An Analog Circuit Implementation of a Huber-Braun Cold Receptor Neuron Model

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***Abstract*— We present the design and implementation of an electronic device that, based on analog discrete components, implements the mathematical model of a cold receptor neuron called Huber-Braun. This model describes the electrical behavior of a certain kind of receptors when interacting with its environment, and it consists of a set of differential equations that has only been solved by numeric simulations. By these means, a chaotic behavior has been found. An analog computer can be relevant for further analysis and validation of the model. The results obtained by means of numeric simulations and through our analog circuit simulator are consistent. The electronic device built allows the observation of all relevant variables and most of the expected behavior (tonic firing, chaotic, burst discharge, subthreshold oscillation and steady state). In addition, bifurcation diagrams were successfully rebuilt for temperature and external current.**

I. INTRODUCTION

Neuronal modeling with electronics circuits can help to understand biological systems. Physical circuit implementation has several advantages. In the first place, circuits that perform specific operations typically operate much faster than general purpose ones. In the second place, it is possible to interface a hardware implementation with biological tissue or operate it with experimentally collected data in real time. In the third place, it enables integration with robotics system [1].

There are several types of models, despite the fact that IF (integrate-and-fire) and QIF (quadratic-integrate-and-fire) have low implementation cost, they have poor biological plausibility. Izhikevich neuron model, recently implemented in sub-threshold VLSI [2], has an excellent trade-of between implementation cost and biological plausibility, but this model is not biophysically meaningful [3].

The Huber-Braun model [4] is a Hodgkin-Huxley conductance-based model [5]. These kind of models are important not only because their parameters are biophysically meaningful and measurable, but also because they allow us to investigate questions related to synaptic integration, dendritic cable filtering, effects of dendritic morphology and other issues related to single cell dynamics [3].

In addition, from the mathematical point of view, the model equations are an interesting and complex problem, that has not been formally resolved. It has not been shown which is the qualitative behavior (part of the complexity is in the high dimension of the system). The behavioral characteristics are just known through numerical simulations and experimental comparison. The variation of the Huber-Braun Model parameters shows different types of behaviors and bifurcations. In particular, there is some evidence that the behavior for some areas of the parameters is chaotic. Since numerical simulations were not able to show this chaotic behavior, it is assumed that an analog implementation could help [6][7][8][9].

This paper presents the development of an electronic device, based on analog discrete components, that simulates the Huber-Braun cold receptor neuron model. It is expected that this analog simulator can be helpful in the biological/neuroscience field, as well as in the mathematical one.

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II. HUBER-BRAUN MODEL

The Huber-Braun [4] is a Hodgkin-Huxley model *Hodgkin-Huxley* of the nerve endings of the skin superficial layer.

The modelated neuron is a cold receptor, which main function is "to respond" to low temperatures. The *temperature* (T) is introduced into the model equations as a parameter. From the physiological point of view, it is interesting to observe the changes of the behavior that arise by varying this parameter. This information is shown in the *bifurcation diagrams*. Another parameter of interest is the *external current* (I_{ext}) that represents the influence of the environment on the neuron.

The full set of equations of the Huber-Braun model are the following:

- **Membrane Potencial**

$$C_M\dot{V} = -g(V - V_1) - I_d - I_r - I_{sd} - I_{sr} - I_{ext}$$

- **Fast Ionic Currents**

$$I_d = \rho g_d a_d (V - V_d); \qquad a_d = a_{d\infty}$$

$$a_{d\infty} = \frac{1}{1+e^{-s_d(V-V_{0d})}}$$

$$I_r = \rho g_r a_r (V - V_r); \qquad \dot{a_r} = \phi\frac{a_{r\infty} - a_r}{\tau_r}$$

$$a_{r\infty} = \frac{1}{1+e^{-s_r(V-V_{0r})}}$$

- **Slow Ionic Currents**

$$I_{sd} = \rho g_{sd} a_{sd} (V - V_{sd}); \qquad \dot{a_{sd}} = \phi\frac{a_{sd\infty} - a_{sd}}{\tau_{sd}}$$

$$a_{sd\infty} = \frac{1}{1+e^{-s_{sd}(V-V_{0sd})}}$$

$$I_{sr} = \rho g_{sr} a_{sr} (V - V_{sr}); \qquad \dot{a_{sr}} = \phi\frac{-\eta I_{sd} - k a_{sd}}{\tau_{sd}}$$

