XII JORNADAS DE INVESTIGACIÓN
16, 17 y 18 de SETIEMBRE 2013
FACULTAD DE CIENCIAS SOCIALES
UNIVERSIDAD DE LA REPÚBLICA

DERECHOS HUMANOS EN EL URUGUAY DEL SIGLO XXI
LIBERTADES
DIVERSIDAD
JUSTICIA

Seguridad Social y retiro en Uruguay

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Agosto 2013

Abstract

We estimate structural life-cycle models for retirement behavior using social security work history records of public employees in Uruguay. We analize two alternative models, with and without life insurance. The estimated coefficient of relative risk aversion is around 1.7, indicating that agents are moderately risk-averse. The estimated discount rate is about 8 percent per annum. The probability of retirement is greater for individuals who have lower propensity to contribute and increases for women and older people. Simulations show a very low impact of moderate policy changes on the retirement age.

Keywords: Social Security, Retirement, Structural model, Maximum Pseudo-likelihood

JEL: H55, J14, J26, D91

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[*]This work is part of project FCE2007_293, financed by ANII. Ianina Rossi decisively collaborated with this paper. Nevertheless, the authors take full responsibility for its contents.

[†]Trabajo presentado en las XII Jornadas de Investigación de la Facultad de Ciencias Sociales, UdelaR, Montevideo, 16-18 de setiembre de 2013)

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1 Introduction

The decline in the age of retirement of men that has taken place in several OECD countries in the last decades attracted considerable attention among policymakers and scholars. Basically two explanations can be found in the literature, namely the increase in wealth that induces individuals to consume more leisure (Burtless and Quinn, 2000) and the implicit incentives in social security (Gruber and Wise, 1999, 2004). This trend, combined with population aging represents a significant challenge for the sustainability of many pension systems.

There has not been a similar decline in the retirement age of Uruguayan workers in the last two decades, with some increase in the case of women (Alvarez et al., 2010). In turn, in 1995 the Parliament passed a law that introduced individual accounts and changed several key parameters in the pay-as-you-go (PAYG) pillar. The law included several measures specifically geared to increasing retirement ages. Different individuals received different treatments, depending on sex, cohort, and income level, so there is much diversity in terms of retirement incentives (Alvarez et al., 2012). This makes Uruguay an interesting case to analyze the impact of pension rules on retirement in a middle income country.

As part of the reform process, the social security administration made a considerable effort to improve work history records. A sample of the work history records was made available to the research community and have been used to analyze the impact of social security reforms on individual behavior and the macroeconomy (Forteza et al., 2009; Bucheli et al., 2010; Alvarez et al., 2010, 2012). Existing studies are based on the estimation of reduced form models, though, and are not immune to the Lucas critique: the policy response parameters being estimated are not necessarily invariant to policy changes. In this paper, using a structural econometric model, we estimate more fundamental parameters -like the coefficient of risk aversion and the discount rate- that are not expected to be affected by policy changes.

We estimate a life cycle retirement and savings decision model, a la Modigliani and Brumberg (1980), in a sample of public employees observed between 1996 and 2004. We assume selfish (or non altruistic) individuals in the sense that no utility is attached to bequests or to the utility of their heirs (Yaari, 1965 and Leung, 1994, 2000). The analysis of retirement decision is similar to the analysis by Crawford and Lilien (1981) and Fabel (1994). We develop two models, with and without life insurance. The model without life insurance is based on the life cycle model with uncertain longevity and credit rationing recently estimated for the Spanish economy by Jiménez-Martín and Sánchez-Martín (2007). The model with life insurance is inspired in Yaari (1965). We estimate the model using maximum Pseudo-likelihood and use the estimated structural preference parameters to simulate several policy changes.

The results are similar in the two versions of the model, so the assumption about life insurance does not seem to have material impact on the results. The estimations of the parameters of preferences are quite precise. The estimated coefficient of risk aversion is about 1.7, indicating moderate risk aversion, and the rate of discount is about 8 percent. Simulations of policy changes do not show a significant impact of social security rules on retirement.

The paper is organized as follows. In section 2 we briefly describe the main social security program of Uruguay, administered by the *Banco de Previsión Social* (BPS). In section 3 we explain the theoretical model, leaving the algebra to the appendix. In section 4 we discuss the strategy for the empirical identification of the model and the estimation method. We describe the database in section 5. In section 6, we present the estimation results and in section 7 the simulations. In section 8, we present some robustness checks. Section 9 concludes.

2 The BPS old-age program

Since the late seventies, two main norms regulate the BPS pension program: the so called *Acto Institutional* 9 (Institutional Act 9), passed in 1979, and law 16.713, passed in September 1995. It was a pay-as-you-go-defined-benefit (PAYG-DB) program under the *Acto Institucional* 9, and a mixed program -with a PAYG-DB and a savings accounts pillars- under law 16.713.

Among other things, the 1995 reform modified pension eligibility conditions. Before the reform, individuals were required to be no less than 55 and 60 years old, women and men, respectively, and to have contributed no less than 30 years. Law 16.713 tightened some of these conditions: 35 years of service and 60 years of age, both sexes, were required to access an ordinary pension. There was a transition, so that the minimum age of retirement for women was gradually adjusted from 55 in 1996 to 60 from 2003 on. In 2008 a new law reduced the required years of service back to 30.

In the PAYG pillar, the initial benefit is computed multiplying the replacement rate (RR) and the average contribution earnings (ACE). In the *Acto* 9, the ACE is computed as the average indexed monthly labor earnings in the last three years before retirement. Law 16.713 extended the period to the last ten or the "best" twenty years - i.e. the twenty years with highest indexed earnings- before retirement (with an upper bound equal to 1.05 times the average indexed monthly earnings in the "best" twenty years). The index used is the average wage index.

The replacement rates range from 60 to 80 percent in the *Acto* 9 norms, depending on years of service and retirement age. The maximum is obtained with 40 years of contribution and 70 of age. Law 16.713 widened the range: 50 to 82.5 percent. More recently, in 2008, law 18.395 widened the range even further: 45 to 82.5 percent, also depending on years of service and age at retirement.

There is a minimum and a maximum pension. With *Acto* 9 norms, the minimum pension was 0.85 times the national minimum salary and the maximum pension was 7 national minimum salaries (or 15 national minimum salaries if the individual had contributed for two or more different jobs). Law 16.713 also set a minimum pension in the PAYG-DB pillar, which is currently about 315 US dollars (approximately 0.75 times de minimum salary). Unlike *Acto* 9, law 16.713 sets a ceiling on insured wages and hence on contributions. This in turn determines a maximum pension equal to the insured wage ceiling times the maximum replacement rate.

3 The retirement model

Consider individuals who are uncertain about their longevity. The probability as of t_0 (first period of working life) of surviving to age t is $S(t)$ and there is a maximum lifetime $\overline{T}$ such that $S(\overline{T}) = 0$. Individuals do not value bequests, and only derive utility from their consumption $c(t)$ and leisure $l(t)$. Utility is assumed to be separable in consumption and leisure and across time. Expected utility is:

$$E\left[U(c,l)\right] = \int_{t_0}^{\overline{T}} S(t)e^{-\delta(t-t_0)}\left[\frac{c(t)^{1-\eta}}{1-\eta} + v(l(t),x(t))\right]dt \qquad (1)$$

where c and l represent the paths of consumption and leisure, respectively, between t_o and $\overline{T}$, δ is the discount rate, $c(t)$ is consumption at t, η is the coefficient of relative risk aversion, and $v()$ is an instantaneous utility function, increasing in $l(t)$, and may also depend on other individual characteristics which are included in $x(t)$.

