Sustainable Growth

Abstract
An overview of the concepts of Growth and Sustainability is developed in the frame of Decision Making. An axiomatic formalization of sustainability is settled and some usual decision criteria are analyzed and criticized under this proposed theoretical frame. The work ends concluding that the best way to solve the environmental (knightian) uncertainty is by using non-additive probabilities and the expectance à la Choquet.

Climate Change: the problem

In 1995, the IPCC[1] concluded "the whole evidence shows that the human body acts in an ostensible way upon the world climate". It is predicted that in the next hundred years the world temperature will grow from 1°C to 3.5°C. The consequences of this change on the human and human habitat are far from being fully predicted. For example, a decrease of 0.5°C in 1570-1730 resulted in "the little ice age" and it forced the European landsmen to abandon their lands.

However, the degree of anthropogenic influence on the warming is not known with certitude. There still exist doubts with respect to some key contributing factors. There is an array of questions pending an answer, such that: what are the exact mechanisms through which the sea influences the level of CO_2, how much coal can be retained by the sea, which is the threshold of global warming that could possibly alter this capacity?

The Evolution of the Growth Concept

The economic progress in the 1950's and 1960's was mainly founded on growth and increase in output with the underlying concept of economic efficiency. By the early 1970's, economic performance demonstrated a great effort in improving income distribution as a mean of progress. Consequently, the development paradigm shifted towards the equitable growth: social objectives were considered as distinct and as important as economic efficiency. At the beginning of the 1980's, the accumulated evidence confirmed that the major barrier to development is the environmental degradation. As a result, the concept of sustainable development has evolved to encompass three major concepts of growth: economic, social and ecological.

"underdeveloped countries cannot advance by retracing the steps of … the developed countries … it would imply repeating those errors that have leave to … deterioration of the environment … The solution … must be based on the creation of a society intrinsically compatible with its environment". (Bariloche model, 1972)

[1] International Panel on Climate Change

"Sustainable development is development that meets the needs of the present without compromising the ability of future generations to meet their own needs" (Brundtland report, 1998)

The Three approaches to Sustainable Development

The Economic Approach

In the economic approach, sustainability corresponds to maximizing the flow of income that can be generated while the stock of assets (or capital) leading to the benefits remains at least at the same level.[2] The concepts of optimality and economic efficiency are obviously underlying this approach. However, as it will be shown later, additional difficulties arise because of uncertainty, irreversibility and catastrophic collapse.

The Ecological Approach

The ecological approach emphasizes that the stability of biological and physical systems as well as protection of biodiversity are at the core of the ecological sustainability. The stress is in reserving the resilience and dynamic ability of the biological and physical systems to adapt to change and not preserving some "ideal" state of nature.

The socio-cultural approach

The underlying concept of sustainability in the socio-cultural approach is the stability of social and cultural systems. Intragenerational equity (or better distribution) and intergenerational equity (care for future generations) are the principal aspects of this approach. The size of the set of opportunities is important. Instead of the preservation of the value of the asset base, biodiversity enables the system to retain resilience by protecting the natural system from external shocks (in the same way as retaining capital contributes to the future consumption).

Incorporating Environment in Economic Decisions

The main goal of Environmental Economics is to identify options for efficient natural resource management. It facilitates incorporation of ecological concerns in the traditional framework of human society.

The socio-economic structure consists of corresponding levels. It starts with a transnational level ("global level"), the down next level are the countries with their corresponding multisectorial macro structures, each sector consisting of different sub-sectors, projects and local schemes.

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[2] See Solow 1986, Maler 1990

The usual decision-making process in the socio-economic structure is based on engineering, technological and financial analysis. Nevertheless, the environmental analysis is not present in this structure. There are global and transnational environmental concerns (climate change, ozone layer depletion), natural habitats (forests), water resources (oceans, aquifers), etc., that impede working in the traditional decision-making economic structure. "For example, a forest ecosystem (like the Amazon) could affect the global climate, span over several countries, and also interact with many different economic sectors within each country".[3]

Traditional Project Evaluation

Identification, preparation, appraisal, negotiations and financing, implementation and supervision, and finally evaluation, are the main steps of the traditional World Bank's approach to this decision problem.

