Analysis and Characterization of Dynamic Spectrum Sharing in Cognitive Radio Networks

Claudina Rattaro, Laura Aspirot and Pablo Belzarena
Universidad de la República
Montevideo, Uruguay
Email: crattaro@fing.edu.uy, laspirot@ccee.edu.uy, belza@fing.edu.uy

***Abstract*—Cognitive Radio Networks have emerged in the last years as a solution of two problems: spectrum underutilization and spectrum scarcity. The main idea is to dynamically reallocate unused licensed frequency bands to secondary users. The focus of this work is on the analysis and characterization of a dynamic spectrum sharing mechanism where primary users have strict priority over secondary ones. We present some tools and criteria that can be used in order to improve the mean spectrum utilization with the commitment of providing to secondary users a satisfactory grade of service and a small interruption probability. Our proposal is based on the application of the fluid limit technique to analyze a stochastic complex system. We support our analysis with representative simulated examples.**

***Keywords*—Cognitive Radio Networks, Fluid Limit, Spectrum sharing**

I. Introduction

Nowadays, with the rapid development of wireless communications, the demand on spectrum has been growing dramatically resulting in the spectrum scarcity problem: unlicensed bands are too crowded while licensed bands are vastly underutilized [1][2][3]. Cognitive Radio Networks (CR) has been proposed as a promising technology to solve that problem by an intelligent and efficient dynamic spectrum access [4][5]. In this new paradigm we can identify two classes of users: primary (PU) and secondary (SU). PUs are the licensed users, they have allocated a certain portion of spectrum. SUs (also called cognitive users) are devices which are capable of detecting unused licensed bands and adapt their parameters for using them.

One challenge today is to distribute the spectrum holes efficiently and fairly. Another goal is to guarantee quality of service (QoS) to the SUs. In this work, we consider a scenario with C subchannels to be distributed between SUs and PUs, and where PUs have strict priority. That is to say, if a PU arrives when all the subchannels are in use, one of the SUs will be deallocated immediately. An example of an application is a cellular network that employs frequency division duplexing where the operator has C frequency bands (subchannels) to be assigned to its users (PUs). Another example is the digital TV spectrum bands. In both scenarios, if there are free subchannels, the SUs could use them with the constraint that their communications can be interrupted at any time. In the last example, each TV channel has its own frequency band, so we assume that when a PU arrives while a SU is using its subchannel, and there are free subchannels, this SU must be moved instantly to another band, without any consequence to its service. If there isn't a free subchannel (the C subchannels are busy), the SU's communication will be interrupted with consequences to its QoS. In a cellular network, PUs can use any of the C subchannels, then, the mobility of the SUs is unnecessary. Our model takes into account only the number of subchannels that are being used by PUs and SUs. Therefore, if there are free subchannels, the case when a PU arrives to a specific subchannel and a SU must be moved instantly to another free subchannel will be modeled as if the PU arrives to a free subchannel.

We are interested in SUs whose service cannot be interrupted with high probability (like a phone call or other interactive services). For these services it is preferable to let the connection be rejected to avoid the situation where the connection is established and then interrupted. These decisions (enter or not) represent a mechanism that can be adopted by the SUs as an admission control policy.

In this paper we analyze two features of these type of systems: the mean spectrum utilization and the probability that the SUs services can be interrupted. Associated with this last issue we analyze a possible admission control policy in order to reduce this probability.

We model the cognitive radio network as a two dimensional continuous time Markov Chain (CMTC). A fluid model approach is used to analyze the stochastic system with an ordinary differential equation (ODE) that approximates it. One of the main results is that the position of the ODE's fixed point is decisive in defining an effective operating point of the system. In addition we show that in many cases, a SU's admission control mechanism is required in order to ensure a low probability of service interruption.

There are some related works that use fluid model approaches applying to CR. Some representative examples are [6][7] and [8]. In particular, in [6] the authors use a fluid model to study SU's queuing delay performance. In [7] they study the coexistence of two wireless networks with different priorities and compare throughput and delay obtained in both networks. On the other hand, in [8], they focus on the collaborative sensing within the SUs and its impact in its QoS. In these papers the authors analyze the delay and throughput in different CR

This work was financially supported by CSIC (I+D Groups) and a CAP-UdelaR scholarship.

scenarios, so their results can be complementary to ours.

The paper is organized in the following way. In section II we describe our model of spectrum sharing in CR networks. Section III presents the fluid model and in section IV we show our analysis and characterization of the behavior of the system. Finally, we conclude and discuss future work in section V.

II. Stochastic Model

In this section we introduce our stochastic model for the number of primary and secondary users in the system. We also model the possibility of admission control decisions when a SU arrives to the system (SUs shall decide, depending on the state of the system, whether to enter or not). Without loss of generality, we associate one user with one channel.

