Community detection by resistance distance: automation and benchmark testing

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Abstract. Heterogeneity characterises real-world networks, where nodes show a broad range of different topological features. However, nodes also tend to organise into communities – subsets of nodes that are sparsely inter-connected but are densely intra-connected (more than the network's average connectivity). This means that nodes belonging to the same community are close to each other by some distance measure, such as the resistance distance, which is the effective distance between any pair of nodes considering all possible paths. In this work, we present automation (i.e., unsupervised) and missing accuracy tests for a recently proposed semi-supervised community detection algorithm based on the resistance distance. The accuracy testing involves quantifying our algorithm's performance in terms of recovering known synthetic communities from benchmark networks, where we present results for Girvan-Newman and Lancichinetti-Fortunato-Radicchi networks. Our findings show that our algorithm falls into the class of accurate performers.

Keywords: Community Detection, Benchmark Tests, Resistance Distance

1 Introduction

A network is an abstract model of the inter-relationships (links) between the elements (nodes) of a data-set [1], which can result in extremely complex structure. However, real-world networks also show the presence of communities; that is, densely connected groups of nodes with sparse inter-group connections [2]. The ability to detect these community structures has many practical applications [3], mainly, because it simplifies the network analysis to clustered subgroups.

Because we lack a strict definition of community, the detection problem may present different solutions (particularly when dealing with a definition that uses macroscopic quantities such as the density of links), leading to the development of many methods with varying degree of success [4,5,6,7,8,9,10,11,12,13,14,15,16,17] A community detection method success requires testing its accuracy on networks where the community structure is known [18,19,20,21]. This can be achieved by selecting benchmark networks, such as the synthetically generated by Girvan 2 J. Gancio & N. Rubido

and Newman (GN) [18] or Lancichinetti, Fortunato, and Radicchi (LFR) [20] benchmark models.

Recently, Zhang and Bu (ZB) [17] proposed a new method for community detection based on the resistance distance. The resistance distance includes more information than the shortest paths, since it also considers every different possible path between any two nodes, weighing them as parallel paths [22,23,24,25]. Although the resistance distance has been widely used for network analysis [26,27,28,29,30,31], its use for community detection has been limited [32,17]. ZB report [17] accurate results on small-sized networks (Zachary 's karate club [33], dolphin social network [34], and the college football network [18]), in comparison to the methods by Kannan, Vempala, and Vetta (KVV) [35] and spectral modularity [10,11]. However, ZB's method lacks benchmark testing and automation.

Here, we adapt ZB's algorithm to detect communities on networks without needing to specify the number of communities beforehand, making it an unsupervised method. We test the accuracy of our modified algorithm's detection rate in GN and LFR benchmarks, which allows us to compare it with previously reported results from other methods. Our findings show that the algorithm's accuracy is comparable – in LFR networks – or better – in GN networks – than methods such as clique percolation (also known as CPM or Cfinder) [9], Markov Clustering algorithm (MCL) [36], hierarchical divisions [5], and exhaustive modularity optimisation [7,8]. Because the resistance distance is unrestricted to these particular networks, we expect that our algorithm can detect communities in weighted, directed, and even, evolving networks.

2 Methods

2.1 Definitions and Notation

A network is a pair of sets, $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where \mathcal{V} is a set of N nodes (vertices) and $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ is a set of M links (edges) from the unordered set of node pairs $\mathcal{V} \times \mathcal{V} = \{(i, j) : i \in \mathcal{V}, j \in \mathcal{V}\}$. \mathcal{G} can be represented by its adjacency matrix, \mathbf{A} , such that $A_{ij} = 1$ if there is a link connecting node i to j, or $A_{ij} = 0$ otherwise. When \mathcal{G} is undirected and unweighted, $A_{ij} = A_{ji}$ and $N-1 \leq M \leq N(N-1)/2$. Also, the node's degree is the number of neighbours of a node, such that $k_i = \sum_{i=1}^{N} A_{ij}$ and $\sum_{i=1}^{N} k_i = 2M = N \langle k \rangle$, making the network's link density be

$$\rho(\mathcal{G}) = \frac{|\mathcal{E}|}{|\mathcal{V} \times \mathcal{V}|} = \frac{2M}{N(N-1)} = \frac{\langle k \rangle}{N-1}.$$
 (1)

