

Decision making in forward power markets with supply and demand uncertainty

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Abstract—The paper studies forward markets of electric energy, where generators and consumers bid for quantities of energy ahead of time, but face uncertainty on their real-time supply or demand, expressed through a probability distribution. The optimal forward decision is derived in both cases, providing extensions to the recent literature on this topic. In particular, the case where demand is elastic in addition to uncertain is addressed in detail. Finally, we analyze the integrated forward market where buyers and sellers of random energy interact with dispatchable sellers to determine a clearing price.

I. INTRODUCTION

Electric power markets of today include economic dispatch decisions made in advance (e.g. a day ahead), with participants that often face *uncertainty* on the quantities they will produce or consume in real time. In particular, electricity demand is subject to unforeseen fluctuations, and supply is also uncertain for most renewable sources (wind, solar, etc.). Since electricity is not a stored commodity, any mismatch between supply and demand must be settled with balancing actions taken in real time, appropriately priced. Imbalance prices in many such markets (see [1], [9], [10]) are such that participants are charged differently depending on whether they are short or long with respect to their previous commitments. This makes the problem of bidding for the forward market non-trivial.

Many researchers have approached such decision problems through numerical optimization, both on the supply side related to wind energy [1], [9], [12], [10], [3], and on the demand side [5], [15]. We are interested here in analytical solutions, which fall in the class of the *newsvendor problem* in operations research (e.g. [13]): Here, a retailer must acquire a certain quantity of a perishable good to face a random demand, knowing that shortages and excesses of stock are priced differently. The optimal risk-neutral solution is to acquire the quantile of the demand distribution that equals a certain price ratio computed from the short and long ex post prices.

These techniques have been recently applied [4], [2] to the supply-side problem of bidding quantities of wind energy. On the demand side, [11] has analyzed optimal day-ahead bids, but with a single price in the regulating market; the two-price, quantile solution for demand is invoked in [14] to analyze a supply duopoly. However, a study of demand bids that incorporate consumption *elasticity* as well as uncertainty

has not, to our knowledge been carried out. This becomes important in emerging networks that will likely incorporate *demand response* [7], [6], but still retain uncertainty.

In this paper we study supply and demand bidding in a common framework, and combine them in a unified analysis of the forward market. After introducing in Section II the relevant prices, in Section III we review the optimal solution from [4] for selling of a random, inelastic quantity of energy, with some additional analysis for the case of correlated supply and real-time prices. In Section IV we consider the demand side, first for the parallel case where demand is random and inelastic, but later incorporate both elasticity and uncertainty through a random utility function. General conditions are derived for the optimal bids. In Section V we integrate both aspects in a forward market where buyers and sellers of random energy interact with dispatchable sellers to determine a clearing price. Conclusions are given in Section VI.

II. FORWARD AND IMBALANCE PRICES

We consider a forward (e.g. day-ahead) market for electricity, in which contracts are established for delivery of energy quantities between suppliers and consumers. Real-time deviations from these quantities (due to demand variability, or supply variations in renewable sources) are compensated in an imbalance market. We define three relevant prices:

- p_F , unit price of energy traded in the forward market.
- p_S (short imbalance price), unit price at which a shortfall of energy (due to shortage in generation, or excess in demand) can be bought in the imbalance market.
- p_L (long imbalance price), unit price at which excess energy (excess generation, or demand below expectations) can be sold in the imbalance market.

We assume market participants are price-takers, they take these signals as exogenous. Provided that

$$p_L \leq p_F \leq p_S, \quad (1)$$

bidders will have incentives to align their forward offers with their true forecasts of generation or consumption. This is precisely the way things are arranged in several markets, like Great Britain [1], [4], Scandinavia [9], the Netherlands [12] or the Iberian peninsula [10]. Imbalance prices are defined ex-post, depending on whether the overall market is long or short. In case of shortage, p_S is typically higher than p_F , reflecting the costs of acquiring the balancing energy, whereas $p_L = p_F$

(long suppliers are actually contributing to balance). If the market is long, $p_L < p_F$ reflects balancing actions for down-generation or increased load, whereas $p_F = p_S$. At the time of the forward bids, the outcome is unknown, yet the inequalities of (1) will hold whichever the case. [4] contains historical data from the British market supporting this fact, which will be assumed henceforth.