One of the principal tools used in projects evaluation is the Cost-Benefit Analysis (CBA) where benefits are defined as a function of the outcome of a project (policy) on the human welfare. The costs are measured by the opportunity costs of the scarce resources.

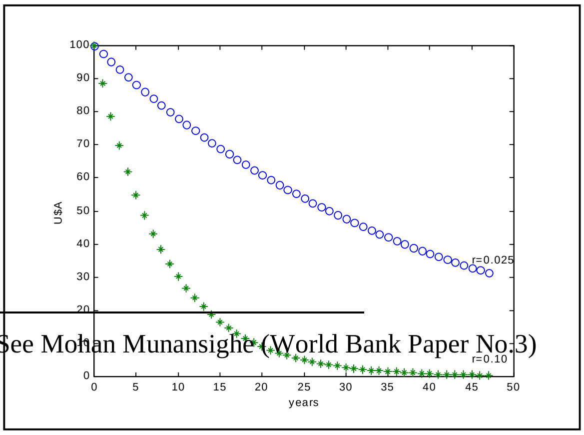
The most basic procedure in accepting a project is the positive value of the Net Present Value (NPV), where the NPV is defined as:

$$NPV = \sum_{t=0}^{t=T} \frac{(B_t - C_t)}{(1+r)^t} \text{, where } \begin{cases} B_t : \textit{net benefits in } t \\ C_t : \textit{net costs in } t \\ r : \textit{discount rate} \\ T : \textit{time horizon} \end{cases}$$

If there is a set of feasible projects, to rank them the NPV is used: the one with the highest value will be preferred. (Obviously, a correction of scale may be needed to establish the comparison).

Another criterion used for the project evaluation is the Internal Rate of Return (IRR). The IRR is the discount rate of return that equalizes the value of NPV to zero. Depending on its relative "position" to *r*, the corresponding project will be accepted or rejected (a *r* less than the *IRR*, implies that the project will be accepted).

It is important to highlight that the discount rate of the project is crucial to determine the project feasibility depending on its lifetime.



The figure in the left shows how different discount rates make an initial amount of $US 100 vanishes in different time horizons. In fact, in 48 years the money disappears when the *r* is 10%, while if it is 2.5%, it is still "alive". Thus, the bigger is *T*, the lower must be the *r*, and so the difficulty in dealing

[3] See Mohan Munansighe (World Bank Paper No 3)

with different time horizon projects (policies).

The discounted utilitarism (the best path that provides the greatest present value of benefits) is consequently particularly controversial for environmental valuations because environmental phenomena are linked to very long run considerations. For example, because of its molecular instability, the lifetime horizon of the steam of water, which is the most responsible of the global change, is not relevant. However, the CFC (chlorofluorinecarbon) and the CO2 are crucial because the former remains in the atmosphere between 60 and 400 years, and the latter for approximately 500 years. Thus, project feasibility decisions (policies) cannot lie only on economic considerations.

Environmental-Economic Decision Making

Incorporation of environmental concerns in economic decision-making implies a comparison of the future scenarios (identifying the possible physical impacts of a given economic activity) and only then a valuation of these physical impacts through adequate environmental-economic analysis tools.

Conceptually, the concept of the value should be modified to include the notions of the use value and non-use value of an environmental good. This broader concept of the value is defined by the Total Economic Value (TEV). The *use value* of an environmental asset can be split mainly in four components. The first is the *direct use value* that is linked to its contribution to actual production and consumption. The second, the *indirect use value* is related to the capability of absorbing and recycling the environment (natural filtration of polluted water or recycling of nutrients). The third is the *option value* of the environmental asset related to biodiversity. It is the premium to guarantee the availability of the good in the future. Finally, the last one is the *existence value* that arises from the satisfaction of just knowing that the asset exists.

Economic Theory has no problem whatsoever to define use values. Much ambiguity appears when going “downstream” (in the sense of trying to compute the “components”) or when considering non-use values. Altruist motives are underlying non-use values in all their forms: intergenerational, interpersonal and the q-altruism (an intrinsic right of a resource to exist). Consequently, the welfare function utilized by policy makers should encompass more than pure human utility.