A community is a subset of nodes, $\mathcal{W} \subset \mathcal{V}$, namely, a partition, that can be defined in terms of its relative link density [2,6,13,15], $\rho(\mathcal{W})$, such that it fulfil

$$\rho(\mathcal{W}) > \rho(\mathcal{G}) > \rho(\mathcal{G} \setminus \mathcal{W}), \tag{2}$$

where $\rho(\mathcal{G} \setminus \mathcal{W})$ is the link density between \mathcal{W} and its complement, $\overline{\mathcal{W}} = \mathcal{G} \setminus \mathcal{W}$. We note that Eq. (2) leads to several ways to define which nodes belong to \mathcal{W} , since small changes can leave it unaffected (such as, depending on the network, the effect of removing or including a node to \mathcal{W}). However, because of the higher density of links within a community, its nodes tend to be topologically closer.

A measure to quantify the topological distance between pairs of nodes is the resistance distance [22,23,24,25]. For nodes *i* and *j*, it can be determined by

$$R_{ij} = \sum_{n=2}^{N} \frac{1}{\lambda_n(\mathbf{L})} \left| [\phi_n]_i - [\phi_n]_j \right|^2 \ge 0,$$
(3)

where $\lambda_n(\mathbf{L})$ is the *n*-th eigenvalue of the Laplacian matrix \mathbf{L} and $[\phi_n]_i$ is the *i*-th component of the corresponding eigenvector, i.e., $\mathbf{L}\phi_n = \lambda_n(\mathbf{L})\phi_n$, with $\{0 = \lambda_1 < \lambda_2 \leq \ldots \leq \lambda_N\}$. $\mathbf{L}(\mathbf{A}) = \mathbf{D} - \mathbf{A}$, where \mathbf{D} is the diagonal matrix containing the node degrees, making $\mathbf{L}(\mathbf{A})$ a positive semi-defined matrix when the network is undirected. It has been shown that nodes within a community have smaller R_{ij} values than nodes belonging to different communities [32]. The reason is that R_{ij} is small $(R_{ij} < 1)$ when there are parallel paths between nodes, but increases with serial paths $(R_{ij} > 1)$, as in a bridge between communities.

2.2 Resistance-Distance-based Community Detection Method

We modify the algorithm by Zhang and Bu (ZB) [17] to automate the partitioning process. That is, we include points 7-10 to the original algorithm.

- 1. Compute $R_{ij}(p)$ from Eq. (3) for all $N_p \times N_p$ pairs of nodes belonging to the *p*-th partition $\mathcal{V}_p \subset \mathcal{V}$.
- 2. Apply a Gaussian transformation to $R_{ij}(p)$: $S_{ij}(p) = A_{ij} \exp\left(-R_{ij}(p)^2/2\right)$, which highlights the different resistance distances between nodes.
- 3. Compute normalised Laplacian matrix of $\mathbf{S}(p)$: $\mathcal{L}_{S(p)} = \mathbf{D}_{S}^{-1/2} \mathbf{L}(\mathbf{S}) \mathbf{D}_{S}^{-1/2}$, where $\mathbf{L}(\mathbf{S}) = \mathbf{D}_{S} - \mathbf{S}(p)$ with $[\mathbf{D}_{S}]_{ii} = \kappa_{i} = \sum_{i=1}^{N_{p}} S_{ij}(p)$ and $[\mathbf{D}_{S}]_{ij} = 0$.
- 4. Find the eigenvector ψ_2 of $\mathcal{L}_{S(p)}$ associated to the algebraic connectivity [37], i.e., the smallest non-zero eigenvalue.
- 5. Order ψ_2 in ascending magnitude: $\psi_2 \rightarrow [\psi_2]_{\pi_1} < [\psi_2]_{\pi_2} < \ldots < [\psi_2]_{\pi_{N_p}}$.
- 6. **Partition** \mathcal{V}_p according to the minimisation of the Cheeger constant [38]:

