Analysis of wake flow models performance in an operating wind farm

José Cataldo*[1] and Bruno López[1]

[1]Instituto de Mecánica de los Fluidos e Ingeniería Ambiental, Facultad de Ingeniería, Universidad de la República, Montevideo, Uruguay

***Correspondence:**
José Cataldo
jcataldo@fing.edu.uy

Keywords: wind energy, wind farm modeling, micro-sitting, power assessment, wake models.

Abstract

An open-source framework has been developed to design and analyze wind farms. This framework can describe the operation of each wind turbine and their interaction with each other through their wakes. The framework must be specified with site topography, wind turbine locations, and roughness length. It must also incorporate hourly or ten-minute meteorological time series, including wind speed, direction, and temperature at two heights. As a result, a production time series is obtained for each wind turbine with the same time interval as the meteorological data. The flow in the wake produced by each wind turbine is described using different analytical models. In this work, the framework is used to analyze the performance of several wake models and compare it with the commercial code WAsP. Actual production data from an operating wind farm located in the northern region of Uruguay is considered for the study. The results show that the correlation between actual and calculated production was slightly lower than 0.6 for each wake flow model considered in the analysis. The empirical probability density function of the difference between actual and calculated power had a mean and mode value of zero but a standard deviation close to 29% of the wind farm power. The calculated annual production of each wind turbine was approximated to the actual production using any of the wake models considered, as well as with WAsP. Although, the best performance was obtained using the open-source framework and the wake model proposed by Jensen.

1 Introduction

The design of a wind farm implies determining the specific locations of wind turbines through a micro-siting study. Such locations are obtained from a trade-off between land use and annual energy production optimization while minimizing the mechanical loads on wind turbines, especially those associated with turbulence. As a result, the wind farm design is an iterative process to obtain maximum power installation and annual energy production and maintain reduced mechanical loads on wind generators (Zhang, 2015). This procedure considers constraints, such as the mean velocity and turbulence intensity at hub height, and the performance curves of the wind turbines. In addition, it is necessary to model the wind turbine wake flow to consider the interaction between wind turbines and the airflow.

This interaction is a multi-scale problem. As the air flows around the wind turbine rotor, interaction forces are applied to the flow, creating a reaction on the blades. This interaction process relies on various factors such as wind speed, air density, and blade geometry. Additionally, turbulence levels and the scale of vortices embedded in the airflow can also impact it. This process is commonly analyzed using tools based on BEM and CFD, as presented in (Burton et al., 2011; Martínez-Tosas et al., 2014), among others.

Due to the forces applied to the flow, a wake is developed downstream of the wind turbine rotor. Several analytical models were developed to describe this region, from the early straightforward model presented by Katic et al. (1987) which modelled only the velocity deficit, through the model introduced by Crespo et al. (1999) where turbulence is described, to more complex models such as those presented by Bastankhah and Porté-Agel (2014) and Grebaad et al. (2014) where the deviation of the wake and the flow in different wake sub-regions is described. A performance analysis of each of these models is presented in (Campagnolo et al., 2019). This paper calibrates the wake models using velocity measurements taken downstream of a wind turbine model in a wind tunnel. The analytical wake flow models are characterized by a low computational cost and ease of use while accurately describing the interaction between the wind and the wind turbine rotor.

The wind farms are designed as clusters of wind turbines to produce power from wind at a large scale. The distance between machines follows a trade-off between wind power availability, wind power directions, topography, and terrain availability. Modelling the interaction between the wake and the wind turbine is relevant at this scale, but accurately estimating the wind farm's annual energy production is the main objective. The micro-sitting tools must include the effect of the terrain on the flow, a wake model and wind turbine performance curves. Examples of such tools include the widely used commercial software WAsP, (2022) and Wind Pro. (2023).

This paper presents an open-source tool to design wind farms and analyze their performance in section 3. In this section, the commercial code WAsP is also introduced. Several wake flow models, tested in the open-source tool, are described in section 2. In section 4, we present results obtained using different models, and in section 5, we analyze their performance.

2 Methods

2.1 Wake flow models

2.1.1 General aspects

When the flow develops around a body, a region with high turbulence and low momentum, named wake, is produced downstream. For a uniform flow with velocity U, it is possible to describe the flow characteristic in a wake based on a theoretical concept as it is presented in (Lesieur, 1997). When a wake with a circular cross-section is produced, as when air flows orthogonally to a circular disk, the momentum deficit distribution could be presented as in Figure 2.1.

