

Decentralized Robust Spectrum Allocation for Cognitive Radio Wireless Mesh Networks

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Abstract

During the last decade we have seen an explosive growth in the deployment of wireless networks in unlicensed frequency bands, mainly driven by the great success of the IEEE 802.11 standard. In addition to its traditional last-hop usage, it has also been widely employed for Internet access infrastructure such as wireless mesh networks (WMNs). A problem that is envisioned in the near future is the spectrum scarcity, which could be a serious threat to cope with the ever increasing demand. Regulators are aware about this problem and they have already started to look for more available spectrum. One of the possibilities that has emerged is to allow secondary assignments in licensed bands, based on the recent cognitive radio networks (CRNs) paradigm. In this context, we focus our work in the analysis of optimum spectrum allocation mechanisms for a cognitive wireless multihop mesh network. We introduce a stochastic model to formulate the problem, considering primary users' activity and a periodically scheduled assignment scheme. To solve the problem we propose a novel robust solution, for which we develop a decentralized algorithm implementation. Furthermore, we evaluate our proposal through extensive simulations, showing for instance its superiority compared with an expectation based approach.

Keywords:

wireless mesh networks, cognitive radio, dynamic spectrum allocation

1. Introduction

Undoubtedly, the deployment of wireless networks in unlicensed frequency bands has increased significantly during the last years, particularly as Internet access technology for end users. The great success of the IEEE 802.11 standard has been one of the keys to this process. Over the last decade we have also witnessed the highest growth in wireless networks traffic [1] and forecasts indicate that this growth will continue [2]. Moreover, the user density is also increasing, resulting in crowded scenarios where the technology is reaching its limits (e.g. classrooms, large conferences, shopping centers or sport events [3]).

Besides these most common scenarios, where we only have a wireless last hop, requirements also increase for the wireless transport networks we found today, also using 802.11-based technology in unlicensed bands. This is the case of the typical wireless mesh network (WMN) solution, used for example in Plan Ceibal [4] as Internet access for schools located in rural or suburban areas. In that case the problem is not about user density, as we only have point to point or point to multipoint links between a few nodes. Instead, we have higher throughput requirements, because we are talking about the network

core. While standards are still evolving, achieving increasingly higher spectral efficiency, we may soon be faced with spectrum scarcity issues to properly cope with traffic demands. Regulators have taken note about this fact and some proposals already exist to extend the available spectrum [5].

Leaving aside traditional spectrum allocation, a new type of spectrum assignment has emerged some years ago: the so-called cognitive radio paradigm [6]. The main idea is to have two types of users; licensed or primary users (PUs from now on), which have the preferential right to use the band; and unlicensed or secondary users (SUs from now on), which can use the band only in the absence of the PUs. This type of spectrum allocation contributes to a more efficient use compared to traditional static assignments, as testified by some recent FCC rulings [7]. Although adoption is not yet massive, much industrial and academic efforts have been dedicated to this kind of technology. For instance, the IEEE 802.22 standard [8] was approved in 2011, which defines a Wireless Regional Area Network (WRAN) based on cognitive radio. Another industrial effort is the 802.11af amendment to enable the operation of Wi-Fi in TV bands, which has been recently published [9].

On the other hand, the development of cognitive radio equipment is still immature, particularly concerning sensing tasks to detect PUs, so the first solutions being deployed are based on databases queries to get the information about the available spectrum [10]. Some major providers such as Google are already authorized in the US to give such spectrum database service [11]. Everything suggests that in the short to medium term

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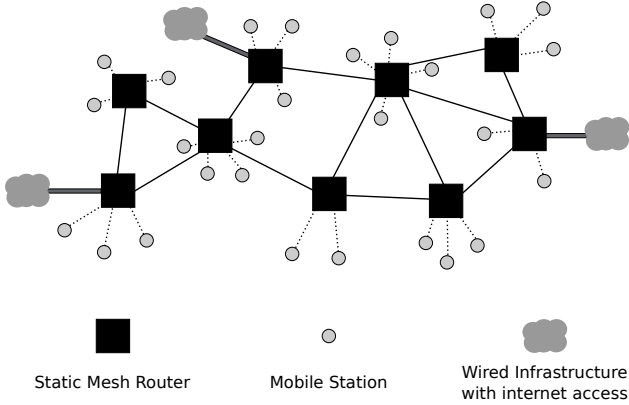


Figure 1: Wireless mesh network typical architecture.

dynamic spectrum allocation will expand, and in a few years we will probably have several standards operating under this paradigm. This enables new possibilities for the development of radio communications equipment, which added to the advances in software defined radio (SDR) technologies, may cause a significant change in the world of wireless communications we know so far.

Our work is focused on WMNs [12, 13], which have emerged in the last years as a cost-efficient alternative to traditional wired access networks. After many years of research, WMNs are no longer just a promise for the future, but a reality today, thanks mainly to the lower prices of radio cards and the operation in unlicensed frequency bands. In particular, outdoor community mesh networks [14] and rural deployments [15, 16] based on IEEE 802.11 have seen tremendous growth in the recent past. An example is Plan Ceibal [17] which provides connectivity to every school in Uruguay, where WMNs are used to reach suburban and rural schools. Lately, even service providers are beginning to use this technology, resulting in an increasing presence of carrier-class equipment in the market [18]. The typical architecture of a WMN is depicted in Fig. 1, which includes one or more Internet gateways and several relay routers. We will concentrate ourselves in the problem of resource allocation for the core of the WMN, that is to say we will only consider the wireless links between the intermediate routers, ignoring then the additional links with the end users (typically in other frequency bands). Moreover, we will develop a decentralized scheme to implement the proposed algorithm, so that the solution properly scales as the size of the WMN grows.

While much research has been recently dedicated to cognitive radio networks and dynamic spectrum allocation, most of the works have mainly focused on the case where there are only licensed bands available [19]. In that case, unlicensed devices can only operate as SUs in the absence of PUs, greatly limiting their possibilities. We believe it is very complex to develop a useful solution in such scenario with high throughput requirements. Several issues arise working only with licensed bands, for example, you need to ensure a control channel to coordinate communication, which is not an easy task without any guaran-

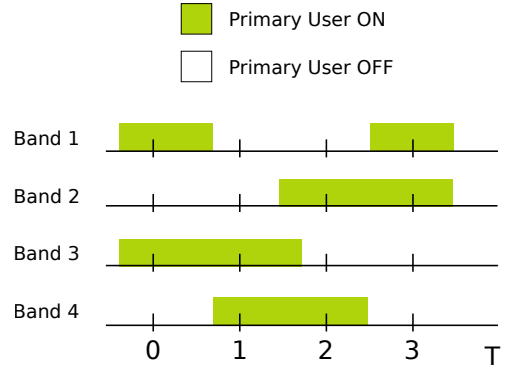


Figure 2: Spectrum occupancy example with four licensed bands.

teed frequency band to use. Moreover, it is possible to have circumstances under which the available spectrum is not sufficient to meet the throughput requirements, as the available capacity strongly depends on the PUs' dynamics. In this paper we work in a mixed licensed and unlicensed scenario, which we believe is more appropriate to support high throughput requirements. This solution has not been deeply explored yet in the literature and we think it is the most suitable model for the equipment and regulations that we may have during the coming years.

This paper bears on the dynamic spectrum assignment in a WMN. That is to say, we will study possible methods to decide which frequency bands may be used by the network devices at any given time. It is worth to highlight that such an assignment means that the bands are *available* for the devices, and are not necessarily used. With this in mind, the natural question that arises is to what purpose this assignment should be performed [20]. In our particular context, examples include minimizing the number of licensed bands assigned [21] or maximizing the user's utility (as a function of the mean rate) [22] without exceeding a maximum interference threshold to other networks. However, in the context of a cognitive WMN, we argue that the most natural objective would be to provide a lower bound to the resulting throughput in each link. The purpose of the spectrum allocation should be thus to ensure a certain effective capacity for each link, independently of the channel conditions and the PU's activity.

The other challenge that these systems pose is the time-scale at which the assignment should be performed. One possibility is to re-assign (and thus re-optimize) every time a band is used or abandoned by PUs, or if significant changes in channel conditions occur. Although this event-driven solution will lead the system to operate with the optimal allocation all the time, it will typically result in a dramatically high signaling overhead. In this sense, we will assume, as many researchers, a periodic optimization every T time units, which leads us to a better performance tradeoff. However, T may include variations in PUs' activity. This fact implies that a licensed band assigned when the period starts might have to be abandoned, resulting in an effective capacity that is less than expected. In Fig. 2 we present an example to clarify this situation. In it we have four licensed bands, with two of them available at the first spectrum assignment at time 0. During the interval between allocations, a PU

starts using band 4, so it is no longer available. The problem occurs again in the second assignment, where bands 1 and 2 are available and the assignment is thus performed, but a PU occupies band 2 during the interval.