III. THE SUPPLY SIDE – UNCERTAIN GENERATION

Energy suppliers are typically of two opposite kinds:

- Dispatchable generation which has no uncertainty as to power availability in the short term, but is elastic to prices due to variable cost (e.g., a fossil-fuel plant) or opportunity cost (e.g. hydro power). These suppliers have no difficulty keeping their forward commitments.
- Non-dispatchable generation such as wind or solar energy sources with no storage or variable cost, therefore no sensitivity to prices in their decision to generate, but uncertainty about the generated quantity. For these suppliers which are the focus of this section, the impact of prices is on the day-ahead planning decision.

While it is possible to conceive a situation that combines price elasticity with uncertainty¹, we will postpone this combination that appears more naturally on the demand side.

A. Optimal offer for inelastic supplier with given prices

The uncertain energy generation in a future time interval is modeled as a random variable of bounded support $[0, M]$, with known cumulative distribution function $F(w) = \mathcal{P}(W \leq w)$, assumed continuous. Suppose first that p_F, p_L, p_S satisfying (1) are given; the decision problem faced by the generator is how much energy y to commit on the forward market.

For a given commitment y , and eventual generation level w , the revenue obtained by the seller is

$$R(y, w) = p_F y - p_S [y - w]^+ + p_L [w - y]^+, \quad (2)$$

where $[\cdot]^+ := \max\{\cdot, 0\}$. It amounts to the revenue from the day ahead sale, minus the imbalance cost of any shortfall energy, plus the imbalance revenue from any excess generation. It can also be written as $R(y, w) = p_F y - C_{p_L}^{p_S}(y - w)$, where the imbalance cost is defined to be

$$C_{p_L}^{p_S}(\xi) := p_S [\xi]^+ - p_L [-\xi]^+ = \begin{cases} p_S \xi & \text{if } \xi > 0; \\ p_L \xi & \text{if } \xi < 0. \end{cases} \quad (3)$$

Taking expectation over the distribution of W we define $\bar{R}(y) := \mathbb{E}[R(y, W)]$. Note that this expected revenue is always less than what one would obtain from selling a *certain* supply equal to the mean $\mathbb{E}[W]$ at day ahead prices. Indeed,

$$\mathbb{E}[R(y, W)] \leq R(y, \mathbb{E}[W]) \leq p_F \mathbb{E}[W].$$

Both inequalities follow from (1): the first from the fact that $R(y, w)$ is (2) is concave in w for fixed y , the second from $\max_y R(y, w) = R(w, w) = p_F w$.

¹For instance, a factory with co-generation may sell excess power depending on uncertain production schedules and current prices.

The (risk-neutral) forward decision problem is to maximize $\bar{R}(y)$ in the committed quantity y . This is an instance of the newsvendor problem [13], we briefly review its solution as documented in [4], [2]. Write

$$\begin{aligned} \bar{R}(y) &= p_F y - p_S \int_0^y [y - w] dF + p_L \int_y^M [w - y] dF \\ &= p_F y - p_S \int_0^y F(w) dw + p_L \int_y^M [1 - F(w)] dw, \end{aligned}$$

which uses integration by parts. The derivative is

$$\bar{R}'(y) = p_F - p_S F(y) - p_L [1 - F(y)],$$

from where we arrive at the optimality condition

$$F(y^*) = \frac{p_F - p_L}{p_S - p_L}. \quad (4)$$

This says it is optimal to bid a quantile of the distribution of the generation W corresponding to the price-ratio on the right-hand side, which is always in $[0, 1]$ under (1).

B. Optimal offer for inelastic supplier with random prices

We consider now the fact that imbalance prices p_S, p_L are uncertain at the time of the forward decision. Modeling them as random variables, the expected revenue is

$$\bar{R}(y) = p_F y - \mathbb{E}[p_S [y - W]^+] + \mathbb{E}[p_L [W - y]^+]. \quad (5)$$

If the random price variables are assumed to be independent of the generation variable W , the problem simplifies to the previous case, leading to a solution (4) with prices replaced by their means $\bar{p}_S = \mathbb{E}[p_S]$, $\bar{p}_L = \mathbb{E}[p_L]$.