In Figure 2.1, the transverse wake dimension is $2\delta(x)$ and the velocity deficit is $\Delta V(x,y) = U-V(x,y)$, where U is the velocity upstream of the rotor and $V(x,y)$ is the velocity at the wake. The velocity deficit in the centerline is defined as $\Delta V_0(x) = U-V_0(x)$. As it is shown in (Lesieur, 1997) for a circular wake, the transverse dimension increases as $x^{1/3}$ and the velocity deficit in the centerline decreases as $x^{2/3}$. Further downstream of the turbine, the flow recovers the characteristics of the free flow.

The transverse distribution of the velocity deficit at a distance x downstream of the turbine could be modelled as shown in equation 2.1.

$$\Delta V(x,y) = \Delta V_0(x) e^{-\left(\frac{y}{\delta(x)}\right)^2} \quad (2.1)$$

The velocity deficit is a result of the thrust force (T) applied to the flow. The non-dimensional expression of this force is called the thrust coefficient, which is defined in equation 2.2.

$$C_T = \frac{T}{\frac{1}{2}\rho A U^2} \quad (2.2)$$

where ρ is the air density and A the rotor swept area.

Different models have been developed to describe the flow in the wake and the force the rotor produces on the flow based on the previous theoretical results.

2.1.2 Model JENSEN

Katic et al. (1987) present a wake flow model assuming a uniform velocity distribution in the wake, as shown in equation 2.3. In the context of this paper this model is named JENSEN.

$$\frac{\Delta V}{U} = 1 - \frac{V}{U} = \frac{1 - \sqrt{1 - C_T}}{\left(1 + 2k\frac{x}{D}\right)^2} \quad (2.3)$$

The velocity deficit is calculated, according to equation 2.3, as a function of the distance ***x*** downstream the rotor with diameter D, the thrust coefficient ***C_T***, the upstream velocity ***U*,** and a coefficient ***k*.** The value of this coefficient differs for the second line of the wind farm (0.075) and the downstream lines (0.11).

2.1.3 Model CRESPO

According to Crespo et al. (1999), the calculation for velocity deficit can be determined using equation 2.4, which is referred to as the CRESPO model.

$$\begin{aligned} &\Delta V = 2.a.U && \text{if } \frac{x}{D} < 2 \text{ or } 3 \\ &\Delta V = k.a.U\left(\frac{D}{x}\right)^n && \text{if } \frac{x}{D} > 2 \text{ or } 3 \end{aligned} \quad (2.4)$$

where ***U*** is the mean velocity upstream of the wind turbine, ***D*** the rotor diameter and ***a*** is de the axial induction velocity coefficient. k and n are parameters that vary between 2 and 4 and between 0.75 and 1.25, respectively.

This model proposes to evaluate the turbulence intensity increment as shown in equation 2.5.

$$\begin{aligned} &I_{ad} = 0.725\, a && \text{if } \frac{x}{D} < 2 \text{ or } 3 \\ &I_{ad} = 0.73\, a^{0.8325}.\, I_{u,eje}^{-0.0325}\left(\frac{D}{x}\right)^{0.32} && \text{if } \frac{x}{D} > 2 \text{ or } 3 \end{aligned} \quad (2.5)$$

where $I_{u,eje}$ is the turbulence intensity upstream the wind turbine.

2.1.4 Model PORTE

Bastankhah et al. (2014) present a model, which in this paper is referred to as PORTE. This model calculates the velocity deficit using equation 2.6.

$$\frac{\Delta V}{U} = \left(1 - \sqrt{1 - \frac{C_T \cos\gamma}{8.\frac{\sigma_y.\sigma_Z}{D^2}}}\right).e^{-\frac{(y-\delta)^2}{2\sigma_y^2}}.e^{-\frac{(Z-Z_h)^2}{2\sigma_Z^2}} \qquad (2.6)$$

The parameters in equation 2.6 are defined in equation 2.7.

$$\frac{\delta}{D} = \theta_{C0}\frac{x}{D} + \frac{\theta_{C0}}{14\cdot 7}\sqrt{\frac{\cos\gamma}{k_y \cdot k_Z \cdot C_T}}\left(2,9 + 1,3\cdot\sqrt{1-C_T} - C_T\right)\ln\frac{\left(1{,}6+\sqrt{C_T}\right)\cdot\left(1{,}6\sqrt{\frac{8\sigma_y\cdot\sigma_Z}{D^2\cdot\cos\gamma}}-\sqrt{C_T}\right)}{\left(1{,}6-\sqrt{C_T}\right)\cdot\left(1{,}6\sqrt{\frac{8\sigma_y\cdot\sigma_Z}{D^2\cdot\cos\gamma}}+\sqrt{C_T}\right)}$$

$$k_y = k_Z = k_a \cdot I_0 + k_b$$

$$\theta_{C0} = \frac{0,3\cdot\gamma}{\cos\gamma}\cdot\left(1-\sqrt{1-C_T\cdot\cos\gamma}\right) \qquad (2.7)$$

$$\frac{\sigma_y}{D} = k_y\frac{(x-x_0)}{D} + \frac{\cos\gamma}{\sqrt{8}}$$

$$\frac{\sigma_Z}{D} = k_Z\frac{(x-x_0)}{D} + \frac{\cos\gamma}{\sqrt{8}}$$

$$\frac{x_0}{D} = \frac{\cos\gamma\cdot\left(1-\sqrt{1-C_T}\right)}{\sqrt{2}\left(\alpha * I_0 + \beta * \left(1-\sqrt{1-C_T}\right)\right)}$$