At the same time, the discount rate plays a critical role in intertemporal decisions about environmental resources in the long run. The interest rate equals the marginal rates of time preference and return on capital in the equilibrium of a perfect market. Nevertheless, the government policies and market failures may lead to discrepancies between these two rates. For example, the scarcity of capital makes the marginal rate of return of capital higher in the developed countries, while the urgency of satisfying basic needs, in the poor countries is usually associated with a higher rate of time preferences. So, the government policies should accommodate such kind of considerations.

As it was discussed before, higher discount rates may discriminate against future generations: projects with high social costs in the long run and with net social benefits in the short run will be favored. Higher discount rates will eliminate projects with benefits accruing in the long run. Thus, future generations may potentially suffer from market discount rates determined by high rates of current generation time preference and/or productivity of capital.

Risk and uncertainty are an inherent part of economic decisions too. While the risk element can be measured by probabilities, the uncertainty cannot be estimated in such a way just by the virtue of its undefined nature. Moreover, the use of an expected value of risk does not have much to say about the variability or the range of possible values of the event. If the future cannot be perceived clearly, suppositions can be stated only to the extent the clarity of a vision is bounded to.

Global Warming is an example. In the past, the greenhouse gas effect of CO2 emissions was not recognized as an ecological risk in the evolution of the concept of growth. Today, the uncertainty concerning its impacts remains. However, a caution is warranted in the respects of its potential effects.

Uncertainty plays an important role in environmental valuation and policy. Option values[4] and quasi-option values[5] surge in economic analysis because of uncertainty. Irreversible damages vanish the opportunity to expand knowledge of the environment, and mainly this is what a quasi-option covers. Both tools may be used in facing this problem. Nevertheless, environmental policy is complicated due the variety of uncertainties underlying: Bromley identified six different types in the case of air pollution resulting from acid deposition.[6] This makes more difficult the analysis of sustainable growth, as we will see later.

Analyzing Sustainability: A Background

There are many approaches to analyze sustainability. Pearce considers sustainable growth conditioned to a constant natural capital stock. The Solow & Hartwick approach is based on a definition and maximization of an intertemporal welfare function. They propose to maximize the welfare of the poorest generation, following a Rawlsian criterion: finding the welfare level of the least-well-off generation of each path and choosing the feasible path giving the greatest value of the minimal level.

If a country invests the same market value of depletable resources that is used, Hartwick, Dixit et al. show that it solves the Rawlsian problem and obtains the higher possible utility level for the "worst" generation. This result means that investing at the level of used or depleted implies that the stock of aggregate capital (physical and natural) would remain constant: there should be then a perfect substitution among natural capital and produced capital.

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[4] The option value corresponds to the premium consumers are willing to pay to avoid the risk of not having something available in the future.
[5] The quasi-option value is linked to preserve options for future use in the expectation that knowledge will improve as time goes by.
[6] More details in Bromley, D.W. "Entitlements, Missing Markets and Environmental Uncertainty", *Journal of Environmental Economics and Management*, vol.17, pp. 1181-1194 (1989)

At this point, it seems relevant to ask the following question (Chichilnisky): does is constitute a “sustainable” policy to substitute trees by buildings of the same market value?

One possible answer, as guise to defend the last position, should be to say that maybe the market, by a price mechanism, should make impossible this substitution.

Another critic to Rawls approach is if it is applied to underdeveloped countries (in which it is desirable that the present generation be the worst one), this criterion indicates that it is not feasible to save for future generations. More than this, to aim a stationary capital should avoid the whole substitution of environmental capital by produced one: nevertheless, in biology, “stillness” determines, on the long run, extinction.

Apparently, the essence of sustainability should be summarized in the basis of two fundamental premises that: i) give a symmetric consideration to the present and the long run future (the very long run must have a positive value) and ii) explicitly recognize an intrinsic value of the environmental assets.

As it was explained before, the environmental goods have an instrumental value as streams of knowledge (they are a stream of knowledge in which refers to biodiversity) and as a life support and maintenance for the human body. These aspects make environmental quality to pursue as a mean and not only as a goal.

On the other hand, the environmental goods have an intrinsic value: they have their own right to exist per se, independently of the anthropogenic value that we can assign to them. This intrinsic value is mainly linked to the concept of sustainability, considered as a permanence of a stock level. Consequently, the utility (or welfare) of the generations should be then a function of consumption, but of the environmental stock level too (Heal).

