ON SOME DISSIPATIVITY PROPERTIES OF A CLASS OF POWER SYSTEM MODELS

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Abstract: This paper studies the dissipativity properties of a class of power system models, characterized by the absence of resistive loads and leaky lines. A Port-Controlled Hamiltonian (PCH) representation is given for each component of the network and its dissipativity properties are shown. The linear model around the equilibrium is shown to meet a convex condition in the frequency domain, able to be exploited in the stability analysis of interconnected systems. The application of this property to a classical example shows that it can be computationally exploited even in the case of non-idealized models.

Keywords: Dissipativity, Power system stability, Multipliers

1. INTRODUCTION

Power systems dynamics is a complex phenomenon characterized by nonlinear, high order differential–algebraic equations (DAE) subject to topology changes and parameter variation (Kundur 1994). The proper concept of stability recognizes a set of different approaches: transient stability, voltage (long term) stability, structural stability, etc.

Typically, the power system stability results from the dynamic interaction of several subsystems, often operated or regulated by different entities (utilities, states,etc.).

This paper addresses the study of some structural properties of basic power system models and its exploitation for the analysis of equilibrium stability. This approach recognizes antecedents in (Varaiya *et al.* 1985) and references therein, and, more recently (Giusto *et al.* 2006). However, the main objective of this paper is to exploit these properties in order to analyze the stability of interconnections of power systems trying to obtain a set of dynamic conditions to be met by each subsystem, sufficient to ensure the overall stability.

The structure of the paper is as follows. Section 2 presents the mathematical model of the various elements comprising the power system and gives their PCH representation. In Section 3 we derive their dissipativity properties under idealized assumptions. Section 4 introduces the corresponding linear model around the equilibrium point and shows how the dissipation inequality can be posed as a convex condition in the frequency domain. Section 5 presents the application of these procedures to the analysis of a classical, two-area example. We wrap up the paper with some concluding remarks.

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2. POWER SYSTEM MODELING

In this section we recall the well–known model for n–machine power systems reported in (Varaiya *et al.* 1985). All network components share as port variables the angle θ_j and the magnitude V_j of the bus voltage phasor:

$$y_j := \begin{bmatrix} \theta_j & V_j \end{bmatrix}^T \in \mathbb{R}^2. \tag{1}$$

The system has n machines. Each machine and its corresponding bus, have an associated identifier $i \in J_M$. The total number of buses, including the n machines, is m. Generic buses are denoted by identifiers j or k with $j,k \in J_B$. Each load and its associated bus are denoted by $l \in J_L \subset J_B$.

Attached to each bus there is a machine and/or a load, and the buses are interconnected through transmission lines. Transmission lines are identified by the double subindex $jk \in \Omega \subset J_B \times J_B$, indicating that the line jk connects the bus $j \in J_B$ with the bus $k \in J_B$—the set avoids obvious repetitions, e.g., if $jk \in \Omega$ then $kj \notin \Omega$. Ω_j will denote the set of buses linked to the bus j.

Associated to each bus are the active and reactive powers entering the machine, the load or the transmission lines, that will be denoted

$$u_i^M = \begin{bmatrix} P_i^M \\ Q_i^M \end{bmatrix}, u_l^L = \begin{bmatrix} P_l^L \\ Q_l^L \end{bmatrix}, u_{jk} = \begin{bmatrix} P_{jk} \\ Q_{jk} \end{bmatrix} \tag{2}$$

respectively. We denote $u_i^e := \begin{bmatrix} P_i^e & Q_i^e \end{bmatrix}^\top$ the external injection of active and reactive power at bus i. We take active and reactive powers as positive when entering their corresponding component.

