A Stochastic Geometry Analysis of Multichannel Cognitive Radio Networks

Claudina Rattaro, Paola Bermolen, Federico Larroca, Pablo Belzarena
Facultad de Ingeniería
Universidad de la República
Montevideo, Uruguay
{crattaro,paola,flarroca,belza}@fing.edu.uy

ABSTRACT

With the explosive development of wireless technologies, the demand of electromagnetic spectrum has been growing dramatically. Therefore, looking for more available spectrum, regulators have already begun to study secondary assignments in licensed bands. In this paper we present a probabilistic model based on a stochastic geometry approach to analyze cognitive radio networks. We focus on those scenarios where more than one band is available, a natural situation in this kind of networks. Quiet surprisingly, and to the best of our knowledge, such scenario has not been deeply explored yet in the literature. In particular we focus our study in the two main performance metrics: medium access probability and coverage probability. We evaluate our proposal through simulations and we present the analytical results of a particular case.

Keywords

cognitive radio networks, stochastic geometry, radio interference, dynamic spectrum allocation, multichannel

1. INTRODUCTION

Nowadays, with the exponential growth in the number of wireless devices in our everyday lives, unlicensed radio spectrum bands are heavily loaded generating a severe interference problem. On the other hand, spectrum utilization measurements have shown that many of the licensed bands are vastly underutilized [1][9][4]. In this context, Cognitive Radio (CR) is considered an attractive technology to deal with that problem by an intelligent and efficient dynamic spectrum access [10]. The idea behind CR is to allow secondary unlicensed users (SUs) to opportunistically access the underutilized spectrum that is licensed to the primary users (PUs). The key requirement is that the PUs must be as little affected as possible by the presence of SUs. It has been proved that CR can reduce the interference in unlicensed bands improving spectrum utilization.

As in [11, 12, 8] we consider the most important performance metrics in these kind of networks: the Medium Access Probability (MAP) and the Coverage Probability (COP). The former is the probability that a user gets access to a channel band. Due to the interference, not every transmission attempt is successful. In this sense, COP measures this probability of success. For instance, given the network and the PUs utilization, a performance metrics of interest here is naturally the MAP of SUs. This value measures the portion of spectrum "wasted" by PUs and which may be leveraged by SUs. In this context, primary COP value gives an idea of the degradation of PU's communications caused by the presence of SUs. COP is also a key value to estimate the throughput obtained by primary and secondary users.

It is demonstrated that stochastic geometry is a powerful tool that allows to define and compute macroscopic properties of a wireless network by averaging over all potential geometrical patterns of the nodes [2, 3]. It is specially useful to model interactions between nodes in large random networks. This randomness may include node positions, node mobility, fading, or traffic (stochastic arrivals and departure). The articles [11, 12] are the most representative examples of the use of this technique in CR networks. In those works, the authors developed a probabilistic model to analyze the performance of different MAC protocols within this context. They concentrated their analysis in a unique cell; that is to say, all users can transmit in the same channel band. We aim at generalize these results in the multichannel case. In particular, we are interested in estimating MAP and COP in scenarios where more than one band is available, which is a natural situation in cognitive radio networks.

A large volume of research has been conducted in the cognitive radio area over the last decade (some of the last examples are [18, 17]). However, it is important to highlight that to our knowledge, a multichannel scenario in CR, which considers geometric characteristics such as nodes positions, has not been deeply explored yet. Motivated by this fact and to help in filling this gap, we choose an approach based on stochastic geometry and we extend the probabilistic framework developed in [11] to that context in CR. In particular, in this paper we consider ALOHA to schedule primary transmissions in each frequency band and we also consider that all SU transmitters are saturated, i.e. have a packet ready to be sent in every time slot. In this situation, MAP and COP give an idea of the possibilities offered by cognitive radio to improve the spectrum utilization. Those metrics are also useful to explore how PUs are affected under the spectrum sharing context.

The rest of the paper is structured as follows. In the next section we present the previous related work and highlight some recent papers. In section 3 we introduce our hypotheses and the notation. In section 4 we present our main results, in particular we show the MAP and COP estimation using an stochastic geometry approach. We present analytical expressions for those metrics in a specific scenario that consists in two channel bands. In section 5 we validate our results presenting numerical examples based on simulations. Finally, we conclude and discuss future work in section 6.