This model assumes a Gaussian velocity deficit for the wake transverse distribution, similar to the theoretical expression presented in equation 2.1, and a misalignment γ between the wake centerline and the wind direction as it is showed in figure 2.2. Parameters k_a, k_b, α^* y β^* must be adjusted in relation to wind exposure.

2.1.5 Model FLORIS

Gebraad et al. (2014) presented a wake model identified in this paper as FLORIS, composed of three regions: near wake, far wake and the mixed layer. Figure 2.3 shows a sketch of the proposed wake model.

This model proposes the evolution of the wake width and the velocity deficit in each region. Also, this model considers misalignment between wind direction and rotor axis. Equation 2.8 presents the growth of the wake width, shown in figure 2.3.

$$\boldsymbol{\delta_1 = D + 2.k_e.m_{e1}.Y_i}$$

$$\boldsymbol{\delta_2 = D + 2.k_e.m_{e2}.Y_i} \qquad (\mathbf{2.8})$$

$$\boldsymbol{\delta_3 = D + 2.k_e.m_{e3}.Y_i}$$

Equation 2.9 shows the velocity deficit proposed in FLORIS model.

$$\boldsymbol{\Delta V = U_\infty - U_w = 2.a.C_k} \qquad (\mathbf{2.9})$$

The coefficients *Ck* are defined in equation 2.10.

$$\begin{array}{ll}
\textbf{If } |\boldsymbol{X_i}| < \boldsymbol{\delta_1/2} & \boldsymbol{C_1 = \left(\frac{D}{D + k_e.m_{U1}.Y_i}\right)^2} \\
\textbf{If } \boldsymbol{\delta_1/2} < |\boldsymbol{X_i}| < \boldsymbol{\delta_2/2} & \boldsymbol{C_2 = \left(\frac{D}{D + k_e.m_{U2}.Y_i}\right)^2} \\
\textbf{If } \boldsymbol{\delta_2/2} < |\boldsymbol{X_i}| < \boldsymbol{\delta_3/2} & \boldsymbol{C_3 = \left(\frac{D}{D + k_e.m_{U3}.Y_i}\right)^2} \\
\textbf{If } |\boldsymbol{X_i}| > \boldsymbol{\delta_3/2} & \boldsymbol{C_1 = 0}
\end{array} \qquad (\mathbf{2.10})$$

m_{Uk} parameters are defines as equation 2.11.

$$\mathbf{m_{Uk} = \frac{M_{Uk}}{\cos(a_U)}} \qquad (\mathbf{2.11})$$

The parameters k_e, m_{e1}, m_{e2}, m_{e3}, M_{U1}, M_{U2}, M_{U3} y a_U must be selected to adjust the model.

2.1.6 Models adjustment

Each model detailed previously includes parameters to be determined. The value of those parameters could be associated with wind exposure, but they could also depend on the wind farm geometry, the topography, or the incident turbulence level. Furthermore, it is important to note that wind turbines experience varying wind characteristics, such as velocity distribution and turbulence, depending on the direction of the wind.

While models JENSEN and CRESPO are applied to analyze wind farms' performance, Campagnolo et al. (2019) show the performance of models PORTE and FLORIS to describe the wake flow downstream of a wind turbine model in a wind tunnel. Although the authors obtained these results for different turbulence intensities upstream of the wind turbine, they could differ from the actual conditions in a wind farm.

Campagnolo et al. (2019) present wind speed measurement in the wake flow and a methodology to optimize the different models' performance and obtain the parameters' values. The following sections analyze the performance of the different wake models to describe the production of an operating wind farm, using a methodology inspired by the work of Campagnolo et al. (2019).

2.2 Micro-sitting models

The performance of the wake models presented in the previous section is analyzed using a dataset obtained from an operating onshore wind farm in Uruguay. With this aim, the models are implemented in an in-house framework developed by the authors in MATLAB. The obtained results are then compared with commercial code WAsP.

