Differences between the operations of the generation power system of Uruguay operated minimizing the Expected Value vs. minimizing the Value at Risk of the future operating costs.

D. Larrosa, E. Coppes , G. Casaravilla and R. Chaer, *Senior Member IEEE*

Abstract -- **Hydrothermal systems optimal operation includes a step of optimizing the resources that are valued system storable. This optimization is performed traditionally by a stochastic dynamic programming in which the objective function to minimize is the expected value of future cost of operation, also known as Bellman function. While in theory, minimize the expected value of future cost of operation is "objective" in practice there are many reasons why the actual operation includes additional precautions, sometimes actually taken by operators who are the ones which have responsibility for the consequences of the operation or sometimes made based on safety considerations were not introduced in the optimization of the operation. This work shows the implementation on the platform Simulation of Electric Power Systems of stochastic dynamic programming algorithm for specifying the objective function cost reduction future with a certain probability of exceedance. This work was performed as part of the draft platform enhancements SimSEE with funding from the Energy Sector ANII. The paper presents the results of the operation to minimize the expected value of future cost and minimizing the risk value of 5% of being exceeded. Both operations are compared both costs achieved as in the qualitative aspects. The results allow evaluating the cost of being introduced by risk averse and also identify situations where there are major differences. It also discusses the impact on the marginal cost of system operation with a slogan risk averse. This value is relevant because it is the basis for calculating the Uruguayan market spot price. These scenarios correspond to the operation in 2017 with high penetration of wind energy in the system.**

.

I. Introduction

The optimal operation of a dynamic system involves the calculation of a function y (t) from knowledge of the system state x (t) and its inputs r (t) determine the value of the control variables u (t) system leading to the optimal path.

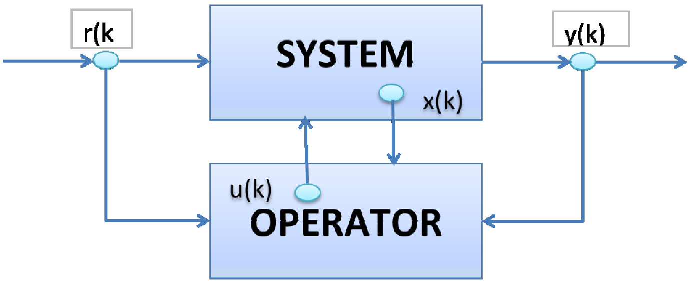

Fig.1. Block diagram of the relationship between the System and the Operator

For dynamical system we mean a system in which "the past matters." Assuming that past information is representable in a vector of state variables, system dynamics can be modeled by an evolution equation as shown in (1):

$$x' = x_{k+1} = f\left(x_k, u_k, r_k, k\right) \qquad (1)$$

Where we have assumed a discretization of time and the vector x_k is the state vector at the beginning of step k by dentition contains all the relevant information system of the past needed to calculate their future paths from the input series $\{u_j\}$, $\{r_j\}$, $\{j\}$ for all j> k.

The series $\{r_j\}$ are the inputs "uncontrolled" usually associated with stochastic processes. By way of example, the wind speed from which energy is generated in the wind farm is part of this set of entries not controlled. Rk will call the "noise vector" that "attacks" the system at stage k.

U_k parameter in (1) is the control input of the system. The system operator must select at each stage the "best" value for that vector control in the space of possible values. The series of vectors {u} is the number of control vectors.

The Operator, considering the system state information and tickets uncontrolled fixed at all times the control vector u_k to guide the system for the optimal path. Explicitly or implicitly, to do this the operator has a policy function or operation as shown in (2):

$$u_k = PO\left(x_k, r_k\right) \qquad (2)$$

dlarrosa@ute.com.uy, ecoppes@fing.edu.uy, gcasaravilla@ute.com.uy, rchaer@fing.edu.uy

Given the evolution equation of state of the system (1) Operation and Operator Policy (2), it is possible to simulate the trajectories of the system from a known initial state x_0 if you know the possible series of entries $\{r_k\}$ controlled for all k> 0. Since the vector rk corresponds to the realizations of stochastic processes, from an initial state given the state evolve through different paths. In each of these possible paths is possible to calculate the cost incurred during the operation of the system and indicators of exposure experienced by the system to go through them. Whether an Operation Policy is better than another depends on what the merit function is used for comparison. The simplest and most commonly used is to minimize the expected value of future cost of operation. The aim of this work is to develop alternatives to the expected value of future cost of operation that allows including the concept of risk aversion in determining Operating Policy.