Each *generator* is described by a set of third order Differential Algebraic Equations (DAEs):

$$\begin{aligned}
\dot{\delta}_i &= \omega_i \\
M_i \dot{\omega}_i &= P_{m_i} - D_i \omega_i + P_i^M \\
\tau_i \dot{E}_i &= -\frac{x_{d_i}}{x'_{d_i}} E_i + \frac{x_{d_i} - x'_{d_i}}{x'_{d_i}} V_i \cos(\delta_i - \theta_i) + E_{F_i} \\
P_i^M &= -\frac{1}{x'_{d_i}} E_i V_i \sin(\delta_i - \theta_i) - Y_{2_i} V_i^2 \sin 2(\delta_i - \theta_i) \\
Q_i^M &= Y_{V_i} V_i^2 - \frac{1}{x'_{d_i}} E_i V_i \cos(\delta_i - \theta_i) - Y_{2_i} V_i^2 \cos 2(\delta_i - \theta_i)
\end{aligned} \tag{3}$$

where we defined $Y_{2_i} := \frac{x'_{d_i} - x_{q_i}}{2 x_{q_i} x'_{d_i}}$, $Y_{V_i} := \frac{x'_{d_i} + x_{q_i}}{2 x_{q_i} x'_{d_i}}$. The *state variables* $x_i := \mathrm{col}(\delta_i,\ \omega_i,\ E_i) \in \mathbb{R}^3$ denote the rotor angle, the rotor speed and the quadrature axis internal e.m.f., respectively, and E_{F_i} is the field voltage. The parameters are denoted as in (Varaiya *et al.* 1985), and they are fairly standard. We will make the physically reasonable assumptions $D_i > 0,\ x_{d_i} - x'_{d_i} > 0$.

For convenience, we will separate the field voltage in two terms, $E_{F_i} = E_{F_i}^\star + v_i$, the first one is constant and fixes the equilibrium value, while the second one is the *control action.*

We have the following simple fact, whose proof follows from (1), (2) by direct substitution.

Fact 1. The model (3) defines an operator $\Sigma_i^M : (u_i^M, y_i)$ described by the implicit PCH system. [2].

$$\begin{cases} \dot{x}_i = (J_i - R_i)\nabla_{x_i} S_i^M(x_i, y_i) + B_{v_i} v_i \\ 0 = -\nabla_{y_i} S_i^M(x_i, y_i) + B_u(y_i) u_i^M \end{cases} \tag{4}$$

with storage function $S_i^M : \mathbb{R}^3 \times \mathbb{R}^2 \to \mathbb{R}$,

$$S_i^M(x_i, y_i) := \frac{1}{2} M_i \omega_i^2 - P_{mi}\delta_i - \frac{E_i V_i}{x'_{d_i}} \cos(\theta_i - \delta_i) -$$

$$-\frac{Y_{2i}}{2} V_i^2 \cos 2(\theta_i - \delta_i) + \frac{Y_{E_i}}{2} E_i^2 - Y_{F_i} E_{F_i}^\star E_i + \frac{Y_{Vi}}{2} V_i^2.$$

We defined the coefficients Y_{E_i}, Y_{F_i} from the model parameters. The matrices are built analogously, see (Giusto *et al.* 2006), and satisfy $J_i = -J_i^\top \in R^{3\times 3}$, $R_i = R_i^\top \geq 0 \in R^{3\times 3}$, $B_{v_i} \in R^{1\times 3}$. Matrix $B_u(y_i)$ and the term $B_u(y_i)u_i$ are given by

$$B_u(y_i) := \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{V_i} \end{bmatrix}; B_u(y_i) u_i = \begin{bmatrix} P_i \\ \frac{Q_i}{V_i} \end{bmatrix}. \tag{5}$$

Denoting the DAE model (4) as an "implicit PCH system" is done with some abuse of notation. See (Van der Schaft 2000) for a precise definition.

Loads are described with constant active power and by the standard ZIP model (Kundur 1994) for reactive power:

$$\begin{aligned} P_l^L &= P_{0_l} \\ Q_l^L &= Q_{Z_l} V_l^2 + Q_{I_l} V_l + Q_{0_l} \end{aligned}, \tag{6}$$

Fact 2. The model (6) defines a (memory–less) PCH operator $\Sigma_l^L : (u_l^L, y_l)$ given by

$$0 = -\nabla_{y_l} S_l^L(y_l) + B_u(y_l) u_l^L \tag{7}$$

with storage function S_l^L:

$$S_l^L(y_l) := P_{0_l}\theta_l + \frac{Q_{Z_l}}{2} V_l^2 + Q_{I_l} V_l + Q_{0_l} \ln(V_l).$$