$$h_G(\mathcal{W}^{\star}_{\pi}) = \min_{\mathcal{W}_{\pi} \subset \mathcal{V}_p} \left\{ \frac{\sum_{i \in \mathcal{W}_{\pi}} \sum_{j \in \overline{\mathcal{W}_{\pi}}} A_{ij}}{\min\left\{ \sum_{i \in \mathcal{W}_{\pi}} k_i, \sum_{i \in \overline{\mathcal{W}_k}} k_i \right\}} \right\},\tag{4}$$

where $\mathcal{V}_p = \mathcal{W}_{\pi} \cup \overline{\mathcal{W}_{\pi}}$ for any $\pi \in \{\pi_1, \pi_2, \ldots, \pi_{N_p}\}$. Equation (4) minimises the ratio between the number of inter-partition links (numerator) and the number of intra-partition links to give the optimal π^* , defining the subset $\mathcal{W}_{\pi}^* = \{\pi_1, \ldots, \pi^*\}$ and its complement, $\overline{\mathcal{W}_{\pi}^*} = \{\pi^*, \ldots, \pi_{N_p}\}$.

7. **Update** tentative community structure: $\mathcal{B}_{new} = \{\mathcal{V}_1 \cup \cdots \cup \mathcal{V}_{p-1} \cup \mathcal{W}_{\pi}^* \cup \overline{\mathcal{W}_{\pi}^*} \cup \mathcal{V}_{p+1} \cup \cdots \cup \mathcal{V}_{N(B)_{new}}\} \leftarrow \mathcal{B}_{old} = \{\mathcal{V}_1 \cup \cdots \cup \mathcal{V}_{p-1} \cup \mathcal{V}_p \cup \mathcal{V}_{p+1} \cup \cdots \cup \mathcal{V}_{N(B)_{old}}\},$ where $N(B)_{new} = N(B)_{old} + 1$ is the number of communities up to this point.

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- 8. Compute modularity [10]: $Q(\mathcal{B}_{new}) = \frac{1}{4M} \sum_{i,j} \left(A_{ij} \frac{k_i k_j}{2M} \right) (s_i s_j)$, where $s_i s_j = 1$ if nodes *i* and *j* belong to the same community, otherwise $s_i s_j = 0$.
- 9. Accept partition if $Q(\mathcal{B}_{new}) > Q(\mathcal{B}_{old})$. Otherwise, discard \mathcal{B}_{new} .
- 10. **Repeat** steps 1 to 8 changing $p \rightarrow p+1$ while p+1 < N(B) < N.

We initialise the community detection by generating an initial partition, \mathcal{B}_{init} , by applying steps 1 to 7 on $\mathcal{G} = \mathcal{V}_0$, which divides the network into 2 subsets based on Cheeger's constant [Eq. (4)]. This means that ZB's method can divide each subset into two new subsets, successively.

2.3 Method Validation: Benchmark Testing and Accuracy

We test our ZB algorithm's ability to correctly detect communities on 100 network realisations of Girvan-Newman (GN) [18] and Lancichinetti-Fortunato-Radicchi (LFR) [20] benchmarks. 4 examples of these reference networks, $\{\mathcal{A}_{ref}\}$, with predefined N(A) communities, are shown in Fig. 1. We generate these benchmarks by freely available codes from Refs. [39,40] (GN) and [41] (LFR).



