

# DIFFERENTIAL SPACE TIME BLOCK CODES FOR HIGH MOBILITY SCENARIOS

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Keywords: OFDM, MIMO, DSTBCs, WiMAX, LTE, Wireless Broadband Mobile Networks.

Abstract: In this paper the advantages of using a particular class of Differential Space Time Block Codes (DSTBCs) in high mobility scenarios are reported. This is a high bandwidth efficiency technique with specially good performance when the mobile terminal velocity is high. For Orthogonal Frequency Division Multiplexing (OFDM) based systems in high mobility scenarios, as the ones that can be considered for Worldwide Interoperability for Microwave Access (WiMAX) and Long Term Evolution (LTE), the analyzed technique reports improvements of up to 14 dB with respect to the use of 64PSK in DSTBCs.

## 1 INTRODUCTION

Space diversity produces well known benefits over the performance of wireless systems. One alternative to achieve it, is by using Space Time Block Codes (STBCs) (Alamouti, 1998), (Tarokh et al., 1999), which were quite successful and very well accepted as an interesting solution for improving the transmission quality in wireless systems. More precisely STBC is one of the three operation modes considered in IEEE802.16e (Matrix A, Matrix B and Beamforming). STBC (Matrix A operation mode) provides in this case a robust transmission mode useful in low SNR scenarios.

But space diversity schemes have a price, as in all Multiple Input Multiple Output (MIMO) systems. The price is a more complex channel estimation process, because more channels are involved in the communication. This complexity grows with the order of the MIMO system. If we consider a MIMO system with two transmit antennas and three receive antennas (MIMO 2x3), then six channels must be estimated and corrected in order to have an acceptable quality in the transmission. Therefore, differential coding schemes as DSTBCs have been proposed (Tarokh and Jafarkhani, 2000; Tao and Cheng, 2001; Xia, 2002; Chen et al., 2003; Hwang et al., 2003; Bauch, 2004;

Bauch and Mengi, 2005), which do not need any information about the radio channel. The information is transmitted in this case as the quotient between two adjacent modulation symbols which are equally affected by the channel. That allows the elimination of the channel influence in the receiver and for this reason is not necessary to cope with complex channel estimation and correction procedures. That become the DSTBC schemes a quite attractive technique to increase the number of antennas in the system in order to obtain better performance.

In (Rodríguez and Rohling, 2006), (Rodríguez and Rohling, 2007) and (Rodríguez, 2007) a new class of DSTBC was proposed and exhaustively analyzed. This paper discusses the system performance of this particular class of DSTBCs for high mobility scenarios.

## 2 DESCRIPTION OF AN IMPROVED DSTBC SCHEME

In all this work a flat radio channel transfer function is considered, which is an acceptable assumption for each subcarrier in a multicarrier system, in particular for OFDM systems.

As it was described in (Rodríguez and Rohling, 2006), the first step is to obtain the information matrix  $S_k$  which contains the two complex valued modulation symbols  $s1_k$  and  $s2_k$ .

$$S_k = \begin{bmatrix} s1_k & s2_k \\ -s2_k^* & s1_k^* \end{bmatrix} \quad (1)$$

Then the rule to obtain the differential modulation is shown in the next equation.

$$C_k = S_k \cdot C_{k-1} = \begin{bmatrix} c1_k & c2_k \\ -c2_k^* & c1_k^* \end{bmatrix} \quad (2)$$

Where  $C_k$  is the matrix to be transmitted.

In the receiver the following matrix  $R_k$  is obtained:

$$\begin{bmatrix} r1_k & -r2_k^* \\ r2_k & r1_k^* \end{bmatrix} = \begin{bmatrix} c1_k & c2_k \\ -c2_k^* & c1_k^* \end{bmatrix} \cdot \begin{bmatrix} h1_k & -h2_k^* \\ h2_k & h1_k^* \end{bmatrix} + \begin{bmatrix} n1_k & -n2_k^* \\ n2_k & n1_k^* \end{bmatrix} \quad (3)$$

in a summarized form

$$R_k = C_k \cdot H_k + N_k \quad (4)$$

Where  $C_k$  is the transmitted matrix,  $H_k$  is the channel matrix and  $N_k$  is the noise matrix -Additive White Gaussian Noise (AWGN) mainly added in the receiver-

The incoherent decoding procedure is performed as indicated in the next equation

$$D_k = R_k \cdot R_{k-1}^{-1} \simeq C_k \cdot H_k \cdot (C_{k-1} \cdot H_{k-1})^{-1} = C_k \cdot H_k \cdot H_{k-1}^{-1} \cdot C_{k-1}^{-1} \quad (5)$$

where the noise influence was ignored.