2. RELATED WORK

In the context of multichannel CR Networks the most representative previous examples are based on the theory of queues and priority queues (see for example [15, 16] and the references therein). In those articles, authors use queuing results to find several network statistics. Generally, queuing models do not consider physical user interactions such as: interference, propagation models, random locations and/or mobility.

Much research has been recently dedicated to CR networks. However, there are few works that capture random features such as user locations, fading/shadowing and path loss; which play a crucial role in network performance. According to that, Stochastic Geometry is the natural tool to be used in these scenarios; even more, when the interest is to analyze the impact of those features.

Some of the most representative previous works in CR area using this technique are [5, 11, 12, 8]. However, only in [8] the authors analyze a multichannel environment. In that paper, each SU previously select a channel and then, if that channel is primary-free it will transmit; therefore the MAP is strictly dependent of the selected channel band. In our case, each SU determines which channels are primary-free and then it selects one of them to transmit. When the interest is to know the opportunities in CR networks, our assumption is more reasonable. Also, in [8] the impact of the SU presence in PUs communications has not been studied.

3. NETWORK MODEL AND PROBLEM FORMULATION

Let us begin by describing our working scenario and introducing the notation, definitions and hypotheses. The location of the nodes of the network is seen as the realization of two point processes [6]. This means that the network can be considered as a snapshot of a stationary random model in the (Euclidean) space and that it is possible to analyze it in a probabilistic way. The time is divided into slots and one slot is needed to transmit a packet for all users. Then, one snapshot represents the nodes spatial distribution in one time slot.

In the particular case of this work, the users of the network are assumed to be a realization of two independent homogeneous Poisson point processes (PPP) $\Phi_p = \{X_i^p\}$ and $\Phi_s = \{X_i^s\}$ with intensities λ_p and λ_s on $\mathbb{R}^2$ respectively. Specifically, $\{X_i^p\}$ and $\{X_i^s\}$ denote the positions of the potential primary and secondary transmitters respectively. We assume that each transmitter has its intended receiver uniformly distributed in a circle of radius r centered in each transmitter location. We define $r(x)$ as the relative location of the receiver of a transmitter located in x.

In order to introduce the multichannel aspect, we define $f = \{f_1, f_2, \ldots, f_n\}$ as the set of channel bands to be used by PUs or SUs. We consider ALOHA in the primary transmissions. We can define a new PPP $\Phi_p^* = \{X_i^p : e(X_i^p) = 1\}$ where $e(X_i^p)$ is a $\{0,1\}$-value r.v. indicating whether X_i^p chooses to transmit in the current time slot or not ($P(e(X_i^p) = 1) = p_e$). Φ_p^* is a PPP with intensity $\lambda_p p_e$, represents the process of the active primary transmitters and it is an independent thinning of the original Poisson. Each PU also has to choose its frequency band for its transmission. We assume that each band f_k has probability p_{f_k} to be selected by any active PU. Therefore we can define Φ_{p,f_k}^*, a PPP with intensity $\lambda_p p_e p_{f_k}$, as the process of the active primary transmitters using band f_k.

A deterministic attenuation $\alpha > 2$ is also assumed; that is, the signal power decays with the distance between two nodes. Given two nodes x and y, the power received from x by y is $P(x, y) = P(x)l(||y - x||)$ where $P(x)$ is the transmission power of node x and $l(||y - x||)$ is the path loss function from x to y which depends on the distance between nodes. Different path loss functions can be considered. For our specific calculus we have considered $l(||y - x||) = ||x - y||^{-\alpha}$ but they are totally adaptable to other models.

In the literature, there are many proposals of spectrum sensing methodologies for cognitive radio networks (for instance, energy detection or/and cyclostationary sensing techniques [7]). We assume that SUs implement a sensing method based on energy detection to detect primary activity. More specifically, a SU y detects the presence of a PU x if $P(x,y) > \rho$ where ρ is a pre-set constant. Each SU has to sense the bands to know the set of primary-free channels. If there is at least one free band of primary users, it can transmit. Analogously to the PUs case, we define Φ_s^* as the process of active SU and Φ_{s,f_k}^* the one of active SUs using band f_k. Note that these processes are dependent thinning of the original Poisson process Φ_s.