Fig. 1. Force-directed layouts of benchmark networks. Top panels show Girvan-Newman [18] networks with N = 128 nodes and average degree $\langle k \rangle = 16$. Bottom panels show Lancichinetti-Fortunato-Radicchi [20] networks with N = 1000 nodes, average degree $\langle k \rangle = 15$, and power-law's exponent for the degree and community-size distributions being $\gamma = 2$ and $\beta = 1$, respectively. Left [Right] panels are networks with mixing parameter $\mu = 0.1$ [$\mu = 0.3$]. Colours indicate the predefined communities.

GN networks are generated from the Erdös-Réyni (ER) random network model [1]. This model sets N(A) = 4 communities with $n_j = 32$ nodes each $(N = \sum_{j=1}^{N(A)} n_j = 128)$, assigning intra-community links with probability p_{in} and inter-community links with probability p_{out} , where $0 < p_{out}/p_{in} < 1$. These probabilities are related to the internal and external node degrees, k^{in} and k^{out} , respectively – the attachment probability, p, in ER networks is such that $p = \rho(\mathcal{G}) = \langle k \rangle / (N-1)$ [see Eq. (1)]. We fix the average node degree to $\langle k \rangle = 16$ and change the mixing of communities by changing p_{in} and p_{out} such that

$$\mu_i = \frac{k_i^{out}}{k_i^{out} + k_i^{in}} < 1.$$
(5)

The mixing control parameter is then the network average $\sum_{i} \mu_i / N = \mu \in [0, 1]$.

LFR networks are generated by assigning each node degree, k_i , from a powerlaw distribution: $P(k) \sim k^{-\gamma}$. The distribution range is set to satisfy the prefixed network-average, $\langle k \rangle$. The size of each community, n_j , is assigned from another power-law distribution: $P(n) \sim n^{-\beta}$, such that $\sum_{j=1}^{N(A)} n_j = N$. Then, nodes are assigned randomly to these communities, as long as the community size is bigger than the internal degree of the node. Finally, links are assigned by various rewiring steps that modify the internal and external degrees of each node, without modifying the node's degree, k_i , until $k^{in} \approx \mu k_i$ and $k^{out} \approx (1 - \mu)k_i$, where the mixing parameter is defined by Eq. (5).

We measure the community detection accuracy by the Normalised Mutual Information, I [19,21], which quantifies the similarity between a reference structure, $\mathcal{A}_{ref} = \{\mathcal{V}_1 \cup \mathcal{V}_2 \cup \ldots \cup \mathcal{V}_{N(A)}\}$ with N(A) communities, and a detected structure, $\mathcal{B} = \{\mathcal{W}_1 \cup \mathcal{W}_2 \cup \ldots \cup \mathcal{W}_{N(B)}\}$ with N(B) communities. Specifically,

$$I(\mathcal{A}_{ref}, \mathcal{B}) = \frac{-2\sum_{i=1}^{N(A)} \sum_{j=1}^{N(B)} C_{ij} \log\left(\frac{C_{ij}N}{C_i C_j}\right)}{\sum_{i=1}^{N(A)} C_i \log\left(\frac{C_i}{N}\right) + \sum_{j=1}^{N(B)} C_j \log\left(\frac{C_j}{N}\right)},$$
(6)

where C_{ij} is the number of nodes in community *i* of \mathcal{A}_{ref} that are also in community *j* of \mathcal{B} , $C_i = \sum_j C_{ij}$, and $C_j = \sum_i C_{ij}$.

3 Results

Figure 2 shows degree (top panels) and spectral (bottom panels) distributions, P(k) and $P(\lambda)$, respectively, for 8 benchmark networks. The 6LFR networks show power-law degree-distributions, which fit the expected exponent $\gamma = 2$, as can be seen by the dotted lines in Figs. 2(a) and (b). Because the Laplacian matrix is closely related to the node degrees, the spectral distributions, $P(\lambda)$, also tend to a power-law with similar exponent, as can be seen from their tails in Figs. 2(c) and (d). On the other hand, GN networks have a narrow degree distribution (not shown), since we fix $\langle k \rangle = 16$, which implies that $P(k) \sim \delta(k - 16)$. Because of the random features, GN spectral distribution tends to Wigner's

semi-circle distribution, as can be seen from the dashed curves in Fig. 2(c) and (d). These characteristics are translated to the resistance distance by means of Eq. (3), where the smaller eigenvalues have the larger influence, particularly, the algebraic connectivity (also known as Fiedler eigenvalue [37]).