Transmission lines are modeled with the standard lumped Π circuit (Kundur 1994). Power entering at bur j is given by

$$\begin{aligned} P_{jk} &= B_{jk} V_j V_k \sin(\theta_j - \theta_k) \\ Q_{jk} &= (B_{jk} - B_{jk}^c) V_j^2 - B_{jk} V_j V_k \cos(\theta_j - \theta_k), \end{aligned} \tag{8}$$

for all $jk \in \Omega$. The transfer conductances were neglected; the power entering at node k, P_{kj}, Q_{kj} can be obtained by a simple change of indexes.

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[2] All vectors in the paper are denoted *column* vectors, even the gradient of a scalar function: $\nabla_x = \frac{\partial}{\partial x}$.

Fact 3. The model (8) defines the implicit (memory–less) PCH operator $\Sigma_{jk}:(u_{jk},u_{kj},y_j,y_k)$:

$$\Sigma_{jk}:\begin{cases}0{=}{-}\nabla_{y_j}S_{jk}(y_j,y_k)+B_u(y_j)u_{jk}\\0{=}{-}\nabla_{y_k}S_{jk}(y_j,y_k)+B_u(y_k)u_{kj}\end{cases}\qquad(9)$$

with storage function S_{jk} given by

$$S_{jk}(y_j,y_k){:=}\frac{B_{jk}{-}B^c_{jk}}{2}(V_j^2{+}V_k^2){-}B_{jk}V_jV_k\cos(\theta_j-\theta_k).$$

At each *bus*, we have the corresponding Kirchoff law, which can be written (see equation (5)):

$$\Sigma^B_j\,:\;0=B_u(y_j)[\sum_{k\in\Omega_j}u_{jk}+u^M_j+u^L_j-u^e_j].\;(10)$$

We are now in position to use the external power injections to *model the interactions between adjacent subsystems*. The system interacts with its environment trough the pairs (u^e,y), i.e. the complex power and the voltage phasor, at the frontier buses. This description is very attractive from an engineering point of view, but the convenience in working with a symmetrical interconnection lead us to describe the power injections in function of the voltage phasors at the frontier buses. Be Ω_e the set of frontier buses, i.e., the buses where the interconnection with adjacent subsystems is done, and denote m_e its cardinality. In each bus $j\in\Omega_e$, the interaction with the adjacent area is described by Σ_{ja}, (see equation (9)). Thus, u^e_j is a function on the variables θ_j,V_j and the variables θ_a,V_a of the frontier bus of the adjacent area. We will denote $z_j:=y_a=[\theta_a,V_a]^\top$ these variables to avoid confusion with any internal bus.

The equation (9) allows us to write the term $\mathcal{B}_u(y_j)u^e_j,\forall j\in\Omega_e$ in equation (10):

$$B_u(y_j)u^e_j=-\nabla_{y_j}S^e_j(y_j,z_j)\qquad(11)$$

where the function $S^e_j:\mathbb{R}^2\times\mathbb{R}^2\to\mathbb{R}$ is given by

$$S^e_j(y_j,z_j)\triangleq S_{ja}(y_j,z_j).$$

The *power system model* results from the models already presented (equations (4), (7), (9), (10), and (11)), and some notational convention.

Let x the state vector of the system, y the link variables, and z the input variables defined as $x:=\mathrm{col}_{i\in J_M}(x_i),\ y:=\mathrm{col}_{j\in J_B}(y_j),z:=col_{j\in\Omega_e}(z_j)$.