However, there is good reason to question the independence assumption: if W is renewable generation, high levels of W will likely correlate with abundance of power in the region surrounding it, and thus *lower* prices in the imbalance market. Following [4], this correlation can be modeled through the conditional expectations

$$\phi_S(w) = \mathbb{E}[p_S | W = w], \quad \phi_L(w) = \mathbb{E}[p_L | W = w].$$

Through prior experience in the market, the seller could have good estimates of these regression functions. The expectation terms in (5) become then

$$\mathbb{E}[p_S [y - W]^+] = \int_0^y (y - w) \phi_S(w) dF(w). \quad (6a)$$

$$\mathbb{E}[p_L [W - y]^+] = \int_y^M (w - y) \phi_L(w) dF(w). \quad (6b)$$

In [4] some further conclusions were drawn by assuming specific expressions for the above regressors.

Here we will carry out some additional analysis from the assumption that $\phi_L(w) \leq \phi_S(w)$ are both *decreasing* in $w \in [0, M]$, which expresses the aforementioned negative correlation. For this purpose, introduce the notation

$$\psi_S(y) := \int_0^y \phi_S(w) dF(w); \quad \psi_L(y) := \int_0^y \phi_L(w) dF(w).$$

Note that $\psi_L(y) \leq \psi_S(y)$, and $\psi_S(M) = \bar{p}_S$, $\psi_L(M) = \bar{p}_L$. Integration by parts in (6) gives the expression

$$\bar{R}(y) = p_F y - \int_0^y \psi_S(w) dw + \int_y^M [\bar{p}_L - \psi_L(w)] dw. \quad (7)$$

Differentiation gives $\bar{R}'(y) = p_F - \psi_S(y) - \bar{p}_L + \psi_L(y)$, and $\bar{R}''(y) = \phi_L(y) - \phi_S(y) \leq 0$. Invoking (1) for the mean prices, $\bar{R}'(y)$ decreases from $\bar{R}'(0) = p_F - \bar{p}_L \geq 0$ to $\bar{R}'(M) = p_F - \bar{p}_S \leq 0$, therefore there exists y^* satisfying the optimal revenue condition $\bar{R}'(y^*) = 0$.

There is no general relationship between this optimal bid and the one obtained for independent prices², but we can compare the optimal revenue in both cases: negative correlation of prices and generation hurts the revenue of the seller.

Proposition 1: $\bar{R}(y) \leq \bar{R}_i(y)$ for every y , where

$$\bar{R}_i(y) := p_F y - \bar{p}_S \mathbb{E}[y - W]^+ + \bar{p}_L \mathbb{E}[W - y]^+,$$

revenue under independence of prices and W .

Proof: We first claim that, with non-increasing conditional prices $\phi_S(w)$ we have

$$\psi_S(y) \geq \bar{p}_S F(y) \quad \forall y. \quad (8)$$

to see this write the inequalities

$$\begin{aligned} \bar{p}_S &= \psi_S(y) + \int_y^M \phi_S(w) dF \leq \psi_S(y) + \phi_S(y)[1 - F(y)]; \\ \phi_S(y)F(y) &\leq \int_0^y \phi_S(w) dF = \psi_S(y). \end{aligned}$$

Combining them with respective factors $F(y)$, $[1 - F(y)]$ leads after simplification to (8). Integration now gives

$$\int_0^y \psi_S(w) dw \geq \bar{p}_S \int_0^y F(w) dw = \bar{p}_S \mathbb{E}[y - W]^+.$$

Similarly we establish that $\psi_L(y) \geq \bar{p}_L F(y)$, and therefore

$$\int_y^M [\bar{p}_L - \psi_L(w)] dw \leq \bar{p}_L \int_y^M [1 - F(w)] dw = \bar{p}_L \mathbb{E}[W - y]^+.$$

Combining these inequalities with (7) gives

$$\bar{R}(y) \leq p_F y - \bar{p}_S \mathbb{E}[y - W]^+ + \bar{p}_L \mathbb{E}[W - y]^+.$$