Fig. 2. Degree and spectral probability distributions, P(k) and $P(\lambda)$, of networks with communities. All panels have Lancichinetti-Fortunato-Radicchi networks [20] with N = 1000 nodes, generated from $P(k) \sim k^{-2}$ (dotted line) and communities with sizes following $P(n) \sim n^{-1}$, where $\langle k \rangle = 15$ (black), $\langle k \rangle = 20$ (blue), or $\langle k \rangle = 25$ (magenta), changing the distribution ranges. Bottom panels have Girvan-Newman networks [18] with N = 128 and $\langle k \rangle = 16$ (red dashed). Panels (a) and (c) [(b) and (d)] show the distributions when the mixing parameter $\mu = 0.1$ [$\mu = 0.3$].

As can be seen from Fig. 3, the shape of the degree distributions from Fig. 2 remain nearly unaltered when analysing node strengths, κ , resultant from the weighted degrees of the Gaussian transformation of the resistance distance (steps 2 ad 3 in Sect. 2.2). Specifically, we find power-law distributions, $P(\kappa) \sim \kappa^{-2}$, for the LFR networks – Figs. 3(a) and (b) – and a narrowly Gaussian-like distribution for the GN networks – Figs. 3(c) and (d). These distributions hold for both mixing parameters, $\mu = 0.1$ and $\mu = 0.3$, which also correspond to the networks shown in Fig. 1. Because the node strength distribution share similarities with the node degree distribution, we also find that the spectral distribution of the Gaussian-transformed resistance distance is similar to that of the initially unweighted network (not shown). This means that ZB's community detection method (Sect. 2.2) keeps the main topological properties of the original network and that its partitioning process can be narrowed to the network's Fiedler eigenvector, which fits into the category of spectral methods.



Fig. 3. Node strength probability distributions, $P(\kappa)$, of benchmark networks. The *i*-th node strength, $\kappa_i = \sum_j S_{ij}(R_{ij})$, is the weighted-degree of the Gaussian-transformed resistance distance, R_{ij} , of the network [Eq. (3)]. Top panels show $P(\kappa)$ for the Lancichinetti-Fortunato-Radicchi networks of Figs. 2(a) and (b). Bottom panels show $P(\kappa)$ for the Girvan-Newman networks in Fig. 1. Mixing parameters, colours, and symbols follow those of Fig. 2.

In spite of ZB's method being (in its core) a spectral partitioning method, the inclusion of the resistance distance [Eq. (3)] as the matrix to use for the partitioning process results in more accurate community detection – as we show in Fig. 4. In particular, the normalised mutual information, I, values we obtain for 100 GN network realisations show excellent results ($I \simeq 1$) up to $\mu = 0.4$, when I starts to decline – this can be seen from Fig. 4(a). On the other hand, results for 100 LFR network realisations show a monotonous decline in accuracy, from $I \simeq 1$ when $\mu = 0.1$ to $I \simeq 0.7$ when $\mu = 0.6$, as can be seen from the remaining panels in Fig. 4. These values are nearly unchanged when considering different network-average node degrees, $\langle k \rangle$, which are shown by differently coloured curves and symbols (following the same pattern as in Figs. 2 and 3). However, in all LFR tests, *I* slightly increases for all μ when $\langle k \rangle$ is increased. Overall, we note that the algorithm perform sufficiently accurate, particularly, performing better than previously reported results [15,20,42] that use Fast Greedy Optimisation [43] or Label Propagation [44], to name a few.



