Cognitive Radio Networks: Analysis of a Paid-Sharing Approach based on a Fluid Model

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ABSTRACT

Cognitive Radio Networks have emerged in the last years as a solution for two problems: spectrum underutilization and spectrum scarcity. In this context we consider a paid-sharing approach where secondary users (SUs) pay for spectrum utilization. We assume a preemptive system where primary users (PUs) have strict priority over SUs; when a PU arrives to the system and all the channels are busy, a SU will be deallocated. This affected SU will then be reimbursed, implying some cost for the PUs service provider. This paper bears on the analysis of the behavior of the system where the number of users is arbitrary large and an admission control policy over SUs is applied. We develop a computationally efficient way to find an accurate estimation of the optimal admission control boundary based on the fluid limit technique. Our results are validated through numerical examples.

Keywords

cognitive radio, fluid model, admission control

1. INTRODUCTION

Nowadays, with the rapid development of wireless communications, the demand on spectrum has been growing dramatically resulting in the spectrum scarcity problem. In spite of this problem, spectrum utilization measurements have shown that licensed bands are vastly underutilized while unlicensed bands are too crowded. Cognitive Radio (CR) has been proposed as a promising technology to solve those problems by an intelligent and efficient dynamic spectrum access [2].

In this new paradigm we can identify two classes of users: primary (PU) and secondary (SU). PUs are those for which a certain portion of the spectrum has been allocated to (often in the form of contractual quality of service (QoS) guarantees). SUs are devices which are capable of detecting unused licensed bands and adapt their parameters for using them. There are roughly two different approaches for this dynamic spectrum sharing: paid-sharing or free-sharing. While there are many problems yet to be solved in this area, one of the most important is how to stimulate the spectrum sharing behavior of PUs. In this context paid-sharing methods seem to be most suitable.

This work is an extension of [4]. In that work we have studied a paid-sharing approach based on admission control decisions over SUs. We have characterized the optimal admission control policy that maximizes the profit of the PU's Service Provider (SP) and we have concluded that the admission control boundary is a "switching curve" (see the definition in [4]). In addition, in [3], we have analyzed the behavior of the system (without considering the economical aspect) using a fluid model approximation. This work joins both previous analyses. The main contribution consists in defining and studying a fluid model of the stochastic one presented in [4] and developing a methodology in order to obtain an approximation of the optimal admission control boundary.

The rest of the paper is structured as follows. In section 2 we sum up the main results of [4, 3], we also introduce the fluid model and present a methodology to obtain an approximation of the admission control policy over SUs. In section 3 we include a numerical example that validates our results. Finally, we draw some conclusions.

2. FLUID MODEL

First of all we present the stochastic model proposed in [4]. Let us note as C the total number of identical channels. Let $x(t)$ and $y(t)$ be the number of PUs and SUs in the system at time t respectively. Let λ_1 and μ_1 be the arrival and leaving rates for PUs (independent Poisson arrivals and exponentially distributed service times). In the same way, λ_2 and μ_2 represent the arrival and leaving rates for SUs.

We consider a paid spectrum sharing mechanism where SUs pay to the PU's SP for the spectrum utilization. Let $R > 0$ be the reward collected for each SU when it is allowed to exploit the PU's resource. We also consider a preemptive system where PUs have strict priority over SUs. This means that a SU can be removed from the system if all the channels are busy and a PU arrives. In this model, this affected SU will be reimbursed with $K > 0$, implying a punishment for the SP. We take into account a discount rate $\alpha > 0$, that is, the rewards and costs at time t are scaled by a factor $e^{-\alpha t}$.

We have associated one user with one channel. We thus have a continuous time Markov Decision Process (MDPs) with state space $S = \{(x, y)|0 \leq x \leq C, 0 \leq y \leq C, 0 \leq x+y \leq C\}$ and transition rates $q((x, y),(x', y'))$:

- $q((x, y),(x + 1, y)) = \lambda_1$, if $x + y < C$
- $q((x, y),(x - 1, y)) = \mu_1 x$
- $q((x, y),(x, y + 1)) = a(x, y)\lambda_2$, if $x + y < C$
- $q((x, y),(x, y - 1)) = \mu_2 y$
- $q((x, y),(x+1, y-1)) = \lambda_1$, if $x+y = C$ and $y \neq 0$ (preemption)

where $a(x, y)$ represents the admission control decision in each state.

