A POWER–SHAPING SOLUTION FOR THE TRANSIENT STABILIZATION OF POWER SYSTEMS

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Abstract— The approach adopted in this paper for the problem of transient stabilization of multimachine power systems sees the entire network as the (structure–preserving) interconnection of the network components, described by well–known models. We first show that, under the assumptions of non-resistive loads and zero transfer conductances on the lines, these models admit a Hamiltonian description and are, then, shown to be cyclo–dissipative with storage functions akin to power. Our main contribution is the identification—in terms of a Linear Matrix Inequality (LMI)—of a class of power systems with resistive loads and leaky lines for which we can design generator excitation controllers that (locally) stabilize the desired equilibrium point. Since the LMI depends explicitly on the controller gain, a decentralized control action can be easily computed. The proposed technique is applied to a classical example.

Keywords— Power systems transient stability, energy shaping, dissipativity.

1 Introduction

Classical research on transient stabilization of power system has relied on the use of *reduced network* models that represent the system as an n–port. Based on these models several *excitation controllers* to enhance transient stability have been reported. The various nonlinear controller design techniques that have been considered include feedback linearization , damping injection, as well as energy–shaping. In the recent paper (Ortega et al., 2005) Interconnection and Damping Assignment Control is used to prove the existence of a nonlinear static state feedback law that ensures asymptotic stability of the operating point for a general n–machine system including transfer conductances. Unfortunately, an explicit expression of the controller can be derived only for the case $n \leq 3$.

In this paper, we follow the dissipativity-based approach first suggested in (Giusto, 2004). By doing this, we abandon the n–port view of the network and propose to leave its structure in its original form. Instrumental towards this end is the use of *structure–preserving models*, see (Varaiya et al., 1985) for a tutorial review. These models allow a more realistic treatment of the loads and it fosters a more natural view of the entire network as the *power–preserving interconnection* of its components.

We first show that, under classical simplifying assumptions, the different network components admit an implicit Port-Controlled Hamiltonian (PCH) description (Van der Schaft, 2000) and are, therefore, *cyclo–dissipative* with respect to some suitable defined supply rate (Willems, 1972).

Our main contribution is the LMI characterization of a class of power systems with nonlinear (so-called ZIP) loads and leaky lines for which we can design generator excitation (decentralized) controllers that locally stabilize the desired equilibrium point.

The structure of the paper is as follows. Section 2 presents the mathematical model of the various elements comprising the power system and give their PCH representation. In Section 3 we derive their cyclo–dissipativity properties under idealized assumptions. Section 4 contains the design of the power–shaping excitation controller. Section 5 includes the application of the proposed technique to a classical example. We wrap up the paper with some concluding remarks.

Notation All vectors in the paper are *column* vectors, even the gradient of a scalar function: $\nabla_x = \frac{\partial}{\partial x}$. For any function $f : \mathbb{R}^n \to \mathbb{R}$, we denote $\nabla_{z_i} f(z) := \frac{\partial f}{\partial z_i}(z)$ and, for vector functions $g : \mathbb{R}^n \to \mathbb{R}^n$, we define the Jacobian $\nabla g(z) := [\nabla g_1(z), \ldots, \nabla g_n(z)]^\top$.

2 Structure–Preserving Modelling: A PCH Representation

In this section we recall the well–known *structure–preserving* model for n–machine power systems reported in (Varaiya et al., 1985). Attached to each bus there is a machine or a load, and the buses are interconnected through transmission lines. Each machine and its corresponding bus, have an associated identifier $i \in J_M := \{1, .., n\}$. Each load and its associated bus are denoted by $l \in J_L := \{n+2,..,n+m+1\}$. Generic buses are denoted by identifiers j or k with $j,k \in J_B := \{1,..,n+m+1\}$. Transmission lines are identified by the double subindex $jk \in \Omega \subset J_B \times J_B$, indicating that the line jk connects the bus $j \in J_B$ with the bus $k \in J_B$—the set avoids obvious repetitions, e.g., if $jk \in \Omega$ then $kj \notin \Omega$.[1]