The model assumptions are the following:

- $X_1(t)$: number of PUs at time t
- $X_2(t)$: number of SUs at time t
- C: total number of channels, therefore, the state space is $\{(X_1,X_2)/0 \le X_1 \le C, 0 \le X_2 \le C, X_1+X_2 \le C\}$
- λ_1, μ_1: arrival and leaving rates for PUs respectively (independent Poisson arrivals and exponentially distributed service times)
- λ_2, μ_2: arrival and leaving rates for SUs respectively (independent Poisson arrivals and exponentially distributed service times).
- $a(X_1,X_2)$: admission decision in each state ($a(X_1,X_2) \in \{0,1\}$). If $a(X_1,X_2)=1$ and a SU arrives, it will enter, and when $a(X_1,X_2)=0$, it will not.

Thus the stochastic process $(X_1(t),X_1(t))$ has transition rates $q((X_1,X_2),(X_1',X_2'))$, from state (X_1,X_2) to state (X_1',X_2'), defined by:

- $q((X_1,X_2),(X_1+1,X_2)) = \lambda_1$, if $X_1+X_2 < C$
- $q((X_1,X_2),(X_1-1,X_2)) = \mu_1 X_1$
- $q((X_1,X_2),(X_1,X_2+1)) = a(X_1,X_2)\lambda_2$, if $X_1+X_2 < C$
- $q((X_1,X_2),(X_1,X_2-1)) = \mu_2 X_2$
- $q((X_1,X_2),(X_1+1,X_2-1)) = \lambda_1$, if $X_1+X_2 = C$ and $X_2 \neq 0$, a PU arrives when all the channels are in use, as a result, one of the SUs is deallocated immediately.

When $\mu_1 \neq \mu_2$ it is not possible to obtain a closed form expression of its stationary distribution (see for example [12], [13] and the references therein). Although it can be computed numerically depending on $a(X_1,X_2)$, we formulate the corresponding fluid limit in order to characterize the system and study the influence of admission control decisions in a more feasible and efficient way.

III. Fluid Model

In this section we formulate a fluid model that approximates the original one and allows us to study and characterize the evolution of the system when the number of channels as well as the arrival rates are arbitrary large.

Using a convenient scaled Markov Chain, let N be the scaling factor, then:

- $\tilde{X}_1^N(t)$: number of PUs at time t
- $\tilde{X}_2^N(t)$: number of SUs at time t
- CN: total number of channels
- $\lambda_1 N$: arrival rate for PUs
- $\lambda_2 N$: arrival rate for SUs
- μ_1: leaving rate for PUs
- μ_2: leaving rate for SUs

Based on [11], we consider the process $(X_1^N,X_2^N) = 1/N(\tilde{X}_1^N,\tilde{X}_2^N)$. This process can be decomposed in the following way:

$$(X_1^N(t),X_2^N(t)) = (X_1^N(0),X_2^N(0)) + \frac{1}{N}\int_0^t Q(\tilde{X}_1^N(s),\tilde{X}_2^N(s))ds + \frac{M^N(t)}{N}.$$

where $Q(\cdot)$ is called the drift and $M^N(t)$ is a Martingale. $Q(\cdot)$ in a generic point (x,y) is $Q(x,y) = \sum_{(x',y')\neq(x,y)}((x',y')-(x,y)).q((x,y),(x',y'))$ where (x',y') are all possible states.

When N goes to infinity, $\frac{M^N(t)}{N}$ converges to zero in probability. Then, $(X_1^N(t),X_2^N(t))$ converges in probability to a deterministic process described by an ODE. Let $(x_1(t),x_2(t))$ be the limit process. In the next section we will show that this fluid model can provide a good approximation to the original system.

IV. Analysis and Characterization

A. Spectrum Sharing without SU's admission control

First of all, we assume that $a(x_1,x_2)=1$ for all (x_1,x_2). The idea is to study the behavior of the system without any intervention (if a SU arrives and there is at least one unoccupied channel, the SU will enter).

We can observe that some transitions of the system (fluid model) have rates that are discontinuous functions. Using the idea of [9], generally it is possible to determine a piecewise-smooth (PWS) system (i.e. a dynamical system in which the vector field is discontinuous in the domain of interest, but with a controlled form of discontinuity). That is to say, considering $\frac{d}{dt}\mathbf{x} = f(\mathbf{x}), f: E \to \mathbb{R}^n, E \subseteq \mathbb{R}^n, \bigcup R_i \supseteq E$ (R_i $i=1\ldots s$ is a finite set of different regions), a PWS system is when f is smooth on R_i and can be discontinuous only on the boundaries of R_i. In [9], they also prove that the sequence of CMTC converges to the trajectories of this hybrid dynamical system.