2.2.1 Open-source micro sitting model

2.2.1.1 Required data

The model analyzes the performance of wind farms composed of several wind turbines. Typically, these turbines are equal to each other, but the model can also assess different wind turbine models simultaneously. For each analyzed machine, it is required to know the rated power, rotor diameter and hub height (Alt), as well as the power and thrust coefficient curves as a function of wind velocity every 0.5 m/s, between 0 and 30 m/s.

The meteorological data are measured from a mast and stored in time series with an hourly or ten-minute time step. The evolution of the wind farm production will be computed using this data time step. The time series include mean wind velocity (V, at one or two heights), wind direction (φ), root mean square of velocity fluctuation (σ) and the temperature (T) at two heights. Also, it must be specified the measurement height (Z_{Ref}), roughness length (Z_0) and zero displacement plan height (d) around the mast for different wind directions every 10°.

For each possible wind turbine site, the coordinates are specified in a geographically indexed frame. The topography is described from satellite information available in the following website **http://srtm.csi.cgiar.org/SELECTION/inputCoord.asp**. Once the Shuttle Radar Topography Mission (SRTM) region is identified, a rectangular zone is defined where the wind farm would be located. As a preliminary result, the contour lines are obtained and verified from the digital information about the topography. Then, the altitude and slope of the terrain are calculated at each possible location of a wind turbine. From this information, a speed-up (SU) factor is guessed for every 10° wind direction using a model developed with this aim.

For each time series component, the temperature gradient is calculated following equation 2.12.

$$\frac{\partial T}{\partial Z} \cong \frac{T(Z_2) - T(Z_1)}{Z_2 - Z_1} \qquad (2.12)$$

The temperature gradient is used to determine Pasquill´s stability class following the criteria specified in (Arya, 1999) and shown in table 2.1. From the stability class and the roughness length, an estimation of the Monin-Obukhov length scale is determined following the relationship shown in figure 2.4 from (Golder, 1972).

In addition, if the velocity at two heights is known, the Monin-Obukhov length scale could be estimated from the Richardson number, defined as equation 2.13, following the work of Stull (1988).

$$Ri = \frac{g}{\theta}\frac{\frac{\partial\theta}{\partial z}}{\left(\frac{\partial U}{\partial z}\right)^2} \qquad (2.13)$$

where θ is the potential temperature. In this work, potential temperature is approximated using the temperature value.

2.2.1.2 Model description

The production of each wind turbine is calculated for each meteorological time series component. Thus, a power time series is obtained with the same time step as the meteorological data.

When defining a wind farm layout, a first micro-sitting guess is proposed considering the wind rose, topography and wind turbine dimensions. A coordinate is then assigned to each machine in the same frame as the topography.

A frame rotation is made for each analyzed wind direction j, so the new frame would have the y-axis pointing upstream, as shown in figure 2.5. The wind turbines are then ordered in decreasing y-value so that the first one in the list, with the greatest y-value, is not affected by the wake of the other wind turbines.

The wind turbines are then processed in the order of the list, from $\ell = 1$, the wind turbine with highest y-coordinate, to $\ell = s$, where s is the number of turbines. For each wind turbine ℓ, a first approximation of the turbulence intensity (I_x) and the mean velocity (V_{nn}) of a flow undisturbed by the wakes are obtained from the hub height, the changes of roughness (Z_0) and the speed-up factor (SU). The extrapolation is made following equations 2.14 and 2.15.

$$\mathbf{I_X} = \frac{\boldsymbol{\sigma}_{\mathbf{EM}}}{\mathbf{V_{EM}}}.\frac{\ln\left({}^{\mathbf{Z_{REF}}}/_{\mathbf{Z_0}(\boldsymbol{\varphi})}\right)}{\ln\left({}^{\mathbf{Alt}}/_{\mathbf{Z}_{0,\ell}(\boldsymbol{\varphi})}\right)} \qquad (2.14)$$

$$\mathbf{V_{nn}} = \mathbf{V_{REF}}.\left(\frac{\mathbf{Z}_{0,\ell}(\boldsymbol{\varphi})}{\mathbf{Z_0}(\boldsymbol{\varphi})}\right)^{0.0706}\frac{\ln\left({}^{\mathbf{Alt}}/_{\mathbf{Z}_{0,\ell}(\boldsymbol{\varphi})}\right) - \boldsymbol{\psi}\left(\frac{\mathbf{Alt}}{\mathbf{L}},\frac{\mathbf{Z}_{0,\ell}(\boldsymbol{\varphi})}{\mathbf{L}}\right)}{\ln\left({}^{\mathbf{Z_{Ref}}}/_{\mathbf{Z_0}(\boldsymbol{\varphi})}\right) - \boldsymbol{\psi}\left(\frac{\mathbf{Z_{REF}}}{\mathbf{L}},\frac{\mathbf{Z_0}(\boldsymbol{\varphi})}{\mathbf{L}}\right)}.\mathbf{SU}_\ell(\boldsymbol{\varphi}) \qquad (2.15)$$

where ψ is a function that depends on atmospheric stability as it is presented in equation 2.16.