To address this issue, the most commonly used approach is to model the availability of licensed bands as random, and optimize the expected value of indicators such as interference or throughput, as discussed before. Although this means that in the long run the objective will be accomplished (e.g. the throughput will be maximized), at shorter time-scales the resulting performance may be far from optimal. In contrast with previous works, we will present a frequency assignment scheme that provides the required throughput, which will hold with very high probability during the whole operating time. Naturally, such guarantee will require a certain degree of overprovisioning, but our simulations indicate that this is usually below 35% of that required by an oracle that knows beforehand the PUs' activities. Moreover, the results show that simply considering an expected value approach leads us to a solution where the throughput requirement is not fulfilled more than 40% of the time.

The rest of the paper is structured as follows. In the next section we present the previous related work and highlight some recent papers. In Sec. 3 we introduce most of the notation used in the paper and the spectrum allocation problem model. The formulation results in a stochastic optimization problem, which is presented in the same section, along with the equivalent deterministic optimization problem which leads to a robust solution. The paper continues in Sec. 4 where we describe the network architecture that enables the implementation of the proposed scheme in a decentralized way. Finally, in Sec. 5 we present the simulation experiments and performance comparison, while conclusions and future work are discussed in Sec. 6.

2. Related work

More than a decade has already passed since the emergence of the cognitive radio networks (CRNs) paradigm, and a large amount of the research done in the area during last years has been dedicated to spectrum assignment. An example of this is the number of papers that can be found as references in the broad survey by Tragos *et al.* [20]. However, as the authors state in the paper, there are still many issues and challenges to be solved, something which is also remarked in [23]. As we previously mentioned, most of the work so far is focused on a scenario with all licensed frequency bands, where cognitive nodes can only access to the spectrum as SUs while PUs are not present, discarding the use of unlicensed bands, available at any time. While this problem is still of interest for certain applications, such as delay-tolerant or sensor networks, it is not suitable for a transport mesh network, with high throughput and high availability requirements. In other cases, the spectrum allocation simply ignores the PUs, or just consider that SUs have a fixed set of available frequency channels, separated from the ones of the PUs.

This latter scenario reduces to the traditional spectrum allocation problem in a WMN, which has been the focus of several

articles. In this problem, different variants arise, such as the number of radios per node, which can vary from a single radio per node [24, 25], to the higher capacity multi radio case [26, 27, 28], which gives name to the multi-radio multi-channel (MR-MC) WMNs. Our work can be seen as an extension to this model, as we consider the same problem but under the paradigm of CRNs, which we believe should be the natural next step in the evolution of wireless multi-hop networks. Furthermore, we consider a novel robust approach, but in this case the uncertainty is not about the channel conditions [29, 30], nor the traffic variations [31], but the PUs' activity. In particular, incorporating licensed bands generates a dynamic resources availability, so one of the requirements of the spectrum allocation is to be robust against such variations.

To the best of our knowledge, very few works have studied the resource allocation in a mixed licensed and unlicensed scenario. In [21] an opportunistic spectrum assignment is proposed in order to alleviate congestion in a WLAN environment. The problem is formulated as a binary linear program, where they seek to minimize the number of assigned bands without exceeding a maximum interference threshold. The proposal is limited to the allocation of a single frequency band for each access point, so channel aggregation is not considered, something already included in newer standards (802.11n and 802.11ac) and which is quite an important limitation in order to increase capacity when needed. A similar problem, but from the PUs' perspective, is studied in [22]. In that case the authors analyze the simultaneous use of both type of frequency bands by a mobile operator, in order to increase the capacity in a femtocell scenario.

We highlight the work in [32] where the authors studied a traffic engineering solution in the context of a multihop cognitive WMN. They considered the combined use of ISM bands and licensed bands in the absence of PUs, and also assumed nodes have cognitive sensing capabilities in order to exploit unused primary bands. The traffic engineering problem is formulated as a network utility maximization, which is solved with a stochastic primal-dual approach, without knowledge of the probability distribution of PUs' activity. The spectrum assignment is not treated directly as it is an underlying problem of the traffic engineering issue addressed in the paper, so they just assume the available spectrum for each link determines its variable capacity. Our work is based on similar assumptions as the ones stated in [32], but we focus on the spectrum assignment problem. The main difference is that we consider a measurement-based approach where we estimate the probability distribution of PUs' activity, based on the nodes' cognitive sensing capabilities. In this work we thus take into account the PUs' activity, something which was not considered in many previous works, as stated in [20].

3. Network Model and Problem Formulation

In this paper we study the spectrum allocation problem in a mixed licensed and unlicensed scenario. In the proposed scheme, devices operate always as unlicensed devices but in two types of frequency bands, licensed ones, where they are

only allowed to operate when there is no presence of PUs, and unlicensed ones, where they can operate all the time. This offers greater flexibility to meet the requirements, given the scarcity of unlicensed spectrum. Furthermore, by having both type of bands, we simplify the protocol design complexity compared to solutions which only use licensed bands, as we can perform control communications through unlicensed bands, which are available all the time. To accomplish this goal we will impose that any possible assignment should include a minimum amount of unlicensed spectrum that guarantees a minimum capacity for control plane traffic (which we shall call w). This way we ensure the control plane connectivity between nodes, which makes possible the proper coordination for the use of the allocated frequency bands.

As in other previous works (e.g. [33]) we will assume that each node has a dedicated interface to enable cognitive sensing capabilities. By this mean, each node is able to keep a record for the PUs' activity on each licensed band. Besides, this interface is used to collect air measurements data, which are used to estimate the available capacity on each band, either licensed or unlicensed. This effective capacity depends on several factors such as channel conditions and other SUs' activity (devices from other networks that are not under our control), but it can be estimated passively through measurements [34][35]. We consider a solution where the assignment is performed every T time units and we will further assume that T is relatively small, so that an accurate estimation of each band's available capacity may be obtained using information from the previous interval. In this work we suppose that such estimation is exact, so as to focus only in the PUs' dynamics. We will also assume that devices can fully exploit the available spectrum (even disjoint available bands), using a PHY layer such as OFDM. We also assume there is a MAC layer mechanism in order to share the spectrum between nodes (e.g. 802.11 MAC layer).

3.1. Single collision domain

In this section we will focus on a single-domain spectrum assignment, that is to say, a network with a unique collision domain, corresponding to the case of just one point to point link between two nodes. In the next section we will present the model extension for a wireless mesh network (WMN) with multiple collision domains. Let $u = 1, \dots, U$, be the set of unlicensed frequency bands (i.e. no PUs, as in ISM bands). Let $b = 1, \dots, B$, be the set of licensed frequency bands (which are assigned to a PU) available at time t (i.e. PUs are not present). We will note as $c_b(t)$ the effective capacity available on licensed frequency band b and $c_u(t)$ the effective capacity available on unlicensed frequency band u . We define as spectrum assignment variables $\alpha_b(t)$ and $\alpha_u(t)$, which belong to $[0, 1]$, assuming partial band assignment is possible (e.g. via OFDMA or TDMA).

Now, we can define the total effective capacity assigned for the interval starting at T as:

$$C_{\text{eff}}(\alpha_T) = \sum_{b=1}^B \alpha_b(T) c_b(T) h_b(T) + \sum_{u=1}^U \alpha_u(T) c_u(T) \quad (1)$$

where $h_b(T)$ is a real number in $[0, 1]$, according to how much time each licensed band was actually available during the interval. We will model h_b as a random variable, whose distribution will be learnt from the previously observed dynamics. As we stated previously the objective is to provide a lower bound to the resulting throughput, so we will set this bound as a problem constraint, and we shall note it as d . This lower bound d is actually the minimum total capacity our system should have considering all nodes. We further define a cost function:

$$C(\alpha_t) = C_{\text{lic}}(\alpha_1(t), \dots, \alpha_B(t)) + C_{\text{unlic}}(\alpha_1(t), \dots, \alpha_U(t)) \quad (2)$$

The cost functions $C_{\text{lic}}()$ and $C_{\text{unlic}}()$ allow us to give different weights for each band, depending on the desired spectrum allocation goal. For example, it is possible to have different costs depending if the band corresponds to a higher or lower frequency, which may imply different transmission power requirements.

After all the stated assumptions, definitions and goals, we can now define an optimization problem which will lead us to the assignment algorithm for the single domain case. This problem should be solved periodically, so we will omit the time index from now on for a matter of clarity. That is to say, each time T we should strive at solving the following problem:

$$\begin{aligned} \min_{\alpha} \quad & C(\alpha), \\ \text{s.t.} \quad & \sum_{b=1}^B \alpha_b c_b h_b + \sum_{u=1}^U \alpha_u c_u \geq d, \\ & \sum_{u=1}^U \alpha_u c_u \geq w, \\ & \alpha_b \in [0, 1], b = 1, \dots, B, \\ & \alpha_u \in [0, 1], u = 1, \dots, U. \end{aligned} \quad (3)$$

The problem above is actually not well defined, as h_b is a random variable. To take into account this fact, the first and, as discussed in the introduction, most common approach, is to use the expected capacity, which leads us to the following equivalent deterministic constraint:

$$\bar{C}_{\text{eff}}(\alpha) = \sum_{b=1}^B \alpha_b c_b E\{h_b\} + \sum_{u=1}^U \alpha_u c_u \geq d, \quad (4)$$

where $E\{h_b\}$ can be estimated from the previous records of the PU's activity. Thus, the problem above is convex (assuming the defined cost functions are convex) and can be solved with standard optimization tools.