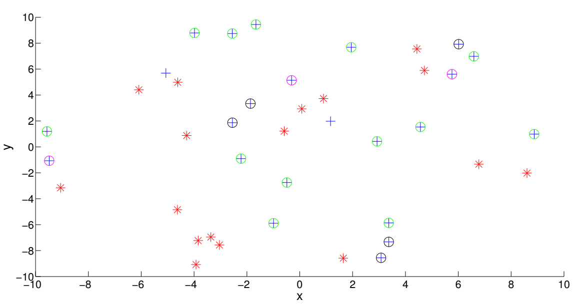

Figure 1: A snapshot of the nodes of the network defined by Φ_p (stars) and Φ_s (crosses) processes using the parameters $\lambda_p = 0.05$, $\lambda_s = 0.1$, two frequency bands $\{f_1, f_2\}$, $p_e = 1$, $p_f = 0.5$, $\alpha = 3$ and $\rho = 0.1$. SUs that are surrounded by a circle are the active SUs. In particular, the ones that have a green circle have detected both primary-free channels. On the other hand, the ones that are surrounded by black and magenta circles have detected only one primary-free band, f_1 or f_2 respectively.

In order to model the random variations of the channel conditions, we consider two infinite symmetric matrices $F = \{F(i,j)_{i,j}\}$ and $F^r = \{F^r(i,j)_{i,j}\}$. The first one represents the fading of the channel from primary transmitter i to secondary transmitter j. It will be used in the sensing mechanism implementation. The second one models the fading of the channel from transmitter i to receiver of transmitter j (being i and j two arbitrary transmitters, PU or SU) being essential for the COP calculus. In our particular case, we consider a Rayleigh fading (suitable when many obstacles are present and there is no line of sight between transmitter and receiver) therefore, the random variables $\{F(i,j)_{i,j}\}$ and also $\{F^r(i,j)_{i,j}\}$ are assumed to be independent and exponentially distributed with parameter μ. Including this new feature, we can say that the power received from y by x is:

$$P(x,y) = P(x)F(x,y)l(||y-x||); \tag{1}$$

analogously if $F^r(x,y)$ is considered.

As usual, we model interference as noise. In this work, we do not consider the co-channel interference. Hence, a transmission in frequency f_k will be successful if the $SINR$ is higher than a certain threshold considering only the signals operating in the same spectrum band. The interference is assumed to be the sum of signal strengths generated by all the other nodes transmitting in the same time slot. Let t_i and r_i be the locations of a pair of primary transmitter and its receiver using band f_k (please note that r_i is the relative location). We will assume that the communication between t_i and r_i will be successful if the following condition is verified:

$$SINR^p(t_i, r_i, f_k) = \frac{P(t_i, r_i)}{N + I_{pp}^{f_k}(r_i) + I_{sp}^{f_k}(r_i)} \geq \gamma \tag{2}$$

where $I_{pp}^{f_k}(.)$ and $I_{sp}^{f_k}(.)$ represents the interference associated to the active primaries and secondaries transmitters using band f_k respectively:

$$I_{pp}^{f_k}(r_i) = \sum_{y \in \Phi_{p,f_k}^* \setminus \{t_i\}} P(y)F^r(y,t_i)l(||y - t_i - r_i||) \tag{3}$$

$$I_{sp}^{f_k}(r_i) = \sum_{y \in \Phi_{s,f_k}^*} P(y)F^r(y,t_i)l(||y - t_i - r_i||) \tag{4}$$

I_{pp} denotes the interference from primary transmitters to a primary receiver. On the other hand I_{sp} denotes the influence of the secondary communications in a specific primary receiver.

Analogously, when t_i and r_i are a pair of secondary users, the transmission will be successful if

$$SINR^s(t_i, r_i, f_k) = \frac{P(t_i, r_i)}{N + I_{ss}^{f_k}(r_i) + I_{ps}^{f_k}(r_i)} \geq \gamma \tag{5}$$

$$I_{ps}^{f_k}(r_i) = \sum_{y \in \Phi_{p,f_k}^*} P(y)F^r(y,t_i)l(||y - t_i - r_i||) \tag{6}$$

$$I_{ss}^{f_k}(r_i) = \sum_{y \in \Phi_{s,f_k}^* \setminus \{t_i\}} P(y)F^r(y,t_i)l(||y - t_i - r_i||) \tag{7}$$

where γ is a selectable threshold and it is strongly related with the receiver sensitivity. Please note that this threshold can be different between classes of users.