All elements share as port variables the angle θ_j and the magnitude V_j of the bus voltage phasor:

$$y_j := \begin{bmatrix} \theta_j & V_j \end{bmatrix}^T \in \mathbb{R}^2. \quad (1)$$

We also consider the presence of an *infinite bus* represented by $y_{n+1} = \begin{bmatrix} \theta_{n+1} & V_{n+1} \end{bmatrix}^T = \begin{bmatrix} 0 & \bar{V}_{n+1} \end{bmatrix}^T$, where $\bar{V}_{n+1}$ is constant. Associated to each bus are the active and reactive powers entering the machine, the load or the transmission lines, that will be denoted

$$u_i^M = \begin{bmatrix} P_i^M \\ Q_i^M \end{bmatrix}, u_l^L = \begin{bmatrix} P_l^L \\ Q_l^L \end{bmatrix}, u_{jk} = \begin{bmatrix} P_{jk} \\ Q_{jk} \end{bmatrix} \quad (2)$$

respectively. We take active and reactive powers as positive when entering their corresponding component.

2.1 Synchronous Machines Model

Each synchronous machine is described by a set of third order Differential Algebraic Equations (DAEs), (Varaiya et al., 1985):

$$\begin{array}{rcl} \dot{\delta}_i &=& \omega_i \\ M_i\dot{\omega}_i &=& P_{m_i} - D_i\omega_i - P_i^M \\ \tau_i\dot{E}_i &=& -\frac{x_{d_i}}{x'_{d_i}}E_i + \frac{x_{d_i}-x'_{d_i}}{x'_{d_i}}V_i\cos(\delta_i-\theta_i) + E_{F_i} \\ P_i^M &=& -\frac{1}{x'_{d_i}}E_iV_i\sin(\delta_i-\theta_i) - \\ && \frac{x'_{d_i}-x_{q_i}}{2x_{q_i}x'_{d_i}}V_i^2\sin(2(\delta_i-\theta_i)) \\ Q_i^M &=& \frac{x'_{d_i}+x_{q_i}}{2x_{q_i}x'_{d_i}}V_i^2 - \frac{1}{x'_{d_i}}E_iV_i\cos(\delta_i-\theta_i) - \\ && -\frac{x'_{d_i}-x_{q_i}}{2x_{q_i}x'_{d_i}}V_i^2\cos(2(\delta_i-\theta_i)) \end{array} \quad (3)$$

where the *state variables* $x_i := \text{col}(\delta_i, \omega_i, E_i) \in \mathbb{R}^3$ denote the rotor angle, the rotor speed and the quadrature axis internal e.m.f., respectively, and E_{F_i} is the field voltage. The parameters are denoted as in (Varaiya et al., 1985), and they are fairly standard. We will make the physically reasonable assumptions $D_i > 0$, $x_{d_i} - x'_{d_i} > 0$.

For convenience, we will separate the field voltage in two terms, $E_{F_i} = E^\star_{F_i} + v_i$, the first one is constant and fixes the equilibrium value, while the second one is the *control action*. Also we define the constants

$$Y_{2i} \triangleq \frac{x'_{d_i} - x_{q_i}}{2x_{q_i}x'_{d_i}},\ Y_{E_i} \triangleq \frac{x_{d_i}}{x'_{d_i}(x_{d_i} - x'_{d_i})},$$

$$Y_{F_i} \triangleq \frac{1}{x_{d_i} - x'_{d_i}},\ Y_{Vi} \triangleq \frac{x'_{d_i} + x_{q_i}}{2x_{q_i}x'_{d_i}},$$

$$J_i \triangleq \begin{bmatrix} 0 & \frac{1}{M_i} & 0 \\ -\frac{1}{M_i} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ R_i \triangleq \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{D_i}{M_i^2} & 0 \\ 0 & 0 & \frac{1}{\tau_i Y_{F_i}} \end{bmatrix},$$

$$B_{v_i} \triangleq \begin{bmatrix} 0 & 0 & \frac{1}{\tau_i} \end{bmatrix}^T, B_u(y_i) \triangleq \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{V_i} \end{bmatrix}. \quad (4)$$

We have the following simple fact, whose proof follows from (1), (2) by direct substitution.