Let us give an informal explanation. If we restrict our attention to a two regions system, we have f_1 and f_2 the velocity vectors, both continuous in R_1 and R_2 respectively and let γ be the boundary between R_1 and R_2. If we are in a point $\mathbf{x}$ of γ and $n(\mathbf{x})$ is the normal vector to γ at $\mathbf{x}$, we find the following behaviors of a solution starting in $\mathbf{x}$ depending on the value of $n^T(\mathbf{x})f_1(\mathbf{x})$ and $n^T(\mathbf{x})f_2(\mathbf{x})$:
- transversal motion: if $n^T(\mathbf{x})f_1(\mathbf{x})$ and $n^T(\mathbf{x})f_2(\mathbf{x})$ are non zero and have the same sign.
- sliding motion: if $n^T(\mathbf{x})f_1(\mathbf{x}) > 0$ and $n^T(\mathbf{x})f_2(\mathbf{x}) < 0$
- tangential crossing: if $n^T(\mathbf{x})f_1(\mathbf{x}) = 0$ or $n^T(\mathbf{x})f_2(\mathbf{x}) = 0$

For the deterministic approximation of our system we note points in the state space by (x_1,x_2), with the state space $\{(x_1,x_2) : x_1+x_2 \leq C\}$. In order to be in the context of [9] it is useful to artificially extend our processes besides the region $\{x_1+x_2 \leq C\}$, assuming that in the region $\{x_1+x_2 > C\}$ the vector field is $(\lambda_1-\mu_1 x_1, -\lambda_1-\mu_2 x_2)$. This leads to a different behavior of the fluid limit in the region $\{x_1+x_2 = C\}$, where the deterministic system is driven by the ODEs:

If $x_1+x_2-C<0$ (R_1):

$$\begin{cases} x_1' = \lambda_1-\mu_1 x_1 \\ x_2' = \lambda_2-\mu_2 x_2 \end{cases}$$

else, if $x_1+x_2-C=0$ (γ) and the sliding motion condition is verified:

$$\begin{cases} x_1' = \lambda_1-\mu_1 x_1 \\ x_2' = -\lambda_1+\mu_1 x_1 \end{cases}$$

and if $x_1+x_2-C>0$ (R_2):

$$\begin{cases} x_1' = \lambda_1-\mu_1 x_1 \\ x_2' = -\lambda_1-\mu_2 x_2 \end{cases}$$

Proposition 1. *Considering $a(x_1,x_2)=1$ for all (x_1,x_2), and let R_1, R_2 be the above defined zones:*

a. *If $\frac{\lambda_1}{\mu_1}+\frac{\lambda_2}{\mu_2}<C$, the ODE's fixed point (x_1^*,x_2^*) will be in R_1 ($x_1^*=\frac{\lambda_1}{\mu_1}$, $x_2^*=\frac{\lambda_2}{\mu_2}$) and the mean system utilization will be $\frac{\lambda_1}{\mu_1}+\frac{\lambda_2}{\mu_2}$.*
b. *If $\frac{\lambda_1}{\mu_1}+\frac{\lambda_2}{\mu_2}\geq C$, the ODE's fixed point will be on γ ($x_1^*=\frac{\lambda_1}{\mu_1}$, $x_2^*=C-\frac{\lambda_1}{\mu_1}$) and the mean system utilization will be C.*

Sketch of the proof: Being f_1 and f_2 the velocity vectors, both continuous in R_1 and R_2 respectively:

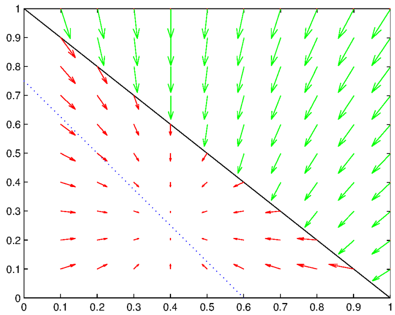
$$f_1(x_1,x_2)=\begin{pmatrix}\lambda_1-\mu_1 x_1\\ \lambda_2-\mu_2 x_2\end{pmatrix},\ f_2(x_1,x_2)=\begin{pmatrix}\lambda_1-\mu_1 x_1\\ -\lambda_1-\mu_2 x_2\end{pmatrix}$$

and let $n(x_1,x_2)$ be the normal vector to the surface γ: $x_1+x_2-C=0$, therefore $n^T=(1,1)$ $\forall(x_1,x_2)\in\gamma$ and it points to R_2.

