p_j^t + F(p_m^t - p_n^t) \qquad F \in [0,1] \tag{3.1}$$

Then, to create the offspring population, Pc_i^t, the crossover operator is applied. In this operation the alleles z of each child are determined, in some cases by inheritance from the parents p_i^t and in others obtained from the trial vector tv_i^t. To determine those cases, a set of alleles indexes ζ is defined. The crossover operation is indicated in Equation 3.2.

$$pc_{iz}^t = \begin{cases} tv_{iz}^t & \text{if } z \in \zeta \\ pc_{iz}^t & \text{if } z \notin \zeta \end{cases} \tag{3.2}$$

The most used methods to define ζ are the binomial crossover and the exponential crossover [170]. In the binomial crossover, the allele indexes are randomly selected from a certain probability CR. The higher the probability, the greater the number of allele indexes and therefore a greater number of elements from the trial vector are used to create the offspring. A low probability causes that many alleles from the parents are used to create the offspring. The exponential crossover operates in a similar way to the 1-point crossover used in GAs, and is currently the most widely used operator by DE [161]. In particular, it selects a sequence of adjacent crossover points from a randomly selected index. In this operator, the list of potential crossover points is treated as a circular list. Also, the selection operation is applied to determine which individuals will participate in the mutation operation (to produce the trial vector) and which of the parents or children will survive to the next generation. The random selection is the most widely used method to select the individuals from which the difference vectors are calculated, and deterministic selection is used to construct the population for the next generation. In the next generation, an offspring replaces its parent if its fitness value is higher, otherwise the parent survive.

3.5.3 Parameters of Differential Evolution

In addition to the population size, the scale factor F and the crossover rate CR are two control parameters which influence the performance of DE by keeping up a proper equilibrium between exploration and exploitation processes. The modification in the new solutions is controlled by CR, and the step size is managed by F during the solution search process.

The size of the population has a direct influence on the exploration capacity of the DE algorithms. The greater the number of individuals in the population, the greater the number of differential vectors, and therefore more directions can be explored. However, it should be taken into account that the computational complexity increases with the population size. Empirical studies suggest that it can be estimated in 10 times the size of the solution [154]. The scale factor F controls the amplification of the differential variations. High values of F should make it easier to explore the solution space, but can cause the algorithm to miss optimal solutions, while small values of F help to explore the solution space precisely. Also, as the size of the population increases, the scale factor should decrease. On the other hand, the smaller the F value, the smaller the mutation step sizes, resulting in a longer convergence time. The crossover probability or crossover rate CR has a direct influence on the diversity of DE. This parameter controls the number of parent alleles that are changed for the next generation. The larger CR, the more variation is introduced into the new population, thus increasing diversity and exploration. Increasing CR often results in faster convergence, while decreasing CR increases search robustness [115].

Chapter 4

Problem Overview

In grassland-based dairy systems, determining how to rotate the cows among fields for grazing, how much concentrate to supply and the correct stocking rate to be used are important decisions that directly impact the efficiency of the system, particularly in milk production, feed consumption and economic results. To carry out the allocation of feed resources, the dairy herd is divided into groups of cows, and each group is distributed among the different feeding options. In many countries (and Uruguay is no exception) this process is carried out following management rules, but also based on the experience, intuition and even traditions of the producers. Considering that a good feed resource allocation leads to good productive results, this is a central decision problem in dairy farming systems. The main goal of this work is to determine how to allocate the available resources in order to optimize the dairy system.

4.1 Milk production components

The problem is presented in terms of supply and demand. The demand structure is defined by the characteristics of the herd while the supply structure is defined by the availability and characteristics of feed resources. The dairy herd is made up of a certain number of cows, and each cow has different characteristics. Cows with similar characteristics are considered as cows of the same type. Each type of cow is differentiated by the following attributes: body weight (BW, kg), genetic potential (GP, liters of milk in 305 days), lactation days (LD) or lactation weeks (LW), and fat (G) and protein (P) content in milk. We consider animals with varying body weight, genetic potential and number of weeks in lactation, while we fix (without loss of generality) the other parameters to the following values: G = 3.6%, P = 3.1%. These values are based on typical Uruguayan farm values [58]. The energy requirements of each cow type are calculated and then feed intake capacity is established. Feedstuff is differentiated into pastures and supplements. Pastures differ in dry matter (DM) productivity (herbage mass in Mg DM per ha), energy content (expressed in net energy for lactation in MCal per kg DM), costs (USD per Mg DM) and distance from the milking parlour. Supplements are a combination of conserved forage and concentrates that differ in their availability, energy content and costs. Supplements are placed in feed bunks located close to the milking parlour to minimize walking distance and machinery logistics. In Uruguay, cows are milked twice a day, which implies the movement of the animals from the place where they are located to the milking room, and the subsequent movement from there to the new feeding option. This new destination can be the same pasture, a different pasture or a place near the milking room where they will receive some supplements.

In this approach, the simplest version of the milk production model is considered, in which the available pastures are a finite resource, and can be used only once. The use of the pastures will not have consequences for its subsequent growth (which would imply a certain cost). Regarding supplements, availability will not be taken into account since these can be obtained practically unlimitedly in the food market, and therefore this resource can be considered as infinite.

Milk production (M; kg per day) is derived from the amount of energy available for lactation, which is calculated as the difference between the energy in consumed feed ($cInt$; MCal) and the requirements for maintenance (BR; MCal), movement (MR; MCal) and grazing (GR; MCal) (Equation 4.1). Requirement calculations are based on the nutrient requirements of dairy cattle as published by the (U.S.) [117]. The amount of net energy needed to produce one kg of milk (eM; MCal kg^{-1}) depends on the fat and protein content of the milk (Equation 4.2).

$$M = \frac{cInt - BR - MR - GR}{eM} \tag{4.1}$$

$$eM = 0.0929 \times G + 0.0547 \times P + 0.192 \tag{4.2}$$

Maintenance requirements depend directly on the metabolic weight ($MW = BW^{0.75}$; see Equation 4.3). Movement requirement is related to the walking distance and BW (Equation 4.4), while energy required for grazing is assumed to be proportional to the maintenance requirement (Equation 4.5).

$$BR = f_B \times MW \tag{4.3}$$

$$MR = 2 \times f_M \times D \times BW \tag{4.4}$$

$$GR = f_G \times BR \tag{4.5}$$

where:
f_B = proportionality constant for energy requirement for maintenance (0.08 MCal kg$^{-0.75}$);
f_M = proportionality constant for energy requirement for movement (0.00045 MCal km^{-1});
D = distance between the milking parlour and feeding area (km);
f_G = proportionality constant for energy requirement for grazing (MCal kg^{-1}).

The feed intake capacity (PC; expressed in kg DM day^{-1}) defines the upper limit of feed consumption per animal per day is proportional to potential milk production and metabolic weight (Equation 4.6).

$$PC_t = (f_p \times P_{GP} + f_i \times MW) \times (1 - e^{-0.192 \times (LW + 3.67)}) \tag{4.6}$$

where:
P_{GP} = potential milk production (liter day^{-1});
f_p = proportionality constant for potential milk production (0.372 kg DM liter^{-1} milk);
f_i = proportionality constant that relates intake to body weight (0.0968 kg DM kg^{-1} metabolic weight).

The gross margin over feeding cost (Ma) is calculated as the revenues from milk production minus feeding costs. In this approach, we consider an empty/un-pregnant dairy cow on neutral energy balance. Future work will include changes in the body condition score or the animal's actual energy balance, as well as pregnancy.

4.2 Computational techniques in agriculture and dairy systems

4.2.1 Computational techniques in agriculture

Various computational techniques have been applied in agriculture [168]. Since the 1950s, many farmers have relied on linear programming to improve economic benefit, particularly in pursuit of an optimal livestock diet design.

One of the first successful mathematical programming applications in agriculture was proposed by Waugh in 1951 [167], who used linear programming models to determine the minimum cost of the livestock ration fulfilling its nutritional requirements. In 1972, the possibility of increasing the efficiency and profitability of milk production per cow was analyzed by Dean [38]. In that work, linear programming models and production functions were combined to develop a system capable of providing feeding programs to optimize the feeding in dairy systems.

Later, a model considering the ingredients prices was studied in order to find the optimal mixing food. To address this problem, models involving different types of food, including grass species that can be grazed directly, but also species that must be harvested mechanically (which adds an additional cost) have been formulated [141, 118, 171]. The minimization of the ration cost was also used in cattle fattening systems [67]. In these systems, some producers seek to achieve adequate fattening from the first stages of cow growth, taking advantage of the fact that the conversion efficiency of young animals is higher than that presented by adults.

In 2001, a model to integrate individual components of a pasture-based system, with the objective of implementing a unique economic model to maximize farm profitability was proposed [142]. In that work, the problem was approached by linear programming, taking into account some constraints such as pasture growth or the conditions to enable or not its use. Also, in 2007, Neal addressed the problem of determining the most cost-effective combination of forage species by developing a linear programming model [119]. This work also studied the impact of using alternative criteria to take advantage of the forage choice and the effect of the progressive elimination of the most profitable forage species among the available options. Some of these alternative criteria consisted of maximizing DM yield or water use efficiency.

More recently, in 2010, Anderson presented a model that incorporates the economic relationship of production factors on a 100 hectares pastoral system [3]. In order to optimize the economic benefit linear programming was used, considering some limitations such as average milk production, the herd replacement rate, cows' death rate and the maximum number of lactations per cow. In 2016, a hybrid EA (a Genetic algorithm that uses a Simulated Annealing to improve the initial population) was proposed to optimize livestock feeding for the dairy industry [4]. The main objective was to find a minimum cost diet from a set of available ingredients while improving the quality of the food.

In 2020, accounting and mathematical modeling methods were combined to analyze the cost structure and economic optimization of dairy production systems. Information on the technical, economic, and production characteristics of several farms was used, and different accounting methods were adapted to mixed integer linear programming models to predict the percentage of feed cultivation and to maximize the economic benefit [148]. In 2021, a study was conducted to explore the impact of changes in feed ingredients' prices and feed ingredients' availability on dairy ration composition, feed cost and predicted methane yield under different levels of milk production. To address this problem, the output of multi-period linear programming models was monthly analyzed to produce a feed blend at a minimum cost in different periods [1].

Recently, in 2023, two studies addressing allocation problems in agriculture using optimization techniques were published. The first study aimed to identify the best way for choosing and distributing land parcels in available areas of the system to optimize food production [69]. This task was posed as a combinatorial optimization problem, and in consideration of its potential scale, two distinct approaches were proposed. The first one addressed the problem using a Mixed-Integer Linear Programming model, whereas the second one employed a Greedy Random Adaptive Search Procedure, representing a global optimization technique. The objective of the second study was to enhance livestock management approaches for maximizing farmers' profits [85]. The study employed a decision-making model for the purchase of heifers and the allocation of calves, which could potentially develop into either fattening heifers or breeding heifers. The researchers aimed to optimize the acquisition of new cattle while considering the system's capacity expansion and determining the distribution of female calves. At the age of seven months, each calf's fate as either a fattening animal or a breeding animal needed to be decided. To represent the overall equilibrium and composition of the herd, a linear programming model was utilized.

Although it is possible to find different models capable of determining the minimum cost rations for cattle or the maximum economic benefit, it is important to differentiate the formulations corresponding to animals being fed in stables from those who graze. In 2007, an interactive computer program (using linear programming) was presented to formulate minimum cost rations for dairy cows, which contemplate the mentioned conditions [60]. Although this work contemplates formulations for animals in stables or grazing, it does not do it for both conditions jointly. In particular, this problem is only oriented to the generation of diets with minimal cost (considering particularly a wide variety of proteins), and also the grouping of animals is not considered in the formulation.

4.2.2 Computational techniques in dairy systems

Operational research techniques have also been used to optimize dairy systems from another perspective, such as individual and total milk production, the management of available feeds, livestock production, the impact of different stocking rates, logistic management, among others.

In 2002, a work to predict the amount of forage that will be available to feed cattle was presented [108]. This work was carried out in the context of the prediction of the dry matter yield of grasslands through climatic variables by using multiple regression and neural networks techniques. Mathematical programming is also an optimization technique that has been used to analyze the multiple components within pastoral systems.

In 2008, a book was presented with the aim of improving the operational efficiency of livestock production [22]. The authors examine the effects of herd breeding or nutrition change, and determine the most efficient method to address this challenge. This work provides simulation models and includes a detailed analysis of livestock production in the context of environmental sustainability. Also in 2008, a study was presented where the effect of the stocking rate on pasture production, milk production and reproduction of dairy cows in pastoral systems was addressed [109]. The objective of this work was to optimize the stocking rate to maximize milk production and the use of pastures, and it was approached by making linear and quadratic analysis. For its resolution, data on production and quality of pastures, milk production, milk components and reproduction were used. In addition, an exponential function was used to adjust milk production during the lactation period.

The problem to determine the optimum replacement policy for dairy herds keeping total milk production constant was addressed in 2010 by Kalantari, where he used dynamic programming [93]. Later, nonlinear programming models were also introduced. In 2012, a detailed nonlinear optimization model of a dairy farming system was presented, where the representation of pasture and cow biology are described in depth [46]. The model includes pasture growth and digestibility, pasture utilization that varies by stocking rate, and different levels of intake regulation. Then, in 2013, the model was validated and its ability to provide reasonable predictions outside of calibrated scenarios was highlighted [47]. Also, the inherent rigidity present in a less-detailed linear programming model is shown to limit its capacity to provide reasonable predictions away from the calibrated baseline.

In 2017, the problem of the collection of raw milk from a network of farms supplying processing plants was presented [134]. To solve the milk collection problem, a model based on integer programming and two approaches were used for solving it. The first approach used a branch-and-cut algorithm for small instances and the second one used a heuristic procedure combining both exact and approximated methods to handle large instances (looking for reasonable execution times). In 2016, the same problem was addressed but following another perspective. In this case, a two-stage method based on an adaptive large neighborhood search was presented. The first phase solves the transportation problem and the second phase ensures that the optimization of the plant assignment is performed [112].

In 2019, a multi-stage stochastic optimization model of a pastoral dairy farm was presented. This stochastic model divides the dairy farming season into 52 weeks (stages) and links these weeks by a system of linear dynamics, helping producers to understand and plan under uncertainty scenarios related to three separate models: a grass growth model, an animal model, and a milk price model [49]. In 2019, a support vector machine (SVM) model was used to analyze the impact of increasing herd size and milk production on electricity consumption in dairy systems. The goal of this study was to highlight the effectiveness of the SVM model for investigating electricity consumption across different infrastructural scenarios during a period of dairy expansion [155].

In 2020, and following a different approach, a model designed to assist producers in the management of dairy cattle to obtain better productivity indexes was presented. The model performs the automation and individualization of the animals feeding by providing the milk production forecasting of each cow, using the autoregressive integrated moving average prediction engine [33]. In 2021 a study was published where the objective was to examine how the decision of dairy farms to either produce their own feedstuff or exclusively purchase it from markets, thus specializing in the dairy enterprise, can influence the future development of the sector. To achieve this, a linear programming model was used to assess the prospects of different farm type and to demonstrate the overall structure of the sector under different external conditions [139].

All these previous works focused on optimizing different aspects of dairy systems, but the main difference with our work is that they developed models to find solutions without considering differences for feed allocation or animal grouping. This is the significant contribution of this research (in addition to the methods evaluated). Also, another important difference with our proposal (apart from how the herd is handled) lies in the definition of the model. Some previous works define the milk production as a constant and then determine the livestock ration in order to maximize the economic benefit. In contrast, in our approach, we maximized the economic benefit by allocating the available resources and then obtaining the total milk production as an output. It is important to note that this work was a continuation of the master's thesis presented by Notte [121]. In that work a first approximation of the problem was presented, where formulations with different codifications including one and several milkings were considered. Exact methods and genetic algorithms were used for its resolution. The master's thesis was a good starting point for this doctoral thesis, despite the great differences between them.

4.2.3 Multi-objective problems in agriculture and dairy systems

Heuristic approaches were also used for situations where multiple objectives were involved and the potential trade-offs and synergies among these objectives were studied.

In particular, EA were used to solve complex multi-objective optimization problems [30, 39] in different contexts such as land-use allocation or dairy systems optimization. Land-use management helps control the allocation of land for specific uses. It can reduce the negative impact of utilization and improve optimal use of resources. In this sense, different approaches for solving land-use problems have been addressed through multi-objective optimization [77, 79, 80, 92, 114].

Problems presented in dairy systems have also been addressed and solved by optimizing multiple objectives. In 2016, a self-organizing migrating genetic algorithm for animal diet formulation was presented [150]. This algorithm provides quick solution and an innovative approach towards successful application of soft computing technique. The problem is addressed with bi-objective models that minimize the diet cost and maximize livestock shelf life, and the objectives were achieved by combining exact methods and GA. In the same year, a mathematical model was presented to solve a multi-objective optimization problem for a dairy system. In this case, the aim was to reduce electricity costs by producing their own energy through the use of solar systems. The objective of the model was to obtain the maximum net profit considering crop production, milk production and different investment options. To solve the computational problem they used the multi-objective tool of Excel [138].

In 2018, a design of mixed crop-livestock farming systems for resource efficiency, economic profitability, and environmental sustainability was addressed. The resulting problem was a multi-objective mixed-integer nonlinear fractional programming problem, where the main goal was to obtain the optimal configuration of mixed crop-livestock farming systems that maximizes economic performance, water productivity and organic matter accumulation. To handle the multi-objective optimization, they apply the ϵ-constraint method to obtain a Pareto surface of the optimal solutions [103].

In 2019, a multi-objective feed formulation problem was presented using a different approach. In this case, the objectives were to minimize feed cost and deviation from the specified constraints. The constraints related to the different nutritional requirements were combined to form an objective function and was simultaneously optimized with the feed cost. The problem was solved using a population-based evolutionary multi-objective optimization algorithm (NSGA-II) [163]. In that work, the authors used Pareto fronts for nondominated solutions to be chosen as optimal if no objective could be improved without compromising at least one objective. The viewing of all the constraints along the Pareto front gave the possibility of displaying the various combinations of nutrient contents and corresponding cost.

Also, in 2019 a multi-objective optimization model was used to obtain the optimal equipment selection, practices management and electricity costs in a dairy farm. In that work, a GA was used to maximize a combined objective function based on a user specified economic and environmental weighting factor [12]. Then, and following the same line of work, in 2020 the authors presented a financial and renewable multi-objective optimization method which used a GA to optimize various combination of the dairy farm equipment and management practices based on a simulated test case farm [14]. In order to respect the policies of the European Union regarding renewable energy contributions, seven sizes of photovoltaic systems were investigated to assess the financial performance and renewable contribution of this technology in a dairy farming context.

In 2020, a study was carried out to determine how much food can be produced in a region while keeping environmental impacts within Germany's policy objectives [20]. To solve this problem they used an optimization approach with five objectives, one to maximize productivity and four to minimize environmental impacts. For this work they decided to use the previously mentioned NSGA-II algorithm, which is widely used in the literature due to the good results presented. Later, in 2021 a multi-objective optimization was used to optimize the milking start times and farm infrastructure setup to maximize a labor utilization function while either maximizing farm net profit or minimizing farm electricity related CO2 emissions, based on a weighting variable. A mathematical model was formulated and a GA was used to solve it [13]. Also in 2021, a sustainable dairy supply chain model under uncertain conditions was presented. In that work, a multi-objective model was proposed to minimize the total costs and environmental impacts, and maximize the social impacts. The model contemplated a multi-period and multi-product chain composed of suppliers, producers, and retailers, which was applied to a dairy company. To solve the proposed model, a Pareto-optimal solution concept was developed by employing a hybrid method based on a heuristic algorithm and the augmented ϵ-constraint method [153].

In 2022, a non-dominated sorting ant colony genetic algorithm was proposed to address the challenge of minimizing the environmental impact and life cycle cost while maximizing the quality of milk in a milk powder spray drying system. The aim was to find the optimal parameters for the system, considering the conflicting nature of these objectives. Additionally, the study explored the diversity of the optimal solutions in this work [169]. In 2023, a work was published that addressed a complex optimization challenge involving multiple objectives in the context of uncertain factors such as crop yields, demand, and precipitation across various locations. The primary objective was to obtain optimal cultivation areas and supplemental irrigation water levels at different sites, taking into account social, economic, and environmental considerations. The authors presented linear and nonlinear regression models to effectively model the relationship between water availability and crop yield. They also suggested employing the simplex algorithm and the interior point method for linear constraints and objectives. For nonlinear aspects, they proposed utilizing either a nonlinear solver or a metaheuristic approach based on Evolutionary methods [2].

Also, in 2023, a many-objective model for agricultural decision-making was published to maximize farmers' earnings by assisting them in making choices regarding crops and their management, considering uncertain factors like crop prices and weather conditions. To incorporate various criteria for satisfactory outcomes, the researchers considered seven distinct objectives that focus on costs, margins, utilities, returns, losses, gains, and regrets, covering a wide range of farmers' goals. To address this problem, they devised a systematic and analytical process that relies on machine learning algorithms to identify alternative options for land allocation. This approach enables farmers to navigate the complexities of decision-making and optimize their agricultural practices based on their specific objectives [73].

As presented above, in the literature there are many studies that employ multi-objective techniques to address problems in in agriculture and dairy systems. However, these studies also do not take into account cow grouping and variations in food allocation across different time periods. In our work, in order to faithfully represent dairy systems and their management methods, a model with five objectives and multiple periods is presented. At the same time, we carry out a detailed study both from the agronomic and computational point of view, thus achieving to identify the strengths and weaknesses of the different solution techniques used.

Chapter 5

Resource Allocation Model: First Mathematical Approximation

Considering the available feed resources in the system and depending on the different conditions presented by each animal to produce milk, a first approximation of the problem consisted in finding a feed resource allocation by grouping and distributing the cows into the feeding options in order to maximize the total milk production or the economic benefit (we refer to economic benefit as the margin over feeding cost for the entire system).

The problem of feed resources allocation to a heterogeneous dairy herd was studied and modeled as a combinatorial optimization problem. Considering that large-scale combinatorial optimization problems usually cannot be solved with traditional exact approaches, not only an Exact Method (EM) but also a GA were used and included in the experimental study. Once the resource allocation results were obtained, we studied the allocation by interpreting the solutions' structure. We also analyzed the quality and the diversity of the solutions obtained by the GA. This particular work was published in Notte et al. [130].