The alternative we propose, which we argue is better to address the problem at hand, is to change the expected effective capacity constraint for a probabilistic one:

$$\text{Prob} \left(\sum_{b=1}^B \alpha_b c_b h_b + \sum_{u=1}^U \alpha_u c_u \geq d \right) \geq 1 - \epsilon, \quad (5)$$

where ϵ is a fixed value (between 0 and 1), which leads us to a convex chance constrained optimization problem [36] (assuming h_b has a log-concave distribution). This approach is more

difficult to solve in the general case and the deterministic equivalent constraint depends on the distribution of h_b . The solution we found suitable for this case, assuming the distribution of h_b is unknown, is to use the distributionally robust deterministic equivalent problem as presented in [37]. This solution is robust as it considers all the possible distributions of h_b with known mean and variance, which in our case can be estimated from the previous records of the PU's activity. This way, we have again a convex optimization problem, but now with a different deterministic equivalent constraint:

$$\bar{C}_{\text{eff}}(\alpha) - \kappa_\epsilon \sqrt{\sum_{b=1}^B (\alpha_b c_b)^2 \text{Var}\{h_b\}} \geq d \quad (6)$$

where $\kappa_\epsilon = \sqrt{(1 - \epsilon)/\epsilon}$. By this equivalence the constraint is no longer linear but a conic quadratic. Thus, the problem is still convex for convex cost functions, so it can also be solved by standard optimization tools. Something that is worth to note is that we assume that PUs' activity in one band is independent from PUs' activity in other bands. This is the case for example in TVWS [9], where the basic frequency band unit corresponds to the spectrum bandwidth of a single TV channel (6, 7 or 8 MHz, depending on the regulatory domain). This assumption allows us to model h_b as independent random variables on each licensed band and simplifies the resultant distributionally robust equivalent, as all the cross terms in the covariance matrix are zeros.

3.2. Model Extension for a Wireless Mesh Network

Now, we extend the previous model to the case of a wireless mesh network (WMN) with L links, where in the general case we may have multiple overlapping collision domains. We will consider for the spectrum allocation only the wireless links in the core of the WMN, assuming that the last hop with end clients is in other non interferent frequency bands. In this case, we can reuse the frequency bands in different links, but to avoid interference, we have to constrain the assignment on each collision domain. Thus, we want to ensure that if a certain frequency band u or b is assigned to a certain link l , then the same band cannot be assigned simultaneously by other links in the same collision domain.

In order to define the collision domains, we will consider interference between links and not between nodes. The reason to do this is that normally when we have communication between nodes, even when the data flows in only one direction, we still have information flowing in the opposite direction (e.g. acknowledgements). So, we will consider that two links interfere with each other if any node of one link is in the same collision domain than any node of the other link. Thus, we first need to know the conflict graph of the WMN, which is an undirected graph, where each vertex represents a wireless link and we have an edge between every pair of links that interfere with each other. Then, to list all the collision domains (noted with q , from 1 to Q), we have to look for all the maximal cliques¹ of

$\alpha_U \in [0, 1]^{U \times L}$	unlicensed spectrum allocation variables
$\alpha_B \in [0, 1]^{B \times L}$	licensed spectrum allocation variables
$C_U \in \mathbb{R}^{+U \times L}$	capacity for unlicensed spectrum bands
$C_B \in \mathbb{R}^{+B \times L}$	capacity for licensed spectrum bands
$H_B \in [0, 1]^{B \times L}$	PUs' activity on licensed bands
$d \in \mathbb{R}^{+L}$	capacity required for data plane traffic
$w \in \mathbb{R}^{+L}$	capacity required for control plane traffic
$A \in \{0, 1\}^{Q \times L}$	link interference matrix (conflict graph)
$\alpha = [\alpha_B, \alpha_U]$	all spectrum allocation variables
$C_{\text{lic}}(\alpha_B) \in \mathbb{R}^+$	cost function for licensed spectrum
$C_{\text{unlic}}(\alpha_U) \in \mathbb{R}^+$	cost function for unlicensed spectrum
$\lambda \in \mathbb{R}^{+Q \times (B+U)}$	Lagrange multipliers
$\mathbf{1}_M$	Column vector of ones of size M

Table 1: Matrix Notation

the conflict graph. Once we have all the Q collision domains in the WMN, we can properly define a binary matrix A to reach the necessary additional constraint:

$$A \cdot \alpha^\top \leq \mathbf{1}_{Q \times (B+U)} \quad (7)$$

using the matrix notation for the spectrum assignment variables defined in Tab. 1.

We are now able to define an optimization problem similar to the previous case of a single collision domain. We will omit again the time index for a matter of clarity. This way, the spectrum assignment in the WMN can be performed solving the following problem:

$$\begin{aligned} \min_{\alpha} \quad & C(\alpha), \\ \text{s.t.} \quad & (\alpha_B \odot C_B \odot H_B) \cdot \mathbf{1}_B + (\alpha_U \odot C_U) \cdot \mathbf{1}_U \geq d, \\ & (\alpha_U \odot C_U) \cdot \mathbf{1}_U \geq w, \\ & A \cdot \alpha^\top \leq \mathbf{1}_{Q \times (B+U)}, \\ & \alpha_B \in [0, 1]^{B \times L}, \\ & \alpha_U \in [0, 1]^{U \times L}. \end{aligned} \quad (8)$$

where \cdot stands for the common vector and matrix product operation and \odot stands for an element-wise matrix multiplication.

As in the previous case, we have to deal with the random variables H_B . To do this we will use the same deterministic equivalents as before, on the one hand based on the expected value of H_B and on the other hand considering the distributionally robust approach. For the first one, the deterministic equivalent constraint is:

$$(\alpha_B \odot C_B \odot E\{H_B\}) \cdot \mathbf{1}_B + (\alpha_U \odot C_U) \cdot \mathbf{1}_U \geq d \quad (9)$$

while for the robust approach is:

$$\begin{aligned} & (\alpha_B \odot C_B \odot E\{H_B\}) \cdot \mathbf{1}_B + (\alpha_U \odot C_U) \cdot \mathbf{1}_U \\ & - \kappa_\epsilon \left\| \alpha_B \odot C_B \odot \sqrt{\text{Var}\{H_B\}} \right\|_2 \cdot \mathbf{1}_B \geq d \end{aligned} \quad (10)$$

where $E\{H_B\}$ is the element-wise expected value of H_B and $\sqrt{\text{Var}\{H_B\}}$ corresponds to a matrix containing the square root of the variance of each element in H_B . Both problems are again

¹A clique is a complete subgraph of at least 2 vertices.

convex if the cost functions are convex, and we have that, while in the first one the deterministic equivalent constraint is linear, in the second one it is a conic quadratic, just as in the single domain case. This ensures that both problems are convex for convex cost functions and can be solved with standard optimization tools. Next, we will develop a decentralized implementation of the algorithm, which is important in order to have a solution that scales properly as the size of the WMN grows.

4. Distributed algorithm architecture

In this section we show how to solve the optimization problem defined in the previous section in a distributed manner. Then, we propose a suitable architecture for the algorithm implementation. Finally, we conclude the section with a discussion on some implementation issues.

4.1. Distributed optimization

In order to find a distributed solution, we will use the dual decomposition of the described problem. This procedure is called *resource allocation via pricing* [38], because the Lagrange multipliers can be seen in a manner equivalent to the price of the resources. In this case the resources correspond to the frequency bands, which are then assigned to minimize the cost of the resulting allocation. The decomposition involves the relaxation of the coupling constraint, which in this case is the one imposed to avoid interference between links. Intuitively, it will be more expensive to allocate frequency bands for those links included in a higher number of collision domains. In turn, those collision domains with a larger number of links will have higher prices for the frequency bands (i.e. the greater the demand, the higher the prices).