Denote $N:=3n$ and define the function $S:\mathbb{R}^N\times\mathbb{R}^{2m}\times\mathbb{R}^{2m_e}\to\mathbb{R}$:

$$S(x,y,z){:=}\sum_i S^M_i(x_i,y_i){+}\sum_l S^L_l(y_l){+}$$

$$+\sum_{jk\in\Omega}S_{jk}(y_j,y_k)+\sum_{j\in\Omega_e}S^e_j(y_j,z_j).\qquad(12)$$

It is possible, through the use of the Hamiltonian description of each component, to get a compact description of the system. Towards this end, it is convenient to group all the algebraic constraints of (4), (7), (9), (10), (11). Define the function $g:\mathbb{R}^N\times\mathbb{R}^{2m}\times\mathbb{R}^{2m_e}\to\mathbb{R}^{2m}$:

$$g(x,y,z):=\ \nabla_yS(x,y,z).\qquad(13)$$

The algebraic constraint –once one eliminates the internal power exchanges– yields

$$g(x,y,z)=0.$$

The introduction of the block–diagonal matrices:

$$J:=\mathrm{diag}\{J_i\},R:=\mathrm{diag}\{R_i\},B_v:=\mathrm{diag}\{B_{v_i}\},$$

$J=-J^\top,R=R^\top\geq0$ allows us to compact the model and rewrite the overall system as

$$\Sigma(x,y,z):\quad\begin{cases}\dot{x}\;=\;(J-R)\nabla_xS(x,y,z)+B_vv\\0\;=\;\nabla_yS(x,y,z).\end{cases}.(14)$$

Notice the control action $v=col(v_i)$.

We need to define the set $\mathcal{D}\in\mathbb{R}^N\times\mathbb{R}^{2m}\times\mathbb{R}^{2m_e}$ where the solutions of the DAE are unique and well defined (Hill and Mareels 1990):

$$\mathcal{D}\triangleq\{(x,y,z)|g(x,y,z)=0\ and$$

$$\nabla_yg(x,y,z) is\ nonsingular\}.$$

3. DISSIPATIVITY PROPERTIES

Equations (4), (7), (9), (10) constitute a set of PCH models for the network components. The subjacent dissipativity property was already established for these models in (Giusto *et al.* 2006), following the lines of (Willems 1972), (Hill and Moylan 1980).

To establish the dissipativity properties of model (14) we make the following assumption.

Assumption A1. The field voltages of the synchronous machines are constant: $E_{F_i}=E^\star_{F_i}$.

Define the function $w^z_j:\mathbb{R}^2\times\mathbb{R}^2\times\mathbb{R}^2\to\mathbb{R}$:

$$w^z_j(y_j,z_j,\dot{z}_j):=\nabla^\top_{z_j}S^e_j(y_j,z_j)\dot{z}_j,$$

and $w^z:\mathbb{R}^{2m}\times\mathbb{R}^{2m_e}\times\mathbb{R}^{2m_e}\to\mathbb{R}$:

$$w^z(y,z,\dot{z}):=\sum_{\Omega_e}w^z_j(y_j,z_j,\dot{z}_j).$$

Proposition 1. If Assumption A1 holds, then for all $(x,y,z)\in\mathcal{D}$ satisfying (14), the following dissipation inequality holds:

$$\frac{dS(x,y,z)}{dt}\leq w^z(y^e,z,\dot{z})\ \forall t.$$

Proof: Compute

$$\frac{dS(x,y,z)}{dt} = \nabla_x^\top S\dot{x} + \nabla_y^\top S\dot{y} + \nabla_z^\top S\dot{z} =$$

$$-\nabla_x^\top SR\nabla_x S + \nabla_z^\top S\dot{z} \leq \nabla_z^\top S\dot{z} =$$

$$= \nabla_z^\top [\sum_{\Omega_e} S_j^e(y_j, z_j)]\dot{z} = w^z(y,z,\dot{z}).$$

4. INPUT-OUTPUT PROPERTIES FOR SMALL SIGNAL MODELS

In this section we will study how the properties considered in Section 3 particularize for small signal models. The equilibrium point, denoted $x^\star, y^\star, z^\star$, will be supposed interior to $\mathcal{D}$. We denote $\tilde{(\cdot)} := (\cdot) - (\cdot)^\star$ the incremental variables.