4. PERFORMANCE ANALYSIS

The aim of this section is to investigate the MAP and COP calculus. For simplicity we do not include here the formal demonstrations of the analytic expressions for the COP in the general case. They can be found in our technical report [13]. Therefore, we will concentrate our efforts in explaining in details the calculus for a particular case (the one that we will test in the validation section, Sec. 5).

Let us first begin with the MAP. In our model, PUs whenever they want has access to the network (as we discussed in the previous section), thus we are only interested in the MAP of a SU.

4.1 MAP of secondary users

For a typical secondary potential transmitter located at 0, in a general way it is possible to define the set of primary contenders in the different frequency bands as:

$$N_0^{p,f_k} = \{y \in \Phi^*_{p,f_k} : F(y,0)l(|y-0|) > \rho\}. \quad (8)$$

That is to say, N_0^{p,f_k} is the set of active primary transmitters using f_k that are detected by the typical SU with the considered sensing mechanism. With this in mind, we can say that the typical SU is in the protection zone of the PUs defined by N_0^{p,f_k}. Without loss of generality we have considered $P(y) = 1, \forall y$ (or $P(y)$ might have been considered constant and included in $F(y,0)$).

Please note that Φ^*_{p,f_k} is a PPP, and a secondary user will transmit if and only if there is at least one primary-free channel; then, the medium access probability can be written as:

$$MAP_s = 1 - \prod_k (1 - e^{-\lambda_p p_e p_{f_k} \bar{N}_0}) \quad (9)$$

where $\bar{N}_0 = \int_{\mathbb{R}^2} e^{-\frac{\mu\rho}{l(y)}} dy$.

It should be noted that the process of active secondary users Φ^*_s is a dependent thinning of the original Poisson process Φ_s but it is not itself a PPP. Moreover, it is a Cox process [14], that is a way to model clustered point patterns (i.e. SUs which fall within a PU protection zone are automatically silenced in that specific primary frequency band). For that reason, the COP calculus for this general case is approximate; more precisely it is possible to obtain analytical expression of an upper bound of the COPs.

In what follows we will add some assumptions in order to obtain more tractable expressions in order to test them with simulations.

MAP_s *for a particular case*

In order to simplify the analysis, we consider only two possibles frequency bands $f = \{f_1, f_2\}$ with probabilities p_f and $1 - p_f$ to be chosen by PUs respectively. We also add a new hypothesis: a SU will transmit if and only if in a ball centered at it with random radius q there is at least one primary-free channel (radius q are considered as i.i.d random variables with $G(q)$ distribution). This is an alternative version of the sensing mechanism: $F(i,j)$, distance between transmitters and path loss function are abstracted into the radius q.

The major consequence of this assumption is that Φ^*_s is now an independent thinning of the original Poisson process. More formally, a typical SU potential transmitter ($0 \in \Phi_s$) will access the medium with probability:

$$\int (1 - P(\Phi^*_{p,f_1}(B(0,q)) > 0) P(\Phi^*_{p,f_2}(B(0,q)) > 0)) dG(q). \quad (10)$$

Φ^*_{p,f_1} and Φ^*_{p,f_2} are PPP in $\mathbb{R}^2$, then $P(\Phi^*_{p,f_1}(A) = 0) = e^{-\lambda_p p_e p_f |A|}$ and $P(\Phi^*_{p,f_2}(A) = 0) = e^{-\lambda_p p_e (1-p_f)|A|}$ where $|A|$ is the area of A. According to that,

$$MAP_s = \int (1 - (1 - e^{-\lambda_p p_e p_f \pi q^2})(1 - e^{-\lambda_p p_e (1-p_f)\pi q^2})) dG(q). \quad (11)$$

It is easy to note that the MAP_s calculus can be easily generalized to the case of n frequency bands ($n > 2$).