In our model, the only working decision is about the age of retirement. Thus, we assume there is bundling in leisure, so $l(t)$ is a binary variable.[1] We normalize $l(t)$ to be equal to 0 and 1, when the individual is working and not working, respectively. Also we do not analyze work interruptions, so we focus on leisure paths such that individuals are active before retirement.

[1] The assumption has a long tradition in the analysis of labor supply. See, among others, Rogerson, 1988, Rogerson and Wallenius, 2007, Ljungqvist and Sargent, 2011.

Therefore, individuals work choice is only about when to retire completely from the labor force.

We consider two alternative environments. One in which individuals cannot insure against uncertain longevity and cannot borrow from pension income and another one in which individuals have access to life insurance in the form of actuarial notes and do not face credit constraints.

3.1 The model without life insurance

We consider a simplified version of the model presented by Jiménez-Martín and Sánchez-Martín (2007). There are two market imperfections in their model: there is no private insurance for life uncertainty and individuals cannot borrow from pension income. The latter implies that assets cannot be negative after retirement. The intertemporal problem can be written as,

$$\max_{c,\tau} E\left[U(c,l)\right] \tag{2}$$

subject to,

$$\begin{aligned} l(t) &= 1,\ if\ t \geq \tau;\ \text{and } 0,\ \text{otherwise} && (3)\\ \widetilde{w}(t,\tau) &= w(t)(1-\varsigma) + b(t,\tau) && (4)\\ \dot{a}(t) &= ra(t) + \widetilde{w}(t,\tau) - c(t) && (5)\\ a(t_0) &= a_0 \qquad a(t) \geq 0\ \forall t \geq \tau && (6) \end{aligned}$$

Individuals choose the paths of consumption and savings and the retirement period (τ) that maximize their lifetime utility subject to the budget constraints. The rate of variation of accumulated savings ($\dot{a}(t)$) is equal to total income minus consumption. Total income is composed of financial and non-financial income. Financial income is equal to the interest rate (r), which is assumed constant, times accumulated savings. Non-financial income ($\widetilde{w}(t,\tau)$) in period t is equal to gross labor income ($w(t)$) times one minus the payroll tax rate ($1-\varsigma$), before retirement, and to pensions ($b(t,\tau)$), after retirement. Labor income is positive before retirement $w(t) > 0 \quad for\ t < \tau$, and zero afterwards: $w(t) = 0 \quad for\ t \geq \tau$. Pensions are positive and grow at the constant rate g in real terms after retirement and after the pension eligibility age $b(t,\tau) = b(\tau)\exp(g(t-\tau)) > 0 \ for\ t \geq \widehat{\tau} = \max(\tau, \widetilde{\tau})$; $b(t,\tau) = 0 \ for\ t < \widehat{\tau}$. The eligibility age $\widetilde{\tau}$ in turn depends on age and years of service. Notice that $b(\tau)$ is the initial pension of someone retiring at age τ, but it is not necessarily the pension to be collected at that age. If the individual is not yet eligible at τ, he will have to wait until $\widetilde{\tau}$ to collect his first pension. The assumption that individuals cannot borrow from pensions implies that accumulated savings cannot be negative after retirement.

Jiménez-Martín and Sánchez-Martín (2007) solve the model by making use of a proposition in Leung (2000) which establishes that the borrowing constraint becomes binding before the maximum life span. They argue that this proposition allow them to transform the original constrained problem in an unconstrained one (for a similar argument, see Crawford and Lilien, 1981; Fabel, 1994). The unconstrained problem includes an additional control variable, called "the wealth depletion time" and denoted by $\bar{t}$, which is the moment the individual exhausts accumulated savings. The model is then solved in three stages. In the first stage, they choose the optimal consumption path for a given retirement age and terminal wealth depletion time. Before the terminal wealth depletion time, the credit constraints is assumed not to be binding so the optimal consumption path is determined by the Euler conditions and the intertemporal budget constraint (see Appendix A). After the wealth depletion time, consumption is equal to current income.

Substituting optimum consumption paths into the utility function, lifetime utility can be expressed as a function of the wealth depletion time and the retirement age. This indirect utility can be maximized in the terminal wealth depletion time to find its optimum. Nevertheless, we skip this computationally demanding step and simply assume that $\bar{t} = \overline{T}$. In so doing, we take advantage of a result in Jiménez-Martín and Sánchez-Martín (2006), who show that optimal retirement ages are not sensitive to the wealth depletion time. They in particular show that assuming that $\bar{t} = \overline{T}$ yields very similar results as optimizing in $\bar{t}$.[2]

In the third stage, Jiménez-Martín and Sánchez-Martín (2007) solve the model for the optimum retirement age. Let $V(\tau)$ denote the maximum utility attainable if the individual retires at τ:

$$V(\tau) = \max_{c} E\left[U(c,l)\right], \text{ subject to } (3,\ 4,\ 5 \text{ and } 6)$$

It can be shown that the marginal utility of postponing retirement in this model is:[3]

$$\begin{aligned} \frac{dV}{d\tau}(\tau) &= \lambda(t_0)e^{-r(\tau-t_0)}y'(\tau) - S\left(\tau\right)e^{-\delta(\tau-t_0)}\Delta v(\tau) && (7) \\ y'(\tau) &= w(\tau)(1-\varsigma) - b(\tau,\tau) + b'\left(\tau\right)\mathcal{A}\left(\widehat{\tau},\bar{t}\right) && (8) \end{aligned}$$

Where $\Delta v(\tau)$ is the utility loss of posponing retirement at age τ due to forgone leisure. $b'\left(\tau\right)$ is the per-period expected increase in future pensions if retirement is postponed at τ. Finally, $\mathcal{A}\left(\widehat{\tau},\bar{t}\right)$ is a factor that determines the expected present discounted value at τ of a flow of 1 per period from τ on.[4]

The individual retires when the marginal utility given by (7) is zero or when it exhibits a discontinuous jump from positive to negative. The latter case arises if the utility function exhibits an angle at the optimal retirement age. This may happen because of discontinuities in $y'(\tau)$, like the ones that

[2] This means that credit constraints have a very small effect in the life cycle model without recursive shocks. In this framework, individuals who forsee at 20 years old a binding credit constraint to take place after retirement can increase savings to avoid the utility loss implied by the constraint.

[3] A detailed derivation is available from the authors upon request. This result was first derived in Jiménez-Martín and Sánchez-Martín (2006).

[4] The formula of $\mathcal{A}\left(\widehat{\tau},\bar{t}\right)$ is $\int_{\widehat{\tau}}^{\bar{t}} e^{(g-r)(t-\tau)}dt + e^{(g-r)\left(\bar{t}-\tau\right)} \int_{\bar{t}}^{\overline{T}} e^{g(t-\bar{t})}e^{-\left(\widetilde{\delta}(t)-\widetilde{\delta}(\bar{t})\right)}dt$, where $e^{-\widetilde{\delta}(t)} = S\left(t\right)e^{-\delta(t-t_0)}$. This expression simplifies to $\int_{\widehat{\tau}}^{\overline{T}} e^{(g-r)(t-\tau)}dt$ under the restriction $\bar{t} = \overline{T}$.

arise if the individual becomes eligible for a benefit when he postpones retirement.