5.1 Problem formulation

In our first approach, a single objective model was proposed, where two independent objective functions were considered. The objectives were to maximize the total milk production (expressed as liters per cow per day; liter cow^{-1} day^{-1})) and to maximize the economic benefit (expressed as the gross margin over the feeding costs per cow per day; USD cow^{-1} day^{-1}).

The model seeks to determine the best way to allocate the cows by grouping and distributing them among the feeding options. In this model, the time was represented by considering several milkings, so the solution to the problem presents an allocation for each of the milkings considered. For each allocation, each feeding option is assigned a group of cows represented by the number of cows of each type. It is possible that there are no cows assigned to a feeding option. This means that the cows will not be fed using that type of food. On the other hand, it is possible that all cows are assigned to the same feeding option. For all allocations, and taking into account the characteristics of the cows and the characteristics of the feeding options, the model calculates the milk produced by each cow. Then, the total milk production obtained for the allocations is calculated as the sum of the milk production obtained by each cow. The solution of the problem can be seen as the interaction of two models: the resource allocation model (cows distribution model), and the milk production model presented in Section 4.1.

The input parameters of the model are: the types of cows and their characteristics, the number of cows of each type, the feeding options and their characteristics, and the number of milkings.

The resulting mathematical formulation from maximizing milk production is shown in Equations 5.1 to 5.4.

$$max \quad \frac{\sum_o \sum_z \sum_t (w_{ozt} \times CL_z - y_{ozt} \times (BR_t + MR_{zt}))}{eM} \tag{5.1}$$

$$sa:$$

$$\sum_z y_{ozt} = C_t \qquad \forall o \in O, \forall t \in T \tag{5.2}$$

$$\sum_o \sum_t w_{ozt} \leq Food_z \qquad \forall z \in Z \tag{5.3}$$

$$w_{ozt} \leq y_{ozt} \times PC_t \qquad \forall o \in O, \forall z \in Z, \forall t \in T \tag{5.4}$$

All different feeding options are represented by the set Z, and the different cow types are represented by the set T. Each cow type is represented by the index t, and each feeding option is represented by the index z. In Uruguay, cows are milked twice a day, so, to identify each milking over several days ($2 \times nbDays$), the index o in the set O is added (with $O = 1,2,...,2 \times nbDays$).

As a consequence, $y_{ozt} \in \mathbb{N}$ represents the number of cows of type t assigned to the feeding option z in milking o. The variable $w_{ozt} \in \mathbb{R}$ represents the total consumption of DM in the feeding option z, for cows of type t in milking o. This model assumes that the food resources available are shared uniformly between the cows assigned to a feeding option. Then, it is enough to know the whole consumption of DM in the feeding option, and it is not necessary to represent the DM consumption for each cow.

The objective function (in this case maximizing milk production) was computed by adding all the energy obtained by cows from their feed consumption (w_{ozt}, multiplied by the calories level for each food type CL_z), and subtracting the energy requirements. In particular, the energy requirements are the group energy basal requirements ($y_{ozt} \times BR_t$) and energy movement requirements ($y_{ozt} \times MR_{zt}$). The basal requirements depend on the animal characteristics, and the movement requirements depend on the distance to each feeding option z. The constraint shown in Equation 5.2 forces the total number of every type of cow in each milking to be equal to C_t (the number of cows for every type of cow). The restriction shown in Equation 5.3 ensures that the food consumed in each feeding option does not exceed the available resources ($Food_z$). Finally, the constraint in Equation 5.4 enforces actual food consumption of each cow not to exceed its potential consumption.

When the objective of the problem is to maximize the economic benefit, the price of each food resource RP_z and the current price of milk MCP must be considered. For this, the new objective function has to calculate the profits obtained for each liter of milk produced and the total economic cost that each kilogram of DM implies.

The objective function for the formulation that maximizes the economic benefit is shown in Equation 5.5.

$$max \quad \frac{\sum_{o}\sum_{z}\sum_{t}(w_{ozt} \times CL_z - y_{ozt} \times (BR_t + MR_{zt}))}{eM} \times MCP - w_{ozt} \times RP_z \qquad (5.5)$$

The constraints are the same as those presented in the formulation that maximizes milk production, Equations 5.2 to 5.4.

The decision variables and parameters definitions are presented in Table 1.

Table 1: Decision variables and parameters definitions.

Acronym	Description
Decision variables	
y_{ozt}	Number of cows of each type assigned to each feeding option in each milking
w_{ozt}	Total intake of DM for each type in each feeding option for each milking
Parameters	
O	Number of milkings
Z	Number of feeding options
T	Number of types of cows
MCP	Milk current price
CL_z	Calories level for each feeding option
BR_t	Basal requirement for each type of cows
MR_{zt}	Movement requirement for each type to each feeding option
C_t	Number of cows for each type of cows
GP_t	Genetic potential for each type of cows
$Food_z$	Available food in each feeding option
RP_z	Food resource price for each zone
ENl	Amount of net energy needed to produce one kg of milk

5.2 Resolution methods

In order to obtain the optimal solution for different scenarios, the mathematical formulation presented in Section 5.1 was solved using an exact method. To program it, we used the GLPK linear programming package (for its acronym, GNU Linear Programming Kit) [110]. This package was designed to solve large scale linear programming and mixed integer programming, among others. To solve those problems, GLPK uses different algorithms, including the simplex method [105], interior point methods [106], and branch and bound [102].

When exact methods are not appropriate, an alternative to solve these problems are genetic algorithms (GA). To implement a GA, it was necessary to define a suitable encoding (see [72, 160]). The encoding must represent an allocation of the dairy herd for each milking for several days, considering different cow types. In this approach, we used an encoding that can be stored in a cube formed by the number of milkings ("days × 2 rows"), the number of feeding options and the number of cow types. Each cube cell corresponds to a feeding option, a milking and a cow type, and its value represents the number of animals of this type that have to be moved to that feeding option in that milking. A major advantage of this encoding is its simplicity. Since there is no theoretical or empirical evidence that a crossover or mutation operator performs better than any other for every optimization problem, we decided to use the operators from the classical Simple Genetic Algorithm with minimal adjustments. Specifically, for recombination and mutation operators the classical "one-point crossover" and "swap mutator" were applied, respectively. Using this encoding and evolutionary operators, it is possible that non-feasible solutions can be generated, since solutions could be created with an unfeasible number of cows. To ensure feasibility, a correcting procedure was implemented as follows: the missing or excess number of cows in each row was determined, then a random feeding option was selected, and finally the number of cows is corrected (by adding or removing cows as appropriate) to the selected feeding option. This process is repeated until the total number of cows in each of the row is corrected.

The pseudocode of this correcting procedure is presented in Algorithm 4.

Algorithm 4 GA - Correcting procedure

```
for each milking o in O do
    if numberCows(o) ≠ totalNumberOfCows then      //non-feasible solution
        difference = |numberCows(o) − totalNumberOfCows|
        z = Random(Z)      //a random feeding option is selected
        update(gene(o,z,difference))      //update the solution by adding or removing a cow
    end if
end for
```

The GA was implemented using the C++ programming language and based on the GALib library [166]. GALib is a library of functions that provides functions and objects for the development of genetic algorithms, providing a wide range of representations and operators. It is possible to define both standard and specific data structures and evolutionary operators. This library also allows to customize the algorithms by extending their classes, in order to incorporate new representations and new genetic operators.

For this work, we performed a parameter calibration. In particular, the crossover probability (CR), mutation probability (M) and population size (N) were calibrated. From the results of some executions, and relying on the literature [37, 74, 151], we decided to carry out the corresponding calibration considering the following values:

- CR: 0.5; 0.65; 0.8; 0.95;
- M: 0.01; 0.1; 0.2; 0.3;
- N: 25; 50; 75; 100;

For all executions, the same numbers of cows were used (100, 300 and 500 cows), and due to a feeding ratio reason it was necessary to use different feeding scenarios. The calibration performed consisted of 20 runs for each parametric combination, and to determine the parameters to be used, the values obtained were studied taking the smallest average gap[1] as a criterion.

Finally, for this first approach to the problem the following values were used: CR (0.8), M (0.2) and N (50).

[1]The gap is the relative difference between the optimal solution and the feasible solution found. If it is not possible to find the optimal solution, the gap is determined by the difference between the upper bound and the solution found.

5.3 Computational experiments

This work is a continuation of the master's thesis presented by Notte [121]. In this thesis it was proven that the exact method required hours to reach the optimal solution, and that the execution time increased as the size of the problem grew. On the other hand, it was found that the genetic algorithm obtained very good solutions (with values very close to the optimal one) in a few seconds. To obtain solutions from the exact method and be able to compare them with those obtained from the genetic algorithm, experiments were carried out with relatively small instances of the problem, although representative of dairy systems.

Based on what was mentioned above, the objective of this work was not to compare execution times between the methods. We focused on three inquiry-based aspects: (1) the solution quality and the solution structure of the GA when the objective was to maximize the milk production, (2) the solution quality and the solution structure of the GA when the objective was to maximize the margin over the feeding cost, and (3) the diversity of the solutions provided by the GA when the objective was to maximize the milk production.

For each aspect, one computational experiment based on real test data was performed. The test data was based on real-life data prepared by one of the directors of this PhD. thesis, Pablo Chilibroste, whose research area is focused on dairy production systems. To evaluate (1), one experiment by running the GA and the EM in a context of maximizing the milk production was performed. Then we compared the milk production results. To evaluate (2), one experiment by running the GA and the EM in a context of maximizing the margin over the feeding cost was performed. Then we compared the margin results. In order to evaluate (3), and to have a measure of the diversity, for each number of cows, 30 executions of the GA in a context of maximizing the milk production were done.

The execution platform was a virtual machine running on a PC with Intel (R) Core (TM) i5-2400 (3.10 GHz CPU with 4 cores and 6 MB of cache) processor and 4 GB of RAM. The virtual machine operating system was Windows XP. That virtual machine uses 50% of the physical machine processor and had allocated 1.5 GB of the total RAM.

The scenarios used for the experiment were determined by the herd description, feeding options information, and herd size. The herd was defined by the BW_t and GP_t of each type of cows. In the experiments, we considered three cow types (T1, T2 and T3) with different values for BW_t and GP_t. Type T1 were cows of 600 kg of BW and 9000 l of GP, T2 were cows of 550 kg of BW and 7000 l of GP, and T3 were cows of 500 kg of BW and 5500 l of GP. For the execution of the experiments, different herd sizes were considered (between 50 and 1500 cows). In each experiment, 50% of the total herd size corresponded to the first type, 30% corresponded to the second type and the remaining 20% corresponded to the third type (leading to constant herd percentage compositions). The information mentioned above is summarized in Table 2.

Table 2: Dairy herd description

Type	BW (kg)	GP (l)	Prop. (%)
T1	600	9000	50
T2	550	7000	30
T3	500	5500	20

Notes: Type = type of cows, BW = body weight per cow in kilograms, GP = genetic potential (milk production potential, liters of milk per cow in 305 days) , Prop. = proportion of the total herd size.

The experiments included five feeding options (Z1, Z2, Z3, Z4 and Z5). Three of these correspond to pastures. The first zone (Z1) was defined with an energy density of 1.4 Mcal ENl / kg DM and with 1100 kg DM available, while the remaining two pastures (Z2 and Z3) were defined with the same energy value, 1.5 Mcal ENl / kg DM and the same available resource amount (1800 kg DM). The distances between the milking room and pastures of type one, two and three were 0.5 km, 1.5 km and 2.5 km, respectively. Additionally, two feeding options that correspond to supplements were included, one (Z4) with high energy density (1.65 Mcal ENl / kg DM) and another one (Z5) with low energy density (1.44 Mcal ENl / kg DM). In both cases with the same availability (4500 kg DM). Considering the feed bunks (where cows receive supplements) are close to the milking room, a distance of 0 km was considered. The information mentioned above is summarized in Table 3.

Table 3: Feed resource information

Feeding option	Description	CL (Mcal)	Distance (Km)	Availability (Kg)	Price
Z1	Pasture	1.4	0.5	1100	20
Z2	Pasture	1.5	1.5	1800	20
Z3	Pasture	1.5	2.5	1800	20
Z4	TMR feeding area	1.65	0	4500	80
Z5	TMR feeding area	1.44	0	4500	60

Notes: Description = description of the feeding option, TMR = total mixed ration, CL = calories or energy density measured like the net energy megacalories per lactation per kilogram of dry matter (Mcal ENl / Kg DM)), Distance = distance to de milking room, Availability = availability of the feeding option, Price = price of the feeding option measured as a percentage of the milk price per kilogram of DM.

In order to evaluate (2), defining milk and food prices was necessary. We considered the milk price as 0.35 USD per liter. The price of pastures were defined as a 20% of the milk price per kilogram of DM, while the prices of supplements were 80% and 60% of the milk price per kilogram of DM for Z4 and Z5, respectively.

To evaluate the diversity of the solutions provided by the GA (3), determining a measure of diversity was needed. The measure considered was the distance between the solutions obtained by the EM and the GA. From the obtained results the gap value and distance to the optimal solution was calculated. In order to calculate the distance between the solutions from the EM and the GA, the Euclidean distance was computed, taking into account the difference between each component of the EM and the GA solutions. The general Euclidean distance formulation is presented in Equation 5.6.

$$\sqrt{\sum_{o,z,t}\left(y_{ozt}^{EM}-y_{ozt}^{GA}\right)^2} \tag{5.6}$$

For all the experiments, a single milking was considered. Due to this, the number of decision variables was small, and could be easily analyzed.

5.3.1 Results

In order to evaluate the performance of the GA, the gap value (computed as a percentage) between the EM and GA results was studied. A small gap value indicates that the results are similar. We considered that a gap value is small when the difference between results is lower than 5%.

Regarding aspect (1), the allocation obtained by the EM is presented in Table 4, while the allocation obtained by the GA is presented in Table 5. Table 4 presents the distribution obtained by the EM for each cow type in the different feeding options. Results showed that when herd size was small, all cows were assigned the feed option with the highest energy density. The last feed option used was the one with the lowest energy density. In feeding options with pasture almost all the cows corresponded to type T1. Most of the cows of types T2 and T3 were sent to feeding options Z4 and Z5 (supplements). Also, the total and individual milk production decreased when the herd size was bigger than 700 cows.

Table 4: Cattle distribution obtained by the EM in an milk production maximization context

	Z1			Z2			Z3			Z4			Z5				
#C	T1	T2	T3	T1	T2	T3	T1	T2	T3	T1	T2	T3	T1	T2	T3	TMP	IMP
50	0	0	0	0	0	0	0	0	0	25	15	10	0	0	0	1843	36.9
210	0	0	0	0	0	0	0	0	0	105	63	42	0	0	0	7741	36.9
290	0	0	0	67	1	3	0	0	0	78	86	55	0	0	0	10255	35.4
350	0	0	0	76	1	0	42	0	0	57	101	66	0	3	4	12093	34.6
560	0	0	0	76	1	0	76	0	0	70	53	104	58	114	8	18496	33.0
600	2	3	0	76	1	0	76	1	0	49	68	115	97	107	5	19707	32.9
700	44	1	3	77	0	0	77	0	0	149	106	0	3	103	137	20372	29.1
800	44	1	3	77	0	0	77	0	0	153	88	138	49	151	19	19041	23.8
1000	44	1	3	77	0	0	77	0	0	300	76	197	2	223	0	16378	16.4
1200	44	1	3	77	0	0	77	0	0	400	136	237	2	223	0	13715	11.4
1500	44	1	3	77	0	0	77	0	0	552	0	260	0	449	37	9721	6.5

Notes: #C = number of cows, Z1;Z2;Z3;Z4;Z5 = feeding options, T1;T2;T3 = type of cows, TMP = total milk production in liters per day, IMP = individual production (average milk production per cow in liters).

Table 5 presents the distribution obtained by the GA for each cow type in the different field zones. By comparing these milk production results with the EM results, we can see that the gap value is very small, but the distributions (structure of solutions) are different. Unlike Table 4, these solutions include cows of types T2 and T3 in feeding options with pasture. There was no correlation between cow types and feeding options.

Regarding aspect (2), the allocation obtained by the EM is presented in Table 6, while the allocation obtained by the GA is presented in Table 7. Table 6 presents the distribution obtained by the EM for each cow type in the different feeding options. Results showed that when the herd size was small, the algorithm avoid the feeding options with supplements (Z4 and Z5). The last food type used was the one in zone Z4 (high price with a low energy density). In feeding options with pasture almost all the cows corresponded to type T1. Most of the cows of types T2 and T3 were sent to zones Z4 and Z5 (supplements). Also, the total and individual earnings decreased when the herd size was bigger than 700 cows.

Table 7 presents the distribution obtained by the GA for each cow type in the different feeding options. The gap value is very small but the distributions (structure of solutions) are different. Unlike Table 6, these solutions include much more cows of types T2 and T3 in feeding options with pasture.

In order to study the diversity of the GA (3), for each number of cows, 30 executions of the GA in a milk production maximization context were performed. From the obtained results, we

Table 5: Cattle distribution obtained by a single representative run of the GA in a milk production maximization context

	Z1			Z2			Z3			Z4			Z5					
#C	T1	T2	T3	T1	T2	T3	T1	T2	T3	T1	T2	T3	T1	T2	T3	TMP	IMP	Gap
50	0	0	0	0	0	0	0	0	0	25	15	10	0	0	0	1843	36.9	0.00
210	0	0	0	0	0	18	0	0	0	105	63	24	0	0	0	7657	36.5	1.09
290	0	0	0	22	27	28	0	0	0	123	60	9	0	0	21	10167	35.1	0.86
350	0	0	0	10	64	3	0	0	2	164	28	0	1	13	65	12045	34.4	0.40
560	0	0	21	32	34	9	0	56	26	130	48	14	118	30	42	18368	32.8	0.69
600	28	4	15	0	67	23	41	9	27	184	3	5	47	97	50	19586	32.7	0.61
700	47	0	0	16	60	15	77	0	0	72	85	103	138	65	22	20162	28.8	1.03
800	31	7	26	45	31	28	11	28	65	149	114	5	164	60	36	18987	23.7	0.29
1000	1	2	61	0	90	0	0	30	74	297	148	4	202	30	61	16346	16.4	0.20
1200	10	1	53	34	56	0	34	56	0	73	247	64	449	0	123	13686	11.4	0.21
1500	17	0	47	55	35	0	77	0	0	256	212	159	345	203	94	9706	6.5	0.16

Notes: #C = number of cows, Z1;Z2;Z3;Z4;Z5 = feeding options, T1;T2;T3 = type of cows, TMP = total milk production in liters per day, IMP = individual production (average milk production per cow in liters), Gap = difference in total milk production between GA and EM solutions (computed as a percentage).

Table 6: Cattle distribution obtained by the EM in an economic benefit maximization context

	Z1			Z2			Z3			Z4			Z5				
#C	T1	T2	T3	T1	T2	T3	T1	T2	T3	T1	T2	T3	T1	T2	T3	TE	IE
50	0	0	0	25	15	10	0	0	0	0	0	0	0	0	0	472	9.5
210	0	6	42	29	56	0	76	1	0	0	0	0	0	0	0	1930	9.2
290	1	52	2	68	7	4	74	0	4	2	28	48	0	0	0	2509	8.7
350	25	24	2	74	0	4	74	0	4	2	81	60	0	0	0	2928	8.4
560	41	1	7	74	0	4	77	0	0	28	166	30	60	1	71	4299	7.7
600	41	1	7	74	0	4	77	0	0	28	166	30	80	13	79	4547	7.6
700	44	1	3	77	0	0	77	0	0	149	106	0	3	103	137	4596	6.6
800	44	1	3	77	0	0	77	0	0	153	88	138	49	151	19	4130	5.2
1000	44	1	3	77	0	0	77	0	0	302	55	197	0	244	0	3198	3.2
1200	44	1	3	77	0	0	77	0	0	402	20	237	0	339	0	2266	1.9
1500	44	1	3	77	0	0	77	0	0	552	0	260	0	449	37	868	0.6

Notes: #C = number of cows, Z1;Z2;Z3;Z4;Z5 = feeding options, T1;T2;T3 = type of cows, TE = total earnings in USD, IE = individual earnings (average earnings per cow in USD).

calculated the gap value and the Euclidean distance between the solutions from the EM and the GA.

For the most representative number of cows, the average and standard deviation values of the solution over 30 different GA runs are presented in Table 8. In particular, the results for milk production, gap and distance are presented.

To perform an analysis considering different perspectives, for each number of cows, three solutions over the 30 different GA runs were chosen: the solutions with the lowest gap value, the lowest distance percentage, and the biggest distance percentage. The structure of the aforementioned solutions, as well as the corresponding values for milk production, gap and distance are presented in Table 9.

Table 7: Cattle distribution obtained by a single representative run of the GA in an economic benefit maximization context

	Z1			Z2			Z3			Z4			Z5					
#C	T1	T2	T3	T1	T2	T3	T1	T2	T3	T1	T2	T3	T1	T2	T3	TE	IE	Gap
50	0	0	0	25	15	10	0	0	0	0	0	0	0	0	0	472	9.5	0.00
210	9	12	26	42	24	11	53	21	3	1	6	2	0	0	0	1907	9.1	1.18
290	44	2	1	38	15	24	24	45	8	39	25	24	0	0	1	2461	8.5	1.93
350	12	24	10	5	41	40	63	14	0	95	25	19	0	1	1	2878	8.2	1.72
560	17	21	9	0	62	27	54	23	0	63	51	75	146	11	1	4231	7.6	1.57
600	1	52	2	27	50	0	10	62	17	131	12	49	131	4	52	4476	7.5	1.58
700	2	26	36	67	23	0	9	81	0	195	0	0	77	80	104	4586	6.6	0.22
800	26	8	33	6	58	42	77	0	0	150	70	56	141	104	29	4122	5.2	0.21
1000	4	4	60	78	0	0	33	33	38	65	234	53	320	29	49	3184	3.2	0.44
1200	13	0	54	77	0	0	77	0	0	154	142	110	279	218	76	2265	1.9	0.06
1500	24	33	8	2	37	65	2	20	82	186	221	130	536	139	15	853	0.6	1.74

Notes: #C = number of cows, Z1;Z2;Z3;Z4;Z5 = feeding options, T1;T2;T3 = type of cows, TE = total earnings in USD, IE - average earnings per cow in USD, Gap = difference in total earnings between GA and EM solutions (computed as a percentage).