Unstable atmosphere $$\boldsymbol{\psi}\left(\frac{Z}{L},\frac{Z_0}{L}\right) = \ln\left(\frac{(1+uu_2)^2.(1+uu_2^2)}{(1+uu_1)^2.(1+uu_1^2)}\right) - 2atg(uu_2) + 2atg(uu_1)$$

Stable Atmosphere $$\boldsymbol{\psi}\left(\frac{Z}{L},\frac{Z_0}{L}\right) = -4.7\left(\frac{Z}{L}-\frac{Z_0}{L}\right) \qquad (2.16)$$

Neutral Atmosphere $$\boldsymbol{\psi}\left(\frac{Z}{L},\frac{Z_0}{L}\right) = 0$$

For unstable atmosphere the magnitudes *uu₁* y *uu₂* are defined in equation 2.17.

$$uu_1 = \sqrt[4]{1 - 15\, {}^{Z_0}\!/_{L}}$$

$$(2.17)$$

$$uu_2 = \sqrt[4]{1 - 15\, {}^{Z}\!/_{L}}$$

For each wind turbine ℓ, the wake interaction produced by the rest of the wind turbines (m) is computed following the ordered list. In the first place, the relative coordinates X_i y Y_i are defined following equation 2.18 for the m wind turbine.

$$X_i = x_m - x_\ell$$

$$(2.18)$$

$$Y_i = y_m - y_\ell$$

If Y_i is negative, then the m wind turbine does not affect the considered wind turbine ℓ and no value is added to the wind deficit *DELTAC*. On the other hand, if Y_i is positive, then there is aerodynamic interference if the rotor has any part inside the wake and the produced deficit following the wake model is added to *DELTAC*. The addition would be linear or quadratic depending on the used wake model.

The velocity in the turbine ℓ is calculated using equation 2.19, from *DELTAC* and V_{nn}, defined in equation 2.15.

$$V_P(\ell) = V_{nn}(1 - DELTAC) \qquad (2.19)$$

Each wake model proposes the modification of the turbulence intensity defining an additional turbulence ($I_{ad,m}$) induced by the wind turbine m on the wind turbine ℓ. Finally, the turbulence intensity is calculated at each wind turbine ℓ as described in equation 2.20.

$$I_u(\ell) = \sqrt{I_x^2 + \sum_m I_{ad,m}^2} \qquad (2.20)$$

When the calculated velocity $V_P(\ell)$ is below zero, greater of cut out or greater than 15 m/s, and turbulence intensity is greater than the reference value for the wind turbine class, the power is considered zero. In other cases, the power is obtained from the performance curve.

If the intensity of turbulence is greater than 10 % a factor (DELTAPT) is applied to the power following equation 2.21.

$$(1 - \ DELTAPT) = \big(1 - \ 1.11(I_u(\ell) - 0.1)\big) \qquad (2.21)$$

Although this correction is inspired by the methodology proposed in code IEC, (2017) to determine the normalized performance curve, a simplification is proposed based on an engineering approach. Finally, a power production time series for all meteorological time series and for each wind turbine is obtained.

2.2.2 WAsP model

The WAsP is a commercial code that can interpolate wind velocity vertically and horizontally across the entire study region where a wind farm is planned for installation. It can also calculate the annual production of each wind turbine, giving an overall estimate of the wind farm's energy production.

WAsP is a hierarchical model consisting of a workspace with several projects. Each project contains information about the topography, location, and dimensions of obstacles, as well as the wind turbines' location and dimensions. It also includes meteorological data such as wind direction, velocity, and height of measurements. The wind farm description includes the location of each wind turbine and the performance curves power-velocity and thrust coefficient-velocity.

The wakes are modeled in the code using the JENSEN model. The swept rotor area proportion affected by the wake is estimated for each wind turbine. Then, a weighting of the affected and non-affected swept area is considered to estimate the velocity downstream of the rotor. The model also accounts for atmospheric stability by adjusting a parameter proportional to the sensible heat from ground to air and applying it to the data.

2.3 Test case

The model's performance was evaluated for a wind farm named JPT, located in the north of Uruguay, composed of 28 NORDEX wind turbines, model N117/2400. Figure 2.6 shows the location and micro-sitting of this wind farm, while in figure 2.7, probability density distribution and wind rose are described.