The decision to postpone retirement impacts on utility through two channels. Firstly, it modifies the individual's budget constraint and hence the optimal consumption path. Secondly, it reduces leisure. The first and second terms on the right hand side of (7) capture these two channels, respectively.

The change in wealth associated to postponing retirement an instant in τ is $y'(\tau)$. It contains three terms. $w(\tau)(1-\varsigma)$ is the wage net of contributions that the individual get when he works another instant at τ. $b(\tau,\tau)$ is the pension loss from postponing retirement. Notice that $b(\tau,\tau) > 0$ only if $\tau \geq \widehat{\tau}$. Finally, $b'(\tau)\,\mathcal{A}\left(\widehat{\tau},\bar{t}\right)$ captures the discounted sum of the expected increase of the pension profile. The wealth increase times the costate variable $\left(\lambda(\tau) = \lambda(t_0)e^{-r(\tau-t_0)} = u'(c_\tau)\right)$ is the impact of postponing retirement that goes through consumption.

The term $\Delta v(\tau)$ is the utility loss of foregone leisure derived from postponing retirement. The individual enjoys utility from leisure $v(1)$ if he retires at τ, but only $v(0) < v(1)$ if he retires one period later. This utility loss is discounted to t_0 using the subjective discount rate and the survival probability.

3.2 The model with life insurance

Uncertain longevity brings the possibility that individuals die in any moment leaving unpaid debts and unintended bequests. Because of this, it is often assumed in the literature that individuals with uncertain lifetime cannot have negative net wealth, i.e. net assests must always be non negative: $a(t) \geqslant 0,\ \forall t$ (Yaari, 1965; Acemoglu, 2009, p 609). These "endogenous credit constraints" can be binding before the terminal wealth depletion time, so some of the consumption paths described in the previous subsection would not be feasible when these constraints are present. Unfortunately, solving the model with these constraints is a non-trivial task. Leung (2000) provides rigurous conditions to determine the optimal terminal wealth depletion time, but this does not rule out the possibility that credit constraints become binding before. In particular, young individuals with low income might be liquidity constrained. One way of analyzing the impact of uncertain longevity without having to deal with endogenous credit constraints is to assume that individuals have access to life insurance.

Following Yaari (1965), we assume in this version of the model that individuals can trade "actuarial notes". Unlike regular (non-contingent) notes, actuarial notes only pay if the individual is alive. With these contingent assets, there is no risk that individuals leave unpaid debts when they die, for creditors can lend in actuarial notes that value zero if the individual dies. With risk neutral insurance companies, actuarial notes would yield the interest rate on regular notes plus the mortality rate if the individual is alive and zero if the individual dies. In the model with life insurance, individuals still solve (2) subject to (3) and (4), but (9) substitutes (5) and (10) substitutes (6):

$$\dot{a}(t) = (r + m(t))\, a(t) + \widetilde{w}(t,\tau) - c(t) \tag{9}$$

$$a(t_0) = a_0 \qquad a(\overline{T}) \geqslant 0 \tag{10}$$

Notice that, unlike in the model without life insurance and with credit constraints, the only constraint on asset acumulation in this version of the model is the transversality condition $a(\overline{T}) \geqslant 0$.

This problem can be solved in two stages. In the first stage, we solve for the optimal consumption and savings plans, given retirement. The consumption path can be computed from the Euler condition and the intertemporal budget constraint (see Appendix A). In this model mortality still reduces the desirability of future consumption (higher effective discount rate) but life insurance reduces the cost of future consumption. With actuarially fair notes, these two effects cancel out. In turn, individuals only have to fulfill a lifetime budget constraint "on average", and not necessarily in each state of nature.

In the second stage, we compute the optimal retirement age. The marginal utility of postponing retirement is given by equation (7) like in the previous model, but $y'(\tau)$ and $\lambda(t_0)$ change. In the model with life insurance $y'(\tau)$ is given by:

$$y'(\tau) = S(\tau)\left[w(\tau)\left(1-\varsigma\right) - b(\tau,\tau)\right] + b'(\tau)\,\mathcal{A}\left(\widehat{\tau},\overline{T}\right) \tag{11}$$

where $\mathcal{A}\left(\widehat{\tau},\overline{T}\right)$ is equal to $\int_{\widehat{\tau}}^{\overline{T}} S(t)e^{(g-r)(t-\tau)}dt$, while in the previous model it was $\int_{\widehat{\tau}}^{\overline{T}} e^{(g-r)(t-\tau)}dt$. In turn, the marginal utility of consumption $\lambda(t_0)$ differs in both models as far as initial consumption is different. Therefore, the impact of postponing retirement on welfare going through consumption may be larger or smaller in the model with life insurance, relative to the previous model, depending on the particular values of preference parameters and the life-cycle profile of wages and pensions.

4 Estimation of the model

The estimation strategy consists of comparing the optimal retirement decision (according to the model) with observed retirement and choose the preference parameters to maximize the likelihood of having observed the sample. Like Jiménez-Martín and Sánchez-Martín (2007), we use a random utility model to derive the likelihood function. We do not observe all the variables that enter the ecomic problem, so we simulate some of them based on auxiliary models. Therefore, we maximize a pseudo-loglikelihood (PL).

In order to write down the PL we assume that the utility loss due to postponement of retirement $\Delta v(\tau)$ has a deterministic component $\Delta v_D(\tau)$, which we assume linear in some observable characteristics, and an unobserved time-invariant individual effect (ε),

$$\Delta v(\tau) = \Delta v_D(\tau) + \varepsilon \qquad \varepsilon \sim F_\varepsilon(.) \tag{12}$$

$F_\varepsilon(.)$ is the distribution function of ε, which is assumed standard normal. We model the deterministic component of the utility loss as a linear function of some observable characteristics (x). Thus, the marginal utility of postponing retirement at τ can be written as

$$\begin{aligned}\Gamma(\tau) &= \frac{\lambda(t_0)\, e^{(\delta-r)(\tau-t_0)}}{S(\tau)} y'(\tau) - x'\pi - \varepsilon \\ &= \Gamma^*(\tau) - \varepsilon \end{aligned} \tag{13}$$

$\Gamma(t)$ must be positive for the individual to be active at t, i.e. for the optimal retirement time τ^* to be larger than t. Therefore, the time-invariant individual component of the utility of leisure ε must be smaller than $\Gamma^*(t)$ if the individual is still active at t. The probability that the individual is still active at t is thus:

$$P(\tau^* > t) = P(\varepsilon < \Gamma^*(t)) = F_\varepsilon(\Gamma^*(t)) \tag{14}$$

The contribution of individual i in period t to pseudo-likelihood is given by:

$$L^{it}(\theta) = \left[1 - \frac{F_{it}}{F_{it-1}}\right]^{d_{it}} \left[\frac{F_{it}}{F_{it-1}}\right]^{1-d_{it}}$$

where d_{it} is an indicator function that takes the value 1 if the individual retires at age t and 0 otherwise, and F_{it}, the probability that individual i is still active in t, can be determined from equation (14) as: $F_{it} = F_\varepsilon(\Gamma^*(t)) = \Phi(\Gamma_i^*(t^i, x_{it}; \theta)$, where $\Phi(.)$ is the standard normal cumulative distribution function of a random variable, Γ_i^* is the function defined by equation (13), t^i is individual i age in t, x_{it} is a set of observable variables of individual i in t and θ is a vector of unknown parameters.