Table 8: Average and standard deviation values obtained by the EM and the GA solutions.

#C	SolAvg (l)	SolSDev	GapAvg	GapSDev	DistAvg	DistSDev
210	7652	5.41	1.15	0.07	12.03	0.15
350	12011	31.80	0.68	0.26	18.68	3.63
560	18372	51.35	0.67	0.28	7.08	1.83
700	20211	100.86	0.79	0.50	8.35	2.05
800	19005	12.10	0.19	0.06	14.93	4.23
1000	16342	14.35	0.22	0.09	23.15	6.53
1500	9691	13.89	0.31	0.14	14.75	13.16

Notes: #C = number of cows, SolAvg = solution average in liters, SolSDev = solution standard deviation, GapAvg = gap average computed as percentage, GapSDev = gap standard deviation, DistAvg = distance average computed as percentage, DistSDev = distance standard deviation.

5.3.2 Discussion

Regarding the first experiment (1), the values presented in Table 4 showed that solutions are generated by distributing as many cows as possible into feeding options with the highest energy density. The algorithm searched for solutions where each cow consume as much food as its potential consumption. As a consequence, for herds of up to 210 cows, animals were sent to Z4, but for slightly larger herds (exceeding the availability of Z4 to give each cow its potential consumption), the available resources in Z2 were also used. If the herd size is even greater, available resources in Z3, Z5 and Z1 were also used in the mentioned order. When the herd size was over 700 animals, resources were not enough to feed all the cows with as much food as their potential consumption, and therefore it was needed to share the food.

It was interesting to analyze the total milk production behavior and how the average individual milk production decreased once the food resources were not enough to feed all the cows with as much food as their potential consumption. When considering herds of up to 700 cows, food resources were enough, so the real consumption of each cow was almost equal to its potential consumption. Naturally, in these cases the total milk production was higher than the production obtained by smaller herds. When the herd size is greater, the individual milk production decreases, to the point that the total milk production of 800 cows is lower than the total milk production of 700 cows. Individual production reached its maximum value when the herd size

Table 9: Results for three solutions over 30 different GA runs: the solutions with the lowest gap value, the lowest distance percentage, the biggest distance. percentage

	Z1			Z2			Z3			Z4			Z5					
#C	T1	T2	T3	T1	T2	T3	T1	T2	T3	T1	T2	T3	T1	T2	T3	Tot	Gap	Dist
210	0	0	0	0	0	18	0	0	0	105	63	24	0	0	0	7657	1.09	12.1
	0	0	0	0	10	7	0	0	0	105	53	34	0	0	1	7649	1.19	11.8
	0	0	0	0	0	18	0	0	0	105	63	24	0	0	0	7657	1.09	12.1
350	0	0	0	0	76	13	7	2	0	167	25	0	1	2	57	12054	0.33	20.3
	0	0	0	51	3	22	45	0	13	79	98	15	0	4	20	11954	1.15	11.3
	0	0	0	18	42	17	0	6	0	157	34	1	0	23	52	12037	0.47	23.8
560	4	0	0	0	88	0	0	0	86	192	0	0	84	80	26	18462	0.18	7.1
	0	0	0	33	38	6	72	0	1	0	129	95	175	1	10	18405	0.49	1.3
	3	0	27	51	2	22	64	3	8	99	63	30	63	100	25	18323	0.94	8.4
700	41	14	0	77	0	0	17	73	0	6	123	140	209	0	0	20356	0.08	5.7
	46	1	1	27	47	16	69	0	8	112	37	111	96	125	4	20131	1.18	3.3
	16	25	24	46	10	48	1	35	68	195	0	0	92	140	0	20322	0.25	10.6
800	6	53	0	78	0	0	0	90	0	99	78	160	217	19	0	19026	0.08	6.1
	6	53	0	78	0	0	0	90	0	99	78	160	217	19	0	19026	0.08	6.1
	31	0	40	78	0	0	30	16	58	201	0	0	60	224	62	19002	0.20	27.7
1000	48	0	0	80	0	0	77	0	0	85	159	143	210	141	57	16374	0.02	26.1
	10	47	0	77	0	0	67	24	0	180	178	115	166	51	85	16353	0.16	12.7
	9	0	56	64	26	0	78	0	0	192	118	0	157	156	144	16360	0.11	35.1
1500	56	0	0	77	0	0	77	0	0	54	450	72	486	0	228	9718	0.03	21.9
	32	24	0	87	0	0	77	0	0	505	182	122	49	244	178	9707	0.15	1.3
	42	21	0	81	0	0	77	0	0	11	149	113	539	280	187	9711	0.10	49.9

Notes: #C = number of cows, Z1;Z2;Z3;Z4;Z5 = feeding options, T1;T2;T3 = cow types, Tot = total milk production, Dist = solution distance percentage, Gap = difference in total milk production between GA and EM solutions (computed as a percentage).

is up to 210 cows, size for which all animals can be fed in Z4 (feeding option which provides the higher energy density). When the herd size was bigger than that, the EM distributed cows in other field zones, so the average energy acquired by the herd was smaller. A smaller amount of energy affected the average individual production causing a decrease in the total milk production. When herd size was over 700 cows, resources became scarce and the individual production average was notably affected.

The values presented in Table 5 show that the solution found with GA behaves similar to the solution obtained by EM. When herd size is small, cows are sent to Z4. When herd size is greater, Z2 was also used. When the herd size is much larger, the use of the feeding options did not follow such a strong pattern as in the case of EM. Despite the herd size, the number of cows in the EM solutions for Z1, Z2 and Z3 remained constant, even for each cow type. Some variations were found for Z4 and Z5.

Total and individual milk production values, as well as behavior, are similar compared to those observed in EM solutions. The gap values showed that GA solutions were almost optimal. The biggest difference is observed when the herd size is 210, where the gap value reached is 1.09%.

This work could not be compared with previous works since the problem is proposed in this thesis, but the milk production values obtained from GA were similar to those obtained with EM, and this was expected thanks to the versatility of GA to solve problems related to search, optimization and machine learning [72].

Regarding the second experiment (2), the values presented in Table 6 show that solutions were generated by distributing as many cows as possible into feeding options with the best relationship between energy density and feed cost. Also, solutions where the real consumption of the cows reaches the potential consumption are intended.

When the herd size is 50, cows are sent to Z2, but when the herd size is up to 210 cows, the available resources in Z1 and Z3 were also used. The information above indicates that the method try to pay as little as possible for feed resources. If the herd size is even greater, available resources in Z4 and Z5 are also used, in the mentioned order. As well as in the first experiment (1), when the herd size is over 700 cows, resources are not enough to reach their potential consumption, and therefore sharing the food was needed. In these cases, the total earnings and the average individual earnings decreased. When the herd size is up to 700 cows, each cow real consumption is equal to their potential consumption. Naturally, in these cases, large herds make higher total profits than small herds. When the herd size is greater than 700 cows, the total earnings decrease. The individual earnings reach their maximum value when the herd is up to 50 cows, where animals obtained the best and cheapest feed considering the relationship between energy density and feed cost. The results showed that until that number all animals are sent to Z2. For larger herds, the EM distributed cows in other feeding options, causing a decrease in the individual earnings. When the herd size is larger than 700 cows, feed resources became scarce and individual earnings were notably affected.

The values presented in Table 7 show that GA solutions behave similar to the EM. When the herd size is small, cows are sent to Z2. When the herd size is greater, Z1, Z3 and Z4 are also used. If the herd is much larger, the use of the feeding options did not follow such a strong pattern as solutions presented by the EM. Despite the herd size, the number of cows in the EM solutions for Z1, Z2 and Z3 remained constant, even for each cow type. Some variations were found for Z4 and Z5.

Total and individual earnings values and behavior were similar compared to those observed in EM solutions. The gap values indicated that GA solutions were close to the optimum value obtained by the EM. The biggest difference was observed in the case of 290 cows, where the gap value reached 1.93%.

Again, this work could not be compared with previous works, but GA obtained results were close to those obtained by the EM.

Another point of interest was to discuss the diversity of the solutions provided by the GA, and therefore another experiment (3) was carried out. A measure of this diversity was the distance between the solutions obtained by the exact method and the genetic algorithm. To measure diversity, for each number of cows, 30 executions of the GA in the context of maximizing milk production were performed. The results obtained were presented in Table 8 and Table 9.

To perform a better analysis considering different perspectives, for each number of cows, three particular solutions over the 30 different GA runs were chosen and compared with EM solution: the solution with the lowest gap value, lowest distance percentage, and biggest distance percentage.

A comparison of the total milk production between the above-mentioned solutions is presented in Figure 7. The results indicate that solutions with the smallest and largest distances to the optimal solution found by the EM had similar gap values. From this it follows that it is possible to find a diversity of good quality solutions.

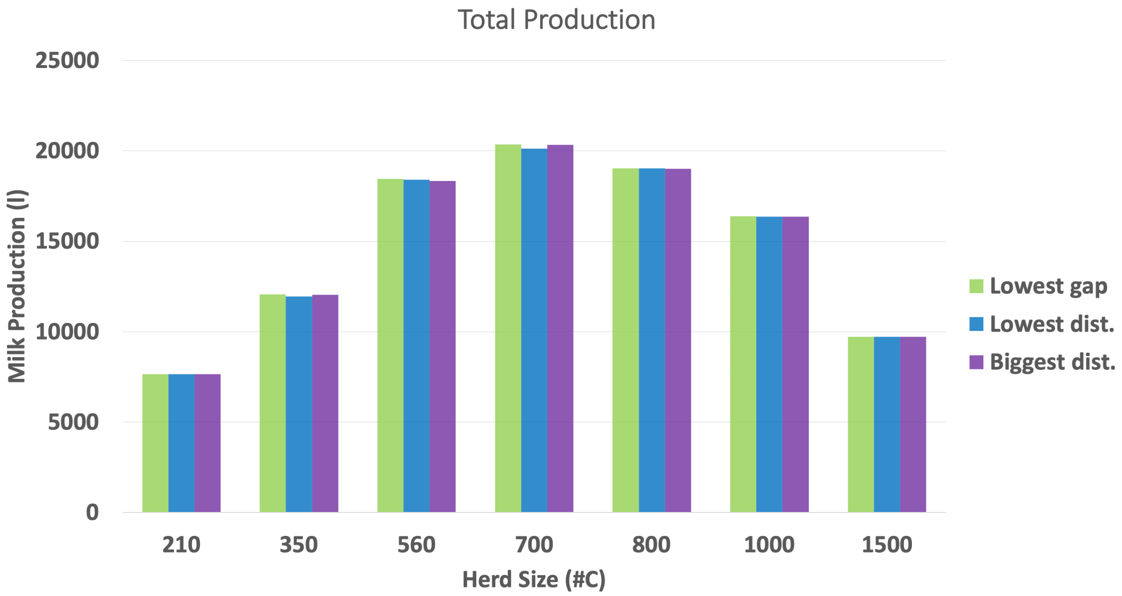

Figure 7: Comparison of total milk production between three particular solutions of the GA and the EM solution

Also, a comparison of the distance between the three particular solutions is shown in Figure 8. From this figure we can see that the solution with the lowest gap value was not related to the solution with the lowest distance. In some cases, the solution with the lowest gap value had a small distance, while in other cases it had a big distance, so that gap and distance values were not necessarily correlated. This also shows that the GA reached a considerable degree of diversity in its solutions.

Finally, a comparison of the gap value between the three particular solutions is presented in Figure 9. This figure shows that the gap values are all lower than 2%, so the solution quality was generally good, with high diversity. It also showed that the biggest gap difference between solutions was presented when the herd size is up to 700 cows. When the herd size is bigger than 700 cows the gap values were similar.

From the results obtained, it is possible to form groups that differ considerably in the number of cows from different types and still maintain solutions near the optimum value (based in the gap values obtained).

5.4 Conclusions

The speed in obtaining solutions is essential in combinatorial optimization problems, since it allows addressing computationally expensive problems and efficiently exploring the search space. Genetic algorithms can be an alternative to obtain good solutions in reasonable times, but it is necessary to evaluate how well they adapt to each domain and try to analyze the quality of the solutions achieved.

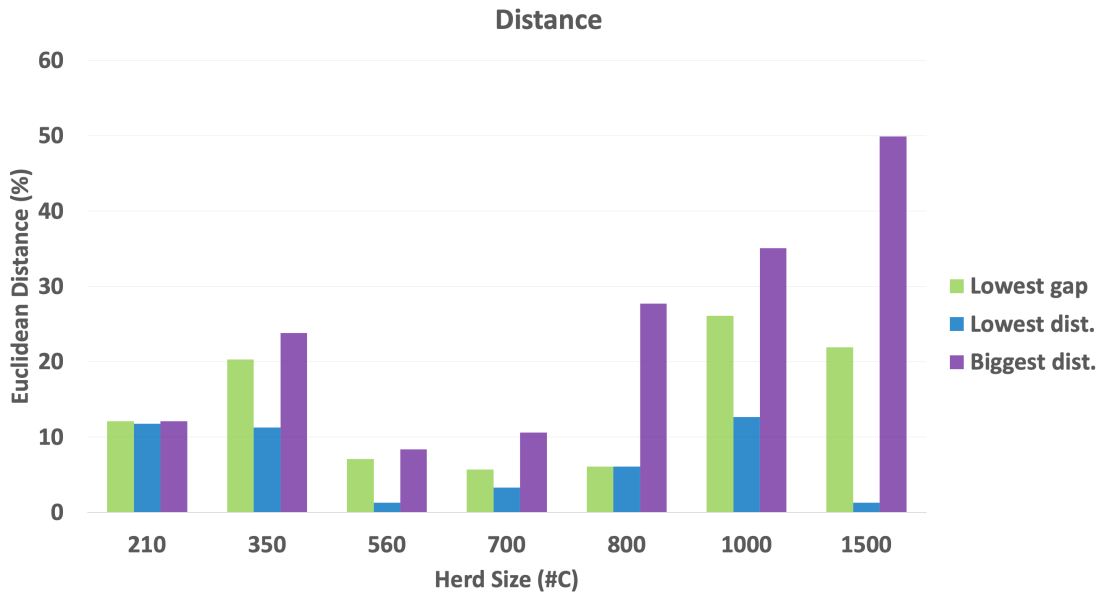

Figure 8: Comparison of the distance between three particular solutions of the GA and the EM solution

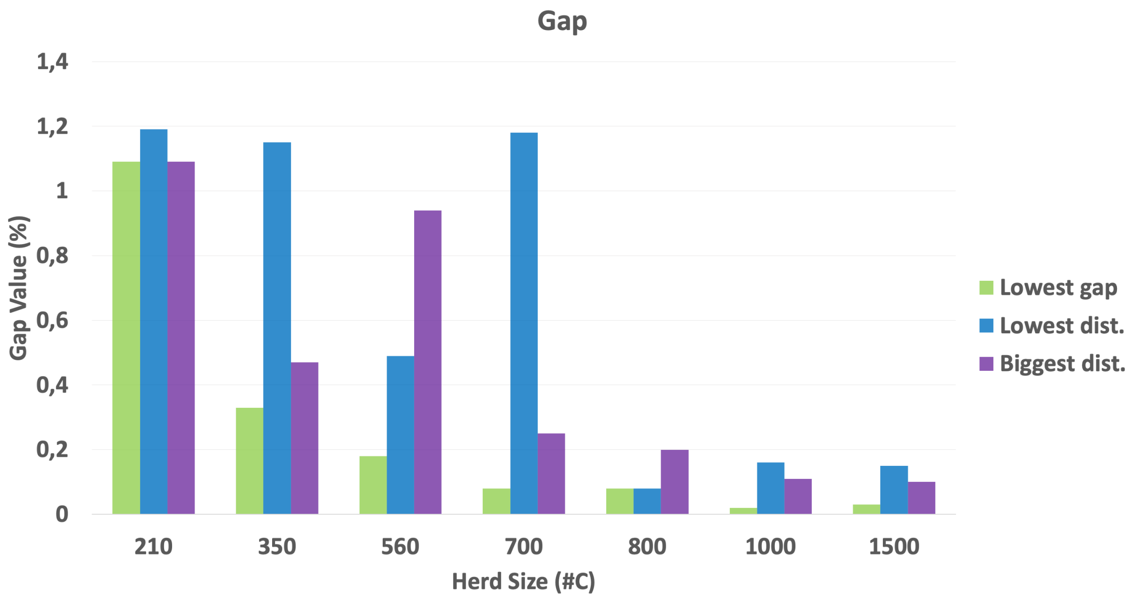

Figure 9: Comparison of the gap value between three particular solutions of the GA and the EM solution

Experiments confirm that the GA was well adapted to the problem, since values obtained were very close to those obtained by the EM. Total and individual milk production values obtained by the GA were similar compared to those obtained by the EM. In particular, the gap value showed that GA solutions were almost optimal, reaching a maximum value of 1.09%, but presenting an average value between the different numbers of cows of 0.50%. Also, total and individual earnings values obtained by the GA were similar to those obtained by the EM. The maximum gap value was 1.93%, but presenting an average value between the different numbers of cows of 0.97%.

Another interesting aspect of the GA is that it can maintain a good diversity of solutions, meaning that the producer can choose good quality feeding strategies with different assignments which can have other desirable properties not necessarily provided by the exact method solutions. Diversity was measured using the Euclidean distance between the structure of the EM and GA solutions. We found several solutions where the gap value was between 0.02% and 1.19% (solutions very close to the optimal one), but with very varied distances and that reach a maximum difference of 49.9%. From these solutions, with a small gap and varied distances, we were able to conclude that the GA can find very good solutions, with a highly varied allocation of food resources, and therefore high diversity.

The solutions obtained by the EM were built distributing as many cows as possible to the feeding options with higher energy density. The obtained results suggest that the animal real consumption must be equal to the potential consumption, otherwise serious losses will occur in economic earnings or milk production process efficiency. When the food resources available were not enough to fulfill the potential consumption of all cows, sharing out those resources was needed. If keeping the entire herd is not a constraint, the best option is not to share the available resources among the whole herd, this means to stop feeding a set of cows (probably meaning selling or disposing these animals, who would otherwise die).

It is also important to mention that the presented model only found the distance (between feeding options) to be a relevant factor when two food types had the same energy density or the same relationship between energy density and feed cost. These conclusions should be considered only for the specific scenario studied in this approach, and can change when using other data instances. Nevertheless, these conclusions show the potential of the model when analyzing a given situation.

Chapter 6

Resource Allocation Model: Multi-objective Approach

In Chapter, 5 a first approximation to the problem of feed resources allocation in dairy systems was presented, where single objective models were used. In this approach, the use of several milkings was contemplated, but they were not framed in a context of annual strategic planning.

As mentioned before, the allocation of feedstuff to intensively managed dairy cows to achieve different objectives is challenging due to the inherent complexity of the system and the combinatorial problem that has to be solved. For feeding purposes the herd is usually split into groups of cows (typically based on parity and/or the level of production), and each group is distributed to different feeding options (or feeing areas). In practice, the composition of each group remains unchanged for a certain period of time, typically one month. After that period new groups of cows are defined, so the distribution process into the feeding areas stars over. This process is repeated over several periods, typically for a year.

The general objective of this approach was to develop a multi-objective and multi-period formulation to address the problem. This particular approach was published in Notte et al. [123].

6.1 Multi-objective optimization

One of the characteristics that differentiate multi-objective and single objective problems lies in the conflict between the objectives. This conflict is reflected when we try to reach the optimal solutions, because the improvement of one objective can lead to the degradation of another one. To define the optimal solutions, the concept of Pareto optimality is generally used: a solution is optimal when there is no other solution that improves one of the objectives without deteriorating another one [176]. The set of optimal solutions forms the Pareto front, which can have a large cardinality. Finding the Pareto front can be computationally difficult or even impossible, so in general, a good approximation is attempted. This approximation consists in a reduced set of solutions, which must be as close as possible to the Pareto front and at the same time must have a uniform distribution within the solution space to prevent unexplored regions.

The concept of multi-objective optimization problem (MOP) can be formally defined as finding vectors $x^* = (x_1^*, x_2^*, ..., x_n^*)$ satisfying the constraints $g_i(x) \leq h_i$ and minimizing the function vector $MinU(x) = (U_1(x), U_2(x), ..., U_k(x))^T$, where the integer $k \geq 2$ represents the number of objectives. If some objective function is to be maximized, it is equivalent to minimize its negative. The multi-objective optimization problem is generally formulated in Equations 6.1-6.3.

$$MinU(x) = (U_1(x), U_2(x), ..., U_k(x))^T \tag{6.1}$$

$$x = (x_1, x_2, ..., x_n)^T \tag{6.2}$$

subject to i constraints:

$$g_i(x) \leq h_i \tag{6.3}$$

The vector $x = (x_1, x_2, ..., x_n)^T$ is the decision variables vector, and the set of all vectors x satisfying the constraints is the feasible solution region. In order to achieve a good approximation of the Pareto front, a set of feasible, non-dominated solutions is usually generated. From the definition of a MOP, we say that a solution $x = (x_1, x_2, ..., x_n)$ dominates a solution $y = (y_1, y_2, ..., y_n)$ if $U_i(x) \leq U_i(y)$ for $i = 1, 2, ..., k$, and there is at least one j $(1 \leq j \leq k)$ such that $U_j(x) < U_j(y)$. In turn, two solutions are mutually non-dominated when neither one dominates the other [50].

6.2 Problem formulation

To optimize the allocation of available feed resources for the dairy herd, it is necessary to achieve multiple objectives of productivity, profitability and efficiency of the system. The objectives of the optimization problem were to maximize milk production (liter cow^{-1} day^{-1}), to maximize the gross margin over the feeding costs (USD cow^{-1} day^{-1}), to maximize the herbage intake from pastures (kg of DM cow^{-1} day^{-1}), to minimize the costs of the ration (USD cow^{-1} day^{-1}) and to minimize the intake of supplements (kg of DM cow^{-1} day^{-1}). The allocation was made by dividing the herd into groups of cows and their subsequent distribution in the available feeding options, which was conducted for one whole year, divided into twelve monthly periods. The lactating cows are milked and fed twice per day, so the feed area allocation is performed twice per day. Then, for each group of cows, the number of allocations for a specific period was calculated as the number of days of that period multiplied by two, resulting in 56 (February) to 62 (e.g., March) allocations per period. Future versions of the model will be extended so that the number of periods and days per period can be entered as input, allowing the user to define the temporal scope of the execution. Also, in this approach, each type of cow (identified by body weight, genetic potential and lactation days) remains unchanged throughout the year, therefore the lactation curve is the same. Future versions of the model will be extended so each type of cow can be discriminated for each period.