At this point, it should clarify the meaning of the word "cost" used in context "cost function" and beyond. The term "cost function" refers to the objective function minimization problem. Outside that context, the cost (e.g. Future Operating Cost System) refers to the cost, expressed in money (usually in constant currency of a given date) and coincides with the meaning that most people assigned to the word "cost". Strictly speaking, these two assignments of meanings for the word "cost" are not divergent but on the contrary, because usually the "cost function" is intended to represent the sum of all costs, both those incurred in a direct and quantifiable in cash and other less direct costs and are able to quantify and add in the cost function.

When the system is generating electricity, the optimal path is usually one that minimizes the expected value of future cost (FC) of operation. This CF is the sum of expenditure on fuel imports and costs assigned to the energy not supplied (costs of shortages or rationing) less revenue in your system from energy sales to other countries (exports).

Theoretically, when the cost function represents all the real components of the cost, minimizing the expected value of said function is the aim of optimization for excellence. In practice there are situations not always well captured in the cost function and lead to you prefer to be "more conservative" or risk averse. One of the reasons for that risk aversion is not to fall into situations which by their low probability weigh little in the expected value of the cost of operation but in the event of a disorder means you may have economic consequences rather than on system administrators or the economy. Of course you can always discuss whether these rare but catastrophic events should not be included reflecting that "catastrophic cost" in a cost function "well formed".

To fix ideas, if the system in question is the power systems of a country like Uruguay, strongly interconnected with Argentina and Brazil that are between two and 70 times larger than Uruguay. Optimizing the Operation Policy with a goal of reducing the expected value of future cost of operation could lead to at some point Uruguay sell all the energy of the lakes at a great price (compared to a prolonged drought of the neighbors who considerably raise their prices) and it was a good deal in expected value, but there is a chance (albeit very low) that it rain not in the short term in Uruguay and therefore have to make cuts power to the country's domestic demand. One could argue that if the costs allocated to power rationing (values of failure costs) reflect the actual cost to the country that the optimizer would not happen "selling water", but this statement is guilty of two errors (or large budgets) . The first of the errors is that the precision of the tools and models used are only approximations to reality, thus determining an optimal policy via a cost reduction in expected value is a mechanism that takes accuracy over what happens in situations of very low probability and very high costs as described. This is because the models and the data available for calibration is based on what happens to most likely and unlikely are well represented "tails of probability." The second assumption is that errors or information you have on the evolution of the variables of interest (e.g. fuel costs) is perfect and what is not and again this effect while impacting on all possible paths in those in which the system goes through situations very high costs, any error in the prediction of cost values is amplified.

This paper documents an implementation on the platform on Simulation of Electric Power Systems - SimSEE for calculating the optimal operating policy of a hydro-thermal generation with a criterion of Risk Aversion.

This work is done in the context of research project ANI_FSE_18/2009 of Faculty of Engineering of Uruguay with funding from the Energy Sector of the ANII (Agencia Nacional de Invertigación e Innovación) of Uruguay.

II. Stochastic dynamic programming with histogram.

This section shows how to modify the recursive algorithm of the stochastic dynamic programming so that instead of calculating the expected value of future operating cost (Bellman function), calculated in each state of the system, at every point of time, probability distribution function of the future cost of operation. This function is denoted by a vector of equi-probable samples arranged in decreasing order. This representation can be thought of as a histogram particularly where the discretization is selected such that all boxes are an example.