The contribution of each individual to pseudo-likelihood is given by

$$\begin{aligned}\prod_{t=55}^{T_i} L^{it}(\theta) &= \prod_{t=55}^{T_i} \left[1 - \frac{F_{it}}{F_{it-1}}\right]^{d_{it}} \left[\frac{F_{it}}{F_{it-1}}\right]^{1-d_{it}} \\ &= \begin{cases} \frac{F_{i,T_i}}{F_{i,55}} & if\ \tau_i > T_i \\ \frac{F_{i,\tau_i-1} - F_{i,\tau_i}}{F_{i,55}} & if\ \tau_i \leq T_i \end{cases}\end{aligned}$$

where we have asssumed that individuals do not retire before age 55 and T_i is individual i age when he is observed for the last time in the database.

We now single out the vector of parameters to be estimated, the vector of parameters defined out of the model and the observable variables. First, the parameters to be estimated are $\theta = (\eta, \delta, \pi)$. The parameters set out of the model include the interest rate (r) which was fixed at 3 percent, and the survival function $(S(s))$, supplied by the Central Bank of Uruguay. Also given are the complete individual wage income profile (which is assumed to be known both by the individual and the econometrician), the pension eligibility rules, the rules for the computation of the average indexed earnings and the replacement rate. The vector of variables is thus $\left(w_{i,t}, b_{i,t}, b'_{i,t}, x_{it}\right)$ with $t = t_0, .., \overline{T}$. In the next section, we explain how we obtained each of these variables and we present some descriptive statistics.

5 Data

Since the pension reform that began in 1996, Uruguay has administrative records of work histories of affiliates to the BPS. In 2004, BPS gave the University of the Republic (dECON-FCS-UDELAR) a random sample of about 80,000 contributors. The sample was chosen in December 2004, including individuals who contributed at least once between April 1996 and December 2004. We thus have a panel with up to 105 monthly records by individual.

The dataset contains valuable information about individuals' work histories (monthly labor earnings, labor category, worked hours, date of initiation and termination in each firm, and the cause for termination). However, it has no information about the amount of pensions paid. As already mentioned, we filled the complete profile of work histories and wages and the corresponding pensions using auxiliary econometric models.[5] Also, little information on individual socio-demographic characteristics is available. However, there

[5] For a detailed explanation of the models used to complete work histories, see Forteza et al. (2009).

is data on some permanent characteristics (date of birth, sex, nationality, among others). There is also information about characteristics of the firms: public or private, number of employees and employers and branch of activity.

We estimated the structural model on a subsample of public employees aged no less than 45 and less than 70 in 1996, who were contributing in April 1996. Most of these individuals are covered by the social security rules of *Acto* 9 and the "transition regime" in law 16.713.

We chose this subsample for several reasons. First, in the private sector there might be underreporting of contribution wages at early stages of the working career and overreporting in the last years, and hence there is considerable error in the measurement of wages. We expect this type of error to be absent or very small in the case of public sector employees. Second, for those who were contributing in April 1996 the date of their first record in Social Security is available. Thus we can recover the number of years of contribution for them. Finally, we focus on the ordinary pension and therefore we considered a sample of individuals up to 70 years old.[6]

5.1 Measuring retirement and pension eligibility ages

Two related but distinct retirement concepts have been used in the empirical literature: exit from the labor force and entry to the pool of pensioners (Alvarez et al., 2009; Börsch-Supan and Schnabel, 1999; Rust, 1990). These concepts have been measured using information from surveys and administrative records from social security (Börsch-Supan et al. 2004). Our theoretical model focus on retirement as exit from the labor force, assuming that individuals claim benefits as soon as they are entitled to do so. We therefore built a proxy for retirement setting the retirement age as the age at which individuals stop contributing for the last time in the window of observation.[7]

[6] In this way we are leaving aside a different path to pensions called the "advanced-age" old age pension, which has different rules.

[7] Boldrin et al. (2004) use several alternative proxies. Their preferred proxies are very similar to ours "exit" measure.

Albeit conceptually clear, this measure may involve some underestimation of the retirement age if there are temporary interruptions. A worker who stops contributing might be retiring or just passing through a temporary interruption. The case is not ambiguous when we have the information about the age the individual starts receiving a pension, but it is more doubtful if the window of observation ends and we do not know whether or when the individual started receiving a pension. Also in institutional environments of high informality, lack of contributions not necessarily imply retirement from the labor force. We expect that these issues are less severe in our sample composed only of public employees than in the whole database that is composed mostly of private sector workers, but we cannot rule them out completely. Because of these issues, we also computed a measure of retirement as "entry" to the pool of pensioners.

About four percent of the individuals in the sample of public employees continue contributing after receiving their first pension. Our model does not allow for partial retirement, so we excluded these cases from our sample.

We present in figure 1 the histogram of ages at which individuals receive their first pension for men and women separately. While a considerable fraction of women retire before 60, only a small fraction of men do it. This is consistent with Uruguayan social security rules in the period: the minimum pensionable age for women was 55 and for men 60 at the beginning of the period of observation. There is also a pick between 60 and 61, for both sexes.

Unlike it is usually reported in PAYG-DB programs, we do not find sharp discontinuities in the rates at which Uruguayan public sector workers withdraw from the labor force at the program's key ages (figure 2). Among others, Gruber and Wise (1999) report that the hazard rates of workers in OECD countries show spikes at key social security ages, like the statutory and the early retirement ages. We do not find similar spikes at the key pension ages in our sample of Uruguayan public employees. The rate of pension claims by Uruguayan public employees is low -but not zero- and increasing between 55 and 59 years of age among women and almost zero among men. The hazard rates fluctuate between 10 and 20 percent between ages 60 and 69. At about 70 and 71 the hazard rates rich their maximums of more than 30 percent among women and about 25 percent among men.

Using a sample of Uruguayan private male workers affiliated to the same pension program as our public employees, Alvarez et al. (2012) report two very different patterns for individuals covered by two different regimes, the *Acto* 9 and the law 16.713 transition regime. Only private workers covered by the "transition" regime exhibit a spike at the ordinary retirement age similar to what it has been reported in OECD countries.

Pension eligibility age also represented a challenge, because the social security administration did not provide this information directly. We computed the moment the individual becomes eligible for a pension using information from the work history database (age and years of service) and the system norms. Since measurement of pension eligibility is crucial for the estimation of the structural model, we used two alternative procedures to compute this variable. We first considered an individual was eligible if he fulfilled the legal requirements in terms of years of service and age. We found that many individuals who were not eligible according to our computation were nevertheless receiving a pension, so we decided to compute a second eligibility indicator that added all those individuals that were receiving pensions to the pool of eligible individuals.