Summing up, sustainability seems to be linked mainly to the concepts of intergenerational equity, of the limitation of resources and to the control of the impact of human activity over the environment.

Formalizing Sustainability: Chichilnisky’s Axioms

Any sustainable welfare criteria should establish a complete order among all the feasible utility paths and should not assign a dictatorial role neither to the present nor to the future.

Assumptions and notations:

- infinite lived world
- g : index of each generation (they may overlap or not) $g = 1,2,\cdots$
- u_g : utility of generation g (neoclassical assumptions)
- C_g : consumption of generation g . ($C_g \in \Re^n$)

$u_g : \Re^n \to \Re^+$ are such that $sup\{u_g\}_{g\geq 1} \leq 1$ (normalization)

The space of all the feasible consumption paths (actions depending on resources disponibility) will be:

$$\Im = \{C : C = \{C_g\}_{g \geq 1}, C_g \in \Re^n\}$$

The space of feasible stream of utilities (feasible outcomes):

$$O = \{\alpha : \alpha = \{\alpha_g\}_{g \geq 1}, \textit{with } \alpha_g = u_g(C_g) \textit{ and } C = \{C_g\} \in \Im\}$$

Due to the normalization, $O \subset l_\infty$, then the supreme norm is applicable in O.

Axiom 1-

The welfare criteria W must be represented by a real valued function defined on l_∞ ($W : l_\infty \to \Re^+$), continue, and monotonically not decreasing.

<u>Definition</u>: A K-cutoff (α^K) of $\alpha \in l_\infty$ is such that $\alpha_i^K = \alpha_i, \forall i \leq K$ y $\alpha_i^K = 0, \forall i > K$

A K-tail (α_K) of $\alpha \in l_\infty$ is such that $\alpha_{Ki} = 0, \forall i \leq K$ y $\alpha_{Ki} = \alpha_i, \forall i > K$

<u>Definition</u>: A welfare function $W : l_\infty \to \Re^+$ gives a dictatorial role to the present if W is insensible to the utilities of all generations, but for a finite number of them. Formally: $\forall \alpha, \beta \in l_\infty, \|\alpha\|_\infty \leq 1, \|\beta\|_\infty \leq 1$:

$$W(\alpha) > W(\beta) \Leftrightarrow \exists N(\alpha, \beta) \textit{ tal que } \forall \gamma, \sigma \in \Omega \textit{ se cumple } W(\alpha^K, \gamma_K) > W(\beta^K, \sigma_K), \forall K > N$$

Axiom 2-

The welfare criteria W must not assign a dictatorial role to the present.

Axiom 3-

The welfare criteria W must not assign a dictatorial role to the future.

<u>Definition</u>: **A sustainable preference must hold axioms 1, 2 and 3.**

<u>Definition</u>: A welfare criteria $W : l_\infty \to \Re^+$ will be level independent if the marginal rate of substitution between the utilities of two generations g_1 y g_2 depends only on their identity and not on their respective utility levels.

Formally: $W : l_\infty \to \Re^+$ is level independent if $\forall \alpha, \beta \in l_\infty, \|\alpha\|_\infty \leq 1, \|\beta\|_\infty \leq 1$,

$$W(\alpha) = W(\beta) \Leftrightarrow \exists \lambda \in l_\infty^* \text{ such that } \lambda = \lambda(W) \text{ verifies } \lambda(\alpha) = \lambda(\beta)$$ [7]

Axiom 4-

The welfare criteria W must be level independent.

<u>Definition</u>: **An independent sustainable preference must hold axioms 1, 2, 3 and 4.**

Analysis of some criteria of sustainability

The discounted sum of utilities: a case of dictatorship of the present

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[7] l_∞^* is the space of the real valued linear functions defined on l_∞ (dual of l_∞)

$W: l_\infty \to \Re^+$ is defined by $W(\alpha) = \sum_{g=1}^{\infty} \lambda_g \alpha_g, \forall \alpha \in O$, where $\{\lambda_g\}_{g\geq 1}$ is such that $\lambda_g > 0, \forall g$ y $\sum_{g=1}^{\infty} \lambda_g < \infty$

In this framework, $\forall \alpha, \beta \in O$, $\alpha \succ \beta$ if and only if $W(\alpha) > W(\beta)$.