■

IV. THE DEMAND SIDE – UNCERTAIN CONSUMPTION

In this section we turn our attention to uncertainty in system demand, for a consumer who participates in a forward market. This could be a large consumer, or a retailer who bids on behalf of a certain group of final customers. Uncertainty in the future consumption is always present; for instance, weather may affect the usage of air-conditioning devices. So when deciding on the amount of energy to reserve in the forward market, the decision must incorporate in addition to the forward price p_F , the imbalance prices p_S, p_L . We assume they satisfy $p_L \leq p_F \leq p_S$ as in (1).

²Under the specific model in [4] it is shown to be lower.

A. Optimal bid, inelastic random demand with given prices

We consider first the traditional situation, where real-time consumption is determined in an inelastic way: consumers are shielded from the spot market and therefore define their energy usage independently of prices. The focus of our analysis is how prices (assumed at first to be given) impact the decision of how much energy to reserve in the forward market.

Given a reservation quantity x , and the actual consumption q , the net user cost is given by

$$\begin{aligned} C(x, q) &= p_F x + p_S [q - x]^+ - p_L [x - q]^+ \\ &= p_F x + C_{p_L}^{p_S}(q - x). \end{aligned} \quad (9)$$

with $C_{p_L}^{p_S}$ from (3). We now model an uncertain demand as a random variable Q with continuous cumulative distribution $G(q)$, and bounded support in $[0, M]$. The expected cost is

$$\begin{aligned} \bar{C}(x) &= p_F x + p_S \int_x^M (q - x) dG - p_L \int_0^x (x - q) dG \\ &= p_F x + p_S \int_x^M [1 - G(q)] dq - p_L \int_0^x G(q) dq, \end{aligned}$$

where the last step follows from integration by parts. Setting $\bar{C}'(x) = 0$ we find the condition

$$p_F = p_L G(x) + p_S (1 - G(x)), \quad (10)$$

which can be solved to give the optimal quantile

$$G(x^*) = \frac{p_S - p_F}{p_S - p_L}. \quad (11)$$

Note the similarity and difference with the supply case. Indeed, this is the complementary quantile to the one obtained in (4).

B. Optimal bid, inelastic random demand with random prices

Analogously to Section III-B, we now address the fact that imbalance prices are also uncertain, modeling p_S and p_L as random variables. Once again, it is natural to consider the correlation between these quantities and the consumer's random demand Q : if consumption goes up for a consumer, it will likely also happen with neighbors, leading to growing prices in the region. This *positive* correlation can be captured by monotonically *increasing* conditional expectation functions

$$\phi_L(q) = \mathbb{E}[p_L | Q = q], \quad \phi_S(q) = \mathbb{E}[p_S | Q = q],$$

with $\phi_L(q) \leq \phi_S(q)$. The expected cost becomes

$$\begin{aligned} \bar{C}(x) &= p_F x + \mathbb{E}[p_S [Q - x]^+] - \mathbb{E}[p_L [x - Q]^+] \\ &= p_F x + \int_x^M (q - x) \phi_S(q) dG - \int_0^x (x - q) \phi_L(q) dG. \end{aligned}$$

The ensuing analysis to find x^* that minimizes $\bar{C}(x)$ can be carried out in a dual way to Section III-B. We state the following result:

Proposition 2: Let $\bar{C}_i(x_i)$ be the mean cost function when prices are independent of Q , and x_i^* the corresponding optimal decision. Then

$$p_F \mathbb{E}[Q] \leq \bar{C}_i(x_i^*) \leq \bar{C}(x^*).$$

This says that the optimal cost is higher (worse) than in the case of independent prices, and the latter is itself higher than the cost of buying the mean demand at day ahead prices. The proof is analogous to Proposition 1.

C. Optimization for elastic random demand, given prices

In emerging power networks with *demand response*, consumers are expected to be sensitive to the price of energy, reducing their consumption accordingly. In some envisaged schemes [7], [6], a load-serving entity (LSE) which buys wholesale power communicates with consumers a day ahead to define their demand as a function of prevailing prices. These references do not, however, include any uncertainty as to the planned consumption in the day-ahead purchase decision.