5.2 Estimation and imputation of flows of wages and pensions

In order to have wages for the whole life cycle of the individuals in the sample $\{w_{i,t}\}_{i=1,..N,t=20,...\overline{T}}$, we estimated auxiliary models. We considered the observed salary in the window of observation and imputed a salary in other periods. We estimated separated models for the imputation for males and females wage profiles, including age and age squared as predictors and allowing for unobserved individual effects. We then computed the individual effects as the mean of the residuals by individual. These individual effects are expected to capture heterogeneity that is not observed in our database, mainly education and hability (Forteza et al., 2009). To obtain a prediction equation, we estimated the model in a second step using the individual effects computed in the first step. In table 1, we present the OLS estimation of the models.[8] Specification 2 includes the individual effects computed from specification 1. As expected, the R-squared in specification 2 is high, due to the inclusion of estimates for the individual effects. This procedure would not be appropriate to do inference, so we do not comment on the values of the parameters, but the estimated models look appropriate for predictive purposes.

Once we have the wage flow for each individual and having information about age and years of service we could impute $b_{i\tau}$, the pension that individual i would obtain if he retired at age τ and hence the present value of that flow. The net present value of the flow of pensions is

$$PV(i,\tau) = \sum_{t=\tau}^{T} \frac{S(t)}{S(\tau)} \left(\frac{1+g}{1+r}\right)^{t-\tau} b_{i\tau} \times \mathcal{I}_{i,t}$$

where g is the expected anual growth of the real value of pensions after retirement, $\mathcal{I}_{i,t}$ is an indicator function that takes value one if indivual i is eligible for pensions at age t, r and $S(.)$ are the interest rate and the survival function, as previously defined. We also computed the present discounted value of the expected change in the pension profile, completing the set of variables we need to proceed with the estimation.[9]

[8] Our database is not top censored as it is often the case in social security datasets.

[9] The present discounted value of the expected change in the pension profile is computed as $\Delta PV(i,\tau) = \left(\frac{S(\tau+1)}{S(\tau)} \frac{1}{1+r}\right) PV(i,\tau+1) - PV(i,\tau) = -b_{\tau,i} \times \mathcal{I}_{i,\tau} + b'_{i,\tau}\mathcal{A}(\tau,T)$

5.3 The utility of leisure

In order to complete the empirical specificacion we need to define the characteristics that enter the reduced form for the utility of leisure in (12). We consider only three arguments for this function because data availability prevent us from considering a richer specification. The first argument is the retirement age. The second argument is a proxy for the individuals preference for leisure obtained from a linear probability model for contributions. In that model, the dependent variable is 1 if the individual contributes and 0 otherwise. The model has unobserved individual effects that can be estimated thanks to the panel structure of the database. We use these estimated individual effects in the linear probability model as an explanatory variable in the retirement model. We interpret this variable (e_i) as a proxy for the willingness of the individual to work and contribute and thus as an indicator of the utility of leisure (the higher e_i, the lower the utility of leisure). Finally, we include a dummy variable to control for sex ($m_i = 1$ if male). Thus, $\Delta v_{Di}(\tau)$, the leisure related deterministic component in equation (12), is $\pi_0 + \pi_1 \; \tau_{it} + \pi_2 e_i + \pi_3 m_i$ and the corresponding vector of structural parameters to be estimated is $\theta = (\eta, \delta, \pi_0, \pi_1, \pi_2, \pi_3)$.

6 Results

In this section, we focus on estimations done with retirement defined as entry to the pool of pensioners, and adopting the more flexible definition of pension eligibility, i.e. assuming that individuals were eligible if they fulfilled the conditions established in legal norms or if they received a pension, even if they were not eligible according to the laws. In section 8, we briefly discuss the results obtained with other options.

We present the results of estimating the structural models in table 2. The structural parameters are estimated with considerable precision (relatively narrow confidence intervals), present the expected signs and are within the values usually reported in the literature. The models with and without life insurance yield similar results.

Our point estimate of the CRRA parameter is about 1.7, with standard deviation between 0.015 and 0.022 in the models with and without life insurance, respectively. Our estimations are within the range of values reported in the literature, but it should be noticed that this range is wide. Jiménez-Martín and Sánchez-Martín (2007), in their preferred estimation, get a point estimate equal to 1 (sd 0.01). Hurd (1989) reports 1.12 (sd 0.074) and Gustman and Steinmeier (2002) report 1.26 (sd 0.03). Attanasio and Weber (1995) and Attanasio et al. (1999) estimate the elasticity of intertemporal substitution and report point estimates of 0.64 (sd 0.33) , corresponding to CRRA of 1.6. French (2005) reports CRRA point estimates between 2.2 and 5.1. Other estimates for the CRRA are 1.6 (Alan, 2006), a range between 0.28 and 2.29 (Gourinchas and Parker, 2002), a range between 1.2 and 1.9 (Alan and Browning, 2010), a range between 1.4 and 6.1 (Alan, 2012) and a range between 1.3 and 2.5 (Sanroman, 2013). Alan and Browning (2010), Alan (2012) and Sanroman (2013) find that the CRRA estimates vary between educational groups, being the less educated the less risk averse.

Our point estimates of the discount rate lie between 8.4 and 7.4 percent, with standard deviation between 0.4 and 0.5 percent, in the models with and without life insurance, respectively. There is a wide range of point estimates in the literature -including negative and positive values- with often large standard errors. French (2005) finds discount rates between -4.0 and 1.9 percent. Jiménez-Martín and Sánchez-Martín (2007) report discount rates in the range of -4.3 (sd 1.5) to -0.7 (sd 0.2) percent. Hurd (1989) reports a discount rate of -1.1 percent (sd 0.2). Alan (2006 and 2012) estimates for the discount rate are substantially higher: about 8.7 percent (sd 0.7) and a range between 6.0 (sd 2.0) and 28 (sd 8.0) percent, respectively. Gan et al. (2004) find discount rates ranging from 0 to 6 percent, with median regressions, and between -5 and -7 percent, with mean regressions (they conjecture that the difference between mean and median regression is due to the large influence of the households at the top of the wealth distribution when the estimation method is mean regression). Samwick (1998) uses the distribution of wealth to income ratios to estimate individual discount rates in the US. His estimated median rates range from 3.2 to 9.8 percent, depending on the indicator used to compute wealth and assumptions on risk aversion and initial assets. To the best of our knowledge, there are no previous estimations of the subjective discount rate in Uruguay.

Our results suggest that, in terms of risk aversion and time preferences, Uruguayan public employees are not very different from individuals covered in the above mentioned studies done in developed countries. It is worth emphasizing, however, that disentangling risk aversion and discounting is usually an issue in the literature, and our study is no exception. The basic stylized fact our model has to explain is why Uruguayan public employees do not seem eager to retire as soon as they are eligible for public pensions, even when the implicit tax on continued work is comparatively high in Uruguay (Alvarez et al., 2012). The model can in principle explain this behavior in two ways: either individuals are patient (low discount rate) or comparatively very risk averse. Patient individuals value much future consumption and are thus willing to work more when they are not too old. Risk averse individuals work harder to save more for precautionary reasons. Therefore, in theory, the same observed retirement pattern could be caused by different combinations of risk aversion and time discounting, with more risk aversion substituting for less discounting.

It is worth noticing that different studies have adopted different strategies to identify risk aversion and time discounting parameters. These different strategies, often dictated by data availability, may have some bearing on the results. Like Jiménez-Martín and Sánchez-Martín (2007) and Rust and Phelan (1997), our identification strategy rests on retirement decisions. Other studies, like Hurd (1989), use information about the accumulation of financial assets. Attanasio and Weber (1995) and Attanasio et al. (1999) exploit information about consumption. Ghez and Becker (1975), MaCurdy (1981), and Browning et al. (1985) exploit information on hours worked by young workers. French (2005) exploits information on asset accumulation and labor supply.