In (1) shows the evolution equation of the system state, where x is the vector of state variables and as such captures all necessary information from the past to calculate the evolution of the system from knowledge of x entries system from a given instant. Tickets r and u are the vectors representing the set of inputs on which we have no control and vector control or to which the operator can impose the values (complying with the restrictions in your system for it) to drive the system by the optimal path. Then (1) can then calculate the state x' at the end of the stage (or time step) k from the knowledge x and state of uncontrollable inputs r at the beginning of stage k.

In Fig 2 shows in schematic form the evolution of the system state from the position x at the beginning of stage k to state x 'at the end of the stage under the influence of uncontrolled inputs r and the control inputs u.

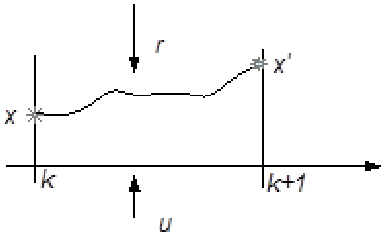

Fig. 2. Evolution of the system state.

In the kind of dynamic system under consideration we assume that the input vector while r 'is not controllable, "is known at the start of the stage and is used as input for the calculation of the control vector u better be applied to the stage during .

We assume that the cost of operating the system is an integral function of the costs incurred at each stage. For example, in the system of power generation (our purpose) consists of expenditure on fuel + imports - exports + rationing costs at each stage.

Given a set of inputs:

$$R_k = \{r_j\};\ j = k, k+1, \ldots$$

And a series of control:

$$U_k = \{u_j\};\ j = k, k+1, \ldots$$

Both from the stage k known system state at the beginning of this stage, the succession of states:

$$X_k = \{x_j\};\ j = k, k+1, \ldots$$

is calculable using the evolution equation (1).

If we have:

$$ce_k(x_k, u_k, r_k, k)$$

The cost of stage k. Future Cost from state x_k can be calculated as:

$$CF(x_k, R_k, U_k) = \sum_{j=k}^{j=\infty} ce_j(x_j, u_j, r_j, j)$$

Sum which can be written recursively as:

$$CF(x_k, R_k, U_k) = ce_k(x_k, u_k, r_k, k) + CF(x_{k+1}, R_{k+1}, U_{k+1})$$

If the system operator has a policy of the form of Operation (2) can eliminate the dependence of the series in the U_k previous recursive equation being:

$$CF(x_k, R_k) = ce_k(x_k, u_k, r_k, k) + CF(x_{k+1}, R_{k+1}) \quad (3)$$

We are assuming that the operator knows $(x_k, r_k,)$ to determine what the control vector u_k that used in stage k that completely ignores the future behavior of uncontrollable inputs R_{k+1}. Thus, the value of future cost in the state of arrival (at the end of step k) is a random variable. The classical method of optimization is to consider the expected value of that variable in the method we are proposing we want to have a representation of the distribution of the variable in order to minimize such things as the value given risk of absence rather than the expected value. To get a representation of:

$$CF(x_{k+1}, R_{k+1})$$

Will store a sample of the distribution function of the random variable CF samples keeping a number of equi-probable. We define a parameter that is the amount of samples stored in this representation will name: N_{mCF} (Number of samples Cost Future).

The optimization algorithm uses Monte Carlo draws in each stage to generate a set of r_k^h with

$$h = 1, 2 \ldots N_{Nsop}$$

possible realizations of the uncontrollable variables. With each of these values, the operator must decide what will be the control vector u_k will use the knowledge that the system will evolve according to (1) and the cost associated with their decision will be that arising from (3). Thus, in the resolution of the step from a known state and for each value x_k and r_k^h obtain a value of costs incurred in stage