4.2 COP of primary users

Considering 0 as the typical primary transmitter (with its corresponding receiver in the relative position $r(0)$), the transmission will be successful with probability:

$$\begin{aligned} COP_p &= P^0_{\Phi^*_p}(SINR^p(0, r(0)) > \gamma) \quad (12) \\ &= \sum_k P^0_{\Phi_p}(SINR^p(0, r(0), f_k) > \gamma) P(f(0) = f_k) \end{aligned}$$

where $P(f(0) = f_k)$ represents the probability that the typical node uses f_k to transmit.

Using the same arguments of [11] we can obtain an analytical expression of an upper bound of the COP.

Next, we present the details of the calculation for the considered particular case. In this scenario we will obtain exact values of the COP_p and COP_s.

COP_p *for a particular case*

Under the same hypothesis that were explained before (i.e. $f = \{f_1, f_2\}$ with probabilities p_f and $1 - p_f$) and given the MAP_s (Eq. (11)), we will compute the COP of primary users. Now, the first question we have is: if a SU finds both bands available, which one is going to be selected? In this work, as a first step, we consider that each band has a fixed probability (p'_f and $1 - p'_f$) to be chosen. Therefore, we can express the probability that a SU transmits in f_1 as:

$$P(f_1) = \int e^{-\lambda_p p_e p_f \pi q^2}(1 - e^{-\lambda_p p_e (1-p_f)\pi q^2}) dG(q) + p'_f \int e^{-\lambda_p p_e p_f \pi q^2} e^{-\lambda_p p_e (1-p_f)\pi q^2} dG(q)$$

complementary, the probability that a SU transmits in f_2 is:

$$P(f_2) = \int (1 - e^{-\lambda_p p_e p_f \pi q^2}) e^{-\lambda_p p_e (1-p_f)\pi q^2} dG(q) + (1 - p'_f) \int e^{-\lambda_p p_e p_f \pi q^2} e^{-\lambda_p p_e (1-p_f)\pi q^2} dG(q).$$

In this context we have

$$P^0_{\Phi_p}(SINR^p(0,r(0),f_1)>\gamma) = P\left(\frac{F_r(0,0)l(|r(0)|)}{N+I^{f_1}_{pp}(0)+I^{f_1}_{sp}(0)}>\gamma\right)$$
$$= P\left(F_r(0,0)>\frac{\gamma(N+I^{f_1}_{pp}(0)+I^{f_1}_{sp}(0))}{l(|r(0)|)}\right) =$$
$$\int\int e^{-\frac{\mu\gamma(N+x+y)}{l(|r(0)|)}} F_{I^{f_1}_{pp}}(dx)F_{I^{f_1}_{sp}}(dy) =$$
$$e^{-\frac{\mu\gamma N}{l(|r(0)|)}}\mathcal{L}_{I^{f_1}_{pp}}\left(\frac{\mu\gamma}{l(|r(0)|)}\right)\mathcal{L}_{I^{f_1}_{sp}}\left(\frac{\mu\gamma}{l(|r(0)|)}\right)$$

where $\mathcal{L}$ is the Laplace transformation.

Please note that we have identified the Laplace transformations of the additive shot noises associated with the point processes of active primary and secondary transmitters using f_1 band. Due to the fact that Φ^*_{p,f_1} and Φ^*_{s,f_1} are PPP with intensities $\lambda_p p_e p_f$ and $\lambda_s P(f_1)$, the corresponding Laplace transformations are:

$$\mathcal{L}_{I^{f_1}_{pp}}(s) = exp\left\{-2\pi\lambda_p p_e p_f\int_0^\infty 1-\frac{\mu}{\mu+sl(|r(0)|)}rdr\right\}$$

$$\mathcal{L}_{I^{f_1}_{sp}}(s) = exp\left\{-2\pi\lambda_s P(f_1)\int_0^\infty 1-\frac{\mu}{\mu+sl(|r(0)|)}rdr\right\}$$

The calculus of $P^0_{\Phi_p}(SINR^p(0,r(0),f_2)>\gamma)$ is analogous and the COP_p is totally determined.