Fact 1 *The synchronous machine model (3) defines an operator $\Sigma_i^M : (u_i^M, y_i)$ described by the implicit PCH system*

$$\left\{ \begin{array}{rcl} \dot{x}_i &=& (J_i - R_i)\nabla_{x_i}S_i^M(x_i,y_i) + B_{v_i}v_i \\ 0 &=& -\nabla_{y_i}S_i^M(x_i,y_i) + B_u(y_i)u_i^M \end{array} \right. \quad (5)$$

with storage function[2] $S_i^M : \mathbb{R}^3 \times \mathbb{R}^2 \to \mathbb{R}$,

$$S_i^M(x_i,y_i) \triangleq \frac{1}{2}M_i\omega_i^2 - P_{mi}\delta_i - \frac{E_iV_i}{x'_{d_i}}\cos(\theta_i - \delta_i) -$$

$$-\frac{Y_{2i}}{2}V_i^2\cos 2(\theta_i - \delta_i) + \frac{Y_{E_i}}{2}E_i^2 - Y_{F_i}E^\star_{F_i}E_i + \frac{Y_{Vi}}{2}V_i^2. \quad (6)$$

Denoting the DAE model (5) as an "implicit PCH system" is done with some abuse of notation. See (Van der Schaft, 2000) for a precise definition.

2.2 Loads Model

Loads are described by the standard ZIP model (Kundur, 1994)

$$\begin{array}{rcl} P_l^L &=& P_{Z_l}V_l^2 + P_{I_l}V_l + P_{0_l} \\ Q_l^L &=& Q_{Z_l}V_l^2 + Q_{I_l}V_l + Q_{0_l} \end{array}, \quad (7)$$

Fact 2 *The ZIP load model (7) defines an operator $\Sigma_l^L : (u_l^L, y_l)$ described by the implicit (memory–less) PCH system*

$$0 = -\nabla_{y_l}S_l^L(y_l) + B_u(y_l)u_l^L - \Psi_l^L(y_l) \quad (8)$$

with storage function $S_l^L : \mathbb{R}^2 \to \mathbb{R}$,

$$S_l^L(y_l) \triangleq P_{0_l}\theta_l + \frac{Q_{Z_l}}{2}V_l^2 + Q_{I_l}V_l + Q_{0_l}\ln(V_l), \quad (9)$$

where

$$\Psi_l^L(y_l) \triangleq \begin{bmatrix} P_{Z_l}V_l^2 + P_{I_l}V_l & 0 \end{bmatrix}^T.$$

[1] To avoid cluttering we reserve the subindex i to variables ranging in the index set J_M without explicit reference to it. The same for indexes $l \in J_L$ and $j,k \in J_B$.

[2] The use of the name "storage function" will be justified in the next section.

2.3 Transmission Lines Model

Transmission lines are modelled with the standard lumped Π circuit (Kundur, 1994):

$$\begin{array}{rcl} P_{jk} & = & G_{jk}V_j^2 - B_{jk}V_jV_k\sin(\theta_j-\theta_k) \\ & & -G_{jk}V_jV_k\cos(\theta_j-\theta_k) \\ Q_{jk} & = & -(B_{jk}+B^c_{jk})V_j^2 + B_{jk}V_jV_k\cos(\theta_j-\theta_k) \\ & & -G_{jk}V_jV_k\sin(\theta_j-\theta_k) \end{array} \tag{10}$$

where $jk \in \Omega$. The active and reactive power entering at node k, P_{kj}, Q_{kj} can be obtained by a simple change of indexes.