Fig. 4. Resistance-distance based community-detection accuracy as the communities becomes mixed. Accuracy is measured by the normalised mutual information, I [Eq. (6)], and community mixing is controlled by μ [Eq. (5)]. Panel (a) show resultant detection accuracy from 100 GN network realisations, where N = 128 nodes and $\langle k \rangle = 16$ average degree. Remaining panels show resultant I from 100 LFR network realisations, where $N = 10^3$ and $\langle k \rangle = 15$ (black), $\langle k \rangle = 20$ (blue), or $\langle k \rangle = 25$ (magenta), as in Figs. 2 and 3. The power-law exponents, γ and β , for the LFR networks' node-degree and community-size are $(\gamma, \beta) = (2, 1)$ in panel (b), $(\gamma, \beta) = (2, 2)$ in panel (c), $(\gamma, \beta) = (3, 1)$ in panel (d), and $(\gamma, \beta) = (3, 2)$ in panel (e). Every symbol represents the median I of 100 realisations and shaded areas the 98% central I values.

In this accuracy analysis we have also used different LFR parameters to explore the effect of changing the degree distribution exponent, γ , and community size distribution exponent, β . For example, when $\gamma = 2$ degree distributions are as in Fig. 2, where there are some hub nodes (given by the distribution tail).

The resultant accuracy for these networks can be seen in Figs. 4(b) and (c), whose difference comes from having more ($\beta = 1$) or fewer ($\beta = 2$) communities with varying size, respectively. Similarly, we analyse the resultant detection accuracy when $\gamma = 3$, where the number of hubs decreases but they increase their degrees. Resultant *I* values are shown in Figs. 4(d) and (e), where differences emerge from changing β as in panels (b) and (c). Overall, Fig. 4 shows that ZB's method for community detection works fairly on GN and LFR networks, complementing their previous results on small-sized real-world networks [17].

4 Discussion and Conclusions

Our works is based on extending Zhang and Bu (ZB) [17] method for community detection to automate its operation and quantify its accuracy to correctly detect communities in benchmark networks. In order to successively partition the network into smaller modules, ZB method follows Kannan-Vempala-Vetta (KVV) bi-sectioning algorithm [35] (steps 2-6 in Sect. 2.2), but it uses the resistance distance of the network (step 2 in Sect. 2.2) instead of its adjacency matrix. We add modularity optimisation to the process (steps 7-10), which makes the resultant algorithm a hybrid method involving resistance distance, spectral partitioning, and modularity optimisation. Consequently, our adaptation allows to iterate the algorithm without needing to specify the number of communities in the network or control its outcomes, making it an unsupervised algorithm.

In order to quantify the method's accuracy, we use Girvan-Newman (GN) and Lancichinetti-Fortunato-Radicchi (LFR) benchmark networks, where N = 128and 1000, respectively. These classes of networks are different in size and overall topology. For example, the mixing parameter, μ , in GN networks relates to the probabilities of intra- and inter-community links. On the other hand, μ in LFR networks relates to the rewiring process by which scale-free degrees and communities are inter-connected. However, according to our results from Fig. 4, the modified ZB algorithm can detect communities in networks with significant mixing, such as $\mu = 0.4$ or 0.5, which as can be seen from the force-directed layout in Fig. 1, the community structure presents serious challenges.

These benchmark tests extend ZB results (where $N \leq 115$) and complements their work. In particular, Fortunato et al. [42] classifies community detection algorithms as a function of μ and their performance on benchmark networks according to 3 categories: a) bad (those that $I \rightarrow 0$ rapidly with increasing μ), b) fair (those that I declines with increasing μ but remains finite), and c) good (those where $I \sim 1$ for $\mu \leq 0.5$). Therefore, ZB's algorithm – with our modifications – falls into the fair category on LFR networks and good category on the GN networks, as it can be corroborated from Fig. 4. Although, we note that there is room for improvement in terms of its the computational complexity, which is the main drawback in most spectral methods. 10 J. Gancio & N. Rubido

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