The input data for the model are the number of periods of time, number of groups of cows, types of cows, number of cows for each type, characteristics of the feeds in the different areas, and prices of feeds and milk. The characteristics of the cows and feedstuff were presented in Section 4.1.

The detailed mathematical formulation of the problem is shown in Equations 6.4 to 6.22, the definition of the parameters is presented in Table 10, and the decision variables are presented in Table 11.

Objective functions

$$max \quad \sum_{s\in S}\sum_{b\in B}\sum_{z\in Z}\sum_{t\in T}\left(\frac{(w_{sbzt}\times CL_z)}{ENl} - \frac{y_{sbz}\times x_{sbt}(BR_t/2+Cte\times D_z\times BW_t)}{ENl}\right) \quad (6.4)$$

$$max \quad PM\times\sum_{s\in S}\sum_{b\in B}\sum_{z\in Z}\sum_{t\in T}\left(\frac{(w_{sbzt}\times CL_z)-y_{sbz}}{ENl}\times \frac{x_{sbt}(BR_t/2+Cte\times D_z\times BW_t)}{ENl}\right) - w_{sbzt}\times RC_z \quad (6.5)$$

$$min \quad \sum_{s\in S}\sum_{b\in B}\sum_{z\in Z}\sum_{t\in T} w_{sbzt}\times RC_z \quad (6.6)$$

$$max \quad \sum_{s\in S}\sum_{b\in B}\sum_{z\in Z}\sum_{t\in T} w_{sbzt} \qquad \forall z\in P/Z=P+Sup \quad (6.7)$$

$$min \quad \sum_{s\in S}\sum_{b\in B}\sum_{z\in Z}\sum_{t\in T} w_{sbzt} \qquad \forall z\in Sup/Z=P{+}Sup \quad (6.8)$$

$$sa:$$

Resource allocation constraints

$$\sum_{z\in Z} y_{sbz} = DA_s\times 2 \qquad \forall s\in S,\forall b\in B \quad (6.9)$$

$$\sum_{b\in B} x_{sbt} = C_t \qquad \forall s\in S,\forall t\in T \quad (6.10)$$

$$\sum_{t\in T} x_{sbt} \geq MinBS \qquad \forall s\in S,\forall b\in B \quad (6.11)$$

$$\sum_{t\in T} x_{sbt} \leq MaxBS \qquad \forall s\in S,\forall b\in B \quad (6.12)$$

$$y_{sbz}\times x_{sbt}\times\frac{PC_t}{2}\geq w_{sbzt} \qquad \forall s\in S,\forall b\in B,\forall z\in Z,\forall t\in T \quad (6.13)$$

$$\sum_{b\in B}\sum_{t\in T} w_{sbzt}\leq v_{sz}-MG_{sz}\times H_z \qquad \forall s\in S,\forall z\in Z \quad (6.14)$$

Available food constraints

$$v_{sz/s=March}=F_z\times H_z \qquad \forall z\in Z \quad (6.15)$$

$$v_{sz} = v_{s-1z} - \sum_{b\in B}\sum_{t\in T} w_{s-1bzt} + RG_{sz} \times DA_s \times H_z \qquad \forall s \in S/s > 1, \forall z \in Z \quad (6.16)$$

Biological constraints

$$\sum_{z\in z} w_{sbzt} \times CL_z \geq MinE_{st} \times x_{sbt} \times DA_s \qquad \forall s \in S, \forall b \in B, \forall t \in T \quad (6.17)$$

$$\sum_{z\in z} w_{sbzt} \times CL_z \leq MaxE_{st} \times x_{sbt} \times DA_s \qquad \forall s \in S, \forall b \in B, \forall t \in T \quad (6.18)$$

$$\sum_{z\in z} w_{sbzt} \times P_z \geq MinP_{st} \times x_{sbt} \times DA_s \qquad \forall s \in S, \forall b \in B, \forall t \in T \quad (6.19)$$

$$\sum_{z\in z} w_{sbzt} \times P_z \leq MaxP_{st} \times x_{sbt} \times DA_s \qquad \forall s \in S, \forall b \in B, \forall t \in T \quad (6.20)$$

$$\sum_{z\in z} w_{sbzt} \times NDF_z \geq MinNDF_{st} \times x_{sbt} \times DA_s \qquad \forall s \in S, \forall b \in B, \forall t \in T \quad (6.21)$$

$$\sum_{z\in z} w_{sbzt} \times NDF_z \leq MaxNDF_{st} \times x_{sbt} \times DA_s \qquad \forall s \in S, \forall b \in B, \forall t \in T \quad (6.22)$$

In this model, each cow type is represented by the index t, and each feeding option is represented by the index z. To identify each group of cows the index b was added (in the set B). Finally, each period (month) is represented by the index s (in the set S).

As a consequence, x_{sbt} represents the number of cows for each type and for each group, y_{sbz} represents the number of times each group is assigned to each zone, w_{sbzt} represents the total consumption of DM for each group in each zone, and v_{sz} represents the available resources in each zone. A minimum level of residual herbage mass per hectare is considered to ensure an adequate growth of pastures.

This model assumes that the food resources available are shared uniformly between the cows assigned to a feeding option, so it is enough to know the whole food consumption of DM for each z, and it is not necessary to represent the DM consumption for each cow.

The objective functions presented from Equation 6.4 to Equation 6.8 are the maximization of the milk production, the margin over feeding cost, the herbage intake and the minimization of the cost and the supplement intake.

The restrictions shown from Equations 6.9 to 6.14 represent the resources allocation constraints. The constraint shown in Equation 6.9 forces the sum of assignments for each feeding option to be equal to the number of days multiplied by 2. The restriction shown in Equation 6.10 ensures that for each type of cows, the sum of cows for each group must be equal to the number of cows of that type. The constraint shown in Equations 6.11 and 6.12 ensure the number of cows in each group to be bigger than the minimum group size and smaller than the maximum group size. The restriction in Equation 6.13 forces the real consumption to be equal or lower than the potential consumption. Finally, Equation 6.14 ensures the real consumption for each zone must be lower or equal than the available amount of food minus the minimum requirement for a controlled regrowth of the pasture.

Equations 6.15 and 6.16 represent the constraints related to food availability. In particular, Equation 6.15 determines the available food in each period, which is calculated considering the food consumption in the previous period and the growth rate per day.

Equations 6.17 to 6.22 represent the biological constraints. Equations 6.17 and 6.18 force the minimum and maximum criteria for energy consumption to be respected. Equations 6.19 and 6.20 force the minimum and maximum criteria for protein consumption to be respected. Finally, Equations 6.21 and 6.22 ensure the minimum and maximum criteria of Neutral Detergent Fiber (NDF) consumption to be respected.

Table 10: Parameters description

Parameter	Description
S	number of periods
B	number of groups of cows
Z	number of feeding options (pastures or feeding places)
T	number of types of cows
MCP	milk current price
CL_z, $z \in Z$	calories level for each zone
BR_t, $t \in T$	basal requirement for each type of cow
MR_{zt}, $z \in Z$, $t \in T$	movement requirement for each type to each feeding option
D_z, $z \in Z$	distance from each zone to the milking room
BW_t, $t \in T$	body weight for each type of cow
ENl	amount of net energy needed to produce one kg of milk
DA_s, $s \in S$	days per period (month)
C_t, $t \in T$	number of cows of each type
GP_t, $t \in T$	genetic potential for each type of cows
PC_t, $t \in T$	potential consumption for each type of cows
MG_{sz}, $s \in S$, $z \in Z$	minimum level of herbage mass per hectare
F_z, $z \in Z$	initial amount of food per hectare in each zone
RC_z, $z \in Z$	food resource price for each zone
RG_{sz}, $s \in S$, $z \in Z$	rate of growth per day per hectare
H_z, $z \in Z$	number of hectares per zone
$MinE_{st}$, $s \in S$, $t \in T$	minimum energy consumption per cow
$MaxE_{st}$, $s \in S$, $t \in T$	maximum energy consumption per cow
$MinP_{st}$, $s \in S$, $t \in T$	minimum protein consumption per cow
$MaxP_{st}$, $s \in S$, $t \in T$	maximum protein consumption per cow
$MinNDF_{st}$, $s \in S$, $t \in T$	minimum NDF consumption per cow
$MaxNDF_{st}$, $s \in S$, $t \in T$	maximum NDF consumption per cow
P_z, $z \in Z$	amount of protein per kg of DM
NDF_z, $z \in Z$	amount of neutral detergent fiber per kg of DM
$MinBS$	minimum batch size
$MaxBS$	maximum batch size

Table 11: Decision variables, objective functions and constraints description.

Acronym	Description
Decision variables	
x_{sbt}, $s \in S$, $b \in B$, $t \in T$	number of cows of each type in each group for each period
y_{sbz}, $s \in S$, $b \in B$, $z \in Z$	number of times each group is assigned to each z for each period
w_{sbzt}, $s \in S$, $b \in B$, $z \in Z$, $t \in T$	total intake of DM for each type for each group in each z
v_{sz}, $s \in S$, $z \in Z$	available feed in each z for each period
Objective functions	
MP	Milk production
Ma	Gross margin over feeding costs
HI	Total herbage intake
SI	Total supplement intake
C	Feeding costs
Constraints	
RAC	feed allocation constraints
AFC	feed availability constraints
BC	biological constraints

6.3 Resolution method

A Pareto-based multi-objective variant of the Differential Evolution algorithm was used to explore the solution space defined by the constraints of the optimization problem [78, 80, 81]. When creating the initial population, each allele is initialized by assigning a random number within the range allowed for individual decision variables. Then, a new generation $t+1$ is created by applying mutation and selection operators on each of the individuals in the population P of the current generation t. For the evolutionary process, a trial population P' that contains a counterpart for each individual in the parent population P is generated by using parameterized uniform crossover of a parent vector and a mutation vector. The mutation vector is derived from three mutually different competitors c_1, c_2 and c_3 that are randomly selected from the population P in the current generation t. The evolutionary process of this type of algorithms was described in detail in Section 3.5. During the evolutionary process, for the next generation, the offspring replaces the parent if its performance (fitness) is better. Here, better performance is interpreted as a better Pareto ranking or a location in a less crowded area of the solution space than the parent genotype [77, 78, 80].

The pseudocode of the detailed DE algorithm is presented in Algorithm 5.

6.3.1 Encoding

The decision variables were represented as an array of integers for each period, that stores the number of cows of each type in each group and the number of times each group is assigned to each feeding option [72, 121, 130, 123, 160]. The size of the array was determined from the number of groups, cow types and feeding options. This encoding is simple but infeasible solutions can be generated due to the randomness used in the evolutionary operators. In the mutation operator, the value of a variable is modified randomly. In the crossover operator, two solutions are randomly selected and two new solutions are generated. Because of this, solutions can be created with an incorrect number of cows, either violating the size of the herd, the number of

Algorithm 5 Pareto-based variant of the DE algorithm

N = Population size; Z = Number of alleles;
t = generation
Create a random initial population
for i = 1 to N **do**
  for j = 1 to Z **do**
    $p_{j,i}^{t=0} = p_j^{min} + rand_j[0,1] * (p_j^{max} - p_j^{min})$
  end for
end for
Evaluate fitness function for each individual of the population
Trial vector generator
for t = 1 to MaxGeneration **do**
  Select randomly c_1, c_2, c_3
  Mutation and Crossover Process
  Selection
end for

cows of each type or the number of cows in each group. Other possible causes for infeasibility are the incorrect number of times each group goes to a feeding option or solutions that do not satisfy the biological constraints. To ensure feasibility, when an infeasible solution was detected, a correcting procedure was repeatedly executed until a feasible solution was found (as presented previously in Algorithm 4).

More details on the Pareto ranking procedure used were presented by Groot et al. [77, 78, 80].

6.4 Computational experiments

In this work, and considering the multi-objective and multi-period approach, we conducted two computational experiments: (1) to assess the performance of the evolutionary algorithm and (2) to determine the influence of different stocking rates (the total number of cows considered in the system divided by the number of hectares) on the trade-offs and synergies among the objectives.

Experiment (1) compared the results of the multi-objective DE optimization with the results obtained from a mono-objective linear programming (LP) model used in an ongoing project using on-farm collected data. Particularly, the results obtained for the objectives to maximize the margin over the feeding cost and minimizing the diet cost were compared. In the LP model milk production levels, characteristics of the herd and the feeding options were defined as inputs, while the feeding cost was defined as the objective to minimize. The gross margin over feeding cost is calculated by subtracting from the net earnings (price of milk multiplied by the liters produced) the cost of food consumed (consumption of pastures, conserved forages and concentrates multiplied by the corresponding price). This model chooses, month by month, the feed combination that maximizes the margin over feeding cost based on the availability of pastures, conserved forages (various types) and concentrates (various types). The biological constraints used in the LP model were the same as those used in the mathematical formulation of this problem presented in Section 5.1. The LP optimization was conducted for two milk production levels, of 25 and 30 liters cow^{-1} day^{-1}, and four stocking rates of 1.1, 1.6, 2.1 and 2.6 cows ha^{-1}, where 128, 185, 247 and 309 cows in the system were considered, respectively. The LP results were compared with the results of four DE optimization runs (one for each stocking rate), of which three solutions were obtained with milk production levels of 25 liters cow^{-1} day^{-1}, and three others of 30 liters cow^{-1} day^{-1}.

The same input data were used for both models, with the exception that milk production is an input value in the LP model and an objective in the multi-objective model. Experiment (2) compared the results of the multi-objective DE optimization for the five objectives at stocking rates of 1.1, 1.6, 2.1 and 2.6 cows ha^{-1}.

In both Experiments (1) and (2) we considered one cow type with body weight BW=580 kg, genetic potential GP=8500 liters of milk in 305 days and 20 weeks of lactation LW. We included eleven feeding options (Z1, Z2, ..., Z11). Nine of these corresponded to equally-sized pastures (Z1, ..., Z9) comprising an area of 117 hectares. The pastures used were prairies, oats, ryegrass and sorghum. Additionally, two corresponded to feeding areas where supplements were supplied (Z10 and Z11). Feed characteristics (energy content, availability throughout the year, cost and the distance in km between the milking room and pastures) are summarized in Table 12. The parameters used for the DE algorithm were crossover amplitude (F=0.7), crossover probability (CR=0.85), number of genotypes in the population (N=1000) and the number of iterations to improve the population (G=1000).

Table 12: Characteristics of the feeding options.

Activity	Description	ED (Mcal ENl / Kg DM)	Distance (km)	Availability (kg DM)	Price (%)
Z1	Pasture	1.4	0.5	72522	-
Z2	Pasture	1.4	1.5	107227	-
Z3	Pasture	1.4	2.5	102026	-
Z4	Pasture	1.4	0.5	89054	-
Z5	Pasture	1.4	1.5	115322	-
Z6	Pasture	1.4	2.5	112608	-
Z7	Pasture	1.4	0.5	27502	-
Z8	Pasture	1.4	1.5	49974	-
Z9	Pasture	1.4	2.5	81357	-
Z10	Mix 1	1.5	0	∞	60
Z11	Mix 2	1.7	0	∞	80

Notes: Activity = food activity, Description = description of the food activity, Mix = mix of concentrate and forages, ED = energy density measured like the net energy megacalories per lactation per kilogram of dry matter, Distance = distance to de milking parlour, Availability = availability of the activity throughout the year, Price = price measured like a percentage of the milk price per kilogram of DM.

The quality of the pasture was defined by the energy content. In this work, we did not represent pasture growth, instead we decided to express the amount of pasture available in each period, which is an input of the model based on pasture growth rate. We did not consider a specific price for the pasture either, instead we considered an average cost of 302 USD per ha to produce the selected pastures during the twelve monthly periods (value obtained from the LP model). The cost of the diet is presented per cow and per day, so the total cost to produce the pastures was evenly distributed per cow and per day. The prices of supplement Z10 and Z11 (mixes of concentrate and forages) were defined as a 60% and 80% of the milk revenue per kilogram of DM, respectively. For these experiments we considered the milk revenue as 0.30 USD per liter. Milk and supplement prices are based on 2019 information received by Uruguayan dairy farmers [88].

6.4.1 LP Model

The linear programming model used to evaluate the multi-objective algorithm was developed in the context of the Competitive Production program developed by CONAPROLE (for its acronym in Spanish: Cooperativa Nacional de Productores de Leche).

CONAPROLE was created in 1936 when Uruguay was going through the world economic crisis started in 1929 by the United States. Six companies were integrated to create this Cooperative that assured all producers the purchase of their milk, providing the population with daily supplies and respecting the requirements established by the national government [31, 111]. The Uruguayan market is mainly dominated by national companies, where CONAPROLE controls the highest percentages of the local market and exports. Its local annual sales are approximately 280 million US dollars, while its exports exceed 500 million US dollars to more than 40 countries. These numbers position CONAPROLE as the main exporting company in the country [32].

The Competitive Production program was born in 2010 as a tool for the monitoring and management of the dairy company, and currently more than 800 producers are participating. It allows to monitor and control the main productive indicators, particularly the margin over the feeding costs. Each producer can observe the monthly evolution of his herd (stocking, production, feeding expenses, operation, and feed margins per cow and per ha). At the same time, it has detailed information on different groups of producers, allowing general analysis or by subgroups with the aim of detecting differences, weaknesses and strengths that serve as input for discussion and definition of strategies to follow. Each producer can access a report with the evolution of the main monthly indicators, and also the accumulated/average of the last twelve months. It provides information on the main processes that define the company's global results, giving every month the opportunity to make the necessary adjustments. At the same time, it allows to analyze the subgroups to observe different situations within the generality and obtain a report of stock of available food, which allows to verify errors or differences due to failures in the administration or accounting of food.

The LP model is part of the aforementioned tool, which allows calculating the margin over the feeding cost from inputs of different dairy systems, such as the characteristics of the herd, feed costs, milk production, among others. This model was developed and validated based on the information provided by the producers that integrate the Competitive Production program, which ensures the correct representation of the different dairy systems existing in Uruguay.

6.4.2 Results

In Experiment (1) the performance of the DE algorithm was evaluated. To perform the evaluation, we compared the results obtained by the LP model and the multi-objective model. Since the LP model is an exact method, it attained the lowest feed costs for the single objective optimization problem minimizing feeding costs.

The results obtained by the LP model are presented in Table 13, while the results obtained by the DE algorithm are presented in Table 14.

By comparing the results presented in Table 13 and Table 14 we can see that the values reached by the DE algorithm were close to those obtained by the LP model. To perform an easier analysis, for each stocking rate and milk production level, the values of the ration cost

Table 13: Dairy project results

SR (cows/ha)	MP (l/cow/day)	MP (l/ha/year)	Cost (US dollars/ha/day)	Margin (US dollars/ha/day)
1.1	25	10037	2.95	5.3
1.1	30	12045	3.52	6.38
1.6	25	14600	4.29	7.71
1.6	30	17520	5.12	9.28
2.1	25	19162	5.63	10.12
2.1	30	22995	6.95	11.95
2.6	25	23725	6.97	12.53
2.6	30	28470	8.99	14.41

Notes: SR = stocking rate = number of cows per hectare, MP = milk production, Cost = cost of the diet, Margin = margin over feeding cost.

Table 14: DE algorithm results

SR (cows/ha)	MP (l/cow/day)	MP (l/ha/year)	Cost (US dollars/ha/day)	Margin (US dollars/ha/day)
1.1	25.27	10147	2.95	5.39
1.1	24.44	9813	2.98	5.18
1.1	25.56	10263	3.03	5.40
1.1	29.63	11897	3.82	5.95
1.1	29.76	11948	3.84	5.97
1.1	29.68	11917	3.85	5.94
1.6	25.10	14662	4.30	7.75
1.6	25.06	14636	4.32	7.70
1.6	24.84	14507	4.30	7.68
1.6	30.15	17609	5.55	8.92
1.6	30.16	17616	5.59	8.88
1.6	29.99	17516	5.64	8.75
2.1	25.30	19397	5.64	10.29
2.1	25.11	19252	5.76	10.05
2.1	25.36	19442	5.80	10.18
2.1	30.11	23081	7.18	11.78
2.1	30.13	23094	7.30	11.67
2.1	29.63	22715	7.04	11.62
2.6	25.02	23743	7.32	12.18
2.6	25.27	23982	7.40	12.30
2.6	25.24	23960	7.36	12.33
2.6	30.09	28562	9.21	14.25
2.6	30.19	28652	9.38	14.16
2.6	30.21	28672	9.41	14.14

Notes: SR = stocking rate = number of cows per hectare, MP = milk production, Cost = cost of the diet, Margin = margin over feeding cost.

and margin over feeding costs obtained from the LP model were compared with the average values of the three solutions obtained from the DE optimization run. The comparison of results is presented in Figure 10.