Production data from each wind turbine was available from SCADA from 2015 to 2019, every 10 minutes, as meteorological data from a nearby mast. Depuration data was made considering: all turbines in operation, operation of all turbines with non-error, and meteorological data available. After this depuration, a time series of 1.37 yearlong was obtained. The capacity factor was 51.09 %.

3 Results

3.1 Open-source model calibration

A model optimization methodology has been developed based on three performance indices. These indices are the capacity factor, a second index called *DifAero*, which is defined in equation 3.1 and evaluates the difference between wind turbine production at each time step, and a third index *DifFarm,* defined in equation 3.2, which assesses the difference between wind farm production at each time step.

$$DifAero = \frac{\sqrt{\sum_{j=1}^{j=L}\sum_{i=1}^{i=N}\left(\frac{P_i(\tau_j)-\widehat{P_i}(\tau_j)}{P_i(\tau_j)}\right)^2}}{N_{Aero}.L} \qquad (3.1)$$

$$DifFarm = \frac{\sqrt{\sum_{j=1}^{j=L}\left(\frac{\left(\sum_{i=1}^{i=N}P_i(\tau_j)\right)-\left(\sum_{i=1}^{i=N}\widehat{P_i}(\tau_j)\right)}{\sum_{i=1}^{i=N}P_i(\tau_j)}\right)^2}}{L} \qquad (3.2)$$

where *N* is the number of turbines (28) and L the length of the time series (72061). $P_i(\tau_j)$ is the actual production of each wind turbine for each time τ_j, while $\widehat{P}_i(\tau_j)$ is the production estimated by the model.

The three indices Capacity Factor, *DifFarm* and *DifAero* are calculated for each parametrization of each wake model.

Each wake model presents several parameters a_i, expressed as a vector κ (see equation 3.3).

$$\boldsymbol{\kappa = (a_1, a_2, a_3, \ldots \ldots, a_1)} \qquad (3.3)$$

The vector dimension is dependent on the model. While for the JENSEN model the vector has dimension one, for the FLORIS model the dimension is eight.

During calibration, the objective is to obtain the parameter vector for each wake model that produces the best fit between the model and the actual production. The optimal model parameterization is obtained by first getting the same capacity factor deduced from the production and then minimizing the *DifFarm* and *DifAero* indices.

A first parameter values guess was obtained from the bibliography (Katic et al., 1987; Crespo et al., 1999; Campagnolo et al., 2019). Such values are presented in table 3.1.

After applying the procedure mentioned above to calibrate the models, the parameter values obtained are presented in table 3.2.

3.2 WAsP calibration

WAsP model version 11.6 was run using the sensible heat mean value of -60 W/m^2 and a dispersion of 100 W/m^2. Table 3.3 presents the results obtained.

4 Discussion

Results presented in tables 3.1 and 3.2 for models JENSEN, CRESPO and FLORIS show that the obtained parameter values are close to the initially guessed ones (given in the bibliography). In contrast, for PORTE model the difference was between three and one order of magnitude. Based on the values presented in table 3.2, the annual production was calculated using the different wake flow models.

Correlation analysis between actual and calculated production for the wind farm and wind turbines 1 and 12 was made. Figure 2.6 shows that wind turbine 1 is on the wind farm EAST bound, where aerodynamic interference is low while wind turbine 12 is in the middle of the wind farm. Table 4.1 presents the correlation coefficient for each case.

In general, the coefficient correlation was relatively low, with the highest value being obtained for the wind farm. Wind turbine 1 had a slightly lower value, and wind turbine 12 had the smallest value. However, the correlation coefficient was not particularly sensitive to the model.

Figure 4.1 shows, for each model, the empiric probability density function of the difference between production and calculated value. The difference was calculated for each time step.

The mean value and the mode value are both zero or very close. An asymmetry was observed with a kurtosis coefficient of -0.1. However, a very high standard deviation was obtained, about 29 % of wind farm power rate.

However, figure 4.2 shows similar annual production calculated with any model and the actual production. The performance evaluation of the model was made with three indices.

The first index consists in the relative difference between the production of each wind turbine and is defined in equation 4.1.

$$\boldsymbol{Relative\ Diference = \frac{1}{N}\sum_{i=1}^{i=N}\frac{P_{cal,i} - P_i}{P_i}x100} \qquad (\mathbf{4.1})$$

where $P_{cal,i}$ is the calculated annual production for the wind turbine i, P_i is the annual production of the same wind turbine and N is the number of wind turbines.