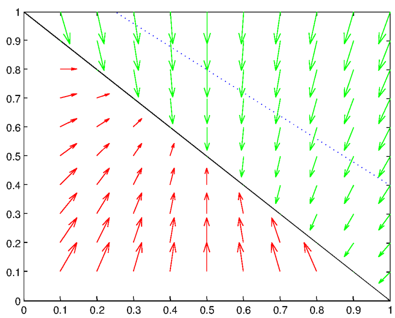
Using the results of [9], $n^T f_1(x)=0 \Leftrightarrow \lambda_1+\lambda_2-\mu_1 x_1-\mu_2 x_2=0$ and $n^T f_2(x)=0 \Leftrightarrow -\mu_1 x_1-\mu_2 x_2=0$. Therefore, for studying $n^T f_i(x)$ we have several cases depending on the position of the line $\lambda_1+\lambda_2-\mu_1 x_1-\mu_2 x_2=0$. It is clear that it depends on the values of λ_1, λ_2, μ_1 and μ_2.

All possible cases, for different values of the parameters, can be categorized in two groups represented by the Cases 1 and 2 showing in Fig. 1a and 1b. In those figures, the continuous line represents γ, the dotted line is $\lambda_1+\lambda_2-\mu_1 x_1-\mu_2 x_2=0$ and the vectors are f_1 and f_2 in R_1 and R_2 respectively. According to the above explanation, $n^T f_1(x)>0$ if $\lambda_1+\lambda_2-\mu_1 x_1-\mu_2 x_2>0$, f_1 and n are tangent in the points over the line $\lambda_1+\lambda_2-\mu_1 x_1-\mu_2 x_2=0$ and $n^T f_1(x)<0$ if $\lambda_1+\lambda_2-\mu_1 x_1-\mu_2 x_2<0$. On the other hand, $n^T f_2(x)<0$ in R_2 independently of the parameters λ_i and μ_i.

In Group 1, represented by Case 1, the ODE's fixed point is located in R_1 (Proposition 1.a). It is easy to note that $x_1^*=\frac{\lambda_1}{\mu_1}$, $x_2^*=\frac{\lambda_2}{\mu_2}$. In Group 2, represented by Case 2, the fixed point is on γ and its value is $x_1^*=\frac{\lambda_1}{\mu_1}$, $x_2^*=C-x_1^*$.



(a) Case 1



(b) Case 2

Fig. 1: Vector field for Case 1: $N=100$, $\lambda_1=2$, $\lambda_2=1$, $\mu_1=5$, $\mu_2=4$ and Case 2: $N=100$, $\lambda_1=2$, $\lambda_2=4$, $\mu_1=4$, $\mu_2=5$. The continuous line represents γ and the dotted line is $\lambda_1+\lambda_2-\mu_1 x_1-\mu_2 x_2=0$

■

In Fig. 2 and 3 we show two examples of Group 1 and 2. In each one, in the left graphic we show the simulation of one trajectory of the scaled Markov process and the trajectory of the ODE. On the other hand, in the right we show for the same simulation the evolution on the plane of the Markov Chain and the ODE. In Fig. 2 the fixed point is in R_1 and in Fig. 3 it is on γ (the boundary between R_1 and R_2). It is important to note that in both cases, for large time values, the scaled number of users in the stochastic process is around the ODE's fixed point (x_1^*,x_2^*), in other words $\lim_{N\to+\infty}(X_1^N(\infty),X_2^N(\infty))=(x_1^*,x_2^*)$, being $(X_1^N(\infty),X_2^N(\infty))$ the system in stationary regime (a related proof of this is done in [10]). As a result, the mean system utilization is $x_1^*+x_2^*$ ($\frac{\lambda_1}{\mu_1}+\frac{\lambda_2}{\mu_2}<C$ and C in Group 1 and 2 respectively).

Let us remark that $\frac{\lambda_1}{\mu_1}<C$ is a necessary condition for sharing the licensed spectrum. If not, most of the time the channels are going to be occupied by PUs and nothing will be available to share with SUs.