The first step of the dual decomposition procedure is to form the Lagrangian by relaxing the coupling constraint. Thus, we shall consider the matrix λ of size $Q \times (B+U)$, with $\{\lambda_{qb}, \lambda_{qu}\} \in \mathbb{R}^+$, to get the following relaxed problem:

$$\begin{aligned} \min_{\alpha} \quad & C(\alpha) - \mathbf{1}_{B+U}^T \cdot (\lambda^T \odot (\mathbf{A} \cdot \alpha^T - \mathbf{1}_{Q \times (B+U)})) \cdot \mathbf{1}_{B+U} \\ \text{s.t.} \quad & (\alpha_B \odot \mathbf{C}_B \odot \mathbf{H}_B) \cdot \mathbf{1}_B + (\alpha_U \odot \mathbf{C}_U) \cdot \mathbf{1}_U \geq \mathbf{d}, \\ & (\alpha_U \odot \mathbf{C}_U) \cdot \mathbf{1}_U \geq \mathbf{w}, \\ & \alpha_B \in [0, 1]^{B \times L}, \\ & \alpha_U \in [0, 1]^{U \times L}. \end{aligned} \quad (11)$$

In the relaxed problem we subtract from the cost function a term which corresponds to the restriction (≤ 0) multiplied by the Lagrangian multipliers (≥ 0), so the resulting solution is a lower bound of the original problem optimum. Then, we have to maximize over λ in order to reach the optimum α^* we are seeking, which results in this two-level optimization problem:

$$\begin{aligned} \max_{\lambda} \quad & \min_{\alpha} C(\alpha) - \mathbf{1}_{B+U}^T \cdot (\lambda^T \odot (\mathbf{A} \cdot \alpha^T - \mathbf{1}_{Q \times (B+U)})) \cdot \mathbf{1}_{B+U} \\ \text{s.t.} \quad & (\alpha_B \odot \mathbf{C}_B \odot \mathbf{H}_B) \cdot \mathbf{1}_B + (\alpha_U \odot \mathbf{C}_U) \cdot \mathbf{1}_U \geq \mathbf{d}, \\ & (\alpha_U \odot \mathbf{C}_U) \cdot \mathbf{1}_U \geq \mathbf{w}, \\ & \alpha_B \in [0, 1]^{B \times L}, \\ & \alpha_U \in [0, 1]^{U \times L}. \\ \text{s.t.} \quad & \lambda \geq 0 \end{aligned} \quad (12)$$

Through this relaxation, we can separate the optimization problem in two levels. We shall call $g(\lambda)$ the solution of the relaxed problem (11) for a given value of λ . At a higher level, we have the master dual problem which corresponds to the update of the Lagrange multipliers λ , variables of the dual problem:

$$\begin{aligned} \max_{\lambda} \quad & g(\lambda) \\ \text{s.t.} \quad & \lambda \geq 0 \end{aligned} \quad (13)$$

Then, at a lower level, and assuming we have a separable cost function, we can decompose the optimization in one sub-problem for each link l . We shall omit from the cost function the constant term in α , so the sub-problem for link l takes the form:

$$\begin{aligned} \min_{\alpha_l} \quad & C_l(\alpha_l) + \sum_{q \in Q_l} \lambda_q \alpha_l \\ \text{s.t.} \quad & \sum_{b=1}^B \alpha_{bl} c_{bl} h_{bl} + \sum_{u=1}^U \alpha_{ul} c_{ul} \geq d_l, \\ & \sum_{u=1}^U \alpha_{ul} c_{ul} \geq w_l, \\ & \alpha_{bl} \in [0, 1], b = 1, \dots, B, \\ & \alpha_{ul} \in [0, 1], u = 1, \dots, U. \end{aligned} \quad (14)$$

where Q_l are the subset of the collision domains where the link l is included, and λ_q is the row q of the Lagrange multipliers matrix λ . It is worth to note that given the value of λ_q this problem can be solved locally by the link, as it has all the other necessary information. That is to say, both the estimation of the h_{bl} distribution parameters as well as the effective capacity values (c_{ul} and c_{bl}) are calculated locally, and they are directly used in the optimization, without need to forward them to any other node.

With this approach we actually solve the dual problem, so it will only work properly if we have strong duality, which holds if the original problem is convex and with strictly feasible solutions (which is commonly known as the Slater's condition [39]). If the function $g(\lambda)$ is differentiable, then the master dual problem can be solved with a gradient method [39]. Thus, the update of the Lagrange multipliers following this method is given by:

$$\lambda_{qb}^{t+1} = \left[\lambda_{qb}^t + \sigma \cdot \left(\frac{\partial g}{\partial \lambda_{qb}} \right)_t \right]^+ \quad (15)$$

where t is the iteration index, σ a positive suitable step-size (sufficiently small), and the projection $[\cdot]^+$ ensures the new

value to be non-negative. Substituting by the corresponding gradient we reach the following:

$$\lambda_{qb}^{t+1} = \left[\lambda_{qb}^t + \sigma \cdot \left(\sum_{l \in q} \alpha_{bl} - 1 \right) \right]^+ \quad (16)$$

In summary, the relaxed problem $g(\lambda)$, for $\lambda \geq 0$, can be decomposed as:

$$g(\lambda) = \sum_l g_l(\lambda) + \mathbf{1}_{B+U}^\top \cdot \lambda^\top \cdot \mathbf{1}_Q \quad (17)$$

where $g_l(\lambda)$ is the subpart of the dual problem corresponding to link l . The dual decomposition results in each link l solving, for the given λ ,

$$\alpha_l^*(\lambda(t)) = \arg \min_{\alpha_l \geq 0} C_l(\alpha_l) + \sum_{q \in Q_l} \lambda_q \alpha_l, \text{ s.t. constraints,} \quad (18)$$

which is unique for strictly convex cost functions [39]. The gradient method ensures the dual variable $\lambda(t)$ will converge to the dual optimal λ^* as $t \rightarrow \infty$. Since the duality gap for the original problem is zero (as Slater's condition is satisfied) and the solution to the subproblems is unique, the primal variable $\alpha^*(\lambda(t))$ will also converge to the primal optimal variable α^* .

4.2. Proposed algorithm architecture

From the distributed optimization presented in the previous section we arrive to a decentralized implementation of the algorithm, according to the architecture described below. We say that it is a decentralized solution following the taxonomy described in [23] where it is stated that the allocation is performed by *more than one but not all of the nodes within the network*. In particular we work with a cluster-based solution where each cluster corresponds to a collision domain in the WMN. Each collision domain has a domain referent which is the head cluster in the proposed algorithm architecture.

In Fig. 3 we can see the proposed hierarchy, where the lower level correspond to links, and the next level to the head clusters, which are the collision domain referents. It is worth to note that one link can belong to one or many collision domains as is shown in the example. In this case, the communication during the optimization should be with all the domain referents corresponding to all the collision domains where it belongs. This way it will receive all the updated prices for the several collision domains where it takes part of. Summarizing, the distributed optimization with the proposed architecture is solved with the following algorithm:

Dual decomposition algorithm for spectrum allocation

- Parameters: each link l estimates local effective capacities c_{bl} and c_{ul} , and local PUs' activity statistics computing the mean and the variance of h_{bl} .
 - Inputs: each link l has its own capacity requirements for data and control traffic, given by d_l and w_l respectively.
 - Hierarchy: each collision domain has a predefined domain referent.
 - Initialization: set $\alpha = 0$ and $\lambda = 0$.
 - 1. Each link locally solves the spectrum allocation by computing the optimum of the corresponding lower level subproblem α_l , which is communicated to each domain referent.
 - 2. Each domain referent receives the pre-computed allocation for each link, updates the prices accordingly, and then, it broadcasts the new prices within the domain.
 - 3. If *stop condition* = *false* go to step 1, else END.
-

4.3. Implementation issues

For the purpose of an actual implementation of the proposed method there are some issues to solve in a real WMN. In this section we will discuss possible solutions to these issues. First, we must resolve the conflict graph construction in order to find all the collision domains in the WMN. We envision several ways to do this, ranging from a planned solution at the deployment phase up to a distributed graph construction solution. Then, the next point which is related to the above, is to define who is the referent node in each collision domain. Finally, we will comment on the possibilities to implement the domain referent assignment, either by one or several nodes in the network, or even without being a physical solution but a distributed communication protocol. This is related to how the collision domain referents communicate with each other.

Starting with the conflict graph construction, on one hand, it is possible to pre-compute it during the network design stage. This graph can also be verified with measurements during the links' installation. This way, it is possible to know a priori all the interference conflicts. On the other hand, we can leverage on the sensing capabilities of the nodes in the network² to detect interferent links and communicate this information to a predefined central entity. With such information from every link centralized in a fusion center, this entity is able to construct the conflict graph.

The next step is to obtain the maximal cliques of the conflict graph, which correspond to the collision domains we are looking for. To solve this problem, which is commonly known as the maximal clique problem, we can use an efficient implementation of the well-known Bron-Kerbosch algorithm [40]. Once we have the complete list of collision domains, we have to proceed to select the referent for each one. To do this we can use as the first selection criterion those nodes that are in a higher number of collision domains, in order to simplify the system architecture, as we will have fewer referents. Then we can simply use an arbitrary criterion, e.g. the higher MAC address or just a pseudorandom selection, just to keep a unique

²Remember we are assuming that each node has a dedicated interface for sensing purposes.

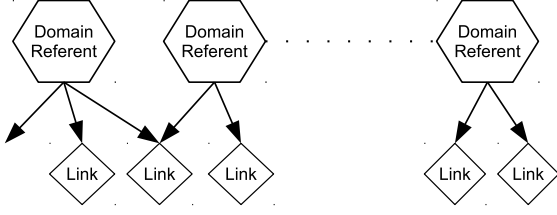


Figure 3: Proposed architecture for the decentralized implementation.

referent per collision domain. Finally, when all the collision domains have the corresponding referent, we are able to carry out the periodic spectrum allocation, following the distributed algorithm described before.