With the help of the Hessian of function S at the equilibrium point:

$$\mathcal{H} = \frac{\partial^2 S(x,y,z)}{\partial(x,y,z)^2}|_\star,$$

we can define $H : \mathbb{R}^N \times \mathbb{R}^{2m} \times \mathbb{R}^{2m_e} \to \mathbb{R}$:

$$H(\tilde{x},\tilde{y},\tilde{z}) := \frac{1}{2}\begin{bmatrix}\tilde{x}\\ \tilde{y}\\ \tilde{z}\end{bmatrix}^\top \mathcal{H} \begin{bmatrix}\tilde{x}\\ \tilde{y}\\ \tilde{z}\end{bmatrix}, \tag{15}$$

and obtain the linear model, valid in a neighborhood of the equilibrium point:

$$\begin{cases} \dot{\tilde{x}} = (J-R)\nabla_{\tilde{x}} H(\tilde{x},\tilde{y},\tilde{z}) + B_v v \\ 0 = \nabla_{\tilde{y}} H(\tilde{x},\tilde{y},\tilde{z}). \end{cases} \tag{16}$$

Denote $y^e := col_{j\in\Omega_e}(y_j)$ the vector of link variables associated to the frontier buses. The matrices $\mathcal{W}_{yz}, \mathcal{W}_{zz} \in R^{2m_e \times 2m_e}$:

$$\mathcal{W}_{yz} := \frac{\partial^2 \sum S_j^e(y_j,z_j)}{\partial y^e \partial z}|_\star, \tag{17}$$

$\mathcal{W}_{zz} := \frac{\partial^2 \sum S_j^e}{\partial z^2}|_\star$, will be very useful to describe the interactions with the adjacent areas. In fact, we define a quadratic supply rate function:

$$\tilde{w}_e^z : R^{2m_e} \times R^{2m_e} \times R^{2m_e} \to R :$$

$$\tilde{w}_e^z(\tilde{y}^e, \tilde{z}, \dot{\tilde{z}}) := (\tilde{y}^{e\top}\mathcal{W}_{yz} + \tilde{z}^\top \mathcal{W}_{zz})\dot{\tilde{z}}.$$

If Assumption A1 holds, $v \equiv 0$ and the dissipation inequality is easily recovered with the help of definitions (15) and (12):

$$\frac{d}{dt}H \leq \nabla_z^\top H\dot{\tilde{z}} = [\tilde{y}^{e\top}\ \tilde{z}^\top]\, \frac{\partial^2 \sum S_j^e(y_j,z_j)}{\partial(y^e,z)^2}|_\star \begin{bmatrix}0\\ \dot{\tilde{z}}\end{bmatrix} =$$

$$= (\tilde{y}^{e\top}\mathcal{W}_{yz} + \tilde{z}^\top\mathcal{W}_{zz})\dot{\tilde{z}} = \tilde{w}_e^z(y^e, z, \dot{\tilde{z}}). \tag{18}$$

Equation (16) also determines an input-output relationship between input $\tilde{z}$ and output $\tilde{y}^e$ for $v \equiv 0$. Denote $\Sigma(s)$ the transfer matrix: $y^e(s) = \Sigma(s)z(s)$; being s the Laplace variable.

Under mild conditions, the dissipation inequality (18) implies a frequency-dependent inequality precisely stated in next proposition.

Proposition 2. If Assumption A1 holds and $\Sigma(j\omega) \in R\mathcal{L}_\infty$, then the transfer matrix $\Sigma(j\omega)$ satisfies

$$\begin{bmatrix} I \\ \Sigma(j\omega)\end{bmatrix}^* \Pi_d(j\omega) \begin{bmatrix} I \\ \Sigma(j\omega)\end{bmatrix} \geq 0 \ \ \forall w \in \mathbb{R} \tag{19}$$

$$\Pi_d(j\omega) := |h(j\omega)|^2 \begin{bmatrix} 0 & -j\omega\mathcal{W}_{yz}^\top \\ j\omega\mathcal{W}_{yz} & 0\end{bmatrix} \tag{20}$$

for all function $h(s)$ real rational stable and strictly proper.