$$ce_k(x_k, u_k, r_k, k)$$

and a state value at the end of step by applying the equation of state evolution r_k^h. In turn, as the future cost in the state of arrival, we have represented by equi-probable N_{mCF} samples and depend only on the future (i.e. are conditioned only state which is reached at the end of the stage and future information not by the particular values (x_k, r_k, u_k) future cost will be the beginning of stage k combinations N_{mCF} future costs in each state of arrival may x_k^h. Thus there are, N_{mCF} * N_{Nsop} equi-probable values for CF (x_k, r_k). To keep the representation of CF (x_k, r_k) with equi-probable N_{mCF} samples, the algorithm proceeds to sample N_{mCF} * N_{Nsop} them for a reduced set of N_{mCF} and to continue the recursive calculation of CF representations. In implementing the sampling switch to N_{mCF} samples from nmCF * NNsop, is done by ordering samples roughly nmCF * NNsop in decreasing order and then selecting samples $N_{mCF,}$ N_{Nsop} apart each beginning with the first as close as possible to half the interval of the first N_{Nsop} as shown in Fig. 2:

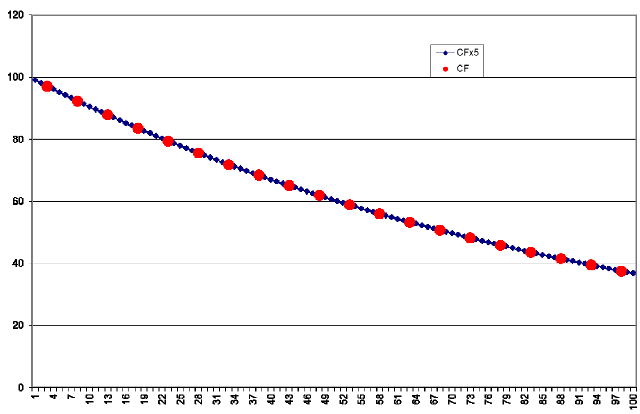

Fig. 2: Resampling from histograms of CF.

In the case of Fig. 2 the amount of points representing the histograms Future Cost is N_{mCF} = 20 and the number of draws of Monte Carlo optimization step is N_{Nsop} = 5. The figure shows for a given starting state, in blue, the equiprobable N_{mCF} * N_{Nsop} = 100 points achieved in the solution of step sorted in decreasing order and the red circles, the N_{mCF} = 20 are selected as representative of the distribution future cost CF (x_k, r_k) to continue the optimization algorithm.

III. VALUE AT RISK VAR.

To give generality to the measure of risk, introduce two measures of risk can be calculated on a random variable which is the value that is exceeded with a given probability of exceedance and the Value at Risk with a certain probability of Leave.

Given a random variable:

$$y \in D_y \subset R^1$$

probability density function $p_y(y)$ and fixed probability value of risk:

$$u \in [0,1]$$

define the VeR (Value and is exceeded with probability u) and VaR (Value at Risk and with probability u) as:

$$VeR = Y \in D_y / P(y > Y) = u \qquad (4)$$

$$VaR = \int_{y=VeR}^{y=+\infty} y \, . \, p_y \, . \, dy \qquad (5)$$

The expected value of y is by definition:

$$VE = \int_{y=-\infty}^{y=+\infty} y \, . \, p_y \, . \, dy$$

Fig. 3 shows an example. Corresponds to 100 equi probable samples arranged in order of decreasing cost. Fixed as a probability of absence for the purposes of the risk values 5%, in Fig vertical line was drawn separating the 5% of the highest values of the rest. The value at risk of 5% is exceeded VeR = 80 MU$S and 5% value at risk is the average value greater than VeR and VaR=88MU$S.

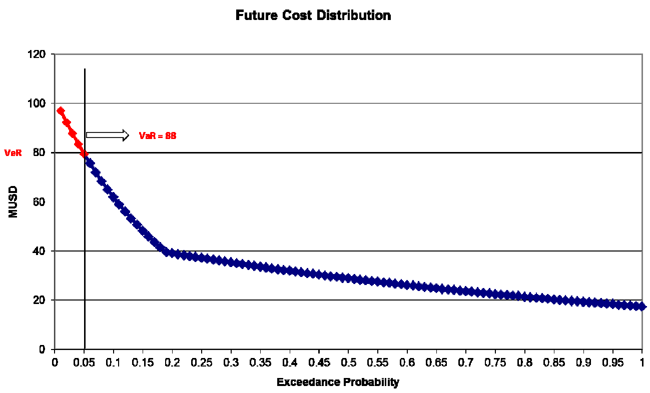

Fig. 3: Example of VeR and VaR whit risk 5%.