4.3 COP of secondary users

In this case, for a typical SU transmitter and its receiver we have:

$$\begin{aligned} COP_s &= P^0_{\Phi^*_s}(SINR^s(0,r(0))>\gamma) \qquad (13)\\ &= \sum_k P^0_{\Phi_s}\frac{(SINR^s(0,r(0),f_k)>\gamma)P(f_k)}{MAP_s}\end{aligned}$$

COP_s for a particular case

Using the same arguments explained before, we can express the probability of a successful secondary transmission as:

$$P^0_{\Phi^*_s}(SINR^s(0)>\gamma) = \frac{P^0_{\Phi_s}(SINR^s(0,r(0),f_1)>\gamma)P(f_1)}{MAP_s}+$$
$$\frac{P^0_{\Phi_s}(SINR^s(0,r(0),f_2)>\gamma)P(f_2)}{MAP_s}$$

where the calculus of the different involved terms are analogous to the COP_p.

5. SIMULATION EXPERIMENTS

In order to illustrate the quality of our results, in this section we will introduce some numerical examples related to the results presented in the previous section. To do that, we implement a set of simulations following the hypotheses of the presented particular case.

The set of transmitters (PUs and SUs) are distributed according of two PPPs in $\mathbb{R}^2$ with intensities λ_p and λ_s. Therefore we generate two Poisson processes in a circle of radio $R=30$ and consider only those points that fall into a circle of radio $R=15$ to minimize border effects (see Figure 2). For each transmitter we simulate each receiver uniformly distributed in a circle of radius 1 centered at each transmitter.

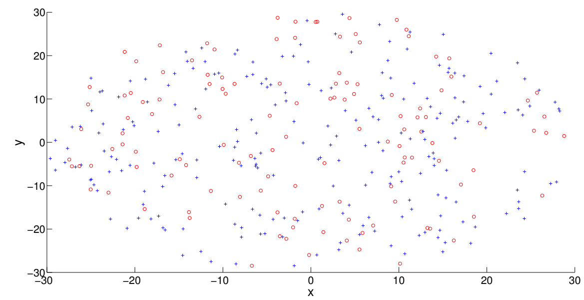

Figure 2: An example of the simulated sets of points of Φ_p (circles) and Φ_s (crosses). Parameters: $\lambda_p=0.05$ and $\lambda_s=0.08$.

The network parameters that we use in the simulations are:

- $\lambda_p=\lambda_p=0.8$
- $p_e=1$: all PUs are active.
- $p_f=0.3$: probability to be selected f_1 band by an active PU transmitter.
- $p'_f=0.5$: probability to be selected f_1 band by a SU when both bands are primary-free.
- $\mu=2$: parameter of the fading random variables
- $\alpha=3$: path loss coefficient
- $\gamma=0.1$: successful detection threshold
- $G(q)$ is an uniform distribution with parameter a,b; $\mathcal{U}[a,b]$, considering $a=0$ and varying b. This is used to implement the sensing mechanism of SUs.

Figures 3, 4 and 5 show the results corresponding to this scenario for different values of the parameter b. For each b value, we run 20 independent simulations and we can observe the analytical results together with the correspondent simulated boxplot representation. Our calculus are validated with the presented numerical examples.

In figures 6 and 7 we analyze the impact of the presence of secondary user. The network parameters are the same that we described before. We can observe in figure 6 the COP value of PU with and without SUs for different b values. Please note that parameter b is used by SU in the sensing algorithm, therefore, COP_p in a traditional network composed only by PUs is independent of b. On the other hand, in the CR scenario, when

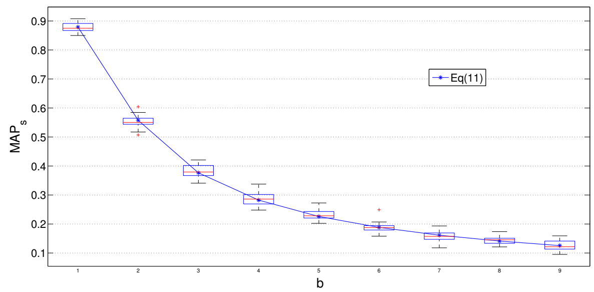

Figure 3: The evaluation of Eq. (11) along with the boxplot of the numerical results of 20 simulations.