When minimizing the feeding costs, the DE algorithm obtained values close to those obtained by the LP model, especially for those solutions of 25 liters cow^{-1} day^{-1} (see Figure 10a). When 25 liters cow^{-1} day^{-1} were considered, for the stocking rates of 1.1, 1.6, 2.1 and 2.6 cows ha^{-1} the values obtained by the DE algorithm were 1.2%, 0.4%, 1.8% and 5.6% higher than the values obtained by the LP model, respectively. When 30 liters cow^{-1} day^{-1} were considered, for the stocking rates of 1.1, 1.6, 2.1 and 2.6 cows ha^{-1} the values obtained by the DE algorithm were 9%, 9.2%, 3.2% and 3.82% higher than the values obtained by the LP model, respectively. Margin values behaved in a similar way to those of cost (see Figure 10b). Let us remember that in the DE algorithm the milk production is an output, while in the LP model is an input. Therefore, the values of milk production in the DE algorithm can bit a little greater (or smaller)

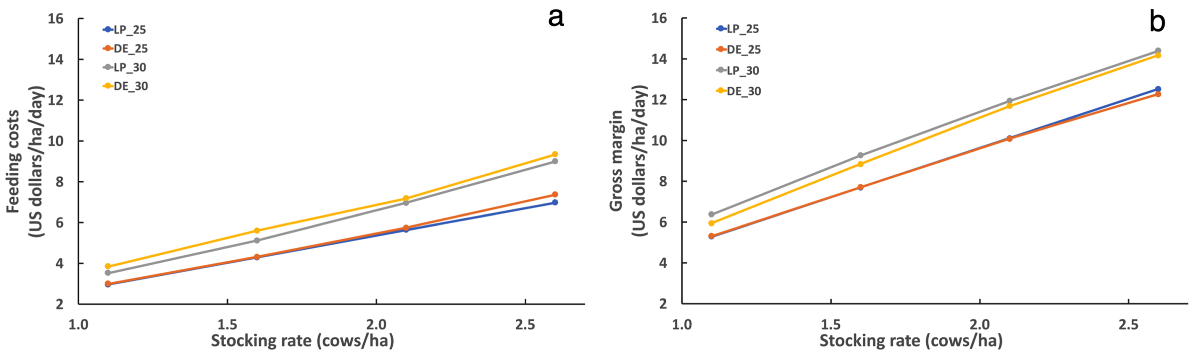

Figure 10: Optimized feeding costs (a) and gross margin over feeding costs (b) as related to stocking rate for dairy cows feeding systems derived from Linear Programming (LP) and Differential Evolution (DE) algorithms, at different production levels of 25 (LP_25 and DE_25) and 30 (LP_30 and DE_30) l of milk per day.

than 25 or 30 (depending on the case). From this, the margin over feeding costs in the DE algorithm may be greater.

In Experiment (2) the influence of different stocking rates by using the multi-objective DE optimization was analyzed. In Table 15 the best attainable solutions for the different objectives are presented for the four stocking rates.

In particular, the execution results for the stocking rate of 1.1 cows ha^{-1} are presented in Figure 11. Each figure shows six plots, where two objectives were compared in each plot. Each dot represents a solution which determines a way to do the feed resource allocation to the dairy herd. Each solution is defined by the values obtained for each objective, and those values are defined by the internal structure of each solution. The internal structure of each solution explains the main characteristics of the feed resource allocation (how to distribute the groups of cows into the different feeding options). In each plot, independently of the objectives compared, the solution that optimize each objective is shown as a larger point.

From Table 15, results showed that when the stocking rate increases from 1.1 to 2.6 (136%), the margin over the feeding cost per hectare also increases for any objective. Particularly, for the objective of maximizing the milk production the increase was 162%, while for the objective of maximizing the herbage intake the margin per hectare increased 104%. At higher stocking rate, the pasture production is used by more cows requiring increased supplement offer. While thus both the feeding cost per hectare and the milk production per hectare increased, the result was that the margins over feeding costs per hectare were greater at higher stocking rates. When maximizing the margin per cow, increasing stocking rate decreased average herbage intake up to 50%, while the supplement intake increased up to 59% per cow. At the same time, the productivity per hectare increased up to 163%, and the margin per unit of area increased up to 141% (Table 15). For the objective of maximizing the herbage intake, by increasing the stocking rate the herbage intake per cow was reduced from 12.9 to 6.7 kg cow^{-1} day^{-1} because the pasture was shared among more cows. The consumption per hectare increased from 5175 to 6386 kg ha^{-1} year^{-1}, which shows a limitation of the DE algorithm when maximizing pasture consumption for a low stocking rate (1.1 cows ha^{-1}). For higher stocking rates the DE algorithm correctly solves the maximization. Maximum margin was achieved at less-than-maximum levels of milk production, so it is not necessary to reach the highest levels of milk production to maximize profit.

We also observed that the milk production values of the solutions at the maximum milk

Table 15: Best attainable values at different stocking rates for the five objectives of the multi-objective optimization with DE. The objective with the highest value is indicated in bold font.

Objective	Stocking rate			
	1.1 cows/ha	1.6 cows/ha	2.1 cows/ha	2.6 cows/ha
Milk production:	**33.9**	**34.3**	**34.7**	**34.7**
Gross margin:	4.84	5.12	5.29	5.37
Feeding costs:	5.32	5.16	5.13	5.02
Herbage consumption:	0.02	0.07	0.04	0.15
Supplement consumption:	19.8	19.7	19.7	19.6
Milk production per hectare:	37.3	54.8	72.9	90.1
Gross margin per hectare:	5.32	8.19	11.1	13.9
Milk production:	27.9	30.1	30.1	31.1
Gross margin:	**5.55**	**5.57**	**5.61**	**5.65**
Feeding costs:	2.83	3.46	3.42	3.67
Herbage consumption:	10.8	7.30	6.79	5.48
Supplement consumption:	8.98	12.5	12.9	14.3
Milk production per hectare:	30.8	48.2	63.2	80.9
Gross margin per hectare:	6.10	8.91	11.8	14.7
Milk production:	10.4	9.87	12.2	12.0
Gross margin:	1.92	1.80	2.15	1.97
Feeding costs:	**1.19**	**1.16**	**1.49**	**1.63**
Herbage consumption:	10.7	9.20	7.40	6.26
Supplement consumption:	2.05	3.05	5.56	6.37
Milk production per hectare:	11.4	15.8	25.5	31.3
Gross margin per hectare:	2.11	2.88	4.51	5.12
Milk production:	19.2	19.5	17.6	19.4
Gross margin:	4.14	4.01	3.38	3.57
Feeding costs:	1.61	1.85	1.89	2.25
Herbage consumption:	**12.9**	**10.4**	**8.36**	**6.73**
Supplement consumption:	3.96	6.23	6.90	9.05
Milk production per hectare:	21.1	31.3	36.9	50.5
Gross margin per hectare:	4.55	6.41	7.09	9.28
Milk production:	10.4	9.87	13.2	12.0
Gross margin:	1.92	1.80	2.36	1.97
Feeding costs:	1.19	1.16	1.58	1.63
Herbage consumption:	10.7	9.20	7.88	6.26
Supplement consumption:	**2.05**	**3.05**	**5.40**	**6.37**
Milk production per hectare:	11.4	15.8	27.6	31.3
Gross margin per hectare:	2.11	2.88	4.95	5.12

Notes: Cows per hectare = cows ha^{-1}, Milk Production = l cow^{-1} day^{-1}, Gross margin = USD cow^{-1} day^{-1}, Feeding cost = USD cow^{-1} day^{-1}, Herbage consumption = kg of DM cow^{-1} day^{-1}, Supplement consumption = kg of DM cow^{-1} day^{-1}, Milk production per hectare = l ha^{-1} day^{-1}, Gross margin per hectare = USD ha^{-1} day^{-1}

production are between 11% and 21% (depending on the stocking rate) higher than the milk production values of the solutions that maximize the margin. Maximum margin is achieved at less-than-maximum levels of herbage intake. Maximum margin over feeding costs are between 34% and 66% higher, depending on the stocking rate compared to margin over feeding costs associated with maximized herbage consumption. Unlike the previous cases, the largest

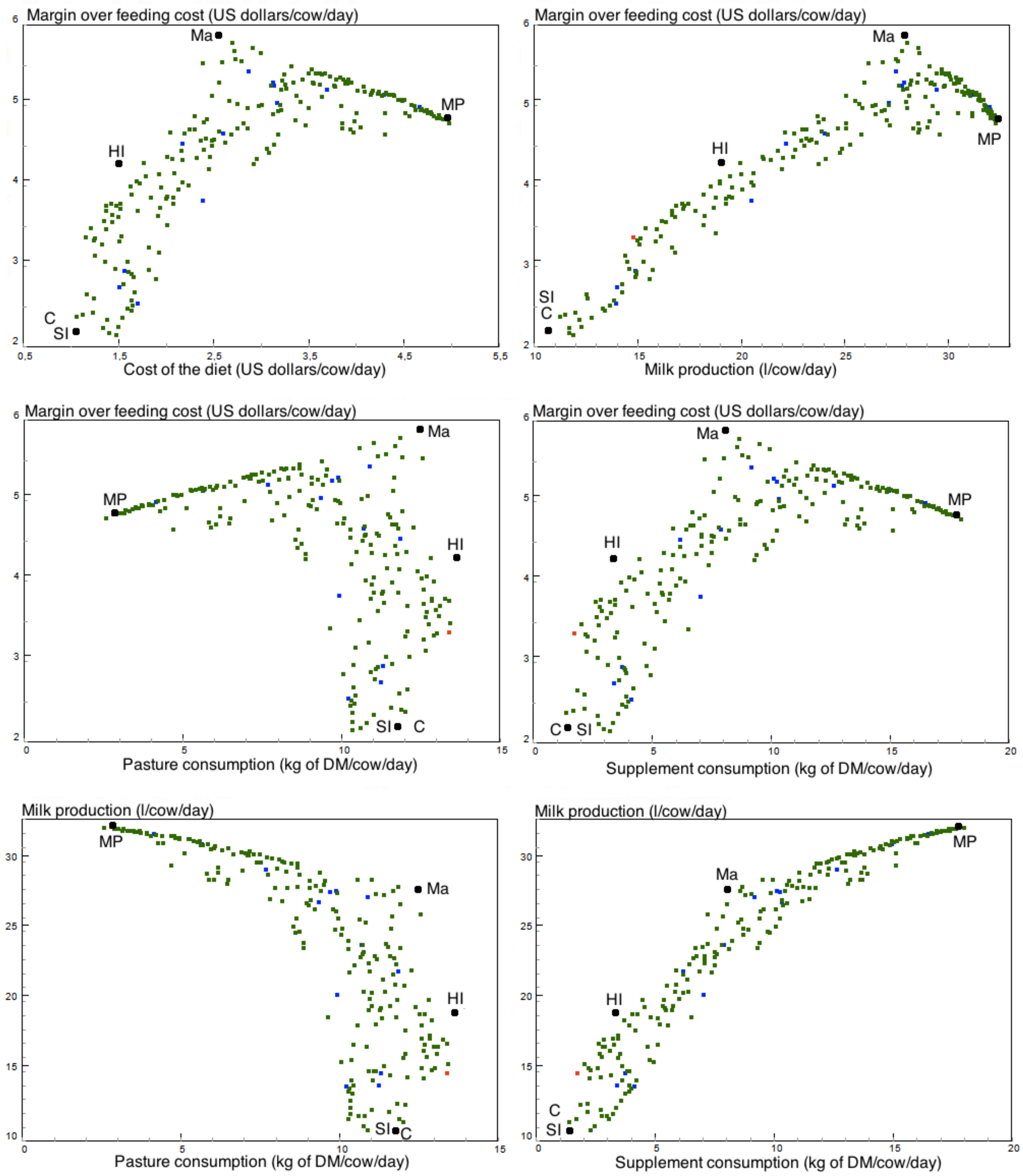

Figure 11: Results obtained using a stocking rate of 1.1 cows/ha.
Notes: Ma = Margin over feeding cost, C = cost of the diet, MP = Milk production, HI = Herbage intake, SC = Supplement intake

percentage difference occurs when the stocking rate is high. For each stocking rate, two comparisons are presented in Figure 12. Milk production values for the solutions that maximize the milk production and the margin over the feeding cost are shown in Figure 12a, while margin over the feeding cost values for the solutions that maximize the herbage intake and the margin over the feeding cost are shown in Figure 12b.

Also, the results considering the five objectives for stocking rates of 1.1 and 2.1 cows ha^{-1} are compared together. In Figure 13 the relationship between the dairy system performance

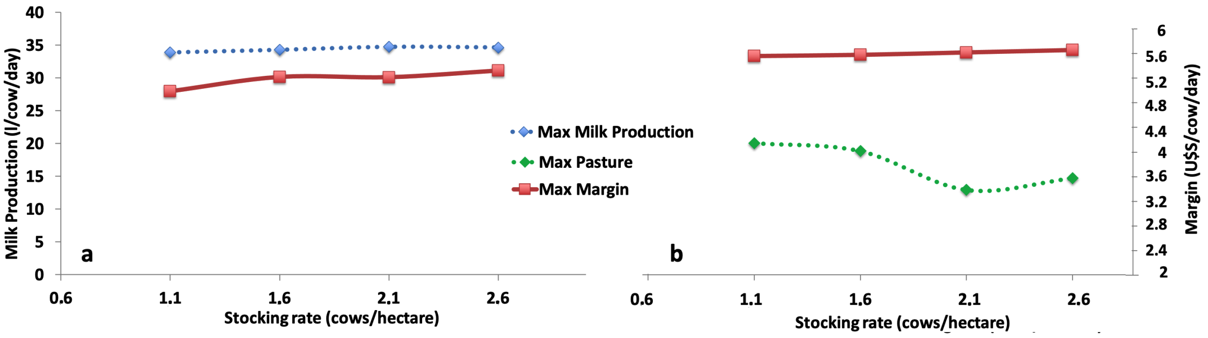

Figure 12: Two comparisons are shown: (a) milk production values when maximizing milk production and margin over feeding cost; (b) margin over feeding cost values when maximizing herbage intake and margin over feeding cost.

solutions, represented by Pareto frontiers after the multi-objective optimization, is presented. The figure is composed of different plots, where two objectives are compared. In each plot, each dot (solution) represents a way to do the feed resource allocation to the dairy herd. The green dots represent the solutions obtained using a stocking rate of 1.1 cows ha^{-1}, while the violet dots represent the solutions obtained using a stocking rate of 2.1 cows ha^{-1}.

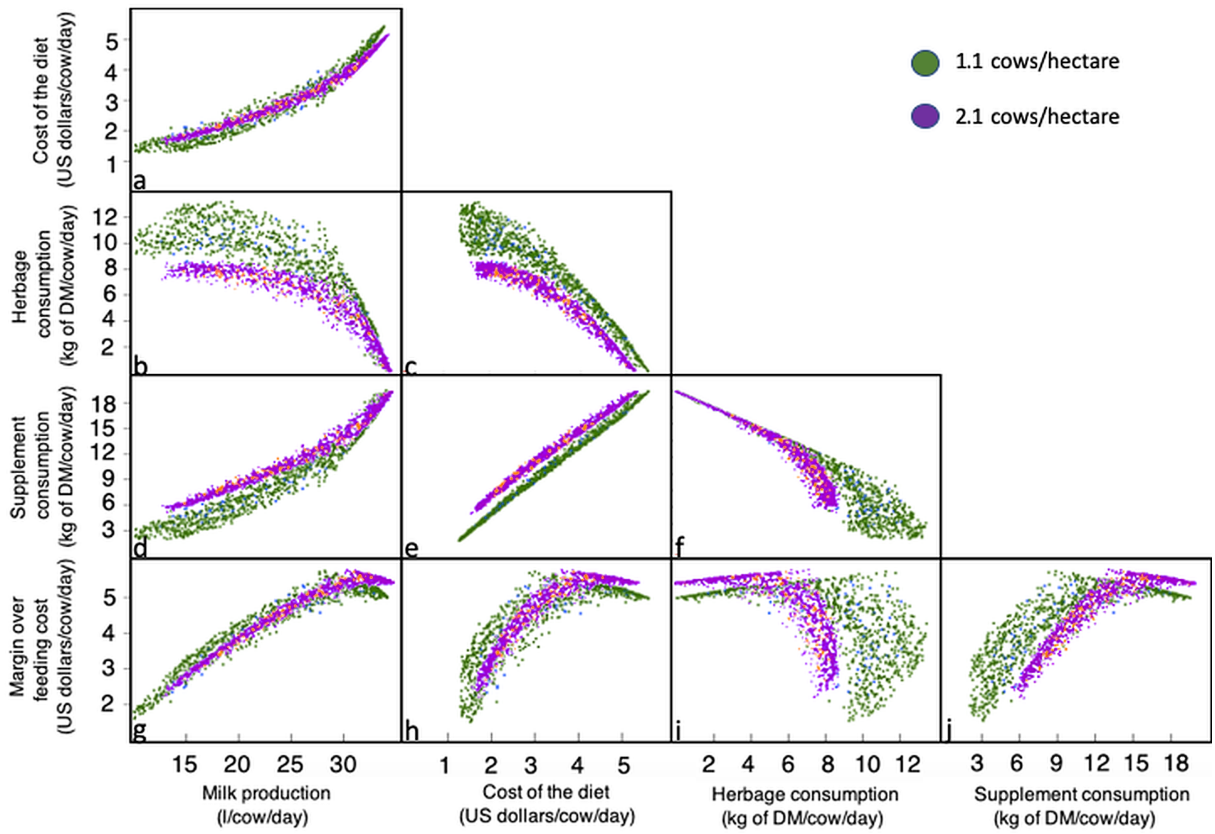

Figure 13: Relationship between the dairy system performance solutions as represented by Pareto frontiers after multi-objective optimization.

For both stocking rates of 1.1 and 2.1 cows ha^{-1}, a clear trade-off was visible between milk production per cow and feeding costs (Figure 13a) due to an increase in the amount of supplements fed (Figure 13d) and a reduction of herbage consumption (Figure 13b) at higher milk production levels. This is due to the existence of a trade-off between herbage consumption and supplement consumption, when one increases the other decreases (Figure 13f). Regardless of the stocking rate, the highest level of milk production was achieved with a diet based on supplements only (Table 15). With increasing cow productivity, the margin over feeding costs increased (Figure 13g). For the stocking rate of 1.1 cows ha^{-1}, the greatest margin was reached at feeding costs of less than 3 USD per day, while for the stocking rate of 2.1 cow ha^{-1}, the greatest margin was reached with costs of almost 3.5 USD per day (Figure 13h; Table 15). For the stocking rate of 1.1 cows ha^{-1}, the highest values for gross margin over feeding costs were obtained when high levels of herbage intake were combined with intermediate supplementary feeding of between 7 and 9 kg cow^{-1} day^{-1} (Figures 13i and 13j; Table 15). In contrast, at higher stocking rate of 2.1 cows ha^{-1}, the highest margin over feeding costs was reached at higher supplementary feeding and lower herbage intake (Figures 13i and 13j; Table 15). By increasing the stocking rate from 1.1 to 2.1 cows ha^{-1}, the number of cows in the system is almost double, but grassland resources remain unchanged. The carrying capacity of the farm is exceeded and external land/feed is used to make this up.

From Figure 13, when comparing both stocking rates, we see that in some cases the solutions for the stocking rate of 2.1 cow ha^{-1} almost completely overlap with those of 1.1 cow ha^{-1} (Figures 13a, 13g and 13h). In these cases the range of solutions obtained by the stocking rate of 1.1 cows ha^{-1} is more diverse than the one obtained by the stocking rate of 2.1 cows ha^{-1}. There are also cases where the overlap is smaller (Figures 13b, 13d, 13f, 13i and 13j) or almost imperceptible (Figures 13c and 13e).

The results obtained from experiments (1) and (2) were achieved from the distribution of the cows among the different feeding options (pastures or supplements) throughout the year, and the values of the decision variables describe how to perform that distribution. In particular, these values indicate, for each period of time (month), how many times the herd must be assigned to each feeding option (or feeding zone).

For each stocking rate in Experiment (1), three solutions with the highest milk production, margin over feeding cost and pasture consumption were obtained. For each solution, the number of times the herd must be assigned to pastures (Z1 to Z9) and supplements (Z10 and Z11) is summarized and presented in Table 16. Considering that cows are fed twice a day and the year has 365 days, there are 730 annual assignments.

Table 16: Number of times the herd must be assigned to pastures (P) or supplements (S).

Objective	Stocking rate							
	1.1 cows/ha		1.6 cows/ha		2.1 cows/ha		2.6 cows/ha	
	P	S	P	S	P	S	P	S
Milk production:	41	689	55	675	11	719	11	719
(l cow^{-1} day^{-1})	45	685	46	684	4	726	6	724
	22	708	9	721	0	730	4	726
Gross margin:	412	318	258	472	286	444	202	528
(USD cow^{-1} day^{-1})	393	337	236	494	254	476	193	537
	431	299	387	343	274	456	245	485
Herbage consumption:	649	81	524	206	475	255	479	251
(kg of DM cow^{-1} day^{-1})	591	139	459	271	470	260	385	345
	597	133	511	219	456	274	499	231

For the stocking rate of 1.1 cows ha^{-1}, the DE algorithm proposed the following distribution of cows: in the solutions with the highest milk production, only between 3.0% and 6.2% of the assignments were on pastures, while most of them were on supplementation, which means that supplements were the main part of the diet; in the solutions with the highest margin over feeding cost, the assignments were balanced between pastures and supplements; in the solutions with the highest herbage intake, most of the assignments were on pastures. For the stocking rate of 1.6 cows ha^{-1}, the DE algorithm proposed the following distribution of cows: in the solutions with the highest milk production, few assignments were on pastures, while most of them were on supplementation; in the solutions with the highest margin over feeding cost, between 32.3% and 53.0% of the assignments were on pastures, while between 47.0% and 67.7% were on supplementation; in the solutions with the highest herbage intake, between 62.9% and 71.8% of the assignments were on pastures and the rest were on supplementation. For the stocking rate of 2.1 cows ha^{-1}, the DE algorithm proposed the following distribution of cows: in the solutions with the highest milk production, at most 1.5% of the assignments were on pastures; in the solutions with the highest margin over feeding cost, between 34.8% and 39.1% of the assignments were on pastures, while the rest of them were on supplementation; in the solutions with the highest herbage intake, between 62.5% and 65.1% of the assignments were on pastures, while between 34.9% and 37.5% were on supplementation. Finally, for the stocking rate of 2.6 cows ha^{-1}, the DE algorithm proposed the following distribution of cows: in the solutions with the highest milk production, the situation was similar to the scenario with a stocking rate of 2.1 cows ha^{-1} (at most 1.5% of the assignments were on pastures); in the solutions with the highest margin over feeding cost, between 26.4% and 33.5% of the assignments were on pastures, while between 66.4% and 73.6% were on supplementation; in the solutions with the highest herbage intake, only between 52.7% and 68.3% of the assignments were on pastures, while between 31.7% and 47.3% were on supplementation.

As the stocking rate increases, the availability of pastures per cow decreases, so it is necessary to add more supplements in the diet. This is clearly reflected in the solutions with the highest pasture consumption.