While the only source of uncertainty in our model is life span, in the real world, individuals face additional sources of uncertainty. This may induce some bias in the estimation of the CRRA.

Our model does not have a bequest motive. This is also dictated by data availability, but we should be aware that in the presence of altruism, failure to recognize the bequest motive may cause a downard bias in the estimated discount rate and an upward bias in the estimation of the CRRA. Altruistic individuals may postpone retirement to save more in order to leave bequests. Lacking this motive, our model accommodates the data through lower discounting or more risk aversion.

The other estimated parameters are meant to capture the impact of several potential determinants on the utility cost of foregone leisure that is associated to postponing retirement. The age parameter is positive and highly significant, indicating that the utility cost increases with age. As expected, individuals find it increasingly hard to continue working as they age. The "propensity to contribute", as measured by the individual effect estimated in the contributing linear probability model, has a negative and significant coefficient. Finally, the utility loss of forgone leisure is significantly higher for females than males.

7 Simulations

Using the parameters estimated in the structural model, we simulated retirement in four different scenarios. The benchmark scenario has the norms in effect in the observed period, with some individuals covered by the norms of the *Acto* 9 and others by the transition regime in law 16.713. We use this scenario as a reference point to assess the impact of the policy changes we simulate in the other three scenarios.

We simulate a second scenario in which we assume there is no reform in 1995: all individuals continue covered by the rules in the *Acto* 9 passed in 1979. In the third scenario, we assume that all individuals switch to the transition regime in 1996. Finally, in the fourth scenario we assume that the years of service required to access a pension remained at 30, rather than increasing to 35 as set in law 16.713 in 1995. This scenario partially mimicks a reform passed in 2008. Each scenario was repeated ten times.

According to these simulations, retirement ages of Uruguayan public employees are not very sensitive to social security rules. Average retirement ages change very little and percentiles 10, 50 and 90 do not change at all across scenarios, relative to the benchmark (table 3). More than 88 percent of individuals do not change retirement age in any of these scenarios, relative to the benchmark.

These results are striking, since the alternative scenarios considered in this study involve considerable changes in incentives. The *Acto 9* regime generates much higher implicit tax on work than the transition regime (Alvarez et al., 2010), and yet, according to our simulations, public employees would not significantly change their retirement age due to this reform. The 2008 reform reduces from 35 to 30 the number of years required to receive a pension, but according to our simulations few public employees would respond to this change reducing their retirement age.

These results are however not at odds with the picture that figures 1 and 2 show. Uruguayan public employees do not seem to retire at one or two preferred ages, as it is often the case in other countries and groups of workers. The hazard rates, in particular, do not show the usual peaks at pension eligibility ages. Therefore, these workers do not seem to decide when to retire based mainly on pension benefits, but rather on other considerations.

Our model capture these alternative forces through the utility associated to leisure. In particular, the disutility of working grows significantly as individuals age, according to our estimated model, and it is this disutility what drives retirement in our simulations. Incentives from pensions seem to be of second order in this population.

However, as Gustman and Steinmeier (2002) warn, this result could be partially driven by the estimation method. They argue that maximum likelihood might give too much weight to observations that are theoretically unlikely but empirically possible. If, for example, the benefit changes dramatically after 20 years of tenure and an individual is observed retiring with 19, the loglikelihood would be dramatically reduced with this observation unless individuals gave economic factors a very low weight in their retirement decision. The counterpart would be an upward bias in the estimation of the coefficient for age, i.e. retirement would seem to be driven mostly by preferences for leisure rather than by monetary incentives. They propose to use an alternative econometric approach where identification rests on matching aggregate moments rather than evaluating individual contributions to the pseudo-likelihood. This looks as a promisory route for future research about the retirement of Uruguayan workers.

8 Robustness

In order to assess robustness of our results, we estimated sixteen variations on the basic model: (i) with and without life insurance, (ii) defining retirement as exist from the labor force and entry to the pool of pensioners, (iii) computing pension eligibility abiding strictly to legal norms and endorsing eligibility whenever we know that the individual is receiving a pension, and (iv) including and excluding the individual effect of the linear probability model as a proxy for willingness to work. In this section, we briefly comment on the results obtained with these alternative specifications.[10]

The models with and without life insurance yield similar results. Hence, our findings do not seem to depend much on these assumptions that are not directly testable.

The estimation with strict application of eligibility rules, rather than the effective or "revealed" eligibility used in table 2, yields similar values in most parameters save for the discount factor that is smaller, and is estimated with much less precision. The estimated standard deviation of the discount factor is about fifteen times larger when eligibility is computed strictly according to the laws. Our preferred estimation is the one presented in table 2 because it takes into account the information contained in the data about pension payments. Indeed, the fact that an individual was receiving a pension, even if he was not entitled to it according to our computations, reveals that he was *de facto* eligible. *(Nota para nosotros: estoy comparando aquí con Table2_Estimacion_entry_causaltipo0_etasi.xls).*

Both the discount factor and the coefficient of risk aversion are estimated with lower precision if retirement is computed as exit from the labor force rather than entry to the pool of pensioners as we did in table 2.[11] While exit from the labor force is closer to the theoretical concept, it has the drawback that it disregards the information contained in pension payments. *(Table2_Estimacion_exit_causaltipo2_etano.xls)*

The exclusion of the proxy for willingness to work has no material impact on the other coefficients. In turn, whenever this variable is included, the associated coefficient has the expected negative sign and is significantly different from zero. (*Table2_Estimacion_entry_causaltipo0_etano.xls*)

[10] These estimations are available from the authors upon request.

[11] We could not found convergence and thus we were not able to estimate the parameters when we computed retirement as exit from the labor force, assumed there is life insurance, used "revealed" eligibility and included the proxy for willingness to work in the regression.

9 Conclusions

In this paper we estimated a structural model for retirement decisions of public employees using administrative records from social security in the main pension program of Uruguay. We used a life-cycle model with and without life insurance. Results do not depend much on these two alternative assumptions.

The estimated CRRA is about 1.7, which means that individuals are moderately risk averse. The discount rate is about 8 percent per year which is higher than the interest rate which was fixed at 3 percent. The estimations also show that the probability of retirement at a given age is higher for individuals with smaller propensity to contribute (measured through a proxy for the utility of leisure), is higher for women than men and increases with age.

We simulated the benchmark case, with the existing norms, and several additional scenarios, including a non-reform scenario, one in which all individuals are immediately switched to the new regime introduced in 1995, and finally one scenario that roughly mimicks the 2008 reform setting the years required to access an ordinary pension at 30. In none of these scenarios do we find important changes in the simulated ages of retirement. According to the model, the retirement decision of Uruguayan public enployees is not very sensitive to pension rules.