- **Temperature Scaling**

$$\rho = 1.3^{(T-T_0)/10^oC}; \quad \phi = 3.0^{(T-T_0)/10^oC}$$

The *Membrane potential* equation includes the membrane capacitance C_M, the ionic currents I_i and a member associated with losses for the transfer of ions, as well as a conductance g and the equilibrium potential V_1. The model includes four membrane potential dependent ionic currents. The depolarization currents are I_d and I_{sd}, fast and slow respectively, and the repolarizing currents are I_r and I_{sr}, fast and slow respectively. The variables a_i are called activation variables of each channel. Those are the ones that determine the dynamics of the opening and closing of the respective channels, and tend to the respective $a_{i\infty}$, which are called asymptotic activation variables. These last ones are *sigmoid* type function and are explained in more detail in section III.

The differential equations system has only been studied by numerical simulations and physical experiments. By these methods it is known that the regime behavior is like an oscillator for most of the parameters values. Furthermore, the system can be seen as composed by two simpler oscillators, one fast and one slow, which are coupled in a non-clear sense. These smaller subsystems arise from considering only the fast or only the slow currents in each case.

The most important variable of the model and through which the different behaviors are displayed is the membrane potential. The neurons transmit information through the spikes that occur in this variable. Therefore, the time between two consecutive spikes, called ISI (Inter Spike Interval) is a relevant magnitude to be measured. Moreover, the bifurcations will be reflected on this magnitude.

There are three different areas depending on the temperature: the *period doubling* area, the *chaotic* area and the *addition of ISI* area.

For the low temperatures area the membrane potential presents a regular regime of a single spike per period called *tonic firing*. As the temperature increases more spikes per period appear, initially with the same time of separation between them. What happens here is that the orbit passes to travel twice the distance at the same speed, doubling the period of the signals. This is because the limit cycle after the bifurcation appears to make two laps near the previous limit cycle before closing. Therefore, also doubles the number of spikes per period, maintaining the value of the ISIs [8]. This type of bifurcations will be called *period doubling*.

At the other end of the temperatures zone of interest a different phenomena is observed. For temperatures above 35^oC the firing ceases and does not get to form spikes, presenting first a *subthreshold oscillation* and tending then to a *steady state*. By decreasing the temperature, initially there is a single spike per period, then the *burst discharges* appears and the number of spikes increases. When a new spike is formed the sum of the intervals between the spikes is kept constant (and equal to the period) [8]. This type of bifurcations will be called *addition of ISI*.

Between the two mentioned areas the *chaotic* behavior is observed. This means that small variations of T produces very perceptible variations in the qualitative dynamic behavior of the system, observed by the ISI. In the chaotic area bifurcations appear to fill out the parameter region (see Figure 7).

By setting the temperature and varying the current I_{ext} the same phenomena appears as varying the temperature, which can be seen in the bifurcation diagram of Figure 8.

III. CIRCUIT DESIGN

The design was made considering all variables and parameters of the model as voltages in the circuit. This led to implement the model equations (section II) based on the following basic blocks: Potencial, Sigmoid, Adder-Subtractor, Amplifier, Integrator and Multiplier.

For the Adder-Subtractor, Amplifier and Integrator the classic implementations with operational amplifiers were used [10].

The multiplication was implemented with the *AD633* of *Analog Devices* which performs this operation with precision and allows wide dynamic range both in inputs and outputs.

For the potential we adapted the design presented in [11] by adjusting the exponent. This design is based on the property: $k\log(a) = log(a^k)$, where the logarithm of the signal is made and then amplified.

A sigmoid function simulates two possible states and the transition between them. The plot presents two asymptotes and tends from one to the other. In this case, the expression is:

$$s(x) = \frac{1}{1+e^{-x}} \tag{1}$$

and the possible states (asymptotes) are 0 and 1. In the model it represents a continuous way of turning ON and OFF the ionic channels; open and close.

To implement this block we used a differential pair, whose response is: $\tanh(kx)$. This is a sigmoide type function, and is related to 1 as follows:

$$s(x) = \frac{\tanh(\frac{x}{2})+1}{2} \tag{2}$$

Starting from 2, a circuit that respondes like 1 can be implemented with a differential pair by adding amplifications and tension references (see figure 3).