Concerning the domain referent assignment, one possibility is to choose a particular node (or a set of redundant nodes) that will in turn select the referent for each collision domain. In any case, its role would only be important at the beginning of the network operation. Then, it would only be necessary to recompute if changes in the network topology occur, which depends strongly on whether the network nodes are fixed or mobile. As the main case of interest for us is with fixed nodes, it is unlikely that the central entity has much activity once the network is operative. At the other extreme, we can think of a completely decentralized solution, starting from a distributed mechanism for the conflict graph construction as presented in [41]. Then, after the conflict graph is known by every node in the network, each node can obtain the domain referents following the same procedure described before. This way, every node in the network will know who the referents are, including the referents themselves, without need to be informed by a central entity.

5. Simulation experiments

In order to test the proposed framework we consider three simulation experiments. In the first case we evaluate the algorithm for a single point to point link, so it is the case with only one collision domain. Then, we test the method for a simple network with four nodes and three links, now with two collision domains. Finally, the last experiment corresponds to the topology of a real network which is part of the Plan Ceibal's [17] rural Internet access deployment. In all the simulations the number of frequency bands used seeks to reflect a real-world situation, taking a quantity of unlicensed spectrum of similar order than the number of 5GHz U-NII bands³, and on the other hand a considerable amount of licensed spectrum, which might correspond to TV or cellular frequency bands. A total of 50 frequency bands is considered for the first simulation, analyzing both the fixed case of 15 unlicensed and 35 licensed bands, and also varying the proportion of bands of each type. For the latter two experiments we sought to simulate a tighter situation with fixed amounts of each kind of spectrum, with 25 licensed bands

³From 6 to 24 non-overlapping WiFi channels in the US, depending on the channel bandwidth considered.

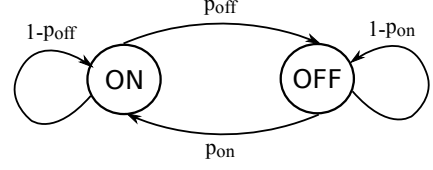


Figure 4: Two-state On-Off DTMC spectrum occupancy model.

and 15 unlicensed, totaling 40 bands. The effective capacities for each band are all drawn from a uniform distribution at the beginning of each experiment, and remain the same during all the simulation.

For the experiments we set as goal to minimize the total assigned spectrum, which might be a suitable objective for the SUs. Thus, if all SUs operate with this objective, it ensures to have a friendly coexistence of multiple devices from different networks, sharing all the available spectrum. This leads us to use the following cost functions:

$$C_{lic}(\alpha_B) = \sum_{l=1}^L \sum_{b=1}^B \alpha_{bl} \quad \text{and} \quad C_{unlic}(\alpha_U) = \sum_{l=1}^L \sum_{u=1}^U \alpha_{ul} \quad (19)$$

Anyway, it is just an example to illustrate the algorithm operation and the proposed framework is more general, enabling to consider other targets of interest that would lead to different cost functions.

In order to model and simulate the PUs' activity in licensed bands, we consider a two-state On-Off discrete time markov chain (DTMC) spectrum occupancy model (see Fig. 4), which has been proved to be suitable [42, 43]. The parameters involved in the model are the transition probabilities p_{on} and p_{off} , which will determine the average busy and non-busy time, $\pi_{on} = p_{on}/(p_{on} + p_{off})$ and $\pi_{off} = p_{off}/(p_{on} + p_{off})$, respectively. While it is not necessary for the implementation of the algorithms, as a measurement-based estimation is sufficient, it is possible with this model to obtain closed-form expressions of $E\{h_{bl}\}$ and $\text{Var}\{h_{bl}\}$ from the model parameters.

All bands are considered of equal spectrum bandwidth, each with a generic value BW. Each simulation is performed for a total time period of 1000 T, where T (also a generic value) is the time interval between spectrum allocations. Finally, we use the DTMC spectrum occupancy model to simulate the PUs' activity, completing a total of 20 transitions during each interval, a fixed value used for all the experiments. The initial occupancy for each licensed band is drawn in all cases from the corresponding stationary distribution π_{on} , in order to start each simulation already at steady state. In order to solve the optimization involved in each method we use CVX [44], working with MOSEK [45] as solver.

As reference results we consider the solution to the proposed problem when the realizations of h_{bl} are known in advance. We shall call this method the fortune teller (FT). We also include as reference another simple approach to solve the problem, which we shall note as CONS (for conservative), and consists of assigning only unlicensed bands to meet the requirements. It is clear that this assignment is the safer one concerning the PUs,

FT: fortune teller (knows h_{bl} realizations in advance)
 EXP: expectation based approach
 CONS: conservative (only uses unlicensed spectrum)
 ROB- ϵ : robust approach (ϵ - value taken by the parameter)
 IND-EXP: individual decision for each link using EXP
 IND-ROB- ϵ : individual decision for each link using ROB- ϵ

Table 2: Algorithms considered for performance comparison.

but it has the disadvantage of missing out on using all the available licensed bands. Furthermore, it cannot solve the problem when the unlicensed spectrum is not enough to reach the throughput lower bound.

To reference the proposed algorithms, we shall call EXP the expectation based approach with a mean value capacity constraint. On the other hand, we shall call ROB- ϵ the one that takes the robust deterministic equivalent constraint, where ϵ indicates the value taken by the parameter. Finally, for the cases with multiple links, we also consider the possibility that each link takes a decision individually. We will note that methods IND-X, where X corresponds to the algorithm that each link uses to perform the spectrum allocation (e.g. EXP or ROB- ϵ). A summary of the aforementioned methods, which will be referenced throughout the simulations, is presented in Tab. 2

For performance comparison we analyze in all cases the spectrum allocated and the average effective capacity resulting from the assignment. We also study the short-term effectiveness (indicated as STE in the results) of the proposed methods, which is the percentage of time intervals where the effective capacity assigned is above or equal to the defined lower bound. Throughout the simulations we will see that although the expected value approach meets the requirements in average, and is the most efficient regarding spectrum usage, robust approaches perform much better at short scales, with a reasonable extra cost in terms of spectrum bandwidth allocated.

5.1. Single domain spectrum allocation

The first example corresponds to the single domain case, which is the suitable model for a single point-to-point link. In this experiment we consider 15 unlicensed bands and 35 licensed ones, and we analyze the algorithm allocation for different values of p_{on} and π_{on} . Then, we vary the proportion of bands of each type, and we study the algorithm allocation for different values of d , now with fixed values of p_{on} and π_{on} . The capacities are taken from a uniform distribution between 5 Mbps and 25 Mbps for unlicensed bands, and values 50% higher for licensed bands. Typically unlicensed bands would be more crowded, so we try to reflect this fact in the selected capacity values for each band.

In Fig. 5 we show an example simulation with parameters $p_{on} = 0.01$ and $\pi_{on} = 0.1$, the same for all licensed bands. As we said before, each simulation lasts 1000 T, while in the example figure we only show 40 T for a matter of clarity. The throughput lower bound d is set at 240 Mbps, somewhat below the total unlicensed bands' capacity which is 248 Mbps. In all the simulated situations for this single domain case, only using unlicensed spectrum is enough to meet the requirements,

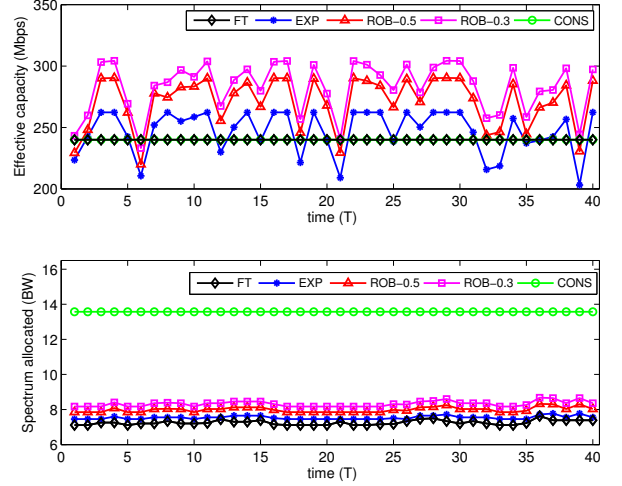


Figure 5: Experiment example with $d = 240$ Mbps, $p_{on} = 0.01$ and $\pi_{on} = 0.1$.

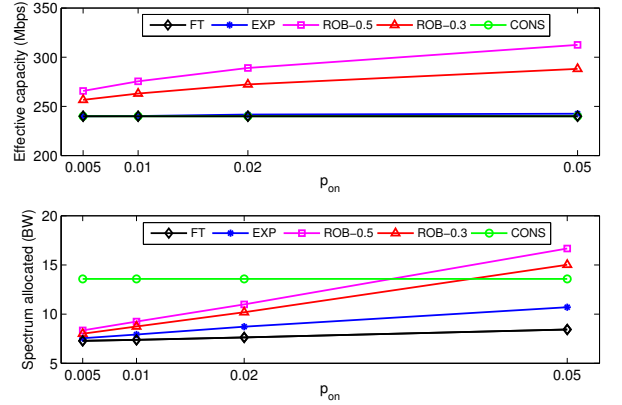


Figure 6: Effective capacity and spectrum allocated as a function of p_{on} when $\pi_{on} = 0.1$ and $d = 240$ Mbps (total unlicensed capacity = 248 Mbps).

which allows to get a solution with CONS. Notice that CONS and FT are superimposed in the capacity plot, as they both solve a deterministic optimization problem and reach the equality in the constraint, assigning exactly the required demand d . Furthermore, it can be seen that ROB-0.3 allocates more spectrum than ROB-0.5, since a smaller ϵ implies more robustness (and thus more spectrum required), and both assign more spectrum than EXP, which is the least robust one.