IV. RISK AVERSE OBJETIVE FUNCTION.

In Section III showed how to construct recursively the histograms of the future cost function. To do this, it was assumed that the system operator has a policy that allows operation to calculate the vector control as a function of state at the beginning of the stage, the realization of uncontrollable input vector and time (step k).

$$u_k = po(x_k, r_k, k)$$

Different functions $u_k = po(x_k, r_k, k)$ lead to different system operations. These operations can be classified into more or less "successful" if it has a measure of credit for the operation. The classical merit function is the expected value of costs incurred in the operation and as mentioned in the introduction the purpose of this paper is to propose an alternative merit function that allows us to take into account the risk aversion likely to have the operator.

In the end, an operator who is VERY risk averse will not look the expected value of future operating costs but will try to minimize the maximum value of that cost. Ie instead of minimizing the average values of the histogram (which is an estimate of the expected value) will attempt to minimize the maximum values of the histograms.

Fixed an exceedance probability for risk measurement (usually 5%) say that the operator is 100% risk-averse if the objective of optimization is to minimize the value at risk VaR and say that is 0% risk-averse when operating minimizing expected value VE.

Defining a Coefficient of Risk Aversion:

$$CAR \in [0,1]$$

we can define the objective function of the operation with that level of risk aversion:

$$J = CAR \, . \, VaR + (1 - CAR) \, . \, VE \qquad (6)$$

Now we show the application of this cost function with risk aversion on the stochastic dynamic programming algorithm for optimal operating policy of a dynamic system minimizing the future cost of operation with a coefficient of risk aversion given.

Having examined the state at the beginning of a time step, and a realization of uncontrolled input vector and applying the procedure described in Section III can be calculated recursively a representation of the distribution function of the future cost for each system state. This representation allows us to evaluate the objective function (6) in the state of arrival

$$x' = f(x, r, u, k)$$

And gain control vector u that achieves minimize the cost of step

$$ce_k(x_k, u_k, r_k, k)$$

more objective point of arrival

$$J(x', k+1)$$

Observe that given a realization r_k minimize the sum

$$ce_k(x_k, u_k, r_k, k) + J(x', k+1)$$

is to minimize the cost of the equation (6).

Knowing the couple (x_k, r_k) and for each value of uk is calculated:

$$ce_k(x_k, u_k, r_k, k)$$

and

$$x' = f(x_k, u_k, r_k, k).$$

V. CASE OF APPLICATION.

For the purpose of verifying the operation of the deployment is executed for optimizing the operating policy corner of Lake Bonete for the last week of November 2017.

This room was chosen as one SimSEE room in which there is an abundance of wind power. This room was calculated operating policy, No Risk aversion and two values of the probability of exceedance PE = 0.05 and 0.10 for the risk measure, we performed the same optimization CAR = 0.00, 0.25, 0.50, 0.75 and 1.00.

The number of points for representation of the histograms was 400.