Remembering the description of the system, if $x_1+x_2=C$ and a PU arrives, a SU will be immediately deallocated giving the channel to the new PU. In this case, the QoS perceived by the SU will be affected because of the interruption of its

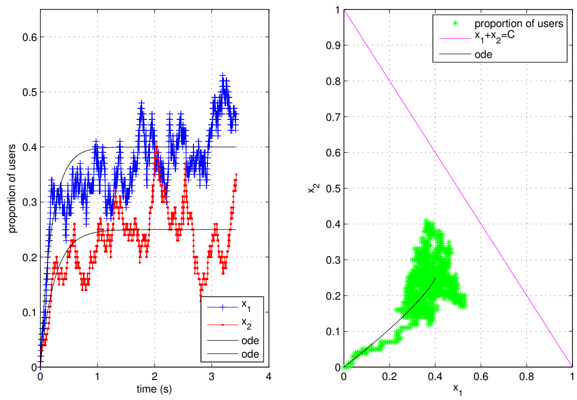


Fig. 2: Group 1, parameters: $N = 100$, $C = 1$, $\lambda_1 = 2$, $\lambda_2 = 1$, $\mu_1 = 5$ and $\mu_2 = 4$

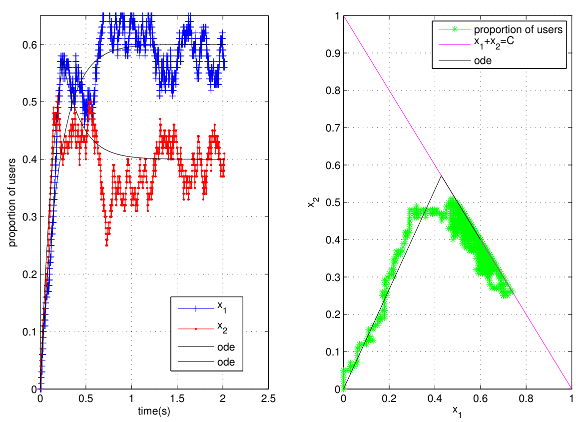


Fig. 3: Group 2, parameters: $N = 100$, $C = 1$, $\lambda_1 = 3$, $\lambda_2 = 4$, $\mu_1 = 5$ and $\mu_2 = 5$

communication. We can make a first conclusion: the fixed point of the fluid limit must be out of γ (it must be in R_1). Even more it has to be far enough from γ to avoid a strong impact on secondary communications, however, it has to be as close as possible to γ to permit more spectrum utilization and a good SU's access probability. Another observation is that the fixed-point's abscissa (λ_1/μ_1) isn't affected with the control action $a(x_1,x_2)$.

In the cases of Group 1, if the ODE's fixed point is far enough from γ, the admission control doesn't make sense. So, the first question to be answered is: how can we determine if it is far enough? Obviously, it fully depends on the QoS requirement for the SU's traffic (for example, a criterion could be to guarantee a low value of probability of service interruption). On the other hand, if it isn't far enough, how can we move the fixed-point? The cases of Group 2 are totally related with this last question. In these cases, the system in stationary state works near γ, so the probability of service interruption is too large. According to that, the analysis in the following subsections is going to be concentrated on answering the above questions.

B. Possible Criteria and Actions

1) Question 1: Is the ODE's fixed point far enough from γ?: Considering Group 1 cases, it is possible to apply the results of [10] together with Kurtz's theorem (see Theorem 2.3 of Chapter 11 in [14]). Let (x_1,x_2) be the trajectory of the PWS dynamical system in R_1 with initial condition $(x_1(0),x_2(0))$, if $\lim_{N\to+\infty} \sqrt{N}[(X_1^N(0),X_2^N(0))-(x_1(0),x_2(0))] = Z(0)$ with $Z(0)$ deterministic, then, $\sqrt{N}[(X_1^N(t),X_2^N(t))-(x_1(t),x_2(t))] \Rightarrow Z(t)$ where $Z(t)$ is a Gaussian process and its covariance matrix can be determined explicitly (see [14]). Obs: $\Rightarrow$ means convergence in distribution.

A possible criterion to determine if the ODE's fixed point is far enough from γ would be to consider a certain confidence region. If the confidence ellipse resulted is entirely inside R_1, certain probability of non-interruption is guarantee. Otherwise, we should try to move the fixed point.

An example is presented in Fig. 4. It is showed the theoretical 95% confidence ellipse determined by Kurtz's theorem and also a simulated confidence one. We simulated $n = 100$ independent samples of $(X_1^N(t),X_2^N(t))$ considering the same large t value. The simulated ellipse is obtained from the empirical covariance matrix. In this particular case, an admission control is not necessary.