We first analyze the results for different values of p_{on} (see Figs. 6 and 7). As we can see all the methods meet the throughput lower bound in average, something we ensure by placing it as a constraint in the problem formulation. Looking at the spectrum assignment, the stochastic approaches clearly outperform CONS, with better spectral efficiency and closer to the FT optimum solution as soon as p_{on} goes to 0. It is clear that for lower values of p_{on} is when these methods make better sense, as it indicates higher possibilities of making profit from licensed

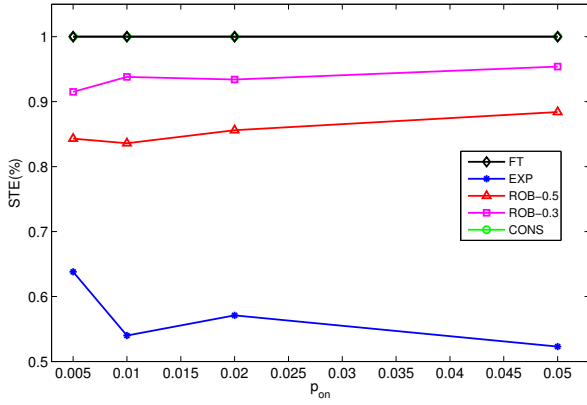


Figure 7: STE as a function of p_{on} when $\pi_{on} = 0.1$ and $d = 240$ Mbps (total unlicensed capacity = 248 Mbps).

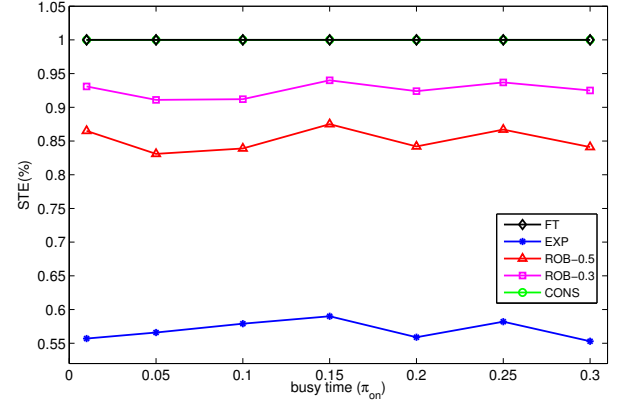


Figure 9: STE as a function of π_{on} when $p_{on} = 0.01$ and $d = 240$ Mbps (total unlicensed capacity = 248 Mbps).

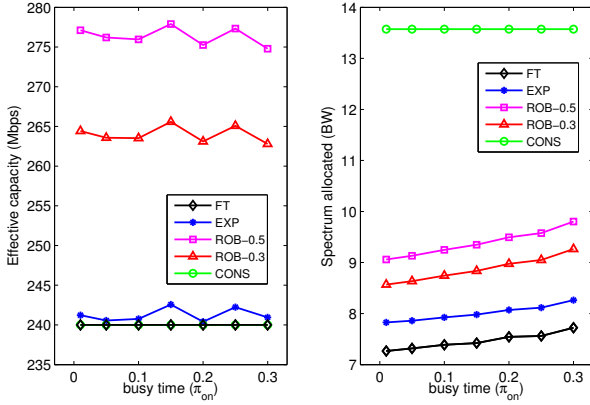


Figure 8: Effective capacity and spectrum allocated as a function of π_{on} when $p_{on} = 0.01$ and $d = 240$ Mbps (total unlicensed capacity = 248 Mbps).

bands. While robust approaches allocate more spectrum than the expected value solution, in exchange they get much better performance at short scale. The average success rate is between 90% and 95% for ROB-0.3 and between 83% and 90% for ROB-0.5, while EXP is always below 65%. The extra spectrum assigned in average by robust approaches implies some average capacity overallocation with respect to the stated throughput lower bound. However, this mild conservatism, allocating not much more spectrum than FT, is what enables a higher probability to meet the throughput lower bound also at the short scale.

Now, we analyze the performance for different busy times (see Figs. 8 and 9). We have again a clear advantage of the stochastic methods against CONS, with less spectrum allocated to meet the same requirements. Furthermore, the advantage is higher for lower values of π_{on} , which are the most interesting situations to benefit from licensed spectrum. Robust approaches present again some average capacity overallocation, which is higher for lower values of π_{on} . In return, their short term effectiveness stands out again, with an average success

rate of 93% and 86%, for ROB-0.3 and ROB-0.5 respectively, against a poor 57% for EXP. This implies that, although the EXP solution meets the requirements in average, more than 40% of the time the effective capacity assigned is below the stated throughput lower bound. Except for particular cases, where an expectation based solution might be sufficient, we argue instead that a robust approach will be more suitable in practice, with much higher short term performance at a reasonable cost in terms of spectrum.

Lastly, we set as fixed values $p_{on} = 0.01$ and $\pi_{on} = 0.1$, and we vary the number of unlicensed bands (from 10 to 20), keeping the same total number of bands (50) for all cases. This way, the total unlicensed bands' capacity changes and we consider for each case a throughput lower bound equal to 90% of its value. In Fig. 10 we can see the spectrum and effective capacity overallocation compared to the FT optimum. The stochastic methods clearly outperform CONS, with higher advantage for lower minimum throughput requirements, which is an expected result, as in this case it corresponds to a situation with more licensed bands. As the proportion of unlicensed spectrum gets higher, the benefit from using available licensed bands is lower, but it is still worth using it for reaching greater spectral efficiency. When we look at the short scale performance (see Fig. 11), we can see again that the proposed robust approach clearly outperforms the expectation based solution. While ROB-0.3 achieves an average success of 92% and ROB-0.5 reaches 85%, EXP only gets a poor 58%. Furthermore, the price for that better performance is only between 6% to 15% more spectrum assigned than EXP, and between 10% to 25% more than the lower bound defined by the FT solution. It is neither too much if we look at capacity overallocation, with only between 8% to 18% more than the optimum FT solution.

5.2. Wireless network with four nodes and three links

The network considered for the second experiment is the one shown in Fig. 12, where links 1 and 2 interfere with each other, and the same happens between links 2 and 3. This gives us a total of two collision domains in the network, the two maximal

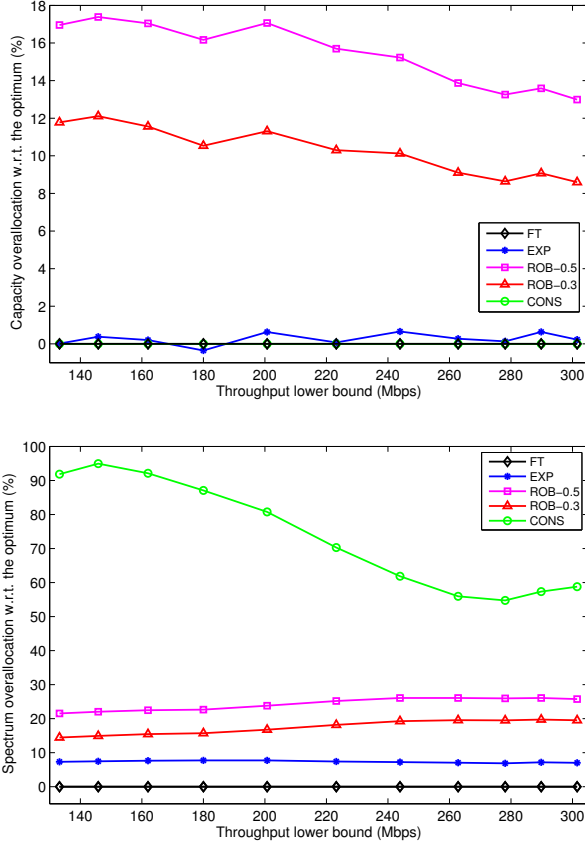


Figure 10: Simulation results for $d = 0.9 \cdot \sum_u c_u$ ($p_{on} = 0.01$ and $\pi_{on} = 0.1$).

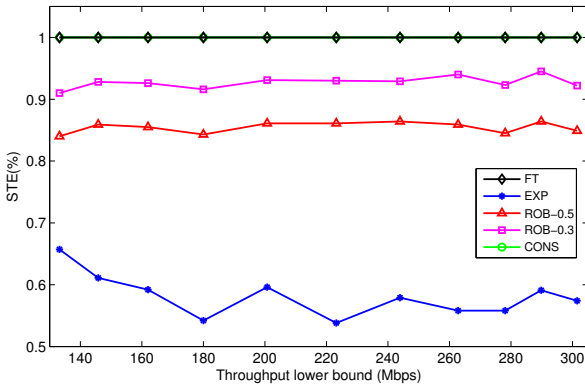


Figure 11: STE for $d = 0.9 \cdot \sum_u c_u$ ($p_{on} = 0.01$ and $\pi_{on} = 0.1$).