Because $\sum_{g=1}^{\infty} \lambda_g < \infty$, then $\exists N$ such that $\forall K \geq N$: $\varepsilon > \sum_{g=K}^{\infty} \lambda_g \geq \sup_g (\alpha_g) \sum_{g=K}^{\infty} \lambda_g \geq \sum_{g=K}^{\infty} \alpha_g \lambda_g$, then the K-tail of $W(\alpha)$ results irrelevant $\forall \alpha \in O$, then the present will determine the preference (the future is irrelevant to adopt the decision).

An extension of Rawls's criteria: a level dependence case

In this case, $\forall \alpha, \beta \in O$, $\alpha \succ \beta$ if and only if $\inf_g \{\alpha_g\}_{g\geq 1} > \inf_g \{\beta_g\}_{g\geq 1}$: the stream of utilities chosen is the one that has the maximum infimum (in considering all the generations). The function $\inf : l_\infty \to \Re$ is not lineal because, in general, $\inf(\alpha+\beta) \geq \inf(\alpha) + \inf(\beta)$, then: $\inf(\alpha) = \inf(\beta) \Leftrightarrow W(\alpha) = W(\beta) \not\Rightarrow \exists \lambda \in l_\infty^*, \lambda = \lambda(W)$ such that $\lambda(\alpha) = \lambda(\beta)$. It suffices to consider the following example to verify it. Let's suppose three periods, $\alpha = \{1; -1; 2\}$ and $\beta = \{-1; 1; 0\}$.

It is clear that $W(\alpha) = W(\beta)$, however if a linear transformation $\lambda(x) = a.x + b$ ($a \neq 0$) is applied to both sequences, this leads to: $\lambda(\alpha) = \{a+b; -a+b; 2a+b\}$ y $\lambda(\beta) = \{-a+b; a+b; b\}$, showing that the equality will hold if and only if $a = 0$.

Until now, it seems that the definition of independent sustainable preferences is an empty one. It should be important to show that at least one criterion exists that verifies all previous axioms.

A level independent and sustainable preferences criterion (Chichilnisky)

$\forall \alpha \in O$, the welfare criterion is defined as: $W(\alpha) = \theta . \sum_{g=1}^{\infty} \delta^g . \alpha_g + (1-\theta) . [\lim \inf \{\alpha_g\}_{g\geq 1}]$, with $0 < \theta < 1$ and $0 < \delta < 1$. In fact, this criterion is well defined in O, it is not decreasing, continuous and a positive function because it is a convex combination of continuous functions. It is neither present dictatorial due to the second term in its definition nor future dictatorial because of its first term. Finally, it is a lineal real function in O.[8]

This last defined criterion proves the existence of the level independent and sustainable preferences. The point is that, until now, uncertainty has not been considered, and it is a particular relevant aspect in taking decisions concerning to the long run.

As it was pointed out before, two kinds of uncertainty underlie the environmental problem: i) the preferences of future generations and ii) future available technologies (information) together with economic restrictions and future resources.

The uncertainty faced in environmental problems is the so called "hard uncertainty", the one that is linked to rational innovation, to "strong" learning, to time irreversibility and in

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[8] More details can be find in Chichilnisky, G.: "Sustainable Development and Social Choice"

the way the agent interacts with the frame (the agent is submitted to the environment, but the environment is modified by the agent too).

Consequently, it is natural to introduce the Knightian uncertainty concept to take into account the capital aspects of the environmental decisions.

Sustainability and Expectance "à la Choquet" or The Knightian Uncertainty

Let's suppose that the current utility is taken as a reference point in each path. Future generations may have different preferences and may value differently the natural resources than we do in the present (pure uncertainty). In the case of sustainable development, this consideration and the presence of asymmetry in the distribution of the possible changes in preferences (that makes positive the expected value of postponing consumption) imply that the decision maker (DM from now on) faces a Knightian uncertainty (that can be modeled by capacities). More than this, due the fact that the environmental capital stock depends on preferences, the volume of it to be preserved in each generation is a source of uncertainty too.