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Appendix A

In this appendix we focus on the procedure to find the solution of the model in stage 1, which correspond to the individual decision on his consumption profile for a given depletion time and retirement age. Jiménez and Sánchez use Leung (2000) to transform the original constrained problem into a new unconstrained one including de "wealth depletion time" ($\bar{t}$) as a new decision variable (see also Crawford and Lilien, 1981; Fabel, 1994):

$$V = \max_{c(t),\tau,\bar{t}} \int_{t_0}^{\bar{t}} S(t)e^{-\delta(t-t_0)}u(c(t))dt \quad + \int_{\bar{t}}^{\overline{T}} S(t)e^{-\delta(t-t_0)}u(b(t,\tau))dt + \int_{t_0}^{\overline{T}} S(t)e^{-\delta(t-t_0)}v(l(t))dt \quad (15)$$

s.t.

$$\begin{aligned}
\dot{a}(t) &= ra(t) + \widetilde{w}(t,\tau) - c(t) \\
\widetilde{w}(t,\tau) &= w(t)(1-\varsigma)+b(t,\tau) \\
l(t) &= 1,\ if\ t \geq \tau;\ and\ 0,\ otherwise \\
\bar{t} &\in \left[\tau, \overline{T}\right) \\
a(t_0) &= a_0 \qquad a(t) = 0\ \forall t \in \left[\bar{t}, \overline{T}\right)
\end{aligned}$$

Given retirement age (τ) and terminal wealth depletion time ($\bar{t}$) the unconstrained problem (15) becomes:

$$V(\tau,\bar{t}) = \max_{c(t)} \int_{t_0}^{\bar{t}} e^{-\widetilde{\delta}(t)}u(c(t))dt + A$$

st

$$\begin{aligned}
\dot{a}(t) &= ra(t) + \widetilde{w}(t,\tau) - c(t) \\
\widetilde{w}(t,\tau) &= w(t)(1-\varsigma)+b(t,\tau) \\
a(t_0) &= a_0 \qquad a(\bar{t}) = 0
\end{aligned}$$

where $A = \int_{\bar{t}}^{\overline{T}} e^{-\widetilde{\delta}(t)}u(b(t,\tau))dt + \int_{t_0}^{\overline{T}} e^{-\widetilde{\delta}(t)}v(l(t))dt$. Notice that A does not depend on $c(t)$.

Optimal control theory allows for a complete characterization of the optimal consumption function.[12] Using that the utility function is of the CRRA type and solving the optimization problem, we get the Euler condition:

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\eta}(r - \delta - m(t)) \tag{16}$$

and the intertemporal budget constraint between t_0 and $\bar{t}$:

$$\begin{aligned}\int_{t_0}^{\bar{t}} e^{-r(t-t_0)} c(t)dt \leq Y(\tau, \bar{t}) = a_0 \quad &+ \int_{t_0}^{\tau} e^{-r(t-t_0)} w(t) \left(1 - \varsigma\right) dt \\ &+ \int_{\tau}^{\bar{t}} e^{-r(t-t_0)} b(t, \tau) dt \end{aligned} \tag{17}$$

The right hand side in (17) is the present value of the flow of income the individual has between t_0 and $\bar{t}$ if he retires at τ. The slope of the consumption path is determined by the Euler condition (16).

Current as a function of initial consumption can be computed integrating (16):

$$c(t) = c(t_0) S(t)^{\frac{1}{\eta}} \exp\left(\frac{(r - \delta)}{\eta}(t - t_0)\right) \tag{18}$$

Substituting back in (17) and using that the budget constraint must be binding in the optimum:

$$c\left(t_0\right) = \frac{Y(\tau, \bar{t})}{\int_{t_0}^{\bar{t}} S(t)^{\frac{1}{\eta}} \exp\left(\frac{(r(1-\eta)-\delta)}{\eta}(t - t_0)\right) dt} \tag{19}$$

The marginal utility of wealth at age t_0, $\lambda(t_0)$ is given by $c\left(t_0\right)^{-\eta}$.

After the terminal wealth depletion time individuals are credit constrained so consumption is equal to current income, which is equal to pensions.

To solve the model with life insurance we follow Yaari (1965). We assume in this version of the model that individuals can trade "actuarial notes". Unlike regular (non-contingent) notes, actuarial notes only pay if the individual

[12] The detailed algebra is available from the authors upon request.

is alive. With these contingent assets, there is no risk that individuals leave unpaid debts when they die, for creditors can lend in actuarial notes that value zero if the individual dies. With risk neutral insurance companies, actuarial notes would yield the interest rate on regular notes plus the mortality rate if the individual is alive and zero if the individual dies. The intertemporal optimization problem of the individual is now:

$$\max_{c,\tau} \int_{t_0}^{\bar{T}} S(t)e^{-\delta(t-t_0)} \left[u\left(c\left(t\right)\right) + v(l(t), x(t))\right] dt$$

subject to:

$$\begin{aligned}
\dot{a}(t) &= \left(r + m\left(t\right)\right) a(t) + \widetilde{w}(t,\tau) - c(t) \\
\widetilde{w}(t,\tau) &= w(t)(1-\varsigma) + b(t,\tau) \\
l(t) &= 1,\ if\ t \geq \tau;\ and\ 0,\ otherwise \\
a(t_0) &= a_0 \qquad a(\overline{T}) \geqslant 0
\end{aligned}$$

We solve this problem in two stages. In the first stage, we solve for the optimal consumption and savings plans, given retirement. In the second stage, we solve for retirement time. Stage 1 is solved as in the previous model (see the suplementary material), but the Euler condition is given by:

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\eta}(r-\delta) \tag{20}$$

and the intertemporal budget constraint between t_0 and $\overline{T}$ is:

$$\begin{aligned}
\int_{t_0}^{\overline{T}} S(t)e^{-r(t-t_0)}c(t)dt \leq Y(\tau,\overline{T}) = a_0 + \int_{t_0}^{\tau} S(t)e^{-r(t-t_0)}w(t)\left(1-\varsigma\right)dt \\
+ \int_{\tau}^{\overline{T}} S(t)e^{-r(t-t_0)}b(t,\tau)dt
\end{aligned} \tag{21}$$

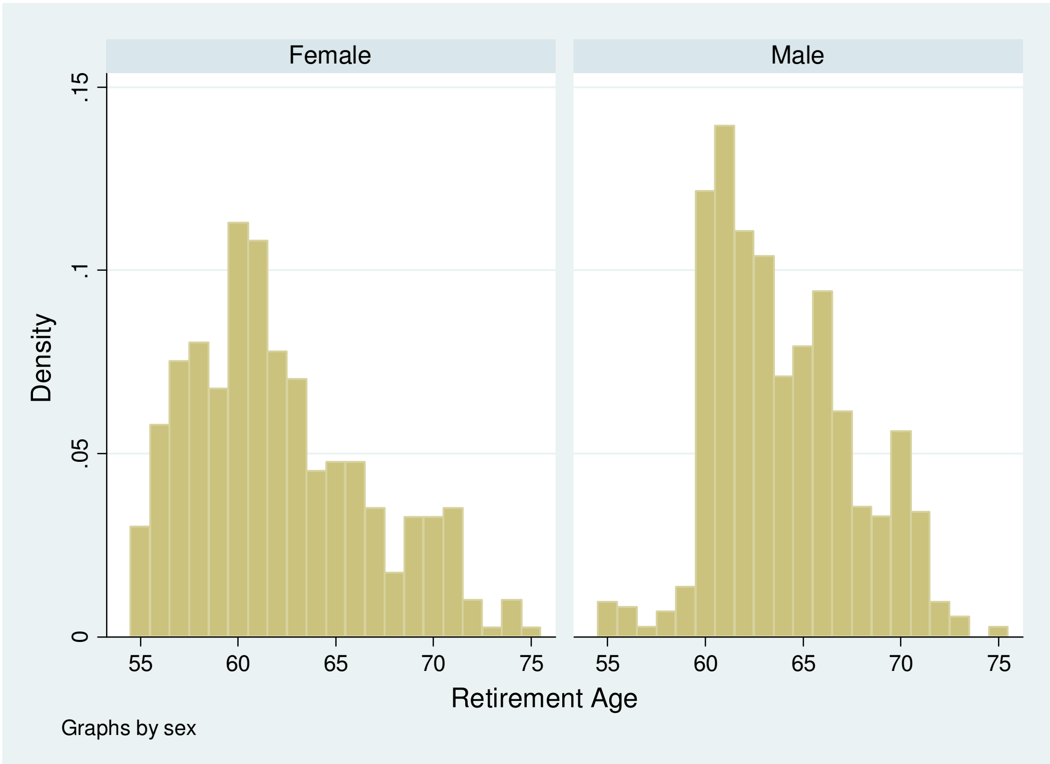
Notice how, thanks to the actuarial notes, individuals only have to satisfy an intertemporal budget constraint in expected terms, with probabilities given by the survival function. The initial consumption can be computed integrating (20):