The objective is to maximize the total expected discounted profit over an infinite time horizon applying admission control decisions over SUs, i.e. we want to find the optimal policy π^* that defines the admission control action $a(x, y) \in \{0, 1\}$ in each state $s \in S$ maximizing the SP's revenue. In [4] we characterized some properties of the optimal policy, in particular we proved that the optimal admission boundary is a "switching curve".

In many scenarios where the number of channels C and the user arrival rates (λ_i) are large, a deterministic fluid model may offer a good approximation to the original control problem [1]. In the direction of introducing the fluid approximation, we make some important definitions. Let $\tilde{x}^N(t)$ and $\tilde{y}^N(t)$ be the number of PUs and SUs in the system considering a scaled version of the original stochastic model. That means that the parameters of this new process are: $\tilde{C} = CN$, $\tilde{\lambda}_i = \lambda_i N$, $\tilde{\mu_i} = \mu_i, i = 1, 2$, being N the scaling factor. In turn, $(x^N(t), y^N(t)) = \frac{1}{N}(\tilde{x}^N(t), \tilde{y}^N(t))$ converges in probability to a deterministic process described by an Ordinary Differential Equation (ODE). Let $(x_f(t), y_f(t))$ be the limit process. A complete explanation of this result is done in our previous work [3].

If the departure rates of both classes of users were equal, the optimal admission control decision would only depend on the total number of occupied channels (see for instance [5]). We can approximate our general system to be in that particular context. That is to say, we will work with a new system were the departure rates of both classes are the same, a μ scaled version of the fluid model. This new system has the following parameters: $\mu_1^s = \mu_2^s = \mu$, $\lambda_1^s = \frac{\lambda_1\mu}{\mu_1}$ and $\lambda_2^s = \frac{\lambda_2\mu}{\mu_2}$.

Considering the scaled system λ_1^s, λ_2^s, μ, the optimal admission control boundary will be a line with equation $x_f + y_f = \delta$ (that is a "switching curve"). Working with the stochastic system, it means that $a(\tilde{x}^N(t), \tilde{y}^N(t)) = 1, \forall(\tilde{x}^N(t), \tilde{y}^N(t))/\tilde{x}^N(t)+\tilde{y}^N(t) < \delta N$ and $a(\tilde{x}^N(t), \tilde{y}^N(t)) = 0$ in other cases. The idea is to approximate the optimal SU access control boundary (the AC boundary of the original system with different departure rates) with a line parallel to γ[1] (solution of the μ-scaled system). Not only is it a "switching curve", it has many advantages (e.g. its practical implementation: it is only necessary to know the number of occupied bands to decide whether the new SU will be accepted in the system or not). Observe that $\frac{\lambda_1^s}{\mu} = \frac{\lambda_1}{\mu_1}$ and $\frac{\lambda_2^s}{\mu} = \frac{\lambda_2}{\mu_2}$.

Let π be a feasible policy and $R_\alpha(k_0, \pi)$ the profit function that we want to maximize. Working with the fluid approximation, we can identify two subsets of system parameters to analyze (Cases of fig. 1).

if $\rho_1 + \rho_2 < C$ (A, B) or $\rho_1 < C \leq \rho_1 + \rho_2$ (C):

$$R_\alpha(k_0, \pi) = \int_0^\infty \lambda_2^s R e^{-\alpha t} dt \quad (1)$$

if $\rho_1 \geq C$ (D):

$$R_\alpha(k_0, \pi) = \int_0^{t_1} \lambda_2^s R e^{-\alpha t} dt - \int_{t_2}^{t_c} \lambda_1^s K e^{-\alpha t} dt \quad (2)$$

where: $\rho_1 = \frac{\lambda_1^s}{\mu}$, $\rho_2 = \frac{\lambda_2^s}{\mu}$, $t_1/x_f(t_1) + y_f(t_1) = \delta$, $t_2/x_f(t_2)+y_f(t_2) = C$, $t_c/x_f(t_c) = C$ and $k_0 = x_f(0)+y_f(0)$.

In the first case, when $\rho_1 + \rho_2 < C$ or ($\rho_1 + \rho_2 > C$ and $\rho_1 < C$), with the fluid analysis we conclude that $\delta < C$ but very closely to C. In terms of the stochastic system, this threshold will be highly dependent on N.

On the other hand, based on Proposition 2 of [3] and incorporating the economical aspect, when $\rho_1 > C$ (case D of figure 1) we can make an additional analysis.