We would like to consider a demand model in which agents adjust both their reservations and their consumption, based on prevailing prices in forward and imbalance markets. One way to combine this elasticity with demand uncertainty is through a utility function $U(q, \theta)$, that depends on the consumption level q and on a parameter θ , itself drawn from a random variable Θ . The consumer agent has two decision variables, x and q , chosen at different times. We analyze these choices backwards in time. Initially we assume all prices are given.

- The real-time decision for q , given x : upon realization of θ , the consumer chooses the surplus maximizing point

$$q^*(x, \theta) := \operatorname{argmax}_q \{U(q, \theta) - C(x, q)\}. \quad (12)$$

Out of the cost given in (9), only the imbalance portion $C_{p_L}^{p_S}(q - x)$ participates in this decision. Assuming the maximum in (12) occurs in an interior q the optimality condition is

$$U'(q^*(x, \theta), \theta) = C_{p_L}^{p_S'}(q^*(x, \theta) - x). \quad (13)$$

- Looking now at the forward decision, the consumer agent must maximize the expected surplus

$$\bar{S}(x) := \mathbb{E}_{\Theta} [U(q^*(x, \Theta), \Theta) - C(x, q^*(x, \Theta))] \quad (14)$$

over the reservation variable x . Computing the derivative $\bar{S}'(x)$ inside the expectation sign, using the structure of the cost and imposing (13) yields the optimality condition:

$$p_F = \mathbb{E}_{\Theta} [C_{p_L}^{p_S'}(q^*(x^*, \Theta) - x^*)]. \quad (15)$$

Thus the optimal x^* equalizes the forward price with the expected marginal imbalance cost of the deviation of demand from the reservation.

We look at two special cases of the above formulation:

1) *Inelastic demand*: The analysis of Section IV-A is recast in this setting as follows. Let Θ be the random inelastic demand, and choose $U(q, \theta)$ that assigns very high marginal utility $U'(q, \theta) = p_i > p_S$ for $q < \theta$, and $U'(q, \theta) = 0$ for $q > \theta$. In this case $q^*(x, \theta) = \theta$ regardless of the prices.

Using the imbalance cost $C_{p_L}^{p_S}(q - x)$ in (3), (15) gives

$$\begin{aligned} p_F &= \mathbb{E}_{\Theta} [p_L \mathbf{1}_{\{\Theta < x^*\}} + p_S \mathbf{1}_{\{\Theta > x^*\}}] \\ &= p_L G(x^*) + p_S (1 - G(x^*)), \end{aligned}$$

which coincides with (10), and thus leads to the quantile reservation in (11).

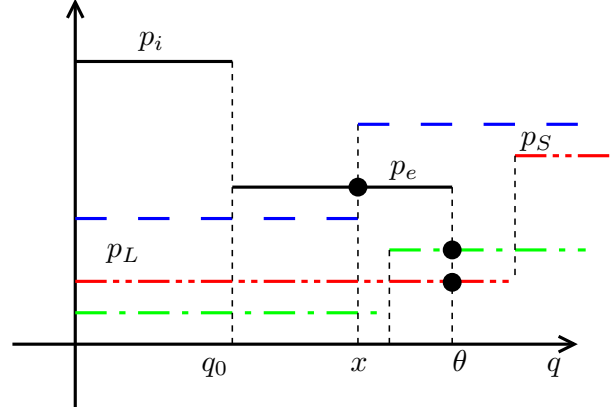


Fig. 1. Marginal utility $U'(q, \theta)$ (solid); marginal cost $C_{p_L}^{p_S}'(q - x)$ for different cases of x and prices (dash, dash-dot and dash-dot-dot).

2) *Elasticity in a portion of the demand*: Let

$$U'(q, \theta) = \begin{cases} p_i & \text{if } q < q_0; \\ p_e & \text{if } q_0 < q < \theta; \\ 0 & \text{if } q > \theta. \end{cases} \quad (16)$$

Assume that $p_i > p_S$ always so the portion q_0 of the demand is firm; the amount of additional consumption will result from comparing p_e with prevailing prices. The saturation point θ is drawn from a random variable Θ with support in $[q_0, q_M]$, and cumulative distribution function $G(\cdot)$.