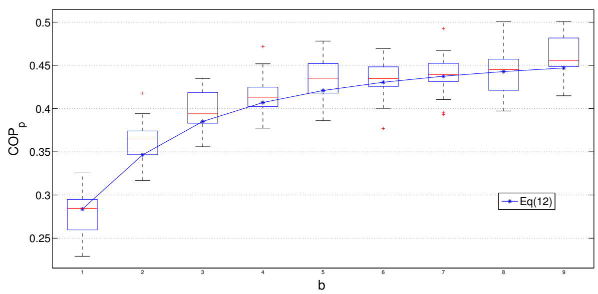

Figure 4: The evaluation of Eq. (12) for the particular case along with the boxplot of the numerical results of 20 simulations.

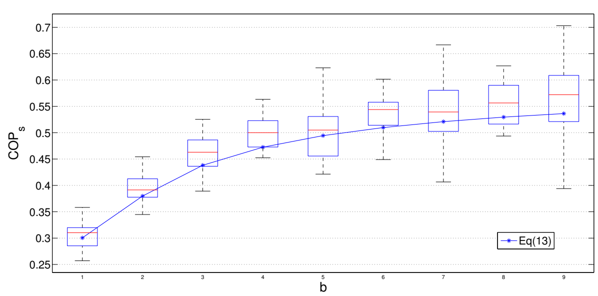

Figure 5: The evaluation of Eq. (13) for the particular case along with the boxplot of the numerical results of 20 simulations.

b increases PUs are less affected by SUs interference. The results obtained are coherent: for large b values, SUs have less probability to access to the network (see figure 3) and also, SUs that reach a channel to transmit have a high probability of being away from PU's protection zones.

In figure 7 it is shown the impact of the successful probability of PUs when the detection threshold varies. In addition we can see the impact of the presence of SUs that will be directly reflected in PU's throughput.

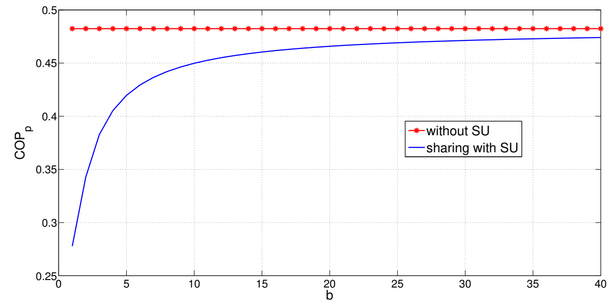

Figure 6: COP_p with and without SUs for different b values.

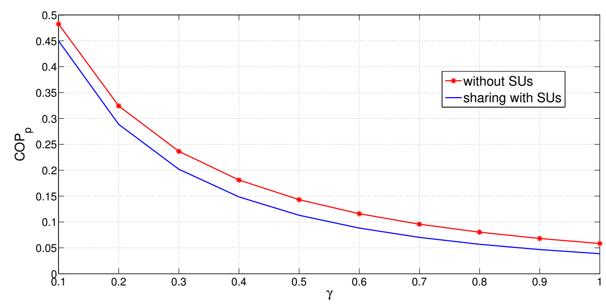

Figure 7: COP_p with and without SUs for different γ values. In this case, we consider a fixed $b = 10$.

6. CONCLUSIONS

We extended the methodology developed in [11] in the particular case of a multichannel cognitive radio environment. We made the first steps in order to analyze a scenario which considers more than one channel together with geometric aspects such as random node locations and path loss functions. We showed analytical results for the calculus of the main performance metrics: Medium Access Probability and Coverage Probability. These parameters give information about the possibilities offered by cognitive radio to improve the spectrum utilization and also they give an idea of how affected are primary users with the presence of secondary ones. We made some simulations in order to show this effect over PU communications.

In our ongoing work, we are investigating the influence of the different system parameters. In particular, we are interested in answer the following questions: what channel band should SU select if there are more then one available? Is it possible to define an optimal parameters configuration in order to maximize the spectrum utilization minimizing the effects over primary communications?

Acknowledgments

This work was financially supported by CSIC (R&D Groups) and a CAP-UdelaR scholarship.

7. REFERENCES

[1] Nprm 04-186 unlicensed operation in the tv broadcast bands. Technical report, Federal Communications Commission (FCC), 2004.