Proof: The proof rests on a classical argument [3] that consists in considering a perfect sinusoidal input $\tilde{z}(t)$ with angular frequency ω and arbitrary spatial direction:

$$\tilde{z}(t) = Re(z_0 e^{j\omega t}), z_0 \in \mathcal{C}^{2m_e}.$$

Obtain the sinusoidal functions $\tilde{x}(t)$ and $\tilde{y}^e(t)$ such that the triple $(\tilde{z}, \tilde{x}, \tilde{y}^e)$ satisfies (16). Naturally, $\tilde{y}^e(t) = Re(\Sigma(j\omega)z_o e^{j\omega t})$ and the supply rate function $\tilde{w}_e^z(t)$ is given by

$$\tilde{w}_e^z(t) = (\tilde{y}^{e\top}\mathcal{W}_{yz} + \tilde{z}^\top\mathcal{W}_{zz})\dot{\tilde{z}} = \frac{d}{dt}\frac{1}{2}\tilde{z}^\top\mathcal{W}_{zz}\tilde{z} +$$

$$+Re(z_o^\top \Sigma(j\omega)^\top e^{j\omega t})\mathcal{W}_{yz} Re(j\omega z_o e^{j\omega t}).$$

If the dissipation inequality (18) is integrated in one period $T = \frac{2\Pi}{\omega}, \omega \neq 0$:

$$\int_{t_0}^{t_o+T} \tilde{w}_e^z(t)dt = \frac{T}{4} z_0^* j\omega[\Sigma(j\omega)^\star \mathcal{W}_{yz} - \mathcal{W}_{yz}^\top \Sigma(j\omega)]z_0 \geq 0$$

Thus, since z_0 is arbitrary, it is necessary that

$$\begin{bmatrix} I \\ \Sigma(j\omega)\end{bmatrix}^\star \begin{bmatrix} 0 & -j\omega\mathcal{W}_{yz}^\top \\ j\omega\mathcal{W}_{yz} & 0\end{bmatrix} \begin{bmatrix} I \\ \Sigma(j\omega)\end{bmatrix} \geq 0\,\forall\omega.$$

The inclusion of the case $\omega = 0$ is immediate since Π_d vanishes. The factor $|h(j\omega)|^2$ is incorporated in order to ensure the boundedness of Π_d for all ω. □□□

Remark 1. Notice that the dependence of Π_d on the system parameters is restricted to the power flow through the transmission lines linking with the adjacent systems, see equation (17).

Remark 2. Equations (19), (20) describe a frequency-weighted passivity condition for system Σ.

[3] In fact, Proposition 2 can be seen as a special case of the classical KYP lemma.

5. APPLICATIONS

Previous sections showed the existence of a multiplier Π_d that satisfies the condition (19) for the linearized models of a class of power system models. This Section is intended to examine the use of Π_d for stability analysis of power systems.

Following subsection briefly introduces the analysis of feedback system through the use of multipliers and the concept of Integral Quadratic Constraints (IQC) (Megretski and Rantzer 1997). The discussion is restricted to linear models.

In subsection 5.2, the application of this technique of analysis to a classical example will be discussed. It is shown that the presence of resistive losses in generators and transmission lines can be easily accommodated through the use of additional degrees of freedom in the multiplier Π.

5.1 Frequency-weighted stability analysis of linear feedback interconnections

Denote $G\star H$ the operator $(e,f)\to(v,w)$ defined by the standard feedback interconnection

$$\begin{cases} v &= Gw + f \\ w &= Hv + e \end{cases}. \tag{21}$$

G and H are two linear, time-invariant operators with transfer functions $G(s), H(s) \in \mathbf{R}\mathcal{L}_\infty^{2m_e\times 2m_e}$.

Define, to facilitate the notation, the function $\sigma : \mathbf{R}\mathcal{L}_\infty^{2m_e\times 2m_e} \times \mathbf{R}\mathcal{L}_\infty^{4m_e\times 4m_e}$ and matrix T:

$$\sigma(H,\Pi) := \begin{bmatrix} I_{2m_e} \\ H \end{bmatrix}^* \Pi \begin{bmatrix} I_{2m_e} \\ H \end{bmatrix}; T := \begin{bmatrix} 0 & I_{2m_e} \\ I_{2m_e} & 0 \end{bmatrix}.$$

The following proposition is a particular formulation of the IQC theorem, (Megretski and Rantzer 1997), specialized for our special case. The well-posedness of the interconnection is assumed.