For the forward decision, it is clear that if $p_F \geq p_e$ the optimal $x^* = q_0$: the forward price is not attractive to make any reservation beyond the firm demand.

We focus then on the case $p_F < p_e$. Note that here $p_L < p_e$, but p_S could fall on either side. We study first the real-time decision for a given (x, θ) , depicted in Figure 1.

In particular:

- If $x > \theta$, then the decision is $q^* = \theta$ and the optimal marginal cost is $C_{p_L}^{p_S}'(q^* - x) = p_L$.
- If $x < \theta$, then the optimal marginal cost is $\min\{p_e, p_S\}$; the minimum price determines whether q^* is x or θ .

The right-hand side of (15) therefore gives

$$\begin{aligned} \mathbb{E} [C_{p_L}^{p_S}'(q^* - x)] &= \mathbb{E} [p_L \mathbf{1}_{\{\Theta < x\}} + \min\{p_e, p_S\} \mathbf{1}_{\{\Theta > x\}}] \\ &= p_L G(x) + \min\{p_e, p_S\} (1 - G(x)); \end{aligned} \quad (17)$$

the optimal quantile decision follows. The following expression summarizes the result for all cases:

$$G(x^*) = \mathbf{1}_{\{p_e > p_F\}} \cdot \frac{\min\{p_e, p_S\} - p_F}{\min\{p_e, p_S\} - p_L}. \quad (18)$$

D. Optimization for elastic random demand, random prices

We now consider the general situation where, in addition to the random parameter Θ in the consumer utility function, there is uncertainty on the imbalance prices p_L, p_S , modeling them as random variables. Note that this uncertainty is *not* removed at the time of the consumption decision, when θ is revealed, since balancing actions matching supply and demand are taken ex post, with the quantities already defined.

Nevertheless, Θ can exhibit correlation to the ex post prices, in a similar manner as the inelastic case; we therefore again introduce the conditional expectations

$$\phi_L(\theta) = \mathbb{E}[p_L|\Theta = \theta], \quad \phi_S(\theta) = \mathbb{E}[p_S|\Theta = \theta].$$

We assume they satisfy $\phi_L(\theta) \leq p_F \leq \phi_S(\theta)$ for all θ ³.

The risk-neutral optimization of surplus from (14) now involves expectations over Θ , p_L , p_S . When we analyze the real-time decision conditioned on $\Theta = \theta$, we have

$$q^*(x, \theta) := \operatorname{argmax}_q \left\{ U(q, \theta) - C_{\phi_L(\theta)}^{\phi_S(\theta)}(q - x) \right\},$$

where $C_{\phi_L(\theta)}^{\phi_S(\theta)}(\xi) = \phi_S(\theta)[\xi]^+ - \phi_L(\theta)[- \xi]^+$. With this modification, the rest of the analysis carries through and the optimal reservation x^* satisfies

$$p_F = \mathbb{E}_{\Theta} \left[C_{\phi_L(\theta)}^{\phi_S(\theta)'}(q^*(x^*, \theta) - x^*) \right]. \quad (19)$$

Note that if prices are assumed independent of Θ , then $\phi_S(\theta) = \bar{p}_S$, $\phi_L(\theta) = \bar{p}_L$, and the problem reduces to the one in the previous section with prices replaced by their means.

We now look at the general, dependent price situation for the utility function considered in (16). Again, it suffices to consider the case where $p_F < p_e$; the analysis of the real-time decision still has two cases (a) and (b) treated before, replacing p_L , p_S by the conditional means $\phi_L(\theta)$, $\phi_S(\theta)$. We therefore have the following generalization of (17):

$$\begin{aligned} \mathbb{E} \left[C_{\phi_L(\theta)}^{\phi_S(\theta)'}(q^* - x) \right] &= \int_{q_0}^x \phi_L(\theta) dG \\ &+ \int_x^{q_M} \min\{p_e, \phi_S(\theta)\} dG. \end{aligned}$$

Noting that $\phi_L(\theta) \leq p_F \leq \min\{p_e, \phi_S(\theta)\}$, the above expression is decreasing in x , from $\mathbb{E}[\min\{p_e, p_S\}] \geq p_F$ at $x = q_0$, to $\bar{p}_L \leq p_F$ at $x = q_M$. Therefore there is a well defined optimal solution satisfying (19).