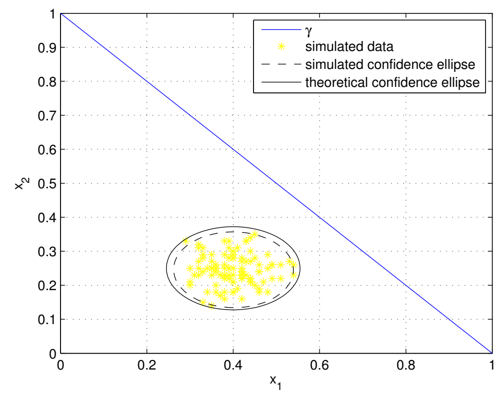


Fig. 4: Simulated and theoretical 95% confidence ellipses of $(X_1^N(t),X_2^N(t))$, parameters: $N = 100$, $\lambda_1 = 2$, $\lambda_2 = 1$, $\mu_1 = 5$ and $\mu_2 = 4$

2) Question 2: How can we move the ODE's fixed point?: A possibility of moving the fixed point is to apply admission control decisions. As a first step, given the simplicity of the analytical description and without loss of generality, we consider the case where the admission control boundary is a line with equation $-x_1 - x_2 + \delta = 0$ for $0 < \delta < C$. That is to say, $a(x_1,x_2) = 1$ if $x_2 \leq -x_1 + \delta$ and $a(x_1,x_2) = 0$ if $x_2 > -x_1 + \delta$.The advantage of this basic case is its simple practical implementation: SUs only need to know the number of occupied bands to decide whether to enter the system or not.

Given the fact that the abscissa of the ODE's fixed point isn't affected with the control action $a(x_1,x_2)$, the objective is to move the fixed point ordinate looking for a better option. The domain where the ordinate could live is the segment $[0,\alpha]$

where α represents the "original ordinate", that is the ordinate when $a(x_1,x_2)=1$ for all (x_1,x_2). In cases of Group 1 $\alpha=\frac{\lambda_2}{\mu_2}$ but in the other group $\alpha=C-\frac{\lambda_1}{\mu_1}=C-x_1^*$.

Proposition 2. *If it is considered $\beta: -x_1-x_2+\delta=0$ as the admission control boundary, the ODE's fixed point will be $\left(\frac{\lambda_1}{\mu_1},\delta-\frac{\lambda_1}{\mu_1}\right)$ then, the mean system utilization will be δ.*

Sketch of the proof: When there is an admission control like we explained above, in the PWS system we identify three zones (R_1, R_2 and R_3) and two surfaces (γ and β), so:

If $x_1+x_2-\delta<0$ (R_1):

$$\begin{cases} x_1'=\lambda_1-\mu_1x_1 \\ x_2'=\lambda_2-\mu_2x_2 \end{cases}$$

else, if $x_1+x_2-\delta=0$ (β):

$$\begin{cases} x_1'=\lambda_1-\mu_1x_1 \\ x_2'=-\lambda_1+\mu_1x_1 \end{cases}$$

else, if $x_1+x_2-\delta>0$ and $x_1+x_2-C<0$ (R_2):

$$\begin{cases} x_1'=\lambda_1-\mu_1x_1 \\ x_2'=-\mu_2x_2 \end{cases}$$

else, if $x_1+x_2-C=0$ (γ):

$$\begin{cases} x_1'=\lambda_1-\mu_1x_1 \\ x_2'=-\lambda_1+\mu_1x_1 \end{cases}$$

and if $x_1+x_2-C>0$ (R_3):

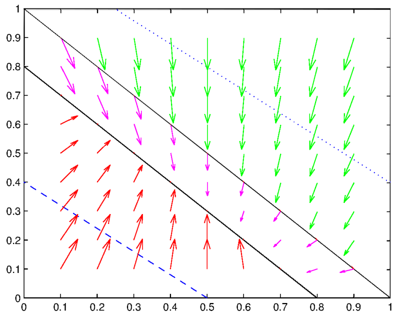
$$\begin{cases} x_1'=\lambda_1-\mu_1x_1 \\ x_2'=-\lambda_1-\mu_2x_2 \end{cases}$$

Defining f_i (velocity vectors) in the same way as in the previous section, and concentrating in a case from the Group 2 defined before (because the problem is more critical than in the other one), the most representative cases of study are shown in Fig. 5a and 5b. They differ in the position of the admission of SUs border. As we explained before, the abscissa of ODE's fixed point is $\frac{\lambda_1}{\mu_1}$, therefore the newest fixed point will be in the intersection of $x_1=\frac{\lambda_1}{\mu_1}$ and β or will be the point $\left(\frac{\lambda_1}{\mu_1},0\right)$. In order to improve the system efficiency, we can conclude that the point $\left(\frac{\lambda_1}{\mu_1},0\right)$ must be included in R_1 zone like in Case A (this restricts the position of β). In this situation the proposition is demonstrated and, looking Fig. 5a it is easy to notice that the fixed point is $\left(x_1^*=\frac{\lambda_1}{\mu_1},x_2^*=\delta-x_1^*\right)$. The other case doesn't make sense when we are interested in guarantee certain access probability to SUs.