By assuming that the channel does not vary in a short time interval, i.e.

$$H_k = H_{k-1}$$

the following result can be obtained (see (2))

$$D_k \simeq C_k \cdot C_{k-1}^{-1} = S_k \quad (6)$$

The used modulation scheme was “4A16PSK” defined in (Rodríguez and Rohling, 2006). This modulation scheme has two subconstellations, a small and a big one. It has 4 amplitude values and 16 phase values for each subconstellation. Each constellation has 2 subconstellations with one amplitude value in common ( $a1 = \sqrt{0.5}$ ); 7 different amplitudes for the whole constellation (see Fig. 1). In “4A16PSK” modulation scheme the set of possible amplitudes  $A$  is determined by a parameter  $a = 1.4$  (optimum value) as follows:

$$A \in \{(\sqrt{0.5}) \cdot [1/a^3, 1/a^2, 1/a, 1, a, a^2, a^3]\}$$

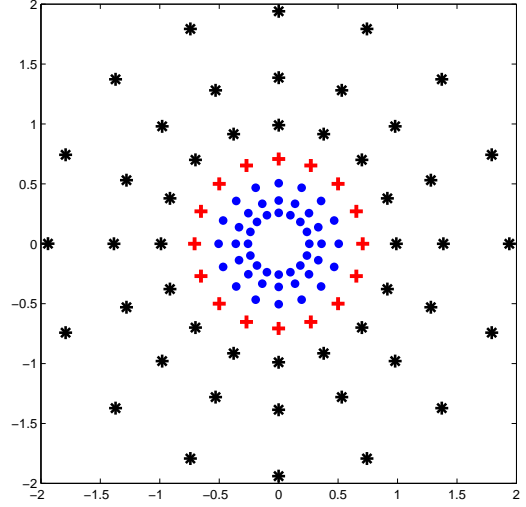


Figure 1: “4A16PSK” modulation scheme used in DSTBC.

(coding [01, 11, 10, 00, 01, 11, 10] respectively) and the phases are 16 equal spaced phase states starting in  $0^\circ$ . These phases map 4 bits onto one modulation symbol in a Gray coding way. The small subconstellation is composed by those constellation points with amplitude smaller or equal to  $a1 = \sqrt{0.5}$  (“+” and “\*” in Fig. 1) and the big one by those constellation points with amplitude bigger or equal to  $a1 = \sqrt{0.5}$  (“+” and “\*” in the same figure).

In the transmitter a Power Control Mechanism (PCM) decides which of both subconstellations should be used in order to control the transmit power. When the small subconstellation is used the transmit power is decreased or maintained. To increase or maintain the transmit power the big subconstellation is used.

In (Rodríguez and Rohling, 2006) it was suggested a PCM (PCM1) based in the observation of the *spectral norm* of the previous transmit matrix ( $\|C_{k-1}\|_2$ ).  $\|C_k\|_2 \equiv (\text{maximum eigenvalue } (C_k^H \cdot C_k))^{1/2}$ , where  $H$  means Hermitian. For matrices as the  $C_k$  matrix defined in (2) it can be written as  $\|C_k\|_2 = (|c1_k|^2 + |c2_k|^2)^{1/2}$ .

This is basically the procedure, which has been proposed in (Rodríguez and Rohling, 2006). Later it was found that better performance can be achieved, by taking the decision of what subconstellation to use (the small or the big one), after calculating the  $\|C_k\|_2$  values that the use of these subconstellations - for mapping  $s1_k$  and  $s2_k$ - would produce (Rodríguez,

2007). This is a PCM (PCM2) based in the observation of the possible values of  $\|C_k\|_2$ , rather than in the observation of the *spectral norm* of the previous transmitted matrix ( $\|C_{k-1}\|_2$ ).

The main difference here is that this PCM is not any more based in  $C_{k-1}$  (the previous transmitted matrix) but in the possible  $C_k$  matrices to transmit. The objective of the PCMs is to maintain  $\|C_k\|_2$  fluctuating around a given value. In (Rodríguez, 2007) and (Jiang et al., 2010) the performance of “4A16PSK” with PCM2 in a Rayleigh fading channel can be observed and compared with 64-PSK in DSTBC.

A variation of these control power mechanisms, based also in two subconstellations, was later considered in (Fellenberg and Rohling, 2009). In this case a 64-QAM scheme is considered and evaluated.

For a comparative evaluation of “4A16PSK PCM2” the reader can check (Xia, 2002), (Bauch, 2004), (Vanaev and Rohling, 2006), and (Fellenberg and Rohling, 2009); there different Amplitude and Phase Shift Keying (APSK) modulation schemes for DSTBC are analyzed.

### 3 SIMULATION RESULTS

Considering typical parameters for a WiMAX system in a Wide Sense Stationary Uncorrelated Scattering (WSSUS) channel, the performance of “4A16PSK PCM2” for different velocities of the mobile terminal was evaluated and compared with the use of 64-PSK in DSTBC.