Some of the implementations of the different equations with these basic blocks can be seen in figures 1 and 2.

Once every basic block was designed we implemented the whole system in a single board, obtaining an analog simulator of the Huber-Braun model (see figure 4).

Finally, it is noteworthy that the design and the implementation developed are flexible in many aspects. Firstly, all the variables of interest can be observed and the parameters can be set in all the specified range (*temperature* between 0 and 36^oC, *external current* between 0.1 and $1.4A/cm^2$). In addition, other parameters that are constant in the model can be modified. Particularly, this is the case of the poles that determine the time scaling (which is an advantage of analog simulators). In this way, the relation between the speed of the

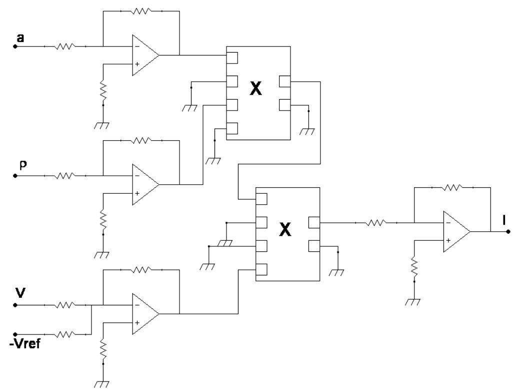

Fig. 1. Implementation of the equations of the ionic currents.

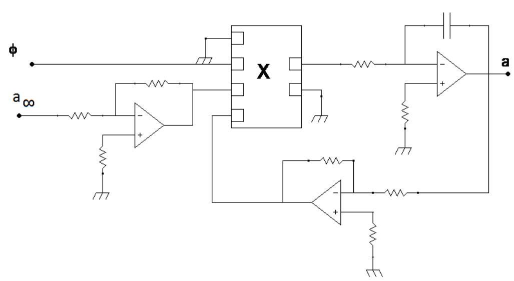

Fig. 2. Implementation of the equations of the activation variables.

simulator and the real system can be adjust; in our case, the simulator can be set up to 100 times faster than the original system.

IV. RESULTS

Measuring the outputs of the circuit, we were able to observe the different behaviors of the membrane potencial by changing the values of the parameters (*tonic firing, chaotic, burst discharge, subthreshold oscillation and steady state*). Figures 5 and 6 show both the measures taken from the circuit and the numeric simulations of the model made in *Matlab* [12] for different states of the neuron. In these figures the difference in the time scale can be seen.

In addition, bifurcation diagrams were successfully rebuilt for both parameters (which can be seen in figure 7 and 8, for T and I_{ext} respectively). In order to do this, the membrane potencial was measured directly from the circuit for a great number of parameter values with a digital oscilloscope, and processed afterwards with *Matlab*.

Regarding the bifurcations, we were able to observe clearly the *addition of ISI*, but because of the noise we could not distinguish the first period doubling before chaos is reached.

Moreover, the implemented system appears to be faster than expected. A small deviation on the values of the poles (of the activation variables) was found but does not seem to explain the difference, neither does the difference in velocity (comparing with the numeric simulations) of the slow and fast oscillators that were analyzed separately. However, the slow oscillator appears to be notoriously bigger in amplitud

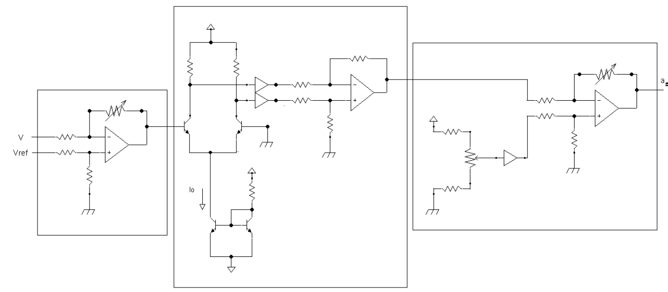

Fig. 3. Implementation of the sigmoid function with a differential pair.