Fact 3 *The transmission line model (10) defines an operator $\Sigma_{jk} : (u_{jk}, u_{kj}, y_j, y_k)$ described by an implicit (memory–less) PCH system*

$$\begin{array}{l} 0 = -\nabla_{y_j} S_{jk}(y_j, y_k) + B_u(y_j)u_{jk} - \Psi_{jk}(y_j, y_k) \\ 0 = -\nabla_{y_k} S_{jk}(y_j, y_k) + B_u(y_k)u_{kj} - \Psi_{jk}(y_k, y_j) \end{array} \tag{11}$$

with storage function $S_{jk} : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$,

$$S_{jk}(y_j, y_k) \triangleq -\frac{(B_{jk}+B^c_{jk})}{2}(V_j^2+V_k^2) + B_{jk}V_jV_k\cos(\theta_j-\theta_k), \tag{12}$$

where

$$\Psi_{jk}(y_j, y_k) \triangleq -G_{jk}\left[\begin{array}{c} -V_j^2 + V_jV_k\cos(\theta_j-\theta_k) \\ V_k\sin(\theta_j-\theta_k) \end{array}\right].$$

Remark 1 The terms Ψ^L_l and Ψ_{jk} play the role of a disturbance that, introducing sign–indefinite terms in the derivative of the storage function, hampers the establishment of the required cyclo–dissipativity property of the respective subsystems Σ^L_l and Σ_{jk}. Notice that $\Psi^L_l = 0$ if the active power can be modeled with constant impedance. Notice also that $\Psi_{jk} = \Psi_{jk} = 0$ if and only if the line conductance $G_{jk} = 0$.

2.4 Bus Equations

At each bus, we have

$$\begin{array}{rcl} 0 & = & \sum_{k\in\Omega_j} P_{jk} + P^M_j + P^L_j \\ 0 & = & \sum_{k\in\Omega_j} Q_{jk} + Q^M_j + Q^L_j \end{array} \tag{13}$$

where $\Omega_j := \{k \in J_B; | \ \exists \ jk \in \Omega\}$, that is, Ω_j is the set of buses that are linked to the bus j through some transmission line.

3 Cyclo–Dissipativity Properties

We adopt in the paper the dissipativity framework proposed in (Willems, 1972), see also (Hill and Moylan, 1980; Van der Schaft, 2000). To establish our results a slight variation of the classical formulation is needed since the supply rate functions that we consider are functions, not only of the port variables (u, y), but also of $\dot{y}$. Another difference with respect to the standard dissipativity framework is that our systems are characterized by DAEs, instead of ODEs and readout maps. But this difference is unessential as we suppose that the algebraic constraints can be solved for the "link variables" (Hill and Mareels, 1990).

Definition 1 *Consider a dynamical system $\Sigma : (u, y)$ represented by the DAEs*

$$\left\{\begin{array}{rcl} \dot{x} & = & F(x, y) \\ 0 & = & G(x, y, u) \end{array}\right. \tag{14}$$

where $x \in \mathbb{R}^n$ is the state and $(u, y) \in \mathbb{R}^p \times \mathbb{R}^p$ are the port variables. Let $w : \mathbb{R}^p \times \mathbb{R}^p \times \mathbb{R}^p \to \mathbb{R}$ be locally integrable along trajectories of Σ. We say that Σ is cyclo–dissipative *with respect to the supply rate $w(u, y, \dot{y})$ if and only if there exists a differentiable function $S : \mathbb{R}^n \to \mathbb{R}$, called storage function, such that, $\forall t_2 \geq t_1$*

$$S(x(t_2)) - S(x(t_1)) \leq \int_{t_1}^{t_2} w(u(t), y(t), \dot{y}(t))dt.$$

The distinction between cyclo–dissipative and dissipative systems is the non–negativity of the storage function. See (Hill and Moylan, 1980) for a deep discussion about this topic.

In this section it will be shown that, *in the absence* of control action and disturbance terms, each device of our power systems model is cyclo-dissipative with respect to the supply rate function $w : \mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$:

$$w(u, y_j, \dot{y}_j) \triangleq u^\top B_u^\top(y_j)\dot{y}_j = P\dot{\theta} + \frac{Q}{V_j}\dot{V}_j, \tag{15}$$

with the u defined in (2) for each of the elements.