[2] F. Baccelli and B. Blaszczyszyn. *Stochastic Geometry and Wireless Networks, Volume I - Theory*, volume 1 of *Foundations and Trends in Networking Vol. 3: No 3-4, pp 249-449*. NoW Publishers, 2009. Stochastic Geometry and Wireless Networks, Volume II - Applications; see http://hal.inria.fr/inria-00403040.

[3] F. Baccelli and B. Blaszczyszyn. *Stochastic Geometry and Wireless Networks, Volume II - Applications*, volume 2 of *Foundations and Trends in Networking: Vol. 4: No 1-2, pp 1-312*. NoW Publishers, 2009. Stochastic Geometry and Wireless Networks, Volume I - Theory; see http://hal.inria.fr/inria-00403039.

[4] V. Blaschke, H. Jaekel, T. Renk, C. Kloeck, and F. Jondral. Occupation measurements supporting dynamic spectrum allocation for cognitive radio design. In *Cognitive Radio Oriented Wireless Networks and Communications, 2007. CrownCom 2007. 2nd International Conference on*, pages 50–57, Aug 2007.

[5] A. Busson, B. Jabbari, A. Babaei, and V. Vèque. Interference and throughput in spectrum sensing cognitive radio networks using point processes. *Journal of Communications and Networks*, 16(1):67–80, Feb 2014.

[6] D. J. Daley and D. Vere-Jones. *An introduction to the theory of point processes. Vol. II*. Probability and its Applications (New York). Springer, New York, second edition, 2008. General theory and structure.

[7] S. Helif, R. Abdulla, and S. Kumar. A review of energy detection and cyclostationary sensing techniques of cognitive radio spectrum. In *2015 IEEE Student Conference on Research and Development (SCOReD)*, pages 177–181, Dec 2015.

[8] S. Lee and G. Hwang. *Throughput Analysis of Multichannel Cognitive Radio Networks Based on Stochastic Geometry*, pages 63–71. Springer International Publishing, Cham, 2016.

[9] M. A. McHenry, D. McCloskey, D. Roberson, and J. T. MacDonald. Nsf spectrum occupancy measurements project summary. Technical report, Shared Spectrum Co., and IIT Wireless Interference Lab Illinois Institute of Technology, 2005.

[10] J. Mitola and G. Q. Maguire. Cognitive radio: making software radios more personal. *IEEE Personal Communications*, 6(4):13–18, Aug 1999.

[11] T. V. Nguyen and F. Baccelli. A probabilistic model of carrier sensing based cognitive radio. In *New Frontiers in Dynamic Spectrum, 2010 IEEE Symposium on*, pages 1–12, April 2010.

[12] T. V. Nguyen and F. Baccelli. A stochastic geometry model for cognitive radio networks. *The Computer Journal*, 2011.

[13] C. Rattaro, P. Bermolen, P. Belzarena, and F. Larroca. Report: A stochastic geometry analysis of multichannel cognitive radio networks. Technical report, Facultad de Ingeniería, Universidad de la República, Montevideo, Uruguay, 2016.

[14] D. Stoyan, W. S. Kendall, and J. Mecke. *Stochastic geometry and its applications*. Wiley series in probability and mathematical statisitics. Wiley, Chichester, W. Sussex, New York, 1987. Rev. translation of: Stochastische Geometrie.

[15] N. Tadayon and S. Aissa. Multi-channel cognitive radio networks: Modeling, analysis and synthesis. *IEEE Journal on Selected Areas in Communications*, 32(11):2065–2074, November 2014.

[16] N. Tadayon and S. Aïssa. Modeling and analysis framework for multi-interface multi-channel cognitive radio networks. *IEEE Transactions on Wireless Communications*, 14(2):935–947, Feb 2015.

[17] G. I. Tsiropoulos, O. A. Dobre, M. H. Ahmed, and K. E. Baddour. Radio resource allocation techniques for efficient spectrum access in cognitive radio networks. *IEEE Communications Surveys Tutorials*, 18(1):824–847, Firstquarter 2016.

[18] D. Zhang, Z. Chen, J. Ren, N. Zhang, M. Awad, H. Zhou, and X. Shen. Energy harvesting-aided spectrum sensing and data transmission in heterogeneous cognitive radio sensor network. *IEEE Transactions on Vehicular Technology*, PP(99):1–1, 2016.