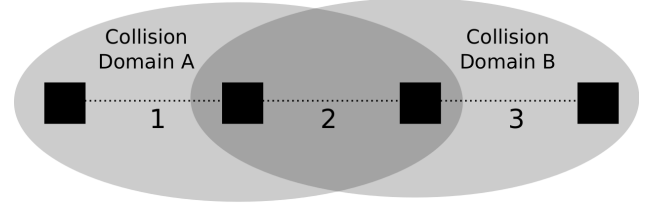


Figure 12: Wireless network with 3 links and 2 collision domains.

cliques of the conflict graph, whose adjacency matrix is shown below:

$$CG = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

In this experiment we consider less spectrum than before, with 15 unlicensed bands and 25 licensed bands, totalizing 40 frequency bands. The capacities are all drawn from the same uniform distribution but with independent values for each link, and biased again with higher values for the licensed bands ($\sim 60\%$ more than unlicensed bands). In this case the throughput lower bound considered for each link is beyond the total capacity of unlicensed bands, so there is no possible solution using the conservative approach. We repeat the analysis from the previous experiment, varying the activity of the primary users through the values of p_{on} and π_{on} .

Now, an important thing to clarify is how to proceed with the evaluation of the STE performance indicator for a WMN. As we now have several links in the network, we can independently reach or not the throughput lower bound in each of them, so we will consider two different STE values. On the one hand we have the average STE (A-STE), which is the average over all the links' STE individual values. On the other hand we have the global STE (G-STE), which is the percentage of time intervals where the effective capacity assigned is above or equal to the defined lower bound on all the links. The difference is that whereas in the former case are counted as successful all the situations when the capacity constraint holds in each link independently, in the latter are only counted as successful the cases when the constraint is accomplished in all the links simultaneously. That said, we can now comment on the results presented in Figs. 13 and 14.

The first thing to notice is that we have again an increasing amount of spectrum allocated as p_{on} or π_{on} rises. If we look at the results for the individual methods, where each link makes a decision on its own, we can see that while the amount of spectrum allocated is similar than the other methods, they have both a null performance considering the STE. The reason for this fact is that no coordination between links is done, so two links in the same collision domain can assign the same frequency band ignoring the other, which results in less capacity than expected for each of them. Thus, with this kind of assignment we are always below the required capacities with a probability close to one.

Comparing the performance of the expected value approach and the proposed robust schemes, it becomes clear again look-

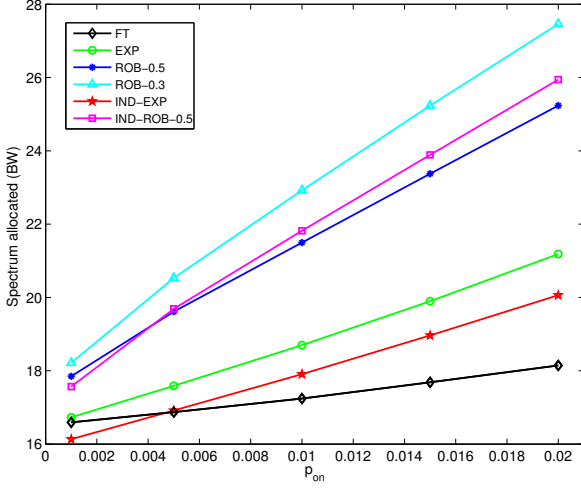


Figure 13: Total spectrum allocated as a function of: p_{on} .

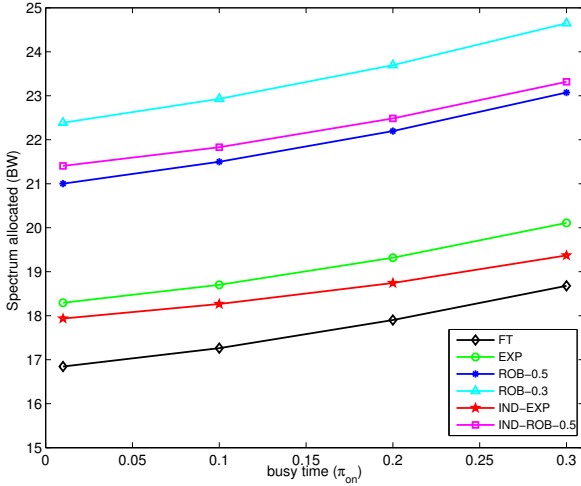


Figure 14: Total spectrum allocated as a function of: π_{on} .

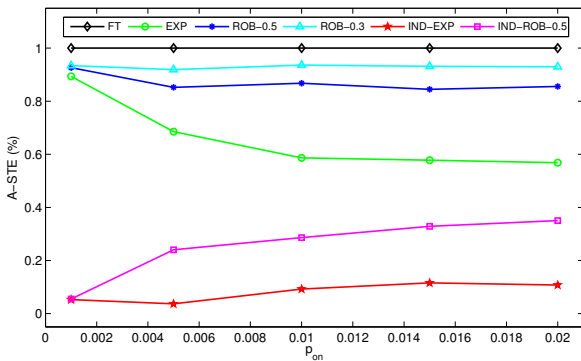


Figure 15: A-STE as a function of p_{on} .

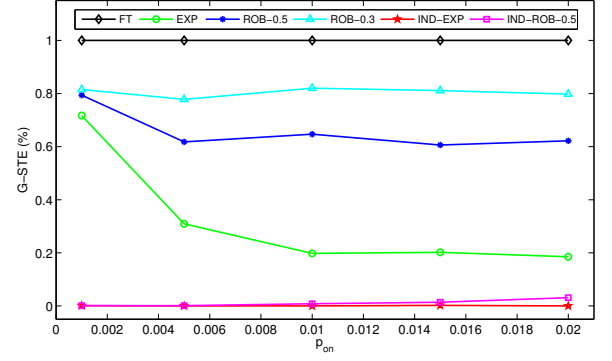


Figure 16: G-STE as a function of p_{on} .

ing at the results in Figs. 15 and 16 the advantages of the latter. In one case, as p_{on} rises, we have a G-STE of 60% and 80% for ROB-0.5 and ROB-0.3 respectively, against a 20% for EXP. On the other hand, the extra spectrum required to reach this robustness is only 20-30% more than what EXP assigns, and 50-60% more than the optimum only reachable by a diviner. Similar results are obtained when we analyze the case where π_{on} varies, hence they are not included in the paper. An important property to note in the algorithms comparison is the performance invariance with respect to the PUs activity parameters, which is noticed when we look at the flatness of the STE curves. This fact implies that when the PUs dynamics statistics are well estimated, the algorithm performance does not depend of it, which is a nice property of the spectrum allocation framework developed.

While the focus of the paper is not the particular optimization algorithm used, some comments regarding its convergence are in order. The number of iterations of each algorithm run depends on the accuracy desired for the assignment variables. As on each iteration the updated values have to be sent to each domain referent, this value will determine the control plane traffic load generated in the network. Based on our simulation experiments, typically 200 iterations are enough to reach an adequate precision in the allocation variables. We consider that this number of iterations is completely reasonable, taking into account that the algorithm is designed to run periodically at a medium to large timescale, and would not overload the network in a real-world implementation.

5.3. Real-world network example

The last simulation experiment corresponds to a real-world network, which is taken from a real deployment of rural Internet access for schools from Plan Ceibal [17]. In this case the network is composed by 13 nodes and 17 links. In Fig. 17 we can see the network topology, while Fig. 18 illustrates the corresponding conflict graph. In this case we have a total of 16 maximal cliques in the conflict graph, which are all the collision domains in the WMN.

For this experiment we consider the same spectrum than for previous case, with a total of 40 frequency bands, 15 unlicensed

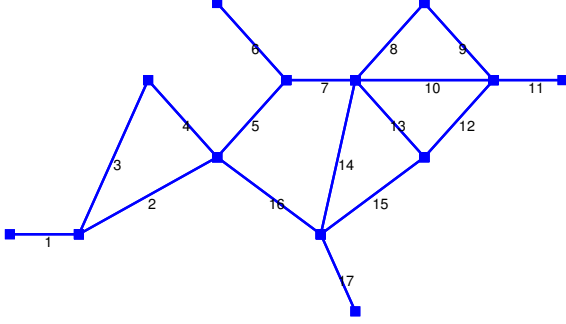


Figure 17: Real-world network example topology.

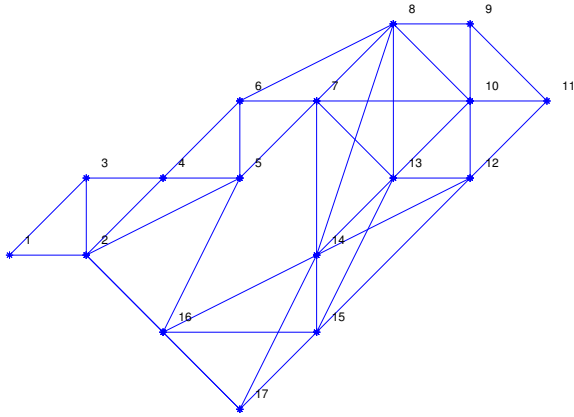


Figure 18: Real-world network example conflict graph.