V. INTEGRATED FORWARD MARKET AND CLEARING PRICE

In this section we consider the global forward market that results from the interaction of the following agents:

- 1) Consumers that demand energy under uncertainty, as analyzed in Section IV. As a function of the forward price p_F , these agents will demand a forward quantity $x^*(p_F)$ that results from the surplus optimization

$$x^* = \operatorname{arg max} \mathbb{E} \left[\max_q \{ U(q, \Theta) - p_F x - C_{p_L}^{p_S}(q - x) \} \right]. \quad (20)$$

Here the expectation is over the utility type Θ , and possibly the imbalance prices p_S, p_L .

- 2) Renewable energy suppliers, as analyzed in Section III. These agents will commit a forward quantity $y^*(p_F)$ that provides the optimal revenue

$$y^* = \operatorname{arg max} \mathbb{E} \left[p_F y - C_{p_L}^{p_S}(y - W) \right]. \quad (21)$$

Here the expectation is over the generation W and possibly the imbalance prices p_S, p_L .

- 3) Dispatchable generators (we assume renewables alone do not cover the demand), jointly characterized by an increasing marginal cost curve $C'(z)$, constructed from price-quantity bids ordered in increasing price. Imposing $C'(z^*) = p_F$ results in an increasing offer curve $z^*(p_F)$.

There could be multiple uncertain consumers and suppliers, with respective bid curves $x_i^*(p_F)$ and $y_j^*(p_F)$. The market clearing condition is

$$z^*(p_F) + \sum_j y_j^*(p_F) = \sum_i x_i^*(p_F). \quad (22)$$

In a classical market equilibrium problem $x_i^*(p_F)$ and $y_j^*(p_F)$ would be monotonic (respectively decreasing and increasing), leading to a well-defined solution to (22). This would indeed happen if the prices p_S, p_L were exogenous, independent of the forward price. In that case we could have to deal, however, with the possibility of the constraint (1) being violated.

But the imbalance and forward markets are not in general independent. If the operator of the forward market sees the reserve bids, before p_F can surpass the cheapest reserve generators, these can be included in the forward dispatch. The remaining reserve bids that are available for short market balancing will therefore be larger than p_F ; symmetrical considerations apply to the long situation. Under this rationale (1) must always hold, but the short and long imbalance prices become dependent on the forward price.

Introduce $\delta_S, \delta_L > 0$, the short and long mean penalties relative to forward price, and their ratio η , by the relations

$$\bar{p}_S = p_F(1 + \delta_S), \quad \bar{p}_L = p_F(1 - \delta_L); \quad \eta = \frac{\delta_S}{\delta_L}. \quad (23)$$

These quantities, which in general could vary with p_F , are now used to analyze the supply and demand bid curves.

A. Properties of bid curves

Consider first the supply bid curves studied in [4], under independence between prices and generation. The optimal offer satisfies the quantile condition (4) with mean prices:

$$F(y^*) = \frac{p_F - \bar{p}_L}{\bar{p}_S - \bar{p}_L}.$$

In terms of the variables of (23) we obtain

$$y^*(p_F) = F^{-1} \left(\frac{1}{1 + \eta} \right).$$

If the ratio η is constant in p_F (e.g, when the penalty fractions δ_S, δ_L are themselves constant) renewable supply which is inelastic in real time, will appear as inelastic in the forward market. But other behaviors are in principle possible. Note that the right hand-side is decreasing in $\eta(p_F)$, therefore if this function increases we will have a un-intuitive, decreasing offer curve $y^*(p_F)$. For further discussion see [4]⁴.

If prices and generation are correlated, the optimal bid is determined through the analysis of Section III-B; while analytical expressions for $y^*(p_F)$ may be hard to obtain, this

⁴In particular, it is mentioned that such un-natural bid curves are forbidden in certain markets.