Proposition 3. Let $G(s), H(s) \in \mathbf{R}\mathcal{H}_\infty^{2m_e\times 2m_e}$ such that the operator $G\star H$ is also stable. Assume the existence of a multiplier $\Pi(j\omega) \in \mathbf{R}\mathcal{L}_\infty^{4m_e\times 4m_e}$ such that

i. $\sigma(H(j\omega), \Pi(j\omega)) \geq 0 \ \forall\omega$
ii. there exists $\epsilon > 0$ such that

$$\sigma(G(j\omega), T\Pi(j\omega)T) \leq -\epsilon I, \ \forall\omega \in \mathbb{R}, \tag{22}$$

iii.
$$\begin{bmatrix} 0 \\ I \end{bmatrix}^* \Pi(j\omega) \begin{bmatrix} 0 \\ I \end{bmatrix} \leq 0, \ \forall\omega \in \mathbb{R}. \tag{23}$$

Then, the feedback interconnection $G\star\mathcal{H}$ is stable for all linear, time invariant, stable operator satisfying

$$\sigma(\mathcal{H}(j\omega), \Pi(j\omega)) \geq 0 \ \forall\omega \tag{24}$$

The proof is straightforward by writing $G\star\mathcal{H}$ as the interconnection of a nominal stable system $G\star H$ with the block $\Delta := \mathcal{H} - H$ and applying the IQC theorem.

Remark 3. Denote $\mathcal{S}_\Pi$ the set of operators $\mathcal{H} \in \mathbf{R}\mathcal{H}_\infty^{2m_e\times 2m_e}$ satisfying (24). It is easy to see that condition (23) implies the convexity of $\mathcal{S}_\Pi$.

5.2 Analysis of a two-area system

Consider the system depicted in Figure 1, (Kundur 1994) . The system is partitioned in two areas, linked by the transmission line $7-9$. The model has the same parameters than (Kundur 1994) (included the non-zero machine resistances and transfer conductances). The generators are modeled with constant excitation and the loads have constant power. This system is unstable in these conditions. The overall active power was scaled by 60% and a small positive damping coefficients D_i were introduced in all machines to ensure a marginal small signal stability.

Linear models for both areas were computed with the program PSAT (Milano 2005). Super-indexes α, β are used to discriminate the quantities of each area. Σ_α has the variables y_9 as inputs, and y_7 as outputs. The situation is the opposite for Σ_β.

Π_d is defined from the Hessian of the storage function associated to the line linking both areas, equations (17), (20). Thus, $\mathcal{W}_{yz}^\alpha = (\mathcal{W}_{yz}^\beta)^\top$ and

$$\Pi_d^\beta = |h|^2 \begin{bmatrix} 0 & -j\omega\mathcal{W}_{yz}^\alpha \\ j\omega(\mathcal{W}_{yz}^\alpha)^\top & 0 \end{bmatrix} = -T\Pi_d^\alpha T \Longrightarrow$$

$$\sigma(\Sigma^\beta, \Pi_d^\beta) = -\sigma(G^\beta, T\Pi_d^\alpha T). \tag{25}$$

Property (25) and the fact of that Π_d satisfies (23) constitute an interesting basis for the application of Proposition 3. However, the positiveness of condition (19) is not strict and Proposition 3 cannot directly be applied to Π_d^α. This difficulty can be easily solved. We can computationally obtain a suitable multiplier Π_1 of the form $\Pi_1 = \Pi_d^\alpha + \Pi_0$, with $\Pi_0 = \Pi_0^\top$ constant, such that conditions *i* to *iii* of Proposition 3 are met by systems Σ_α and Σ_β. Since Π_d^α was analytically computed a priori, the computation of Π_1 is a convex problem, see (Megretski and Rantzer 1997). The addition of term Π_0 is intended to impose the strict sign definition at $\omega = 0$ and to suitably accommodate the non-dissipative perturbations associated to resistive losses. It is worth to note that this was possible in spite of the very poorly damped modes of the systems, which exhibits damping factors smaller than 1.6%, see Figure 2. Figure 3 shows the eigenvalues of $\sigma(\Sigma^\alpha(j\omega), \Pi_1(j\omega))$ and $\sigma(\Sigma^\beta(j\omega), T\Pi_1(j\omega)T)$ for $h(s) = \frac{50}{s+50}$.