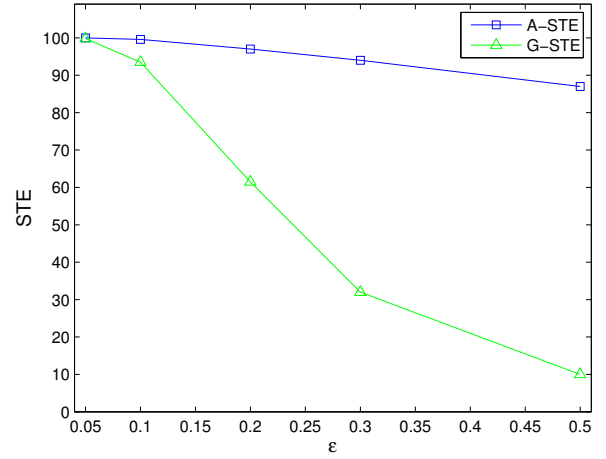


Figure 19: A-STE and G-STE for different values of the parameter ϵ .

Method	small bias		large bias	
	A-STE (%)	G-STE (%)	A-STE (%)	G-STE (%)
FT	100	100	100	100
ROB-0.05	99.988	99.8	99.96	99
ROB-0.1	99.6	93.5	99	92
ROB-0.2	97	61.5	96	50
ROB-0.3	94	32	92	26
ROB-0.5	87	10	85	6
EXP	74	1	64	0

Table 3: A-STE and G-STE comparison for the different methods.

and 25 licensed. In this case the parameters of the PUs' activity are fixed, with $p_{\text{on}} = 0.01$ and $\pi_{\text{on}} = 0.1$. We use again a uniform distribution for the capacities, from which independent values were drawn for each link, and we also maintain the bias in favour of licensed bands, but now two different situations are considered. In the first case licensed bands have $\sim 60\%$ more capacity than unlicensed bands, while in the second case the bias is larger, reaching $\sim 160\%$ extra capacity. As in the previous case the total capacity of unlicensed bands is not enough to reach the throughput lower bound considered for each link, so there is no possible spectrum allocation using the conservative approach. In this experiment we also evaluate other values of ϵ , the robust algorithm parameter, in order to gain knowledge on how one should choose its value.

In Tab. 3 the results are summarized for both cases, the one with smaller and the one with larger bias against unlicensed bands capacities. As we can see, in both cases the performance increases as ϵ decreases, and the expected value approach is even worse than all of them. We also note that the fall of G-STE is much higher than that of A-STE, due to its exponential dependence on the number of links. In Fig. 19 the comparison is illustrated for the case of a smaller bias. Based on these results one should choose the parameter ϵ according to the performance required in the specific network operation.

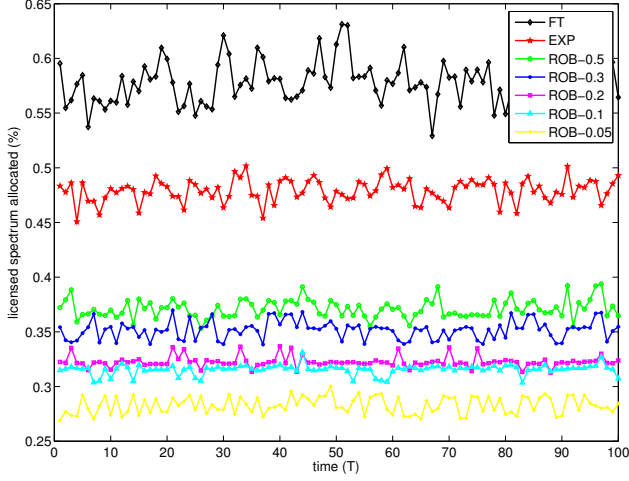


Figure 20: Proportion of licensed spectrum allocated for the different methods, when we have a smaller bias against unlicensed band capacities.

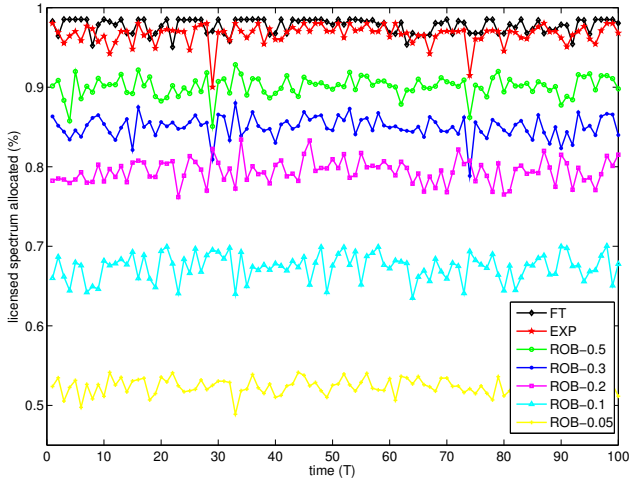


Figure 21: Proportion of licensed spectrum allocated for the different methods, when we have a larger bias against unlicensed band capacities.

Finally, it is worth to note that for a larger bias, and taking into account the same value of ϵ , the performance is worse. This may seem counterintuitive at first, but it becomes clear when we look at the percentage of licensed and unlicensed spectrum assigned in each case. What happens is that when the bias is larger, it is more convenient to assign a greater proportion of licensed spectrum, but it also implies a higher risk, because these bands may become occupied. In Figs. 20 and 21 we can see the evolution (100 first T of the simulation) of the licensed spectrum proportion assigned, which are clearly higher for the larger bias case. There is a compromise between the increased use of licensed spectrum, and the risk it takes to assign these bands. This is also clear when we look at the variation of the licensed spectrum proportion assigned with respect to the different values of ϵ . The more robust is the allocation, the lower the proportion of licensed spectrum assigned, and therefore higher the proportion of unlicensed spectrum. The latter cannot be *occupied* by a PU and hence the effective capacity is 100% available all the time. Remember that the effective capacity considered for each unlicensed band, already takes into account the time sharing with other interferent networks.

6. Conclusions and Future Work

The spectrum allocation in a cognitive wireless mesh network (WMN) was studied, considering a mixed scenario with both type of frequency bands, licensed and non licensed. The problem was analyzed from the perspective of SUs, which might use licensed bands whenever available and unlicensed bands all the time. We developed a general stochastic formulation considering a periodically scheduled assignment. We proposed a novel robust approach to solve the problem and analyzed the advantage against an expectation based solution, comparing their performance by extensive simulations. We also presented a decentralized implementation of the proposed framework, allowing the algorithm to scale properly. We believe the proposed solution is suitable for WMN Internet access solutions, as the one aforementioned from Plan Ceibal's schools, in order to meet the capacity requirements they will face in the coming years.

The results show that the proposed solution presents much better performance than the expectation based approach, with not much additional spectrum allocated. The robust approach guarantees the required throughput in each link with very high probability, while with a mean value solution the requirements are not accomplished more than 40% of the time. The algorithm performance was analyzed in depth, for different situations of the PUs' activity, and the simulations experiments indicate that the extra spectrum required to guarantee the algorithm robustness, is usually below 35% of that required by an oracle that knows beforehand the PUs' activities. Finally, we simulated the proposed scheme in a real-world network, analyzing the operation for different values of the algorithm parameter ϵ and looking how it should be chosen to meet certain previously specified requirements.

In this paper we considered a spectrum allocation framework, with fixed *a priori* requirements for each link in the WMN. In

our future work, we would like to extend this framework, considering a cross-layer approach, which integrates this framework, with automatic selection of dynamic requirements on each link, based on real-time network demand measurements. It would be also interesting to compare the periodically scheduled allocation proposed with an event-driven solution. We could analyze which is the threshold in the PUs' dynamics when signaling overhead of the latter becomes tolerable to get a more efficient spectrum allocation. We believe that each approach may have advantages and disadvantages, so it would be worth to do a thorough comparison of the two alternatives. Another point that can be explored in depth is the choice of different cost functions for specific requirements. This paper presents a general framework, which is then simulated for a particular case. It would be interesting to look for other possible cost functions, according to different practical requirements, to analyze how the same framework could be applied and what are the results we can obtain.

A couple of additional points that could also be analyzed in the future, one more theoretical, the other more practical, are on the one hand the robust equivalence and on the other hand the optimization algorithm. While we considered an approach which is robust with respect to the distribution of the PUs' activity, it could be possible to find better equivalences if we know more about it. Maybe in a particular case, with an adequate statistical model of the PUs' dynamics, it is possible to take advantage of this information for a better solution. About the optimization itself, this article was not focused on it and we just used a gradient descent algorithm. Hence, it is possible to look for efficient alternatives, particularly adapted to the proposed scheme.

Finally, the next stage in our line of research would be to implement the algorithm in a real field deployment, for example using WiFi bands as unlicensed spectrum and TV bands as licensed spectrum (e.g. TVWS technologies).

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