It is interesting to note that in (Ortega et al., 2003) we have identified classes of nonlinear RLC circuits that are dissipative with respect to supply rates of the form $\dot{V}^\top I$ or $\dot{I}^\top V$—which is in the spirit of the results that follow. This antecedent and the fact that several terms of the storage function can be interpreted in terms of reactive power motivated the paper's title.

To establish the cyclo–dissipativity properties we make the following (temporary) assumption.

Assumption A1. The field voltage of the synchronous machines are constant: $E_{F_i} = E^\star_{F_i}$; the ZIP model for the loads fulfills $P_{Z_l} = 0$, $P_{I_l} = 0$; and the transfer conductances are zero: $G_{jk} = 0$, $\forall \ jk \in \Omega$.

The following two propositions can be demonstrated from the PCH-like models derived in previous section. Proofs are omitted by brevity.

Proposition 1 *If Assumption A1 holds the synchronous machine model Σ^M_i, the loads model Σ^L_l and the transmission lines model Σ_{jk}, equations*

(5), (8), (11) are cyclo–dissipative with respect to their respective supply rate functions $w(u_i, y_i, \dot{y}_i)$. More precisely,

$$\frac{dS_i^M(x_i, y_i)}{dt} \leq w(u_i^M, y_i, \dot{y}_i),$$

$$\frac{dS_l^L(y_l)}{dt} \leq w(u_l^L, y_l, \dot{y}_l),$$

$$\frac{dS_{jk}(y_j, y_k)}{dt} = w(u_{jk}, y_j, \dot{y}_j) + w(u_{kj}, y_k, \dot{y}_k).$$

Finally, the bus equations (13) may be written

$$0 = \sum_{k \in \Omega_j} w(u_{jk}, y_j, \dot{y}_j) + w(u_j^M, y_j, \dot{y}_j) + w(u_j^L, y_j, \dot{y}_j)$$

An immediate corollary of Proposition 1 is the following well–known result, (Varaiya et al., 1985).

Proposition 2 *Let $x := col(x_1, \dots, x_n)$, $y := col(y_1, \dots, y_n, y_{n+2}, \dots, y_{(n+m+1)})$ and define*

$$S_0(x, y) := \sum_i S_i^M(x_i) + \sum_l S_l^L(y_l) + \sum_{jk \in \Omega} S_{jk}(y_j, y_k).$$

If Assumption A1 is satisfied then,

$$\frac{dS_0(x, y)}{dt} = -\sum_i [D_i \omega_i^2 + Y_{F_i} \tau_i \dot{E}_i^2] \leq 0.$$

4 Main Stabilization Result

It is clear that, when the simplifying Assumption A1 is violated, the cyclo–dissipativity properties are lost and Proposition 2 is not verified anymore. To the best of our knowledge, no global energy function is known for power system models in such conditions. The main message of this paper is that, with the addition of field control action v_i, it is possible to assign to the *non–ideal linearized* system a storage function that, under some conditions captured by an LMI, achieves a minimum at the desired equilibrium point hence qualifies as a *bona fide* Lyapunov function to assess stability of the equilibrium. Towards this end, it is convenient to group all the algebraic constraints of (5), (8), (11), (13) into the function $g : \mathbb{R}^{3n} \times \mathbb{R}^{2(n+m)} \to \mathbb{R}^{2(n+m)}$:

$$g(x, y) := \nabla_y S_0(x, y) + \Psi(y), \tag{16}$$

where $\Psi : \mathbb{R}^{2(n+m)} \to \mathbb{R}^{2(n+m)}$ is given by $\Psi(y) := \text{col}(\Psi_j(y))$ with

$$\Psi_j(y) = \sum_{k \in \Omega_j} \Psi_{jk}(y_j, y_k) + \Psi_j^L(y_j).$$

A compact description of the system is obtained defining $S : \mathbb{R}^{3n} \times \mathbb{R}^{2(n+m)} \to \mathbb{R}$:

$$S(x, y) := S_0(x, y) + y^\top \Psi(y^\star), \tag{17}$$

with $y^\star \in \mathbb{R}^{2(n+m)}$ the equilibrium value of the link variables y, and the block–diagonal matrices

$$J := \text{diag}\{J_i\},\ R := \text{diag}\{R_i\},\ B_v := \text{diag}\{B_{v_i}\}.$$