6.4.3 Discussion

From the experiments carried out we see that the DE algorithm was effective in solving the single objective optimization problem and allowed exploration of the relations among objectives for the multi-objective optimization problem. Among other things, we can highlight that the increase in the stocking rate caused big differences in the results. The multi-objective model scenarios demonstrated that increasing the stocking rate would enhance milk production per unit of area and gross margin per unit of area, while the feed ration would shift from pastures to a large proportion of supplements. The DE algorithm generated a great range of solutions that showed the trade-offs among the objectives, reaching extreme values close to the values obtained by the LP model in the minimization of costs and maximization of the margin over the feeding cost. This closeness of the values can be seen in Figure 10a, where the results of the costs are shown, and in Figure 10b where the results of the margins are shown. In Experiment (1) we verified that the lower feeding costs obtained by the DE algorithm were within 0.23% and 1.24% higher than those obtained by the LP model (optimal), while the highest margins obtained by the DE algorithm were less than 2.1% lower than those obtained by the LP model, which demonstrates the good performance achieved by the DE algorithm. In Experiment (2) we found that increasing the stocking rate resulted in higher milk production and margin over feeding cost. As the number of cows ha^{-1} increased, the potential herbage intake per cow exceeded herbage availability, and therefore the supplement intake and the cost of the diet were higher.

At low stocking rates, solutions with high productivity and gross margin were identified, whose diet was based on a high herbage intake and limited supplement intake.

In most multi-objective problems it is not easy or not possible to obtain an exact description of the Pareto front set because it can cover a very large or infinite number of points. Although in theory it is possible to find these points exactly, it is computationally difficult and expensive [52]. The solutions obtained by the DE algorithm were validated and considered representative of the system by an expert in the dairy sector. Furthermore, after this validation the model can be expanded to include other objectives that are difficult and expensive (e.g. environmental variables) and sometime impossible (e.g. social variables) at farmlet level.

From the results obtained, the observed trends in productivity and profitability per animal and per unit of area with increasing stocking density are in line with local [58, 133] and international [7] research. For the objective of maximizing the milk production, the herbage intake values were significantly lower than the values obtained when maximizing the herbage intake (for any of the stocking rates considered) and the supplement intake values were significantly higher than the values obtained when minimizing the supplement intake. The high intake of supplements in solutions that maximize milk production was mainly due to the fact that the supplements provide a higher energy density than pastures, and milk production is directly related to the energy acquired from the feed consumption. However, we have observed that for both objectives, minimizing the intake of supplement and minimizing the feeding cost the model converged to very low production levels (within 28.8% and 38% of maximum production) according to the breed and animal live weight used in the experiments. For the objective of maximizing the margin over feeding cost, by increasing the stocking rate from 1.1 to 2.1 cows/hectare, herbage intake levels cannot be maintained because the available pastures do not produce enough dry matter to provide a sufficient amount of feed. Due to this, the model proposed feeding strategies with higher consumption of supplements and lower herbage consumption. This strategy directly impacts on the feeding cost, since the relationship with it and the supplement consumption is almost linear. Minimizing the feeding cost seems reasonable or even necessary in many dairy systems, but this does not ensure greater profitability. Although each system has its peculiarities and limitations, from this work we can appreciate that it is possible to optimize profitability through a high milk production combined with a controlled cost of feeding, which is achieved when the diet is balanced in herbage and supplements consumption.

In dairy systems it is increasingly important and necessary to use models that represent the systems for several reasons: (i) in order to deal with the complexity they present, which includes many components and variables, (ii) to address various objectives, limitations and opportunities that are relevant to the producer, and (iii) to anticipate and respond to the constant change that dynamic systems such as the dairy system presents. These types of tools, which allow incorporating and combining different indicators or objectives, generate alternatives that allow opportunities for discussion and analysis from different points of view. For example, from the results presented, it can be clearly seen that for some systems, by maximizing milk production or minimizing the cost of the diet, the economic benefit will not necessarily increase. Also, this type of tools is very powerful to simulate different production scenarios and analyze the different variants to consider in those cases. For example, scenarios that incorporate adverse climatic factors can be simulated, and therefore analyze what would happen to a productive system that could eventually have a lower amount of grass in a given period of the year. In this particular work, by exploring trade-offs among objectives, the DE algorithm was used to show different options for decision makers on how to do the food resource allocation, letting them to choose the one that fits better for their productive system. Through this tool it is also possible to explore different alternatives to those commonly used by producers. In turn, it allows to analyze the behavior of different production systems when some parameters are modified. In particular, from Figure 13 it is possible to evaluate how the responses and interactions in the dairy system can change as stocking rates increase.

6.5 Conclusions

The DE algorithm proved to be effective in representing the addressed problem; it had the capacity to handle the different constraints of the dairy system, the identified objectives and the existing trade-offs.

Experiments confirmed that the DE algorithm obtained high quality numerical solutions, since solutions reached by the LP model were approached. When minimizing the feeding costs, the average values of the DE solutions of 25 liters/cow/day were 1.2%, 0.4%, 1.8% and 5.6% higher than the values obtained by the LP model for the stocking rates of 1.1, 1.6, 2.1 and 2.6 cows/ha, respectively. The average values of the DE solutions of 30 liters/cow/day were between 3.2% and 9.2% higher than the values obtained by the LP model.

was also observed that it is possible to increase the stocking rate and improve the gross margin per hectare, as long as a large proportion of supplements is used to cover the pasture deficit. In particular, for stocking rates of 1.1, 1.6, 2.1 and 2.6 cows/ha, gross margins of 6.1, 8.9, 11.8 and 14.7 USD/ha/day were obtained, respectively. These values were calculated by multiplying the stocking rate by the gross margin, and reflect significant improvements as the stocking rate is increased, but also requires greater investment in feed resources and management. Taking this into account, for systems like those presented in this work, moving from 1.1 to 2.6 cows/ha could increase the gross margin up to 140%.

Beyond the limitations our simulation may have, the results suggest that increases stocking rate can potentially result in economic benefit. At the same time, it is not necessary to reach the highest levels of milk production or herbage intake to improve the profits, since the solutions that maximize the margin over feeding cost are reached at suboptimal levels of milk production and herbage intake. In particular, for stocking rates of 1.1, 1.6, 2.1 and 2.6 cows/ha, when maximizing the gross margin, the milk production values were 17.7%, 12.2%, 13.2% and 10.4% lower than the values obtained when maximizing the milk production, respectively.

As future work, it would be useful to extend the resource allocation model. Also more experiments to analyze the decision variables and the composition of the solutions can be considered, including more scenarios with several groups of different types of cows.

Chapter 7

Performance Evaluation of Multi-objective Evolutionary Algorithms

The objective of this chapter is to provide a performance evaluation of various multi-objective evolutionary algorithms, specifically focusing on determining the most suitable one for addressing the feed resource allocation problem. Part of this work was published in Notte et al. [128].

7.1 Multi-objective evolutionary algorithms

As mentioned in Section 6.1, there are different problems that require the search for solutions that simultaneously satisfy multiple performance criteria or objectives, which may be contradictory. In this case, the problem is known as a multi-objective optimization problem (MOP). Evolutionary Algorithms (EA) have proven to be especially suitable for multi-objective optimization. The literature reports a large number of Evolutionary Algorithms for Multi-Objective Optimization (MOEA). The first suggestion of applying EA on MOPs was proposed by Rosenberg in 1967, but the first MOEA was implemented in 1984 by Schaffer [151]. Until the 1990s there was very little progress in the area of MOEAs, and in general the proposals were characterized by the use of single objective EAs that incorporated techniques on multiple objective functions. As the interest of MOEAs in real optimization problems increased, it was necessary to improve their design to ensure their applicability to problems of increasing complexity. Concepts of parallelism on the set of solutions were incorporated, allowing to find in each execution a set of approximate solutions to the Pareto front. One of the first proposal was the Nondominated Sorting Genetic Algorithm (NSGA) presented in 1994 [156]. The main features incorporated were the concept of Pareto dominance and the use of sharing to provide diversity. Later, new generations of EA have been presented [157, 176], which differ mainly by the incorporation of elitism in the algorithms (through the use of a selection operator or the use of an external population).

Evolutionary Algorithms have proven to be an adequate alternative to solve multi-objective optimization problems, but in the literature there are many variants proposed which have different performances depending on the problem being solved. Then it is interesting to evaluate some of the most successful algorithms to address the feed resource allocation optimization problem presented in Chapter 6. Considering that a correct choice of the algorithms and their parameter-setting are needed to enhance the behavior of the model, we decided to evaluate the performance of four different evolutionary algorithms; aiming to choose the most appropriate algorithm and its corresponding parametric configuration to determine different strategies for animal grouping and food resource allocation considering multiple objectives.

The rest of this chapter is organized into five sections. In Section 7.2 the four different evolutionary algorithms evaluated are presented. In Section 7.3 the criteria for analyzing the quality of the solutions obtained is explained. Then, in Section 7.4 the experimental setting used to evaluate the algorithms is described. In Section 7.5, we present the obtained results according to the criteria used. Finally, in Section 7.6 we highlight the main conclusions of the evaluation performed.

7.2 Multi-objective evolutionary algorithms evaluated

As mentioned in Section 3.3 there are different types of EA; two of the most popular ones are the Genetic Algorithms (GA) [72] and the Differential Evolution algorithms (DE) [157]. In the context of our work, we decided to evaluate the performance of two advanced GA from the state of the art, such as NSGA-II and SPEA-2 and two advanced DE algorithms, such as GDE-3 and a variant of the Pareto-based DE to generate an approximation of the Pareto front, associated to different sets of evolutionary parameter values. NSGA-II, SPEA-2 and GDE-3 stand out for their good performance in solving discrete multi-objective optimization problems ([50, 175, 176, 173]). NSGA-II is an extension of NSGA, which shows a significant improvement in performance compared to its predecessor. SPEA-2 is a newer algorithm, which has shown a very good performance compared to other genetic algorithms and therefore it has been a reference in various studies [98]. GDE-3 is also an improved extension of the generalized DE. On the other hand, the Pareto-based algorithm is the multi-objective variant of the DE presented in Section 6.3.

The Non-dominated Sorting Genetic Algorithm II (NSGA-II) is a multi-objective genetic algorithm, proposed in [42]. It is an extension and improvement of NSGA [156]. This method incorporates elitism and reduces the complexity of the non-dominance quick order procedure. In each generation, a new population P' is created by creating new individuals after applying the evolutionary operators to P. Then a new population P^+ is created by joining both populations (P and P'). The procedure incorporates the calculation of a crowding distance, an operator used to maintain the diversity of the population, in order to prevent the use of the σ_{share} in the fitness sharing of the NSGA. The P^+ population is ordered according to a ranking procedure and the crowding distance. Finally, the population P^{t+1} is updated with the best individuals in P^+. The pseudocode of the NSGA-II is presented in Algorithm 6.

SPEA-2 is a genetic algorithm that preserves the best individuals (non-dominated solutions) in an external population P_E, based on an improved fitness value, and uses this population to select the parents of new solutions [174]. In this algorithm, the fitness assignment function is improved taking into account the number of individuals dominating and dominated by each candidate solution. This scheme also adds a population density estimate, where the size of P_E is fixed, unlike the original Strength Pareto Evolutionary Algorithm (SPEA), in which the size of P_E was variable and bounded. The clustering technique, in charge of maintaining the diversity of the population in SPEA, is replaced by a truncation procedure, which prevents eliminating

Algorithm 6 NSGA-II

N = Population size
Create a random initial population $(P^{t=0})$
//Evaluate fitness function for each individual of the population
for t = 1 to MaxGeneration **do**
  for i = 1 to N **do**
    $parents = selection(P^t)$
    $offspring = Crossover(parents)$
    $offspring = Mutation(offspring)$
    $evaluationFitness(offspring)$
    $insert(offspring, P')$
  end for
  $P^+ = P \cup P'$
  $rankingCrowdingDistance(P^+)$
  $P^{t+1} = bestIndividuals(P^+)$
end for

the extreme solutions from the set of non-dominated solutions. The selection is made through a binary tournament, considering the fitness of each individual as a criterion for comparison. After the selection process, a new population P^{t+1} is generated by applying the recombination and mutation operators. Then the whole evolutionary process is repeated. The pseudocode of the SPEA-2 is presented in Algorithm 7.

Algorithm 7 SPEA-2

Create a random initial population $P^{t=0}$
Create empty external population $P_E^{t=0}$
for t = 1 to MaxGeneration **do**
  Evaluate fitness function for each individual in P^t *and* P_E^t
  P_E^{t+1} *= all nondominated individuals in* P^t *and* P_E^t
  If the size of P_E^{t+1} *exceeds the capacity, then use the truncation operator to remove elements*
  If the size of P_E^{t+1} *is lower than the capacity, then use dominated individuals in* P^t *to fill* P_E^{t+1}
  Perform binary tournament selection with replacement on P_E^{t+1}
  Apply recombination and mutation operators and set P^{t+1}
end for

The Generalized Differential Evolution (GDE) is an extension of DE for optimization with any number of objectives and constraints. GDE-3 [100] is the third edition of the GDE algorithm, which improves the distribution of solutions considering the diversity in the objectives at the time of replacement. In each generation, a temporary population is generated with the descendants of the current population (using the differential evolution operators). The current population for the next generation is updated using both populations. After each generation, if the number of solutions increases, the size of the current population is reduced to its original size by applying similar methods to those used in the selection step of NSGA-II, where the individuals are ordered based on the dominance factor and the crowding distance, eliminating the worst solutions. The difference is that GDE3 modifies the crowding distance of NSGA-II to solve difficulties when working with more than two objectives. The pseudocode of the GDE-3 is presented in Algorithm 8.

Algorithm 8 GDE-3

1: *Create a random initial population* $P^{t=0}$
2: **for** t = 1 to MaxGeneration **do**
3: *Create a population* P_D^t *from descendants using the differential evolution operator*
4: *Evaluate fitness function for each individual in* P_D^t
5: $P^{t+1} = Update(P^t, P_D^t)$
6: *If the size of* P^{t+1} *exceeds the capacity, then use the truncation operator to remove elements*
7: **end for**

7.3 Criteria for analyzing the quality of the solutions obtained

Solving a multi-objective problem involves generating a set of optimal solutions, generally known as the Pareto front. Since the optimal Pareto front for the problem presented in this work is unknown, an approximation to the Pareto front was generated with all the non-dominated solutions that were found from all the carried out executions. Then, this approximation was used to represent the optimal reference points.

The literature has discussed a number of performance metrics for evaluating the performance of multi-objective optimization methods. In this work, to measure the quality of the algorithms, we considered three criteria: the ability of each algorithm to obtain the optimal values for each objective, a comparison of the non-dominated solutions of each algorithm against the Pareto front approximation, and a comparison of different performance metrics. Particularly, several performance metrics have been defined for this purpose [132, 143, 177]. Metrics consider mainly three aspects: minimize the distance of the Pareto set obtained by the algorithm to the optimal Pareto front (convergence), maximize the extension of the solution set of the algorithm over the optimal Pareto front to obtain a distribution as uniform as possible (diversity), and maximize the number of non-dominated solutions found by the algorithm (cardinality). To measure the quality, another criterion lies in the number of solution sets that will be evaluated by the metric. Two types of metrics are defined: unary metrics (only one set is evaluated considering the Pareto front) and binary metrics (two sets are compared). Unary metrics consider one or more of the three aspects mentioned above and then generate a single value from them, while binary metrics compare the two solution sets in terms of dominance to determined which one is the best. In this work, we have focused on four usually employed, state-of-the-art, unary metrics: Hypervolume, Generational Distance, Inverted Generational Distance Plus and Spread.

For some metrics it is necessary to normalize the non-dominated solutions, particularly for the metrics that depend on the scaling of the values of the objective function. The normalization carried out consists of calculating the maximum and minimum values found in the approximation to the Pareto front (obtained with the solutions of all the algorithms), and then mapping all the values to the interval [0,1]. To perform a consistent analysis, the aforementioned normalization was carried out and the metrics were applied in those values. The metrics used are briefly described below.

Hypervolume (HV) [176] is a unary metric that measures three aspects: convergence, diversity and cardinality, on a given Pareto front. This metric calculates the volume (in the objective space) covered by the Pareto set P_s obtained by an algorithm, where all objectives must be minimized. Mathematically, for each solution s belonging to P_s, a hypercube hc_s is generated with a reference point RP and the solution s that define its diagonal. The RP point can be obtained with the worst values of the objective functions. The union of all hypercubes defines the hypervolume. Algorithms that achieve higher values for HV are better. This metric is possibly the most used in the literature, even though its main disadvantage is that there is no known algorithm that determines the value in polynomial time in the number of objective functions, so its use may be limited for some problems. Its computational complexity is $O(n^{k+1})$, where k is the number of objective functions.

Generational Distance (GD) [164] is a unary metric that measures only convergence, on a given Pareto front. GD takes a set of solutions P_s and calculates how far it is from the Pareto front. This calculation consists of averaging the Euclidean distance between the solutions of P_s and the closest solution of the Pareto front. Then, it is desirable to obtain the lowest possible values. It is simple to calculate but very sensitive to the number of points found by a given algorithm. If the algorithm identifies a single point on the Pareto front, the generational distance will be equal to 0. If an algorithm does not consider a complete portion of the Pareto front, it is not penalized. This favors algorithms that obtain non-dominated solutions close to the Pareto front over those that obtain a more distributed representation of the Pareto front. GD is one of the most used techniques in the literature.

Inverted Generational Distance Plus (IGD^+) is an alternative version of IGD. IGD is a unary metric that shows how far a set of solutions P_s is from the Pareto front. It is an inverted variation of GD but it presents significant differences: calculates the minimum Euclidean distance (instead of the average distance) between P_s and the Pareto front, and uses as reference the solutions in the Pareto front (not the solutions in P_s) to calculate the distance. A result of 0 means that all the solutions obtained by the algorithm are found on the Pareto Front and cover its entire extension. Any other value indicates a divergence between both fronts. Apart from measuring convergence, another difference with GD is that if enough solutions of the Pareto front are known, IGD could measure diversity [172]. A limitation of IGD is that it is not Pareto-compliant [8]. This means that the ranking that establishes for different algorithms (based on their non-dominated solutions) may contradict the Pareto optimality concept. Furthermore, the IGD may consider that two fronts are very similar if the main difference is that one of them contains poor values for an objective. Also, the IGD value is sensitive to changes in the size of the reference front, so a change in that front can alter the previous conclusions. The average Hausdor distance [152] is a variant of IGD that attempts to correct some of the aforementioned limitations, except for lack of Pareto-compliant. Thus, IGD^+ [91] has been proposed as a Pareto-compliant variant of IGD, being very similar to IGD and as robust as the average Hausdor distance against different sizes of reference fronts.

Spread (Δ) is a unary metric that measures only diversity on a given Pareto front. This metric uses the information of the distance to the extremes of the Pareto front to have a more accurate measure of the distribution and extent of spread achieved among the solutions (measure that determines how much of the Pareto front is covered). Initially, the Δ' metric was presented [40] for bi-objective problems considering the Euclidean distances between consecutive points. This metric can be misleading if the Pareto front approximation is piecewise continuous and does not consider the extent of the Pareto front approximation. Then, Δ metric [42] extended Δ' to take into account the extent of the Pareto front approximation with more than two objectives and considering the distances between extreme points of the front. The lower the spread value, then the better the distribution of the solutions. In some problems, this metric value can be related to the number of solutions obtained.

7.4 Experimental setting

To evaluate the algorithms, two experiments (based on real test data) with different stocking rates (1.1 cows/hectare and 2.1 cows/hectare) were performed. For the experiments we kept the same characteristics of feeding options that we used in Section 6.4. We considered the same eleven feeding areas, where nine of these corresponded to equally sized pastures comprising an area of 117 ha. Pastures were considered as a finite resource while supplements were considered as an infinite resource. Once again, we did not considered a specific price for the pasture, instead we considered an average cost of 302 USD per ha to produce the selected pastures during the twelve monthly periods. The cost of the diet was managed per cow and per day, so the total cost to produce the pastures was evenly distributed per cow and per day. The prices of supplement were defined as a 60% and 80% of the milk revenue per kilogram of DM, respectively. We considered the milk revenue as 0.30 USD per liter. Milk and supplements prices were based on last years information received by Uruguayan dairy farmers [88]. In both experiments we considered one cow type, which is described in Table 17. The main difference between the scenarios was the number of cows used (128 and 245), corresponding to the stocking rates mentioned above. It was relevant to evaluate such different scenarios, as a higher stocking rate with unchanged pasture availability implies lower pasture consumptions per cow and higher supplement consumptions.

Table 17: Description of the cow type used in both experiments.

Body Weight (kg)	Genetic Potential (l)	Lactation weeks
580	8500	20

Notes: Genetic Potential = liters of milk produced in 305 days.

For each experiment, we considered different evolutionary parameter values, but the size of the problem remains unchanged (156 decision variables derived from the number of feeding options and types of cows). For the GA (NSGA-II and SPEA-2), and based on the most recommended parameter values in the literature ([54, 41]), we decided to use different values for the mutation rate (0.01 and 0.1) and crossover rate (0.7, 0.8 and 0.9). For the DE algorithms (GDE-3 and Pareto-based DE), and based on the most recommended parameter values in the literature [157, 82], we decided to keep the amplification factor value unchanged (0.9) and we used different values for the crossover rate (0.5 and 0.9). Different variants are known for the NSGA-II algorithm, but we used the original one [42]. Regarding the GDE-3 algorithm, for the crossover operator we used the "rand/1/bin" variant [100]. For all four algorithms we used the same the population size values (150, 450 and 750) and five different values for the number of generations (1000, 2000, 4000, 8000 and 16000). A summary of these values is presented in Table 18. Finally, for each algorithm and each combination of parameter values (ninety and thirty combinations for the genetic and differential algorithms respectively), fifty executions were performed.

As discussed before, solving a multi-objective problem involves generating the set of optimal solutions (Pareto front). Since the Pareto front for the problem instances used in this work is unknown, an approximation was generated by selecting the non-dominated solutions from the union of all the experiments carried out.

To measure the performance of the algorithms, we considered four criteria: a) the ability of each algorithm to obtain the optimal values for each objective; b) the proximity of the non-dominated solutions found by each algorithm to the Pareto front approximation; c) the four

Table 18: Parameter values used in each operator for the different types of algorithms.