If at $t = 0$ the system is in the state k_0 (i.e. $x_0 + y_0 = k_0$), then for a feasible control policy π, the total discounted revenue will be:

$$R_\alpha(k_0, \pi) = \int_0^{t_1} \lambda_2^s R e^{-\alpha t} dt + \int_{t_2}^{t_c} -\lambda_1^s K e^{-\alpha t} dt \quad (3)$$

[1] $\gamma : x + y - C = 0$

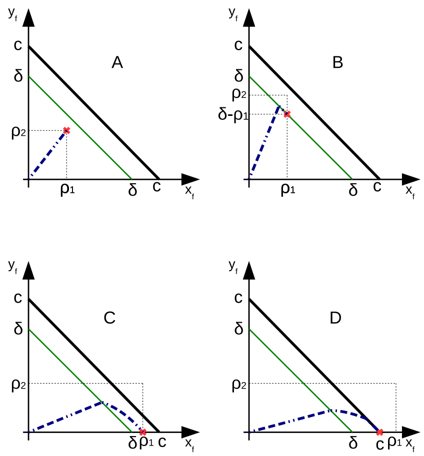

Figure 1: Admission Control with boundary $-x_f - y_f + \delta = 0$. Examples of $(x_f(t), y_f(t))$ are represented for different system parameters. Cases A, B: $\rho_1 + \rho_2 < C$; Case C: $\rho_1 + \rho_2 > C$ and $\rho_1 < C$; and Case D: $\rho_1 > C$

where: $t_1 = \frac{1}{\mu} ln \left[\frac{\rho_1+\rho_2-k_0}{\rho_1+\rho_2-\delta} \right]$, $t_2 = t_1 + \frac{1}{\mu} ln \left[\frac{\rho_1-\delta}{\rho_1-C} \right]$ and $t_c = \frac{1}{\mu} ln \left[\frac{\rho_1-x_0}{\rho_1-C} \right]$ (See the system differential equations in [3])

After working with eq.(3), the explicit form of $R_\alpha(k_0, \pi)$ is obtained. Then, the objective consists in solving the deterministic problem: $\max_\delta R_\alpha(k_0, \pi)\ s.t.\ 0 \leq \delta \leq C$. In the next stage we will present some numerical results that show the performance of this methodology.

3. NUMERICAL RESULTS

The idea is to compare the results of Modified Policy Iteration Algorithm (MPI, one of the best known practical algorithms for solving infinite-horizon MDPs, it was used in [4]) with the solution provided by the deterministic approximation. We have compared both methods in several cases and we present the case of fig. 2 as an example.

The approximation obtained by the fluid model is closely to the optimal admission control boundary (obtained using MPI). We have used $\mu = \mu_1$ in order to obtain the μ-scaled approximation. The reason is because the second term of equation 2 has an important effect in $R_\alpha(k_0, \pi)$ calculation when $\rho_1 > C$, therefore, in the direction of obtaining a good approximation of the original system, λ_1^s must be equal to λ_1 then $\mu = \mu_1$.

In order to test how close to the optimal is the approximation of the admission control boundary, we made several experiments ($n = 30$) with both boundaries (the optimal and its approximation) and computed the profit of the SP. Each experiment consisted in one realization of the continuous time markov chain using the appropriate AC boundary. In each transition the discount profit of the SP was computed.

We built the reward confidence intervals (0.95 level of confidence) for MPI ($[0.021, 0.029]$) and for the fluid approximation ($[0.022, 0.031]$). Based on these results, we can conclude that the performance of the approximated boundary has a good accuracy.

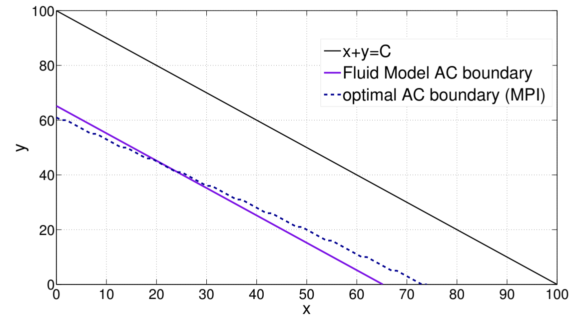

Figure 2: Parameters:$\lambda_1 = 20$, $\lambda_2 = 2$, $\mu_1 = 10$ (μ), $\mu_2 = 1$, $C = 1$, $N = 100$, $\alpha = 50$, $R = 1$ and $K = 3$.

4. CONCLUSIONS

We have studied the behavior of the system when an admission control is applied using a fluid approximation of the stochastic model. This study can be useful for many proposals, as an example, we have developed a computationally efficient way to find an estimation of the admission control boundary. It is important to remark that all the contributions have been evaluated through extensive sets of simulations.

5. ACKNOWLEDGMENTS

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