Second index, similar to the previous one but using the absolute difference according to equation 4.2.

$$\boldsymbol{Absolute\ Relative\ Diference = \frac{1}{N}\sum_{i=1}^{i=N}\frac{|P_{cal,i} - P_i|}{P_i}x100} \qquad (\mathbf{4.2})$$

A third index is the relative difference between the calculated wind farm production and actual production defined as equation 4.3.

$$\boldsymbol{Wind\ Farm\ Relative\ Difference = \frac{\sum_{i=1}^{i=N} P_{cal,i} - \sum_{i=1}^{i=N} P_i}{\sum_{i=1}^{i=N} P_i}x100} \qquad (\mathbf{4.3})$$

Table 4.2 presents the index values obtained with the different wake models and WAsP.

In conclusion, an open-source framework for wind farm design and analysis has been developed that accurately describes the operation of each wind turbine and their interaction with one another through their wakes. The calculated annual production of each wind turbine was approximated to the actual production using any of the wake models considered, as well as with WAsP. Although, for annual wind farm production assessment, the JENSEN model shows the best performance. This model obtained the smallest values for Relative Difference and Wind farm relative difference indices. While the Absolute Relative Difference had the smallest value for the PORTE model, the other models showed values of similar order.

5 Author Contributions

The first author, José Cataldo, was responsible for writing the original draft, directing the research, developing the methodology, and creating the essential software tools for the study. On the other hand, the second author Bruno López conducted extensive revisions and edits to the manuscript, contributed to the research process, and effectively managed the project administration tasks.

6 Acknowledgments

The authors wish to acknowledge financial support from the National Agency for Research and Innovation (ANII) of Uruguay. This work was supported by ANII research project FSE_1_1_2018_1_152951.

7 References

Arya, S.P. (1999). Air pollution meteorology and dispersion (Vol. 310). New York: Oxford University Press.

Bastankhah, M. and Porté-Agel, F. (2014). A new analytical model for wind-turbine wakes. Renewable energy, 70, pp.116-123.

Burton, T., Jenkins, N., Sharpe, D. and Bossanyi, E. (2011). Wind energy handbook. John Wiley & Sons.

Campagnolo, F., Molder, A., Schreiber, J. and Bottasso, C.L. (2019). Comparison of analytical wake models with wind tunnel data. In Journal of Physics: Conference Series (Vol. 1256, No. 1, p. 012006). IOP Publishing.

Crespo, A., Hernandez, J. and Frandsen, S. (1999). Survey of modelling methods for wind turbine wakes and wind farms. Wind Energy: An International Journal for Progress and Applications in Wind Power Conversion Technology, 2(1), pp.1-24.

Gebraad, P.M., Teeuwisse, F.W., Van Wingerden, J.W., Fleming, P.A., Ruben, S.D., Marden, J.R. et al. (2016). Wind plant power optimization through yaw control using a parametric model for wake effects—a CFD simulation study. Wind Energy, 19(1), pp.95-114.

Golder, D. (1972). Relations among stability parameters in the surface layer. Boundary-Layer Meteorology, 3, pp.47-58.

IEC, (2017) Wind energy generation systems – Part 12-1: Power performance measurements of electricity producing wind turbines, IEC 61400 – 12 – 1.

Katic, I., Højstrup, J. and Jensen, N.O. (1986). A simple model for cluster efficiency. In European wind energy association conference and exhibition (Vol. 1, pp. 407-410). Rome, Italy: A. Raguzzi.

Lesieur, M. (1997). Turbulence in fluids. Springer.

Martínez-Tossas, L.A., Churchfield, M.J. and Leonardi, S. (2015). Large eddy simulations of the flow past wind turbines: actuator line and disk modeling. Wind Energy, 18(6), pp.1047-1060.

Stull, R.B. (1988). An introduction to boundary layer meteorology (Vol. 13). Springer Science & Business Media.

WAsP, (2022), WAsP version 1.16, https://www.wasp.dk/

Wind Pro, (2023), https://www.emd-international.com/windpro/

Zhang, M.H. (2015). “Wind Resource assessment and micro-siting. Science and Engineering”, China Machine Press, John Wiley and Sons.

8 Figure captions

Figure 2.1. Wake flow sketch.

Figure 2.2. Misalignment wake centerline sketch.

Figure 2.3. FLORIS wake model sketch, composed of three regions: near wake, far wake and the mixed layer.

Figure 2.4. Relation between roughness length, stability class and Monin-Obukhov length scale (Golder, 1972).

Figure 2.5. Wind farm sketch, frame, and rotation.

Figure 2.6. Location and micro-sitting JPT Wind farm.

Figure 2.7. Wind velocity pdf and wind rose.

Figure 4.1. Probability density curve of difference between production and calculation.

Figure 4.2. Net annual production.

9 Tables

Table 2.1. Pasquill´s stability class (Arya, 1999).