When the simulations are performed for a single subcarrier ( $f_0$ ), the time variance is described as follows (Hoeher, 1992)

$$h(\tau, t) = \frac{1}{\sqrt{P}} \cdot \sum_{p=1}^P \delta(\tau - \tau_p) \cdot e^{j(2\pi f_{D,p}t + \theta_p)} \quad (7)$$

by making a Fourier transform of (7) in the direction of  $\tau$  and using the time as discrete ( $t = nT$ ), (8) is obtained

$$H(f, nT) = \frac{1}{\sqrt{P}} \cdot \sum_{p=1}^P e^{j2\pi f_{D,p}nT} \cdot e^{j\theta_p} \cdot e^{-j2\pi f\tau_p} \quad (8)$$

then by evaluating (8) in  $f = f_0$  the used equation is obtained

$$H(f_0, nT) = \frac{1}{\sqrt{P}} \cdot \sum_{p=1}^P e^{j2\pi f_{D,p}nT} \cdot e^{j\theta_p} \cdot e^{-j2\pi f_0\tau_p} \quad (9)$$

Where  $\theta_p$ ,  $\tau_p$  and  $f_{D,p}$  are obtained by using their respective *probability density functions*.

The set of parameters used in order to perform the simulations was inspired in a WiMAX standard and is contained in Table 1.

Table 1: Simulation parameters.

Parameter	Value
Carrier Frequency	$f_c = 5 \text{ GHz}$
Bandwidth	$B = 10 \text{ MHz}$
Number of subcarriers	$N_{FFT} = 128$
Subcarrier spacing	$\Delta f = \frac{B}{N_{FFT}} = 78125 \text{ Hz}$
Symbol Duration	$T_s = 12.8 \mu\text{s}$
Guard interval	$T_G = \frac{T_s}{8} = 1.6 \mu\text{s}$
Symbol interval	$T_{S+G} = T_s + T_G = 14.4 \mu\text{s}$
Number of paths	$P = 30$
Number of clusters (groups of paths)	$N_c = 1$
Maximum time delay	$\tau_{max} = 1 \mu\text{s}$
Mobile velocity	$v = 5, 50, 100, 150, 200, 300 \text{ km/h}$
Maximum Doppler shift	$f_{Dmax} = f_0 \cdot \frac{v}{c} \approx 23, 231, 463, 694, 926, 1389 \text{ Hz}$ using $f_0 = f_c$
Time Delay distribution	$b = \frac{\tau_{max}}{\ln(1000)} = 0.1448 \mu\text{s}$

For successive transmitted matrices ( $C_k$  and  $C_{k-1}$ ), successive samples of the WSSUS channel ( $H_k$  and  $H_{k-1}$ ) were used. Then it is valid  $H_k \approx H_{k-1}$  instead of  $H_k = H_{k-1}$  (sometimes used), which is a much more realistic assumption.

In Fig. 2 the performance of the average of 64-PSK in DSTBC with the one corresponding to “4A16PSK PCM2” in DSTBC for a mobile terminal velocity of 100 km/h is compared. There, it can be observed that the improvement for “4A16PSK PCM2” is approx. 4.82 dB at  $BER = 1 \times 10^{-1}$ .

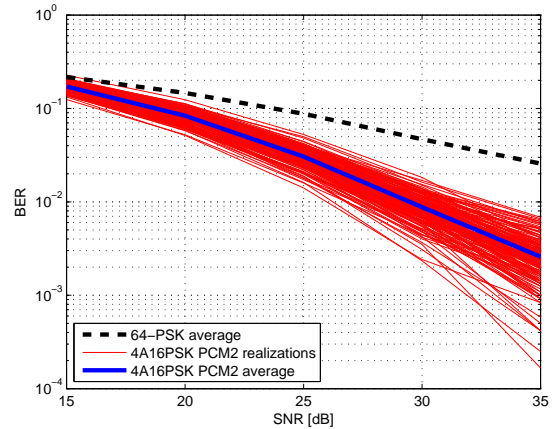


Figure 2: Performance of “4A16PSK PCM2” used in DSTBC under WSSUS channels ( $H_k \approx H_{k-1}$ ,  $v = 100 \text{ km/h}$ ).

In Fig. 3 the results obtained for the reference system (64-PSK in DSTBC) and for “4A16PSK

PCM2”, when the mobile terminal velocity is 150 km/h, are shown. As it can be observed in Fig. 3, the degradation due to the velocity increment is not equal for both systems, being higher for the first one. That increases the improvement obtained by using “4A16PSK PCM2” instead of 64-PSK in DSTBC; it is approx. 5.71 dB at  $BER = 1 \times 10^{-1}$  for this case.