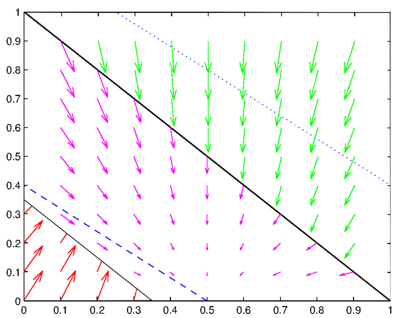
■

In Fig. 6a and 6b it is possible to observe that the ODE's fixed point changes its position, in particular, its ordinate. The Fig. 6a represents the case (of Group 2) when there isn't an admission control algorithm, however in Fig. 6b we can identify some states of the Markov model where SUs won't enter to the system and other states where they will. In this last case, it is possible to note that the mean spectrum utilization is $\delta=0.8$.

The previous proposition can be extended to other types of admission control boundaries. As a general result, the ODE's



(a) Case A, ODE's fixed point is $(0.5,0.3)$, $\beta_1: x_1+x_2-0.8=0$



(b) Case B, ODE's fixed point is $(0.5,0)$, $\beta_2: x_1+x_2-0.35=0$

Fig. 5: $N=100$, $\lambda_1=2$, $\lambda_2=4$, $\mu_1=4$, $\mu_2=5$. Dashed line is $\lambda_1-\mu_1x_1-\mu_2x_2=0$ (where $(1,1)^T f_2(x_1,x_2)=0$), and dotted line is $\lambda_1+\lambda_2-\mu_1x_1-\mu_2x_2=0$.

fixed point, with an arbitrary access boundary defined by an equation $\theta(x_1,x_2)=0$, is going to be located at (x_1^*,x_2^*) with $x_1^*=\frac{\lambda_1}{\mu_1}$ and $\theta(x_1^*,x_2^*)=0$.

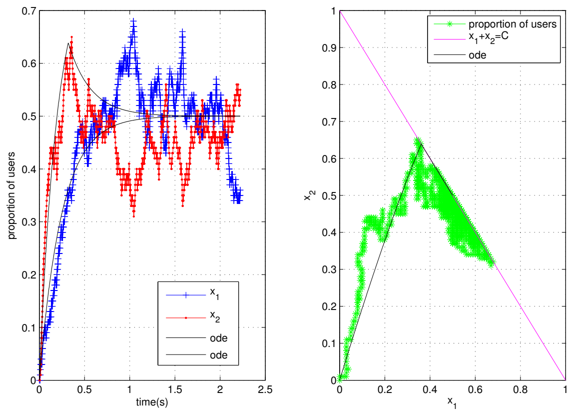
Now, continuing the example of β as the admission control border, the question is: what is a reasonable value of δ? In order to answer this, an option is to make a confidence ellipse for a large t for different values of δ and try to find the one that is tangential to the surface γ. In this case, the hypotheses of Kurtz's theorem are not verified, so the confidence ellipse would only be calculated using simulations. As an example we made different sets of simulations changing the value of δ (see Fig. 7). The Gaussian assumption was tested for each set using Mardia's multivariate skewness and kurtosis test (significance level=0.05).

Note: These last analysis were done with a case of Group 2, but is totally applicable to Group 1 cases when it is necessary to move the ODE's fixed point.

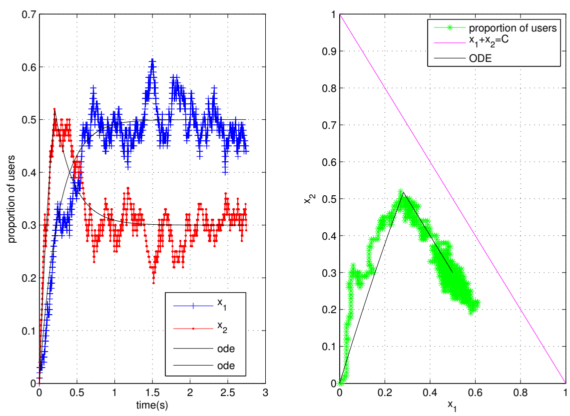
The case with arbitrary admission control border will be addressed in future work.

V. Conclusions and Future Work

The main contributions of this work are the analysis and characterization of a possible model of spectrum sharing in



(a) Without Admission Control



(b) With Admission Control, boundary β_1.

Fig. 6: Comparing the same system ($N = 100$, $\lambda_1 = 2$, $\lambda_2 = 4$, $\mu_1 = 4$, $\mu_2 = 5$) without and with admission control decisions. For each case we show the fluid limit and one trajectory of the corresponding Markov Chain

cognitive radio networks. We present some tools and approaches that can be used to improve the average utilization of the spectrum while ensuring a small probability of interruption to the secondary users.