³ θ being a general parameter, there is no monotonicity in $\phi_L(\cdot)$, $\phi_S(\cdot)$.

function can be evaluated at specific points as part of an iterative procedure to find the clearing price.

Turning to the demand side, the analogous situation is when demand is inelastic and independent of prices; here (11) leads to bid function

$$x^*(p_F) = G^{-1} \left(\frac{\eta}{1 + \eta} \right), \quad (24)$$

with $\eta(p_F)$ as before. This right-hand side is now increasing in η , yielding again un-intuitive forward demand curves if the ratio is increasing in price. Once more, forward demand is inelastic for constant η . Similar considerations apply to the extension to correlated prices and demand.

Let us now look at a case where real-time demand is elastic to price, as considered in Section IV-C. For the utility in (16) we had arrived at the quantile condition (18), which becomes

$$G(x^*) = \mathbf{1}_{\{p_e > p_F\}} \cdot \frac{\min\{p_e - p_F, \delta_S p_F\}}{\min\{p_e - p_F, \delta_S p_F\} + \delta_L p_F}.$$

Suppose δ_S, δ_L are constant with p_F . In this case, the previous identity has two regions:

- (a) For $p_F < \frac{p_e}{1 + \delta_S}$, x^* is constant, with the expression (24).
- (b) For $\frac{p_e}{1 + \delta_S} < p_F$, the optimal bid satisfies

$$G(x^*) = \mathbf{1}_{\{p_e > p_F\}} \cdot \frac{p_e - p_F}{p_e - (1 - \delta_L)p_F}.$$

which is decreasing in p_F ; here the elasticity of the real-time demand appears in the forward bid.

B. Social welfare considerations

Classical economic theory (see [8]) shows that under standard assumptions on consumer utilities and supplier costs, the market clearing price achieves *social welfare*, maximizing the aggregate utility minus cost across all agents. We now briefly take a look at our forward market in this light, assuming for simplicity there is a single demand and renewable supply.

For a social planner with global information, a natural welfare optimization could be

$$\max_z \left\{ \mathbb{E}_{\Theta, W} \left[\max_q \left(U(q, \Theta) - C_{imb}(q - W - z) \right) \right] - C(z) \right\}; \quad (25)$$

This means choosing z in advance, and q upon revelation of uncertain variables Θ, W , to maximize expected utility minus the cost of balancing actions, minus the cost of generation committed in advance. Can this decision be decoupled between agents as discussed before, by appropriate choice of prices?

Let $q^*(z, \Theta, W)$ achieve the inner maximum in (25); the optimality condition

$$C'(z^*) = \mathbb{E}_{\Theta, W} [C'_{imb}(q^*(z^*, \Theta, W) - W - z^*)], \quad (26)$$

can be obtained by differentiating the objective with respect to z under the expectation sign, and applying the envelope theorem [8]. If a solution z^* is found, then choosing $p_F = C'(z^*)$ induces the correct quantity of dispatchable demand.

It is not obvious, however, how imbalance prices could decouple the joint condition (26) in $C'_{imb}(q^* - W - z^*)$, into functions of $q^* - x^*$ and $y^* - W$, as would be required in (15)

and its correlate for supply. In particular there would appear to be a gap in the situation where supply and demand deviations cancel out. Perhaps there is an inevitable inefficiency in the way asymmetric imbalance penalties are assigned, unless one can somehow factor in the cost of availability of reserves that need not be dispatched. Analyzing these issues remains open for further research.

VI. CONCLUSION

We have analyzed power markets with a forward stage and a correction for imbalance, from the point of view of agents who face uncertainty in their demand or supply. The optimal bidding for these agents is characterized in terms of the probability distributions of their uncertainty and the forward and imbalance prices, the latter being possibly random as seen ex ante by the bidding agents. Our results generalize previous research which had considered each side of the market separately, mainly on the demand side where we allow for consumers that are both uncertain and elastic to price. The resulting bid curves interact with dispatchable sources to define a clearing price, leaving questions about the efficiency of the market equilibrium to be studied in future research.

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