This allows us to rewrite the overall system as

$$\left\{ \begin{array}{rcl} \dot{x} & = & (J - R)\nabla_x S(x, y) + B_v v \\ 0 & = & -\nabla_y S(x, y) - \Psi(y) + \Psi(y^\star). \end{array} \right. \tag{18}$$

Notice the presence of the control action $v = col(v_i)$ and the "non-dissipative" terms $-\Psi(y) + \Psi(y^\star)$.

In order to formulate the control problem we need to define the set $\mathcal{D}_g \in \mathbb{R}^{3n} \times \mathbb{R}^{2(n+m)}$:

$$\mathcal{D}_g \triangleq \{(x, y) | \nabla_y S(x, y) + \Psi(y) - \Psi(y^\star) = 0\}.$$

Assumption A4. There exists an isolated *open loop equilibrium* $(x^\star, y^\star)$ of the system 18.

Assumption A5. $\nabla_y g(x^*, y^*)$ is non–singular.

As a consequence of Assumption A5 and the Implicit Function Theorem there exists, locally around $(x^\star, y^\star)$, a function $\hat{y}(x)$ such that $g(x, \hat{y}(x)) = 0$. Consequently, we can write

$$\dot{y} = M(x, y)\dot{x} \tag{19}$$

with $M(x, y) \in \mathbb{R}^{2(n+m) \times 3n}$.

Energy Shaping Problem: Consider the system (18) satisfying Assumptions A4 and A5. Find a control law $v = \hat{v}(x, y)$, and a function $S_a : \mathbb{R}^{3n} \times \mathbb{R}^{2(n+m)} \to \mathbb{R}$, such that, for some set $\mathcal{D} \subset \mathbb{R}^{3n} \times \mathbb{R}^{2(n+m)}$, with $(x^\star, y^\star) \in \mathcal{D}$,

$$S_d(x, y) := S(x, y) + S_a(x, y) \tag{20}$$

satisfies

C1. $(x^\star, y^\star) = \arg\min_{(x,y) \in \mathcal{D} \cap \mathcal{D}_g} S_d(x, y)$,

C2. $\frac{dS_d(x,y)}{dt} \leq 0, \quad \forall (x, y) \in \mathcal{D} \cap \mathcal{D}_g$.

Consequently, $(x^\star, y^\star)$ is a stable closed–loop equilibrium with Lyapunov function $S_d(x, y)$.

We propose the control action

$$v = K\nabla_x S(x, y),\ \ K \in \mathbb{R}^{n \times 3n} \tag{21}$$

with K to be determined. Let us compute

$$\frac{dS_d}{dt} = \frac{dS_a}{dt} + \nabla_x^\top S\dot{x} + \nabla_y^\top S\dot{y} = \frac{dS_a}{dt} +$$

$$\left[\begin{array}{c} \nabla_x S \\ \nabla_y S \end{array} \right]^\top \left[\begin{array}{c} J - R + B_v K \\ M(x, y)(J - R) + M(x, y)B_v K \end{array} \right] \nabla_x S.$$

where we have used (19). We will investigate the negativity of this function around the equilibrium point, when (21) is replaced by its linear approximation and we choose

$$S_a(\delta, \omega) = \frac{1}{2} \left[\begin{array}{c} \delta - \delta^\star \\ \omega \end{array} \right]^\top P \left[\begin{array}{c} \delta - \delta^\star \\ \omega \end{array} \right] \tag{22}$$

where $P = P^T$, is a matrix to be computed.

We show that the existence of matrices P and K ensuring conditions C1 and C2 above are satisfied, can be recasted as a LMI feasibility problem.