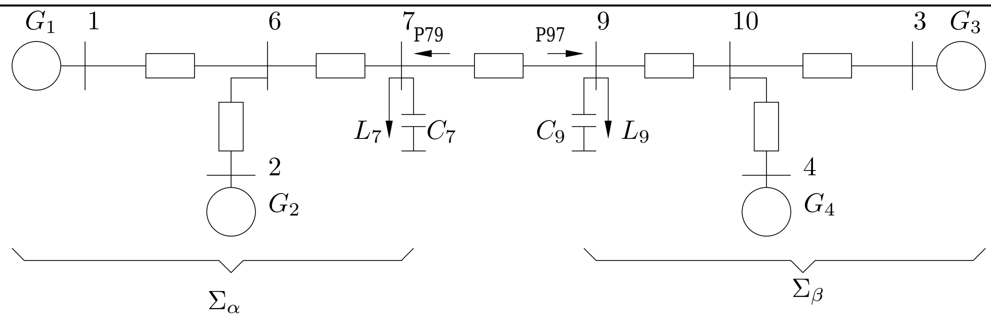

Fig. 1. Two-area system.

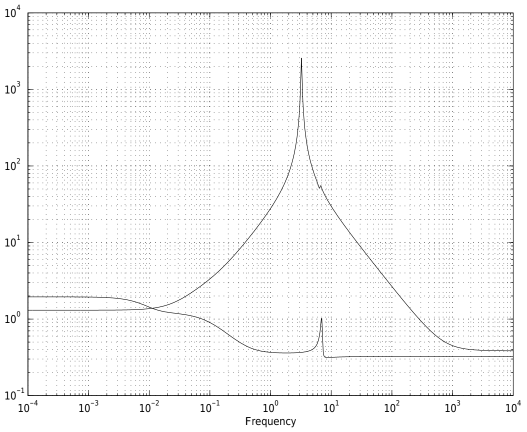

Fig. 2. Singular value plots for areas Σ_α and Σ_β.

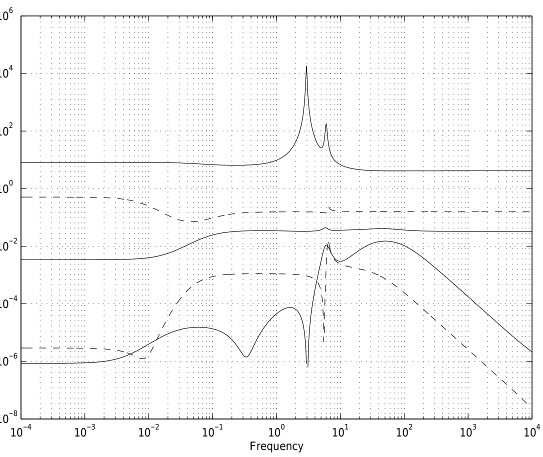

Fig. 3. Frequency condition for area α (continuous) and β (dashed).

In this way, convex sets $\mathcal{S}_{\Pi_1}$ and $\mathcal{S}_{\Pi_2}$ are determined for the respective areas Σ_α,Σ_β which characterizes a robustness condition. Excitation controllers can be designed to meet these constraints, at least in the frequency band where the electromechanical modes are significant.

6. CONCLUDING REMARKS

Dissipative properties of a class of structure-preserving power system models have been precisely stated and demonstrated. The use of this structural property for stability analysis of power system interconnections was illustrated through the use of frequency weighted multipliers and the IQC theorem to a classical two-areas example. Dissipativity provides us with a convex frequency domain condition which is met with independence of the value of a broad set of parameters. The exploitation of these structural properties for control purposes is currently under research.

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