Stability class	Temperature gradient (°K/km)
A	< -19
B	-19 to -17
C	-17 to -15
D	-15 to -5
E	-5 to 15
F	15 to 40
G	> 40

Table 3.1. Initial guess parameter for the different wake models, obtained from bibliography.

Model	Parameters Vector							
JENSEN	$k = 0.11$							
CRESPO	$k = 3$	$n = 1$						
PORTE	$k_a = 0.054$	$k_b=0.025$	$\alpha^* = 2.32$	$\beta^* = 0.154$				
FLORIS	$k_e= 0.0655$	$m_{e1}=-0.5$	$m_{e2}=0.22$	$m_{e3}=1$	$M_{U1}=0.5$	$M_{U2}=1$	$M_{U3}=5.5$	$a_U=5$

Table 3.2. Optimized parameters for each model.

Model	Parameter Vector

JENSEN	$k = 0.23$							
CRESPO	$k = 2$	$n = 1.25$						
PORTE	$k_a = 0.5$	$k_b=0.4$	$\alpha^* = 4.97$	$\beta^* = 0.1$				
PORTE	$k_a = 0.5$	$k_b=0.1$	$\alpha^* = 4.94$	$\beta^* = 0.4$				
FLORIS	$k_e= 0.028$	$m_{e1}= -0.7$	$m_{e2}=0.5$	$m_{e3}=0.843$	$M_{U1}=0.4$	$M_{U2}=0.9$	$M_{U3}=4$	$a_U=10$

Table 3.3. Results obtained with WAsP.

	X[m]	**$Y[m]$**	**Elev. [m]**	**Velocity [m/s]**	**Gross Annual Production [GWh]**	**Net Annual Production [GWh]**	**Wake loss [\%]**
Met mast	0	0	165.3	7.72	-	-	-
JP 1	1362	-1950	153.5	7.49	11,336	10,967	3.26
JP 2	1398	-2341	163.6	7.58	11,532	11,083	3.90
JP 3	1451	-2730	167.0	7.54	11,441	11,156	2.49
JP 4	1575	-3739	182.2	7.71	11,876	11,794	0.69
JP 5	468	-3193	182.7	7.77	11,976	11,538	3.65
JP 6	-217	-2731	170.7	7.68	11,810	11,219	5.01
JP 7	-401	-2204	180.0	7.79	12,063	11,395	5.54
JP 8	-498	-1811	174.1	7.69	11,813	11,080	6.21
JP 9	-589	-1451	169.7	7.65	11,721	10,984	6.29
JP 10	-672	-1021	164.9	7.64	11,677	11,034	5.51
JP 11	-770	-595	158.2	7.58	11,524	10,897	5.44
JP 12	-821	-110	159.0	7.63	11,646	11,054	5.08
JP 13	-851	293	154.6	7.61	11,605	11,008	5.14
JP 14	-879	700	156.5	7.67	11,773	11,251	4.43
JP 15	-1000	1100	147.7	7.58	11,556	11,150	3.52
JP 16	-2079	-1641	169.8	7.68	11,775	11,024	6.38
JP 17	-2193	-1264	163.1	7.61	11,618	10,695	7.95
JP 18	-2243	-561	152.0	7.52	11,383	10,587	6.99
JP 19	-2184	1932	145.9	7.59	11,566	11,073	4.26
JP 20	-2211	2284	147.2	7.59	11,566	10,950	5.33
JP 21	-2250	2676	144.5	7.55	11,447	10,839	5.31
JP 22	-2300	3073	147.4	7.62	11,644	11,081	4.84
JP 23	-2346	3467	152.4	7.74	11,926	11,493	3.64

JP24	-3678	3193	141.8	7.52	11,412	10,629	6.87
JP 25	-3736	2797	159.4	7.79	12,045	11,070	8.10
JP 26	-3875	2441	151.0	7.6	11,590	10,578	8.73
JP 27	-3983	2029	159.1	7.71	11,830	10,934	7.58
JP 28	-4209	1662	155.7	7.65	11,681	11,001	5.82

Table 4.1. Correlation coefficient between time series.

	Wind Farm	**Wind turbine 1**	**Wind turbine 12**
JENSEN	0.580	0.561	0.497
CRESPO	0.589	0.556	0.542
PORTE	0.581	0.565	0.544
FLORIS	0.581	0.559	0.504

Table 4.2. Performance model indices.

Model	**JENSEN**	**CRESPO**	**PORTE**	**FLORIS**	**WAsP**
Relative difference (%)	0.08	2.80	0.20	0.12	3.07
Absolute relative diff (%)	4.90	3.69	3.15	4.67	4.05
Wind farm relative diff (%)	-0.01	2.70	0.15	0.05	2.92