We define the matrices

$$M^\star := M(x^\star, y^\star);\ K_\psi := \frac{\partial \Psi}{\partial y}(x^\star, y^\star)$$

$$\mathcal{F} := \frac{\partial^2 S}{\partial x^2}(x^\star, y^\star) + \frac{\partial^2 S}{\partial x \partial y}(x^\star, y^\star) M^\star,$$

with which we obtain the linear approximations

$$\tilde{y} \simeq M^\star \tilde{x},\ \nabla_x S \simeq \mathcal{F}\tilde{x},\ \nabla_y S) \simeq -K_\psi M^\star \tilde{x},$$

where $\tilde{(\cdot)}$ denotes incremental variables. Now, in view of the definition of $\tilde{x}$, there exists a constant matrix $\mathcal{U}$ such that $[\tilde{\delta}^T\ \ \tilde{\omega}^T]^T = \mathcal{U}\tilde{x}$ and thus

$$\frac{dS_a}{dt} = \tilde{x}^\top \mathcal{U}^\top P \mathcal{U}\dot{\tilde{x}} = \tilde{x}^\top \mathcal{U}^\top P \mathcal{U}(J - R + B_v K)\mathcal{F}\tilde{x}$$

$$= \tilde{x}^\top \mathcal{U}^\top P \mathcal{U}(J - R)\mathcal{F}\tilde{x},$$

since $\mathcal{U}B_v = 0$. We also define the (fixed) matrices

$$\mathcal{W} := -M^\top K_\Psi^\top M(J-R)\mathcal{F},\ \mathcal{V} := \begin{bmatrix} B_v^\top M^\top K_\Psi M \\ \mathcal{F} \end{bmatrix},$$

$$\mathcal{Z} := \begin{bmatrix} \mathcal{U} \\ \mathcal{U}(J - R)\mathcal{F} \end{bmatrix}.$$

Using this notation it can be written

$$\frac{dS_d}{dt} \simeq \frac{1}{2}\tilde{x}^\top Q(P, K)\tilde{x},$$

$$Q(P, K) := \mathcal{F}^\top[-2R + B_v K + K^\top B_v^\top]\mathcal{F} + \mathcal{W} + \mathcal{W}^\top$$
$$-\mathcal{V}^\top \begin{bmatrix} 0_n & K \\ K^\top & 0_{3n} \end{bmatrix} \mathcal{V} + \mathcal{Z}^\top \begin{bmatrix} 0_{2n} & P \\ P & 0_{2n} \end{bmatrix} \mathcal{Z}. \quad (23)$$

Let us check now positivity of the low order approximation of $S_d(x, y)$. By construction

$$\nabla_x S_d(x^\star, y^\star) = 0,\ \ \nabla_y S_d(x^\star, y^\star) = 0.$$

On the other hand, the Hessian evaluated at the equilibrium, that is, $\frac{\partial^2 S_d(x, M^\star x)}{\partial x^2}|_{x^\star}$, is also a linear function of P that we denote $\mathcal{H}(P)$

$$\frac{\partial^2 S(x, M^\star x)}{\partial x^2}|_{x^\star} + \mathcal{U}^\top P \mathcal{U} =: \mathcal{H}(P). \quad (24)$$

We are in position to present our main stabilization result whose proof follows from the calculations above and Lyapunov's direct method.

Proposition 3 *Assume the LMI*

$$Q(P, K) < 0, \quad \mathcal{H}(P) > 0,$$

is feasible for some $P = P^\top$ *and* K*, where* $Q(P, K)$ *and* $\mathcal{H}(P)$ *are defined by (23) and (24), respectively. Then, a solution to the energy shaping problem is provided by the control* $v = K\mathcal{F}\tilde{x}$ *and function* $S_a(\delta, \omega)$ *given by (22).*

Since the computation of a stabilizing control law is formulated as an LMI on P and the controller matrix K, several interesting performance requirements can be considered. We mention only two: decentralized control action and pole allocation in a prescribed region in the complex plane.

Decentralized control

The implementation of the control law (21) requires full information of the overall system in each machine, which is not realistic in industrial applications. If we wish enforce the use of only local information x_i, y_i in the control law v_i we must restrict our matrix K to be block diagonal. This restriction can be easily added to Proposition 3. Details are omitted by brevity.