$$c(t) = c(t_0)\exp\left(\int_{t_0}^{t}\frac{1}{\eta}(r-\delta)ds\right) = c(t_0)\exp\left(\frac{(r-\delta)}{\eta}(t-t_0)\right) \tag{22}$$

and substituting back in (21) and using that the budget constraint must be binding:

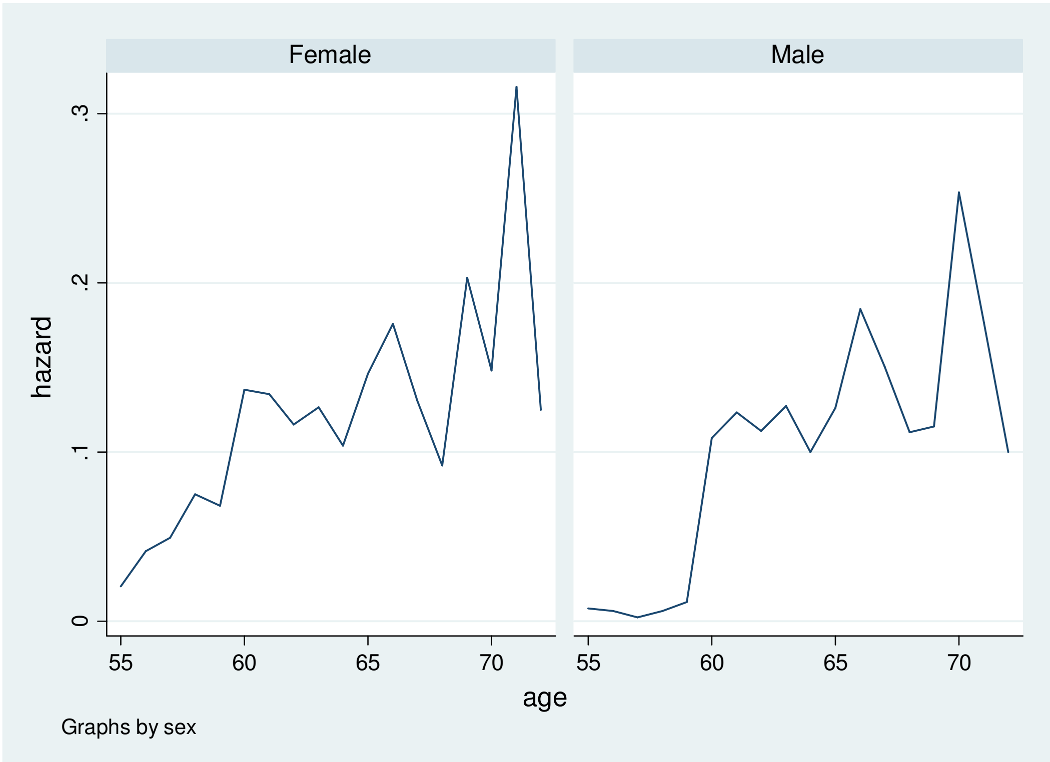
$$c\left(t_0\right) = \frac{Y(\tau,\overline{T})}{\int_{t_0}^{\overline{T}} S(t)\exp\left(\frac{(r(1-\eta)-\delta)}{\eta}(t-t_0)\right)dt}$$

Figure 1: Retirement age



Source: Authors computations based on BPS data base.

Figure 2: Retirement hazard rates



Source: Authors computations based on BPS data base.

Table 1: Wage prediction equation

	Males		*Females*	
	(1)	**(2)**	**(1)**	**(2)**
Age	0.130***	0.128***	0.160***	0.155***
	[0.00338]	[0.00149]	[0.00349]	[0.00159]
Age squared/10	-0.0134***	-0.0130***	-0.0166***	-0.0159***
	[0.000373]	[0.000164]	[0.000405]	[0.000185]
Individual effect		1.000***		1.000***
		[0.00284]		[0.00302]
Constant	3.239***	3.250***	2.359***	2.431***
	[0.0739]	[0.0325]	[0.0729]	[0.0333]
Observations	29,716	29,716	28,805	28,805
R-squared	0.058	0.818	0.100	0.813

Standard errors in brackets
** significant at 5%; *** significant at 1%
Dependent Variable: Real wages (in logs, deflated by IMS May 1995=100)

Source: Authors computations based on BPS data base.

Table 2: Estimates from the structural model [/1]

	With life insurance	Without life insurance
CRRA	1.675***	1.728***
	[0.0152]	[0.0218]
δ	0.0836***	0.0736***
	[0.00431]	[0.00520]
e_i [/1]	-1.326***	-1.448***
	[0.142]	[0.160]
Age	0.193***	0.168***
	[0.00664]	[0.00767]
Male	-0.562***	-0.544***
	[0.0708]	[0.0756]
Constant	-12.03***	-10.35***
	[0.427]	[0.505]
Nobs	2043	2043
Log-likelihood.	-2269	-2187

Standard errors in brackets
** significant at 5%; *** significant at 1%
[/1] e_i is a proxy for the willingness of the individual to work.
Source: Authors computations based on BPS data base.

Table 3: Simulated retirement age in several scenarios

	Mean	P10	Median	P90	No changes [1]
All sample					
Benchmark	63.42	57.00	64.00	69.00	
Only "*Acto* 9"	63.37	57.00	64.00	69.00	88%
Only "Transition"	63.44	57.00	64.00	69.00	96%
Benchmark + 2008 reform	63.42	57.00	64.00	69.00	95%
Females					
Benchmark	62.28	56.00	62.00	68.00	
Only "*Acto* 9"	62.26	56.00	62.00	68.00	88%
Only "Transition"	62.31	56.00	62.00	68.00	95%
Benchmark + 2008 reform	62.28	56.00	62.00	68.00	95%
Males					
Benchmark	64.12	58.00	65.00	69.00	
Only "*Acto* 9"	64.05	58.00	65.00	69.00	88%
Only "Transition"	64.14	58.00	65.00	69.00	97%
Benchmark + 2008 reform	64.11	58.00	65.00	69.00	95%

Note: [1] Percentage of observations that do not change relative to the benchmark

Source: Authors computations based on BPS data base.

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