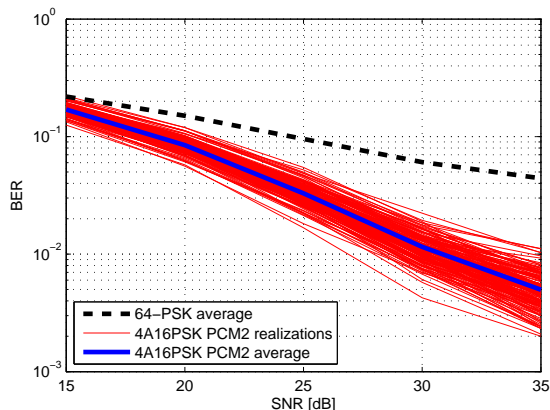


Figure 3: Performance of “4A16PSK PCM2” used in DSTBC under WSSUS channels ( $H_k \approx H_{k-1}, v = 150$  km/h).

In Fig. 4 the performance for 64-PSK in DSTBC and “4A16PSK PCM2” in DSTBC were evaluated for a mobile terminal velocity of 200 km/h. By comparing the average results for both techniques, an improvement of 6.65 dB at  $BER = 1 \times 10^{-1}$  for “4A16PSK PCM2” technique is obtained. For 005 km/h, 050 km/h and 300 km/h, the improvements are 4.73 dB, 4.76 dB and 14.06 dB respectively. That means that the improvement is significantly increased with the increment of the mobile terminal velocity. It shows that “4A16PSK PCM2” is particularly convenient for high mobility scenarios.

In Fig. 5 the variation of the improvement for “4A16PSK PCM2” with the velocity of the mobile terminal is summarized. Observe that the previous discussion is about the improvement in performance of “4A16PSK PCM2” with respect to 64-PSK in DSTBC, not about the absolute performance of “4A16PSK PCM2”. For sure it diminishes when the velocity of the mobile terminal is increased; but it diminishes less than for 64-PSK in DSTBC.

## 4 CONCLUSION

In this paper the performance of a new technique reported in (Rodríguez and Rohling, 2006) is analyzed for different mobile terminal velocities. The results

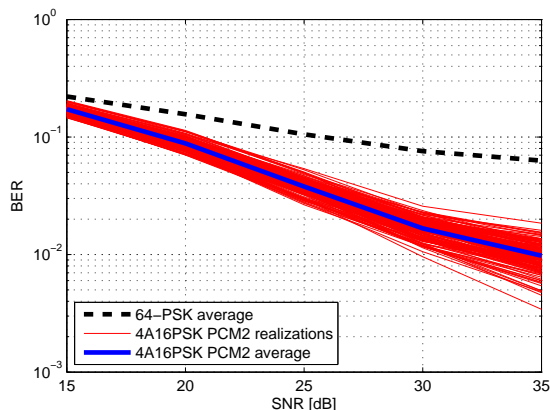


Figure 4: Performance of “4A16PSK PCM2” used in DSTBC under WSSUS channels ( $H_k \approx H_{k-1}, v = 200$  km/h).

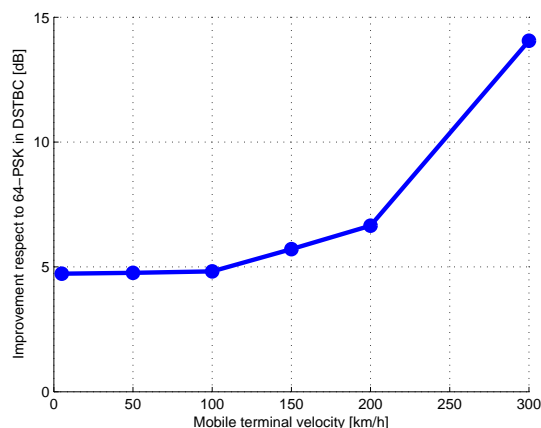


Figure 5: Relative improvement of “4A16PSK PCM2” with respect to 64-PSK -used in DSTBC- versus mobile terminal velocity, under WSSUS channels ( $H_k \approx H_{k-1}$ ).

show that this technique is particularly convenient for high mobility scenarios. It is also known that this interesting characteristic is also shared, at least, by other APSK technique (“2L-APSK”) (Vanaev and Rohling, 2006), (Rodríguez, 2007).

In (Rodríguez and Rohling, 2007) the improvement obtained for “4A16PSK PCM2” by using receive diversity is reported, which is a simple alternative to improve the performance of this technique.

It is important to note the low system complexity of “4A16PSK PCM2” used in DSTBCs, even when receive diversity is used.

Finally, it is also important to highlight the advantages of OFDM, MIMO and DSTBC associated to this technique. By being applicable to OFDM tech-

niques, the robustness in multipath fading scenarios is assured. By being a MIMO technique, better bandwidth efficiency can be achieved, and as a differential modulation technique a low computational complexity can be maintained. As a summary, this is a technique with very high potential for high mobility scenarios.

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