Pole placement

There is a vast set of convex regions which admit an LMI formulation (Chilali and Gahinet, 1996). The requirement that all eigenvalues of the closed loop system belong to the region defined by

$$R(\alpha, r) := \{\lambda \in C | \mathcal{R}e(\lambda) < -\alpha, |\lambda| < r\} \quad (25)$$

is equivalent to the feasibility of a LMI that can be easily considered with Proposition 3.

5 A Benchmark Simulation Example

We consider here the classical 3-machines, 9-buses system considered in (Sauer and Pai, 1998) and depicted in Figure 1. We assume that the active components of the loads have constant power characteristics and the reactive components have constant impedance. Since this system has no infinite bus, the procedure we described in previous sections had to be slightly modified in order to cope with the non-isolated equilibrium point. The details are fairly standard, see (Willems, 1974), and they are omitted. Computations were done with the software package PSAT (Milano, 2005). The LMI condition in Proposition 3 was the tool

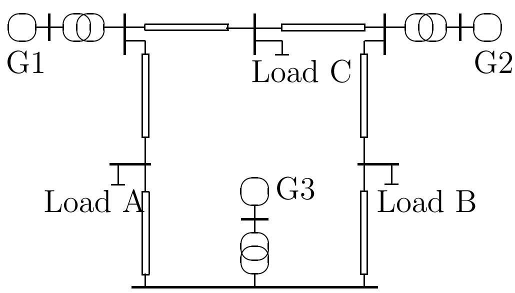

Figure 1: *Three-machines, nine-buses system.*

for the synthesis of the controller gain K. The parameters of the region $R(\alpha, r)$ were chosen $\alpha = 0.008$ and $r = 25$. An additional constraint on the controller gain, namely $\|K\| < 10$, was also imposed. A decentralized control law was also computed. The pole patterns of the open and closed loop systems are given in Table 1.

A zero eigenvalue, associated with the manifold of equilibrium points, is present in all cases.

Table 1: Pole patterns.

Open loop	full control	dec. control
$-0.29 \pm j11.5$	$-2.23 \pm j10.9$	$-0.39 \pm j12.4$
$-0.13 \pm j8.2$	$-1.39 \pm j8.0$	$-0.24 \pm j8.6$
-0.19	-11.54	-0.18
-0.16	-1.71	-0.17
-0.01	-0.17	-0.04
-0.01	-0.01	-0.01

Figure 2 depicts the transient response of the linearized open and closed–loop systems to an initial condition $\omega_1(0) = 0.05$. Closed–loop responses for both full and decentralized controller are shown. As it can be seen, the full controller is able to significantly improve the system's stability and provide damping. The damping factors of the electromechanical modes were increased approximately 8 and 11 times. The decentralized controller also increased the damping of the electromechanical modes by 25 and 75 percent. However its performance is relatively modest in comparison with the full controller. This is not unexpected because the decentralized scheme, albeit more realistic, uses only local variables for each machine.

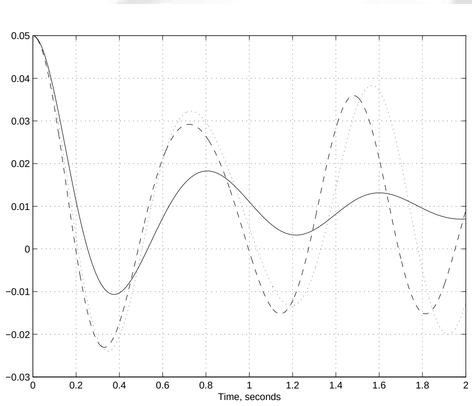

Figure 2: *Speed* ω_1. *Open loop (dotted), decentralized control (dashed), full control (continuous).*

6 Conclusions

We have provided a solution to the transient stabilization problem structure–preserving models of power systems. The analysis is based on the linear approximation and shows that a linear state feedback controller ensures stability of the equilibrium provided an LMI is feasible. The LMI is given in terms of the controller gain and the weighting matrix of the added energy function. The usefulness of the technique for synthesis was illustrated with its application to a classical example.

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