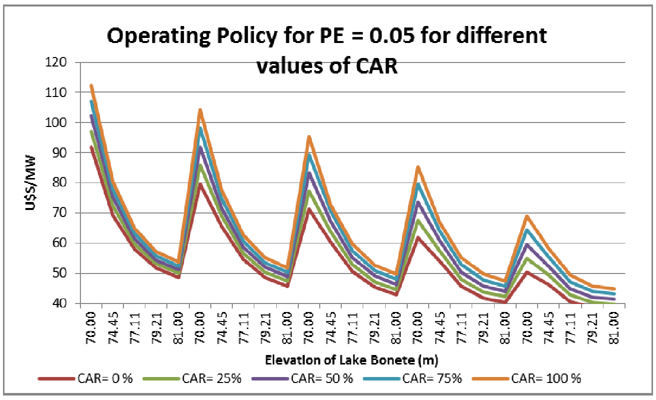

Fig. 4: Politics of operation for PE = 5% for different values of CAR

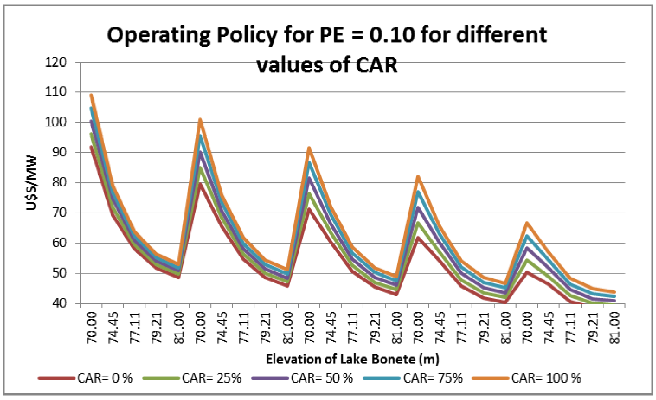

Fig. 5: Politics of operation for PE = 10% for different values of CAR

As you can see the value of the power plant's Rincon del Bonete grows inversely proportional to PE and directly proportional to the CAR as expected, since the operator to be more risk averse is expected to assume greater costs.

It also presents the average level evolves plant Rincon del Bonete for CAR = 100% for different values of PE over 2017.

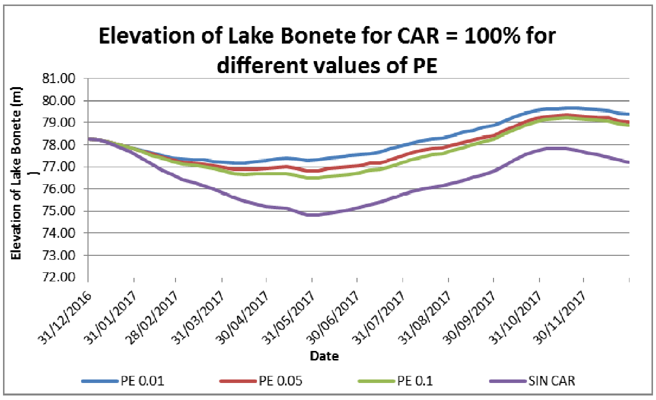

Fig. 6: Politics of operation for PE = 10% for different values of CAR

We interpret this graph that the lower PE, the higher the average level of the plant, since the operator to be more adverse to the tail of probability using other resources (usually more expensive) before using the water from this plant that has seasonal capacity.

VI. CONCLUSIONS.

The implementation SimSEE optimize the possibility of an electrical system being risk averse, it works correctly in all cases analyzed.

It is easy to see the increased costs associated in operating policies as well as conservative in the use of short-term lake system is the more risk averse operation becomes.

VII. REFERENCES

[1] EFECTO DE LA CONSIDERACION DEL RIESGO EN LA PLANIFICACIÓN DE LA EXPANSIÓN PARA LA GENERACIÓN DE ENERGIA ELÉCTRICA, Marisa León, Nicolás Castromán, Ruben Chaer. 8º Encuentro de especialistas en Energía Potencia Instrumentación y Medidas de la IEEE-Uruguay, Montevideo-Nov.2010. http://iie.fing.edu.uy/epim2010/myreview/myPapers/p93.pdf

[2] SimSEE: Simulador de Sistemas de Energía Eléctrica. Proyecto PDT 47/12, Gonzalo Casaravilla, Ruben Chaer, Pablo Alfaro. Technic Report 7, Universidad de la República (Uruguay). Facultad de Ingeniería. Instituto de Ingeniería Eléctrica, Number 7 - dec 2008. http://www.fing.edu.uy/iie/publicaciones/2008/CCA08a/CCA08a.pdf

[3] Modelo de series correlacionadas CEGH. R. Chaer-Octubre 2005. http://iie.fing.edu.uy/simsee/curso2010/Cap9_sintetizadorCEGH.pdf