Genetic Algorithms				DE Algorithms			
M	CR	N	G	F	CR	N	G
0.01	0.7	150	1000	0.9	0.5	150	1000
0.1	0.8	450	2000		0.9	450	2000
	0.9	750	4000			750	4000
			8000				8000
			16000				16000

unary performance metrics discussed in Section 7.3, namely: hypervolume, generational distance, inverted generational distance plus and spread; and d) the execution times (computational effort).

In the next section, we present the analysis of the computational experiments. We note that the performance of the algorithms depended significantly on the parametric combinations. Due to space consideration, we present here a short summary of the principal aspects that arose from the experimentation when selecting the best parameters for the performance evaluation stage. For NSGA-II and SPEA-2, the best results were obtained when using a low value in M, except for the Spread metric. For the GDE-3 algorithm, a low value in N achieves better values for the HV and GD metrics, while for the IGD^+ and Spread metrics a high value in CR is recommended. Finally, for the Pareto-based DE algorithm, the best results are usually obtained by using a high value in N and CR, except for the Spread metric where a low value in CR is recommended.

7.5 Results

In this section, we present the performance of the algorithms at each of the criteria previously mentioned, and is organized into five subsections. In Subsection 7.5.1, we analyze the ability of each algorithm to obtain the optimal values for each of the individual objective functions. In Subsection 7.5.2, we show the non-dominated solutions found by each algorithm and compare them with the Pareto front approximation. In Subsection 7.5.3, we present the results of the four metrics used to evaluate the quality of the algorithms. In Subsection 7.5.4, we analyze the best parametric combinations obtained by the different algorithms. Finally, in Subsection 7.5.5, we present the execution time of the algorithms.

7.5.1 Optimal values

To analyze the ability of each algorithm to obtain the optimal values for each of the individual objective functions, for each experiment (128 and 245 cows), we plotted the best values (normalized in scale [0,1]) reached by each algorithm for each objective. The plot for the experiment of 128 cows is presented in Figure 14, while the plot for the experiment of 245 cows is presented in Figure 15.

From Figure 14 we can see that, in the 128 cows case, the NSGA-II only achieves optimal values for the objectives that minimize supplement intake and cost, while for the remaining

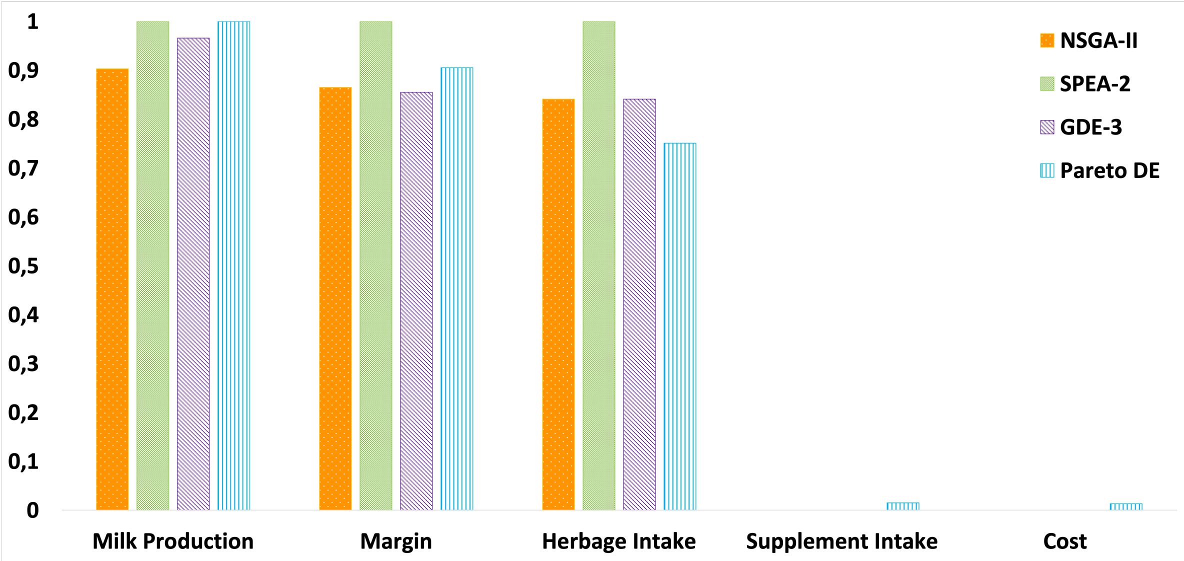

Figure 14: Normalized values (in scale [0,1]) reached by each algorithm for each objective in the experiment with 128 cows.

objectives (maximization) it reaches values between 0.84 and 0.90. On the other hand, the SPEA-2 achieves optimal values for all objectives. The behavior of the GDE-3 is similar to the NSGA-II, reaching optimal values for the minimization of supplement intake and cost, and reaching values between 0.84 and 0.97 for the remaining objectives. Then, the Pareto-based DE algorithm only reaches the optimal value for maximizing milk production. The values achieved for the objectives that minimize the cost and supplement intake are between 0.16 and 0.20, while the values achieved for the objectives that maximize herbage intake and margin are between 0.75 and 0.90.

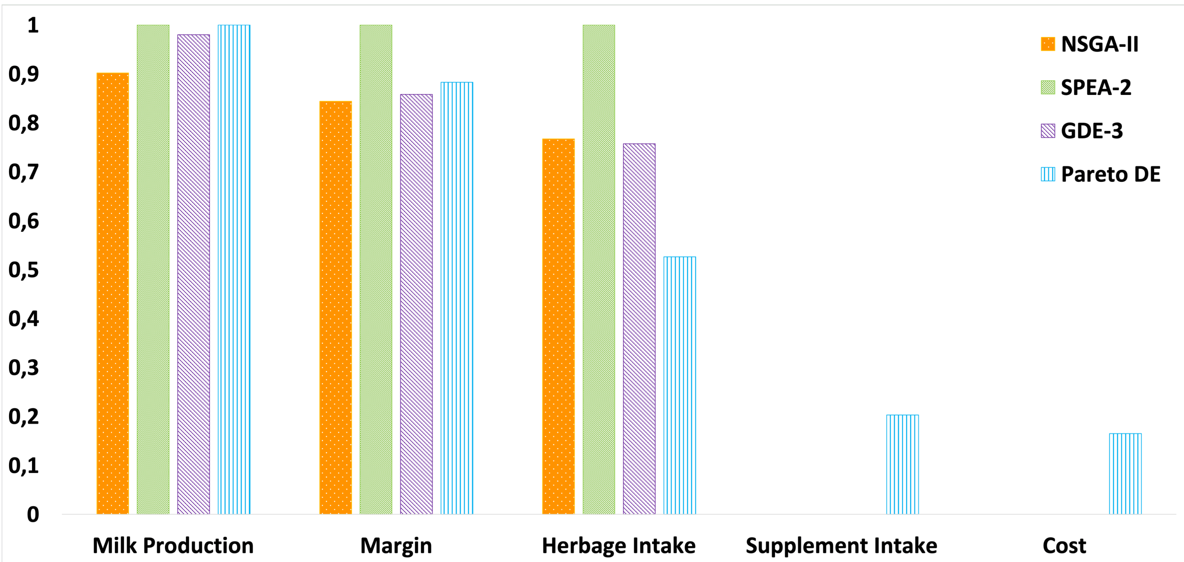

Figure 15: Normalized values (in scale [0,1]) reached by each algorithm for each objective in the experiment with 245 cows.

From Figure 15 we can see that, for the experiment with 245 cows, the behavior of the algorithms is the same as that presented for the experiment with 128 cows, although some significant differences in the values are observed. For the objective of maximizing herbage intake, the maximum value was lower (0.52), and for the objectives of minimizing the cost and supplement intake, the minimum values were higher (0.16 and 0.20, respectively).

7.5.2 Pareto front comparison

To visually analyze how the non-dominated solutions found by each algorithm behave in comparison with the (approximated) full Pareto front reached by the non-dominated solutions of all four algorithms, we show the five sets of solutions (Pareto front and the sets of each algorithm) in two-dimensional plots (comparing two objectives at a time). Figure 16 shows the comparison of the six most important objective combinations for the 128 cow experiment, while Figure 17 shows the same comparison but for the 245 cow experiment.

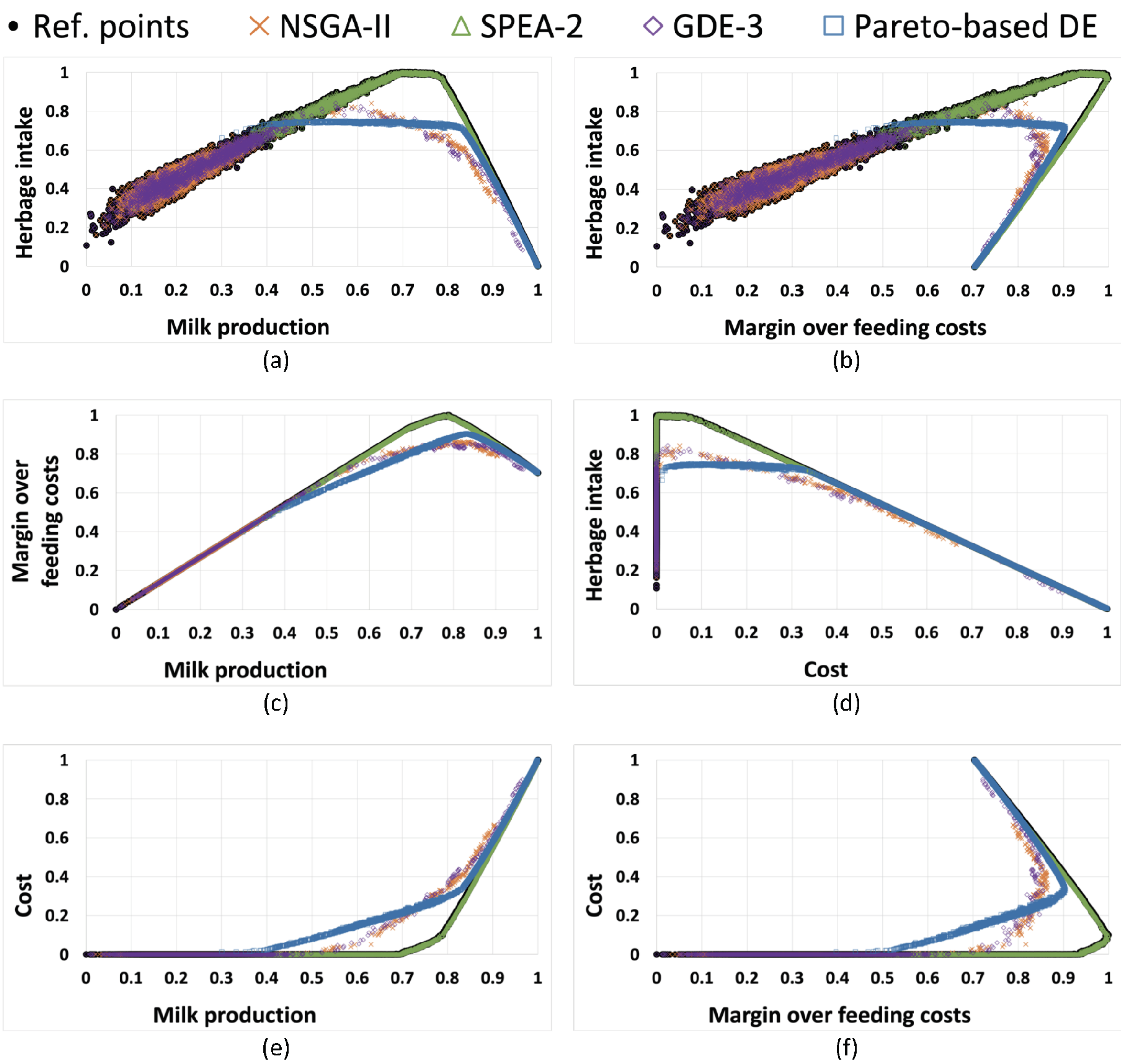

Figure 16: Comparison of the six most important objective combinations for the 128 cow experiment.

From Figure 16, and in general, we see that the SPEA-2 solutions are part of the Pareto front approximation (set of non-dominated solutions generated from the solutions obtained by the

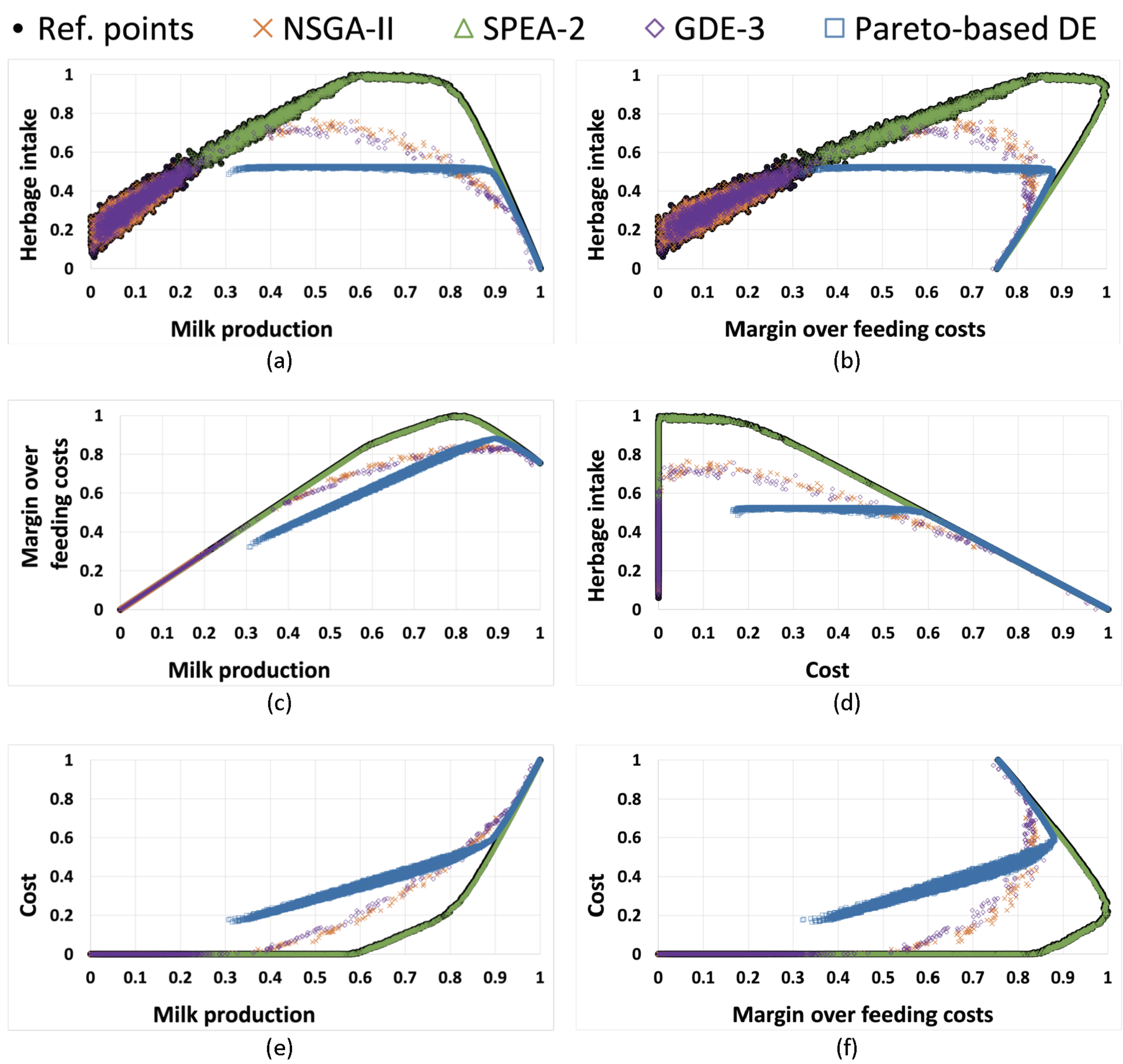

Figure 17: Comparison of the six most important objective combinations for the 245 cow experiment.

four algorithms), while a significant percentage of the solutions corresponding to the remaining algorithms are far from the Pareto front approximation. On the other hand, the NSGA-II and the GDE-3 obtained solutions belonging to the approximation that the SPEA-2 and the Pareto-based DE were not able to obtain. NSGA-II and GDE-3 showed a similar distribution, although at first sight we observe that the NSGA-II presented more solutions than the GDE-3. The Pareto-based DE algorithm presented many solutions, but we can clearly see that in some regions it got stuck in local optima, preventing it from reaching global optima. From Figure 16a, we appreciate that for the objective of maximizing milk production, the NSGA-II is the only algorithm that failed to generate solutions that exceed the 0.9 barrier. Also, we can see that the NSGA-II, GDE-3 and Pareto DE did not generate solutions that consume as much pasture as in SPEA-2, which directly impact on the margin over the feeding cost. This can be seen in Figure 16b, where they did not exceed the value of 0.9 for the margin objective. From Figure 16a we also observe that SPEA-2 and Pareto-based DE did not generate solutions with lower values than 0.6 for herbage intake. From Figure 16c we can clearly see that the solutions that maximize the margin are not the ones that maximize milk production. The margin is maximized for a value of 0.77 in milk production, a value where there are significant differences in the margin obtained by the SPEA-2 compared to the rest of the algorithms. In Figure 16d we see that the solutions that minimize the cost are those that maximize the herbage intake. With the exception of the Pareto-based DE, the rest of the algorithms provide solutions that minimize the cost of the diet, although the NSGA-II and the GDE-3 solutions presented sub-optimal values of pasture consumption. This behavior was maintained for solutions in which the cost of the diet has values lower than 0.3, since from that value the relationship between the cost of the diet and the herbage intake was similar for the four algorithms. From Figure 16e we can see that the cost of the diet increased significantly when milk production exceeded the 0.8 value. The cost of the diet is directly proportional to the supplement consumption, so the increase in cost is due to the fact that it is not possible to define a pasture-based diet only and it is necessary to supplement it with concentrates. From Figure 16f we see that the best margin was obtained when the cost of the diet was approximately 0.1. As in the previous figures, the difference between the solutions obtained by the SPEA-2 and the other algorithms is reflected. One particularity is that for diet costs above 0.20 values, the Pareto-based DE provides solutions with a higher margin than the NSGA-II and the GDE-3, but when the cost of the diet is lower than that value, its solutions provide a lower margin.

Figure 17 presents the results obtained for the experiment with 245 cows. By increasing the number of cows (more cows per hectare) and maintaining the same feed supply, it is necessary to change the resource allocation strategy because the availability of pastures is not enough to feed all the cows. Although Figure 17 shows a similar behavior to the one observed in Figure 16, the limitations of the NSGA-II, GDE-3 and Pareto-based DE stand out even more. The limitations of the NSGA-II, GDE-3 and Pareto-based DE to maximize herbage intake are further highlighted. Particularly for the Pareto-based DE, a more pronounced plateau is observed than the one seen in Figure 16. In any case, and despite this, the Pareto-based DE achieved better margin yields when milk production exceeded 0.8 (Figure 17c).

7.5.3 Performance metrics values

The metrics analyzed were: Hypervolume, Generational Distance, Inverted Generational Distance Plus and Spread. The values of those metrics (attained by each algorithm using its best parametric combination) for the experiments of 128 and 245 cows are presented in Table 19. In particular, we present the average value and the standard deviation in percentage, for each algorithm and each metric.

To compute the Hypervolume it was necessary to determine the reference point, which was determined by the best value achieved for each objective. The best values obtained for each objective in each experiment are presented in Table 20.

From Table 19, in the experiment of 128 cows, regarding the HV metric, we can see that the volume covered by SPEA (0.80) almost doubles the one covered by the rest of the algorithms. There is also a small difference in favor of the Pareto-based DE compared to NSGA-II and GDE-3. We also see a significant difference between SPEA-2 and the rest of the algorithms considering the GD metric, although in this case the difference with NSGA-II and GDE-3 is greater than with the Pareto-based DE. SPEA-2 also obtained the best value for the IGD^+ metric, while NSGA-II the worst one. The values obtained by the DE algorithms are similar, although there is a small difference in favor of the Pareto-based DE algorithm. For the Spread metric, the values obtained by the four algorithms were similar, being the Pareto-based DE value the best, followed by the SPEA-2 value. Finally, and regarding the standard deviation, the values obtained for the GD and IGD^+ metrics present a large variability, particularly for the

Table 19: Average values (AVG) and standard deviation in percentage (SD%) obtained for each metric for the best parametric combination of the algorithms in both experiments.

Experiment	Algorithm	Metric							
		HV		GD		IGD$^+$		Spread	
		AVG	SD%	AVG	SD%	AVG	SD%	AVG	SD%
128 cows	NSGA-II	0.42	4.1	0.02108	21.2	0.1017	23.0	0.706	8.6
	SPEA-2	0.80	3.0	0.00016	92.7	0.0006	100.0	0.617	4.5
	GDE-3	0.41	4.7	0.02165	16.1	0.0697	21.0	0.664	6.5
	Pareto DE	0.43	0.7	0.00413	12.0	0.0396	5.9	0.574	3.0
245 cows	NSGA-II	0.29	6.3	0.02974	25.8	0.1210	27.9	0.629	6.7
	SPEA-2	0.75	2.6	0.00036	72.5	0.0008	100	0.546	8.9
	GDE-3	0.28	5.6	0.02709	15.5	0.0687	18.1	0.550	7.5
	Pareto DE	0.17	0.4	0.01142	1.0	0.1711	1.5	0.546	2.8

Table 20: Best value achieved for each objective in both experiments.

Exp.	Milk Production	Margin	Herbage Intake	Costs	Supplement Intake
128 cows	33.349	7.439	18.345	0	0
245 cows	33.349	6.958	16.157	0	0

SPEA-2 algorithm, indicating that the data is spread over a wide range of values. For SPEA-2, the mean values are very small, so small changes (in absolute terms) cause a large variation. For the other algorithms the values are slightly larger and therefore the standard deviation is not as affected. It is also important to note that the lowest standard deviations occurred for the Pareto-based DE algorithm.