We considered a Markov Chain that represents the population of the different types of users in the system. We formulated the associated fluid model and we studied its solutions. We proposed a simple admission control criteria that, for a system with a large number of users, guarantees with high probability that secondary users in the system will not have service interruptions. This criteria is suggested by a theoretical analysis and supported by simulations.

In our ongoing work, we are investigating extensions of these tools to other scenarios of cognitive radio networks.

References

[1] FCC, NPRM 04-186, Unlicensed Operation in the TV Broadcast Bands, May 2004.

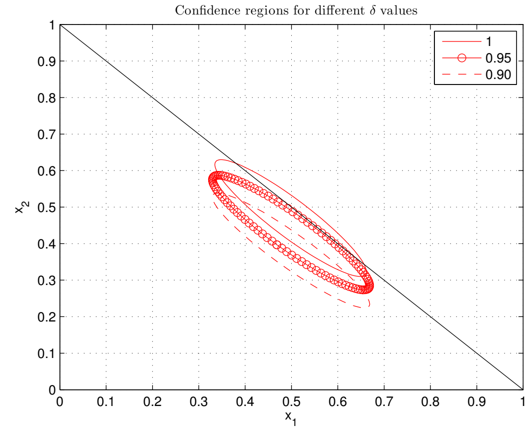


Fig. 7: Simulated 95% confidence ellipses of $\sqrt(N)[(X_1^N(t),X_2^N(t)) - (x_1^*,x_2^*)]$ for three different values of δ, considering a large t value ($N = 100$, $\lambda_1 = 2$, $\lambda_2 = 4$, $\mu_1 = 4$, $\mu_2 = 5$)

[2] M. McHenry, "NSF Spectrum occupancy measurements project summary," Shared Spectrum Co., tech. rep.,Aug. 2005

[3] V. Blaschke, H. Jaekel, T. Renk, C. Kloeck, F.K. Jondral, "Occupation measurements supporting dynamic spectrum allocation for cognitive radio design," CrownCom 2007. 2nd International Conference. on, Publication Year: 2007 , Page(s): 50- 57

[4] S. Haykin, "Cognitive radio: brain-empowered wireless communications", Selected Areas in Communications, IEEE Journal on , vol.23, no.2, pp.201,220, Feb. 2005 doi: 10.1109/JSAC.2004.839380

[5] B. Wang, K.J.R. Liu, "Advances in cognitive radio networks: A survey," Selected Topics in Signal Processing, IEEE. Volume:5 , Issue: 1 Publication Year: 2011 , Page(s): 5- 23

[6] S. Wang, J. Zhang, L. Tong, "Delay analysis for cognitive radio networks with random access: A fluid queue view," INFOCOM, 2010 Proceedings IEEE , vol., no., pp.1,9, 14-19 March 2010

[7] C. Yin, L. Gao, and S. Cui, "Scaling laws for overlaid wireless networks: A cognitive radio network versus a primary network," IEEE/ACM Transactions on Networking, vol. 18, no. 4, pp. 1317-1329, August 2010.

[8] C. Zhang, X. Wang, X. Guan, H. Chen, "Quality-of-Service in Cognitive Radio Networks with Collaborative Sensing," Global Telecommunications Conference, 2009. GLOBECOM 2009. IEEE , vol., no., pp.1,5, Nov. 30 2009-Dec. 4 2009

[9] L. Bortolussi, "Hybrid limits of continuous time Markov Chains," Quantitative Evaluation of Systems (QEST), 2011 Eighth International Conference on , vol., no., pp.3,12, 5-8 Sept. 2011 doi: 10.1109/QEST.2011.10

[10] L. Aspirot, E. Mordecki, and G. Rubino, "Fluid limits applied to peer to peer network analysis," In QEST '11, Proceedings of the Eight International Conference on Quantitative Evaluation of Systems, pages 13–20, Los Alamitos, CA, USA, 2011. IEEE Computer Society. (doi:10.1109/QEST.2011.11)

[11] R.W.R. Darling, "Differential equation approximations for Markov chains," Probability Surveys, Vol. 5 (2008) 37–79, 2008

[12] Z. Zhao, "A preemptive connection pool manager for web-based application collaboration," Collaborative Computing: Networking, Applications and Worksharing (CollaborateCom), 2012 8th International Conference on , vol., no., pp.326,334, 14-17 Oct. 2012

[13] Q. Yan, "Queueing models for capacity changes in cellular networks," PhD thesis, 2013

[14] S. N. Ethier and T. G. Kurtz, "Markov processes: Characterization and convergence," ser. Wiley Series in Probability and Statistics. New York: John Wiley & Sons Inc., 1986.