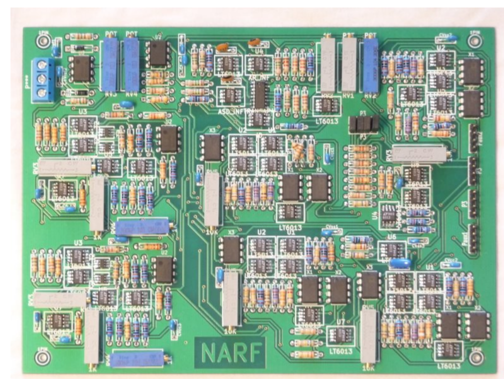

Fig. 4. Implemented board

than expected. We estimate that this can lead to an early reach of the threshold level, increasing therefore the frequency of the spikes when simulating the whole system.

V. CONCLUSION

We developed an electronic device, based on analog discrete components, that simulates the Huber-Braun cold receptor neuron model. The results obtained by means of numeric simulations and through our analog circuit simulator are consistent. The electronic device built allows the observation of all relevant variables and most of the expected behavior (tonic firing, chaotic, burst discharge, subthreshold oscillation and steady state). In addition, bifurcation diagrams were successfully rebuilt for T and I_{ext}.

The signals of interest (including the membrane potencial and the parameters) are presented as voltages in output pins, and may be observed with an oscilloscope or a PC by means of a data acquisition board. Furthermore, it is possible to vary both parameters with presets in the specified range. Through the membrane potencial we were able to identify the different states of the neuron and most bifurcations.

Calibration presets were included to make an accurate adjustment of the factors over which the system is more sensitive. This makes it possible to study the dynamics when changing values other that T and I_{ext}. In addition, it is possible to analyze the fast and the slow oscillators separately by disconnecting the corresponding currents. Finally, the time scaling is also configurable with presets allowing the

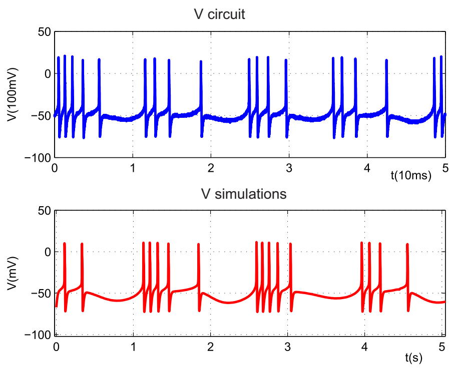

Fig. 5. Chaos. Comparison between Matlab simulations (bottom) and measures from the circuit (top).

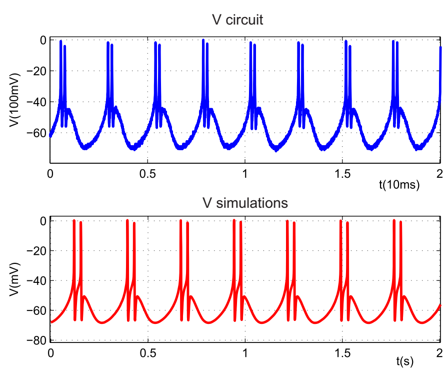

Fig. 6. Burst discharges. Comparison between Matlab simulations (bottom) and measures from the circuit (top).

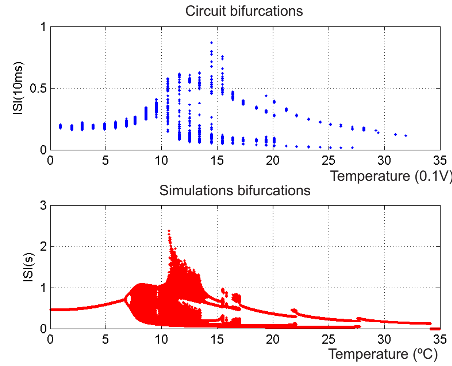

Fig. 7. Bifurcation diagram varying T. Matlab simulations (bottom) and measures from the circuit (top).

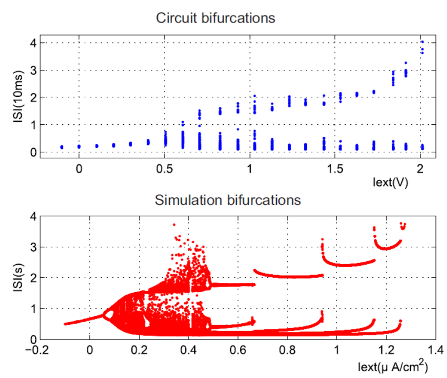

Fig. 8. Bifurcation diagram varying I_{ext}. Matlab simulations (bottom) and measures from the circuit (top).

device to simulate the neuron dynamics in real time or faster.

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