In the experiment of 245 cows, the volume covered by SPEA (0.75) for the HV metric almost triples those covered by NSGA-II and GDE-3, and quadruple the volume covered by the Pareto-based DE. In the case of the GD metric, a similar behavior to the one seen for the 128 cow experiment was obtained, i.e., a significant difference between SPEA-2 and the rest of the algorithms was obtained, although the difference with NSGA-II and GDE-3 was greater than with the Pareto-based DE. Regarding the IGD$^+$ metric a similar behavior to the one seen for the experiment of 128 cows in all the algorithms was obtained, except for the Pareto-based DE, which presented a worse performance (0.17). For the Spread metric, SPEA-2, GDE-3 and the Pareto-based DE obtained almost the same value (0.55), but the NSGA-II value is slightly worse. Finally, and regarding the standard deviation, a similar behavior to the one seen for the 128 cow experiment was obtained.

7.5.4 Best parametric combinations according to quality metrics

To analyze the best parametric combinations obtained by the different algorithms according to the quality metrics, we performed a ranking for each of them. For each metric we present a table with the best three parametric combinations for each algorithm and each experiment. From Table 21, according to the HV metric, the most influential parameters for the NSGA-II are M (0.01) and N (150), while it is possible to vary CR and still maintain good results. For the SPEA-2, the most influential parameter is M (0.01), while it is possible to use different values for CR and N and still maintain good results. For this algorithm, in the experiment of 128 cows, we noticed that the combination located in third place considered 8000 generations, which shows that it is not necessary to reach 16000 generations to obtain good results. On the other hand, the GDE-3 and the Pareto-based DE presented differences in this sense. For the Pareto-based DE it is important to maintain high values of CR and N, while for the GDE-3 (in most cases) it is convenient to use low values. In particular, in the case of the Pareto-based DE, and in the 245 cow experiment, using high values for CR and N, of 0.9 and 750, respectively, is essential, since better results were found with this combination even for low G values.

From Table 22, according to the GD metric, the most influential parameter for the NSGA-II and the SPEA-2 is M (0.01), although for the SPEA-2 a high value for N is also recommended. As for the GDE-3 and the Pareto DE, the main difference is that low N values are suggested for the first one, while high values are suggested for the second one. As in the previous metric, for the Pareto-based DE, and in the experiment of 245 cows, using high values for CR and N, of 0.9 and 750, respectively, is essential.

From Table 23, according to the IGD$^+$ metric, we can see different values for all parameters, but low N values are suggested. The most influential parameter for the SPEA-2 is M, and high N values are suggested. For the DE algorithms, the most influential parameter is the CR (0.9 in all combinations), and a similar behavior to that seen in the previous metrics is observed for the N value. In this sense, it was more important to maintain low N values for the GDE-3, even with 8000 generations, than a higher value for N and 16000 generations. The same behavior is appreciated for the Pareto-based DE, but in this case it was more important to maintain a high value of N.

From Table 24, according to the Spread metric, the most influential parameter for the NSGA-II is M, but in this case with a higher value (0.1). On the other hand, the most influential parameters for SPEA-2 are M and N, but unlike in the previous metrics, in this case a high value of M and a low value of N stood out. In the DE algorithms, a radical difference between the CR values is appreciated, since the GDE-3 presented better results for a high CR value, while the Pareto-based DE presented better results for a low CR value. Something peculiar for this metric and the Pareto-based DE is that the best results are obtained at low values of G. One possible explanation for this behavior is related to how the solutions of this algorithm evolve, since the metric considers the distance to the extremes of the Pareto front to achieve a more precise measure of the solutions distribution.

Table 21: The best three parametric combinations (in order) according to the Hypervolume metric for each algorithm.

Exp.	NSGA-II				SPEA-2				GDE-3				Pareto-based DE			
	M	CR	N	G	M	CR	N	G	F	CR	N	G	F	CR	N	G
128	0.01	0.7	150	16000	0.01	0.8	750	16000	0.9	0.5	150	16000	0.9	0.9	750	16000
	0.01	0.9	150	16000	0.01	0.8	450	16000	0.9	0.9	150	16000	0.9	0.9	450	16000
	0.01	0.8	150	16000	0.01	0.7	150	8000	0.9	0.5	450	16000	0.9	0.9	750	8000
245	0.01	0.8	150	16000	0.01	0.7	150	16000	0.9	0.5	150	16000	0.9	0.9	750	16000
	0.01	0.7	150	16000	0.01	0.9	150	16000	0.9	0.5	450	16000	0.9	0.9	750	8000
	0.01	0.9	150	16000	0.01	0.7	450	16000	0.9	0.5	150	8000	0.9	0.9	750	4000

Table 22: The best three parametric combinations (in order) according to the Generational Distance metric for each algorithm.

Exp.	NSGA-II				SPEA-2				GDE-3				Pareto-based DE			
	M	CR	N	G	M	CR	N	G	F	CR	N	G	F	CR	N	G
128	0.01	0.7	150	16000	0.01	0.8	750	16000	0.9	0.9	150	16000	0.9	0.5	750	16000
	0.01	0.9	150	16000	0.01	0.9	750	16000	0.9	0.9	450	16000	0.9	0.9	750	16000
	0.01	0.9	750	16000	0.01	0.8	450	16000	0.9	0.9	150	8000	0.9	0.9	750	8000
245	0.01	0.9	750	16000	0.01	0.8	750	16000	0.9	0.9	150	16000	0.9	0.9	750	16000
	0.01	0.7	150	16000	0.01	0.7	750	16000	0.9	0.9	450	16000	0.9	0.9	750	8000
	0.01	0.8	150	16000	0.01	0.7	450	16000	0.9	0.5	150	16000	0.9	0.9	750	4000

Table 23: The best three parametric combinations (in order) according to the Inverted Generational Distance Plus metric for each algorithm.

Exp.	NSGA-II				SPEA-2				GDE-3				Pareto-based DE			
	M	CR	N	G	M	CR	N	G	F	CR	N	G	F	CR	N	G
128	0.01	0.7	150	16000	0.01	0.8	750	16000	0.9	0.9	450	16000	0.9	0.9	750	16000
	0.1	0.7	150	16000	0.01	0.9	750	16000	0.9	0.9	150	16000	0.9	0.9	450	16000
	0.01	0.9	150	16000	0.01	0.8	450	16000	0.9	0.9	150	8000	0.9	0.9	750	8000
245	0.01	0.7	150	16000	0.01	0.7	750	16000	0.9	0.9	150	16000	0.9	0.9	750	16000
	0.01	0.8	150	16000	0.01	0.8	750	16000	0.9	0.9	450	16000	0.9	0.9	450	16000
	0.01	0.7	450	16000	0.01	0.7	450	16000	0.9	0.9	150	8000	0.9	0.9	750	8000

7.5.5 Execution times

The Pareto-based DE was implemented using the Landscape IMAGES framework [76, 78, 80], and the remaining three algorithms were implemented using JMetal [51], a Java-based framework for multi-objective optimization. The execution platform was a PC with Intel Core i5-2400 (3.1 GHz. CPU with 6 MB of cache) processor and 8 GB of DDR3 RAM.

Executions times showed significant differences between the algorithms. A summary with approximate values (in seconds) for representative combinations of parameter values can be seen in Tables 25 and 26 for the genetic algorithms and the DE algorithms respectively.

Table 25 shows a large difference between the execution times of the NSGA-II and SPEA-2 algorithms. The NSGA-II presented execution times that ranges between 7 to 23 seconds. Naturally, for the same algorithm, as the population size and the number of generations increase, the execution times also increases. On the other hand, the SPEA-2 presented much higher execution times values, that ranges between 105 and 28400 seconds. Similar behavior is seen for DE algorithms. Table 26 shows a large difference between the execution times of the GDE-3 and the Pareto-based DE algorithms. The GDE-3 presented execution times that ranges between 5 to 22 seconds, while the Pareto-based DE presented execution times values that ranges between 300 and 28000 seconds. It is important to clarify that the original Landscape IMAGES framework includes a component that shows the evolution of the solutions through a dynamic graph as the number of generations increases (until the maximum number of generations is reached). The original framework was modified by removing this component so that the comparison of the execution times between the different algorithms makes sense, since the JMetal framework does not include a component with these characteristics. Taking into account that we do not know the programming of the Landscape IMAGES framework in detail, we are aware that there may be some other factor that influences the solutions, particularly in the high execution times obtained.

Table 24: The best three parametric combinations (in order) according to the Spread metric for each algorithm.

Exp.	NSGA-II				SPEA-2				GDE-3				Pareto-based DE			
	M	CR	N	G	M	CR	N	G	F	CR	N	G	F	CR	N	G
128	0.1	0.7	150	16000	0.1	0.9	150	16000	0.9	0.9	750	16000	0.9	0.5	750	1000
	0.1	0.9	150	16000	0.1	0.8	150	16000	0.9	0.9	450	16000	0.9	0.5	750	2000
	0.1	0.9	750	16000	0.1	0.8	150	8000	0.9	0.9	150	8000	0.9	0.5	450	1000
245	0.1	0.7	150	16000	0.1	0.7	150	8000	0.9	0.9	750	16000	0.9	0.5	750	1000
	0.1	0.7	450	16000	0.1	0.8	150	4000	0.9	0.9	450	16000	0.9	0.5	450	1000
	0.1	0.9	150	16000	0.1	0.9	150	16000	0.9	0.9	150	8000	0.9	0.5	750	2000

Table 25: Average execution times (in seconds) for different combinations of parameter values in the genetic algorithms.

			NSGA-II		SPEA-2	
M	CR	N	G		G	
			1000	16000	1000	16000
0.01	0.7	150	8	16	105	1680
0.01	0.9	750	19	23	1775	28400
0.1	0.7	150	7	15	105	1620
0.1	0.9	750	18	21	1500	24000

7.6 Conclusions

In this chapter, the performance of different evolutionary algorithms was evaluated when using an optimization model with five objectives. We considered two genetic algorithms (NSGA-II and SPEA-2) and two differential evolution algorithms (GDE-3 and a variant of a Pareto-based). As a main conclusion, we can highlight that the algorithms were well adapted to the feed resource allocation optimization model for dairy systems presented in previous chapters.

To evaluate the performance of the algorithms, were considered the quality and the execution time in two different scenarios (contemplating 128 and 245 cows, respectively). We also normalized the values obtained on a scale [0,1], where the optimal value was represented by 1 in the maximization objectives and by 0 in the minimization objectives. The evaluation carried out allowed us to identify the advantages and disadvantages of each one of them, showing interesting properties that can be taken into account in different real-world problems.

To measure quality, one of the criteria was the ability of the algorithms to reach optimal values. When maximizing milk production, all algorithms had very good results for both experiments (between 0.902 and 1), although only SPEA-2 and Pareto-based DE reached the optimal value. By maximizing the gross margin, very good results were also obtained for both experiments (between 0.844 and 1), being SPEA-2 the only one that reached the optimal value. When maximizing herbage intake, all algorithms presented good results, except Pareto-based DE in the experiment with 245 cows (0.526). Again, the only one that reached the optimal value was SPEA-2. For the objectives corresponding to the minimization of cost and supplement intake, all the algorithms reached the optimal value, except Pareto-based DE, but which in any case reached very good values (between 0.013 and 0.015 for the experiment with 128 cows, and between 0.165 and 0.203 for the experiment with 245 cows).

Table 26: Average execution times (in seconds) for different combinations of parameter values in the DE algorithms.

			GDE-3		Pareto-based DE	
F	CR	N	G		G	
			1000	16000	1000	16000
0.9	0.5	150	5	10	300	4800
0.9	0.5	750	9	17	1650	26400
0.9	0.9	150	6	14	330	5280
0.9	0.9	750	16	22	1750	28000

By comparing the non-dominated solutions of each algorithm against the Pareto front approximation, we identified that the SPEA-2 solutions represented a great part of it, while an important portion of the solutions generated by the remaining algorithms deviate from the approximation. On the other hand, NSGA-II and GDE-3 have successfully produced solutions within the approximation, a feat that SPEA-2 and Pareto-based DE failed to achieve. Also, the Pareto-based DE algorithm obtained many solutions, but it got stuck in local optima, preventing it from reaching global ones. In general, the cost of the diet, which is directly proportional to the supplement intake, increased significantly when milk production was higher than 0.8. Also, the best gross margin was obtained when the cost of the diet was higher than 0.1. In particular, for the experiment of 128 cows, when maximizing milk production, all the algorithms obtained values higher than 0.9, except the NSGA-II. When maximizing herbage intake, the only algorithm which obtained values higher than 0.9 was SPEA-2. In line with conclusions already presented previously, solutions which maximize the gross margin are not the ones that maximize milk production. In this experiment, the margin is maximized for values of 0.77 in milk production. In the second experiment, by maintaining the same feed supply and increasing the stocking rate (more cows per hectare), the algorithms changed the resource allocation strategy because the availability of pastures was not enough to feed all the cows. In any case, we observed a similar behavior to the one seen in the previous experiment.

Regarding the four quality metrics, in general, SPEA-2 obtained the best results, being inferior to the Pareto-based DE algorithm for the Spread indicator in the experiment of 128 cows (SPEA-2 obtained 0.617, while Pareto-based obtained 0.574). For the remaining quality metrics, significant differences were observed compared to the rest of the algorithms in both experiments. Among the other algorithms, Pareto-based DE is the one that obtained the best results in the 128 experiment, but its performance decreased in the 245 cow experiment, being exceeded in the HV and IGD^+ metrics. On the other hand, the NSGA-II and GDE-3 algorithms obtained similar values, and they were close to those obtained by the Pareto DE. Regarding the parametric combinations, significant differences were observed according to the algorithm and the metric to be used. For both algorithms, the NSGA-II and the SPEA-2, the importance of using a low value in M is highlighted, except for the Spread metric. For the GDE-3 algorithm, a low value in N stands out for the HV and GD metrics, while for the IGD^+ and Spread metrics a high value in CR is recommended.

Regarding the parametric combination, in general, no clear pattern is observed, except for the Pareto-based DE algorithm, where the importance of using a high value in N and CR is highlighted (750 and 0.9 respectively) for most metrics. The exception occurred for the Spread metric, where a low value in CR is recommended (0.5).

When comparing execution times, the algorithms showed very large differences. Low values for the NSGA-II and the GDE-3 (between 5 and 23 seconds), and very high values for the SPEA-2 and the Pareto-based DE (between 105 and 28400 seconds) were obtained. The SPEA-2 is the algorithm that presented the highest values. Despite this, in terms of convergence, diversity and cardinality, it is the one that presented the best results.

As future work, and considering that the SPEA-2 is the algorithm that presents the best results in terms of convergence, diversity and cardinality, it is interesting to evaluate alternatives that allow reducing its execution time. It would be very interesting to rate the algorithms in relation to the FLOPS they demand, thus presenting another point of view with greater power of analysis. Also, it is proposed to study other variants of the NSGA-II and GDE-3 algorithms. In particular, there is a variant of the NSGA-II where three variation operators (SBX crossover, polynomial mutation and differential evolution) are selected randomly to create new individuals, and another one that works like the previous one but the operators are selected adaptively. Regarding the GDE-3 algorithm, variations with different DE strategies can be easily created simply by using different equations for crossover and mutation. Finally, it would be interesting to study how different attributes of solutions combine to affect performance. In this line of work, a different type of algorithm, known as Multi-dimensional Archive of Phenotypic Elites, was proposed in the literature to provide a holistic view of how high-performing solutions are distributed throughout a search space [116].

Chapter 8

Conclusions and Future Work

This chapter summarizes the conclusions of this thesis work presented in the previous chapters. Together, the main lines of research that deserve to be addressed in the future are presented.

8.1 Conclusions

Determining the correct allocation of food resources in dairy systems is a complex problem, as there are various factors that need to be considered, such as the spatial heterogeneity and nutritive value of available foods (nutrient density and dry matter intake), herd characteristics, different cow management practices, the cost and availability of food throughout the year, milk production goals, among others. Throughout this study, in its different stages, we set multiple objectives with the aim of collaborating with producers in addressing problems that can be solved through the analysis and discussion of different scenarios that may arise. To accomplish this, we developed models that allow us to address the identified issues, including the constraints and limitations of the systems. These models were applied and evaluated, taking into account both a computational and agronomic approach. A general evaluation allows us to conclude that the main goals set out in this work were successfully achieved, and we can help producers by providing assistance in decision making.

The feed resource allocation problem in dairy systems was studied and modeled as a combinatorial optimization problem. Different solution techniques were used and evaluated through several experiments, which were based on specially designed scenarios contemplating real data from Uruguayan dairy systems. From the experiments we can conclude that the quality of the results obtained was satisfactory, both from the agronomic and computational point of view. Throughout the work, we proposed combinatorial optimization models for both single and multiple objectives, which solve specific scenarios of a system as well as the planning of medium and long-term strategies.

From the model of a single objective (milk production or economic benefit), we determined the best alternative to group the cows and distribute them among the different available feeding options. The time was represented by considering several milkings, so the solution to the problem presents an allocation for each of the milkings considered. To obtain the optimal solution for different scenarios, a mathematical formulation was presented and initially solved using an Exact Method (EM). The solutions obtained by the EM were built distributing as many cows as possible to the feeding options with higher energy density. From the distribution of the cows, it was evidenced that the real consumption of the cows must at least reach their potential consumption, otherwise there will be serious losses in economic gains or efficiency in milk production.

Since EM are not appropriate for large scale problems, we implemented a Genetic Algorithm (GA) as an alternative technique. Suitable representations were proposed. They are simple and facilitate the use of operators, however corrections must be done to maintain the feasibility of the solutions. The structure of the GA solutions was interpreted, and their quality were analyzed. The experiments confirmed that the GA solutions were very close to those obtained by exact method. For various dairy herd management reasons, it is possible that the solutions provided by the EM may not be applicable in real systems, so we also analyzed the diversity of the GA solutions. To evaluate the diversity we used the Euclidean distance between the solutions obtained by the EM and the GA. From the obtained results the gap value and distance to the optimal solution was calculated. The GA reached a good diversity of solutions, which provides producers various good quality alternatives of feeding strategies.

Since the previous model does not contemplate the feed resources allocation considering various relevant objectives in dairy systems, and was not framed in a context of annual strategic planning for feed allocation, we then successfully extended the model to cover multiple objectives and multiple time periods.

This model was first implemented using a differential evolutionary (DE) algorithm, which proved to be adequate to addressed the problem. Experiments confirmed that the algorithm reached high quality solutions, which where validated and considered representative of the system by experts in the dairy sector. The solutions were also validated using an agronomic linear programming (LP) model, which confirmed the proximity to the Pareto front. The DE algorithm also generated a great range of solutions that showed the trade-offs among the objectives, reaching extreme values close to the values obtained by the LP model. The trade-offs presented by the different objectives were analyzed, which present interesting answers that can be decisive and influential in decision making at farm level. Among other things, and based on the Pareto front comparison, we can highlight that increasing the stocking rate caused big differences in the results. The multi-objective model scenarios demonstrated that increasing the stocking rate would enhance milk production and gross margin per unit of area. It is worth noting that maximizing profit does not necessarily requires achieving the highest levels of milk production or forage intake. Instead, solutions that maximize the margin were attained at sub-optimal levels of milk production and forage consumption. The productive and economic results from this increase are aligned with national and international research work.

Finally, taking into account the importance of selecting appropriate algorithms and setting their parameters to improve the performance of the multi-objective model, we conducted a performance evaluation of four different evolutionary algorithms (NSGA-II, SPEA-2, GDE-3 and a Pareto-based DE), carrying out two computational experiments based on different sets of evolutionary parameters that allowed us to analyze the performance of the solutions reached by each algorithm. To evaluate the performance of the algorithms, we took into account the execution times and the quality. The algorithms showed significant variations in execution times, with NSGA-II and GDE-3 attaining low values, while SPEA-2 and the Pareto-based DE presented high values. Despite the higher execution times, SPEA-2 yielded the best results in terms of convergence, diversity, and cardinality. In both experiments, significant differences were observed between SPEA-2 and the other algorithms, outperforming them across all four performance indicators, except for the Spread indicator in one experiment where the Pareto-based DE surpassed it. The Pareto-based DE performed well in one experiment but showed decreased performance in the second experiment, particularly when using the HV and IGD+ indicators. NSGA-II and GDE-3 produced similar results, which were comparable to those of the Pareto-based DE. The choice of parameter combinations showed significant differences depending on the algorithm and performance indicator. For both NSGA-II and SPEA-2, using a low value for M was crucial, except for the Spread indicator. In the case of GDE-3, a low value for N was important for the HV and GD indicators, while a high value for CR was recommended for the IGD+ and Spread indicators. Lastly, for the Pareto-based DE algorithm, employing a high value for N and CR was essential, except for the Spread indicator where a low value for CR was recommended.

8.2 Future Work

In the first place, and thinking on short-term planning, it is proposed to carry out experiments (also based on real data) that contemplate more complex scenarios. Among them, different types of cows, food availability or cost differences that represent other realities of the dairy systems can be contemplated. In this sense, and considering that SPEA-2 is the algorithm that presents the best results in terms of diversity, cardinality and convergence, we find it interesting to evaluate alternatives for reducing its running times, like parallel evaluation of solutions, among others. Thinking about medium-term planning, we propose to continue expanding the dairy model presented, incorporating other factors that might influence the decision making process and the results in it´s different dimensions (production, economics, environment, etc). Among these factors we can highlight the incidence of climatic factors in the available resources, the influence of real consumption in the energy balance of the animal, the incidence of different types of paths to reach the pastures, among others.

In this line of research, a recent academic study introduced a novel approach utilizing machine learning methods to diminish the number of objectives in a given problem. It proposes an objective reduction method based on data mining that can be applied regardless of the type and size of the data and the shape of the Pareto optimal front. This method is independent of the choice/definition of the algorithm parameters, it considers relationships between different decision variables and between different objectives, and can provide results that are easy to understand for a person who needs to participate in decision making and is not an expert in optimization or machine learning [68]. Also, a recent study [94] has conducted a comprehensive and contemporary literature review on multi and many-objective problems, examining and analyzing 32 of the most significant algorithms in detail. All of these methods were evaluated using state-of-the-art quality measures. Furthermore, the study discussed various aspects, including the historical motivations behind the use of evolutionary algorithms and the currently prevalent techniques in the field.

Finally, and considering that in this work the availability of feed in each zone is assumed to be known, another line of future work consists of studying alternatives that automatically estimate the amount of available feed to automate the entire decision-making process. In Uruguay, there are ongoing studies exploring alternatives to estimate pasture availability through image processing techniques. The hypothesis of these studies establishes that it is possible to obtain the height of the pasture through image processing, and from the height, calculate the availability of dry matter.

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