In the proximities of the current present (current generation), the DM (contemporaneous) will be risk neutral, because the world will seem to him familiar and known. Beyond the present, far in the future, this world will not seem so familiar. Then, depending on his attitude facing risk (optimistic or pessimistic), will be the type of capacity that the DM will use in moments of computing the Choquet's expected value that will enable him to establish an order over the feasible utility paths and to determine the optimal intertemporal welfare function.

For each path, $\Im$ is defined as the set of all the possible sequences of the states (consumptions) that the DM plans to do in each generation, because a finite horizon is considered $(T < \infty)$, $\Im \subset \Re^T$. Then, the streams of utility considered by the DM will be: $\{u_g\}_{1 \leq g \leq T}$, and, consequently, the order to establish will be in $\Re^T$.

On the other hand, the model is supposed to be sensible to the irreversibility of decisions that are sequentially adopted and to the information that will be revealed as time pass away. Then, the utility considered in each generation will be a function of consumption[9] and of the environmental stock available (this last will reflect the irreversible decisions that had been adopted by previous generations): $u_g = u(C_g, s_g)$, where:

- C_g : consumption corresponding of generation g
- s_g : environmental stock available on generation g

It is supposed that:

- u_g : the utility of generation g, $u_g = u(C_g, s_g)$ (neoclassical assumptions)
- $u_g : \Re^2 \to \Re$ such that $sup\{u_g\}_{T \geq g \geq 1} \leq 1$ (normalization)

[9] The environmental resource will be the only good in the economy: then, consumption will be used in a widely sense

The stock of capital is evolving $s_{g+1} = r(s_g) - C_g + s_g$ [10], where:

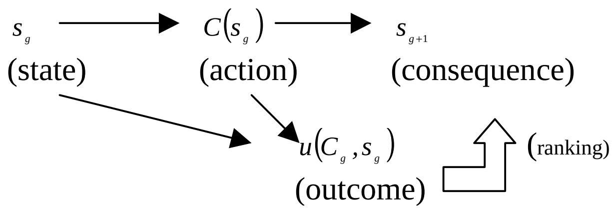
- r: intrinsic renewal of the resource (it is assumed it depends only on s_g and do not change over time[11]) that verifies:
  - $\frac{\partial r}{\partial s_g} \geq 0 \; ; \forall s_g$
  - $\frac{\partial r}{\partial s_g} = 0 \; ; \forall s_g \Leftrightarrow$ the resource is non renewable.

The higher are the consumption level and the environmental capital stock, the higher is the utility level. However, the more is the consumption level, the more is the volume environmental capital depleted.

The considerations formulated concerning *r* imply that consumption can be expressed as: $C_g = r(s_g) + s_g - s_{g+1}$ Then, in controlling consumption today (the action), the DM controls indirectly the environmental stock tomorrow (the consequence). The streams of utility to compute Choquet's expectancy will be defined in the basis of the streams of consumption's levels $\{C_g\}_{1 \leq g \leq T}$. The expectancy will be used in order to define the preferences and the consequences of the different possible policies.

Formally, for each path, let $\Omega = \{s_1, s_2, \cdots s_N\}$ be a non-empty set of states of the nature (environmental stocks today) and let $S = 2^{\Omega}$ the set of all the possible events over which the capacity will be defined. An action (consumption today) will be a function that will assign a consequence (an environmental stock tomorrow) to each state of each path.

The set of feasible actions will be: $\Im = \{C : C = \{C_g\}_{1 \leq g \leq T}, C_g \in \Re\}$. Because we're assuming a finite time horizon ($T < \infty$): $\Im \subset \Re^T$. Comonotonicity of Consumption must hold[12]. The subset of all the comonotonic acts (consumption functions) is denoted by $\chi \subseteq \Im$.



Let's suppose that for each action (path of consumption today), consequences are ranked $s_1^+ \geq s_2^+ \geq \cdots s_T^-$ (remember that $s_{g+1} = r(s_g) - C_g + s_g$), according to their corresponding utilities net of the initial utility level ($u_0 = u(C_0, s_0) = u(s_{0+1})$ the current generation's one)[13].

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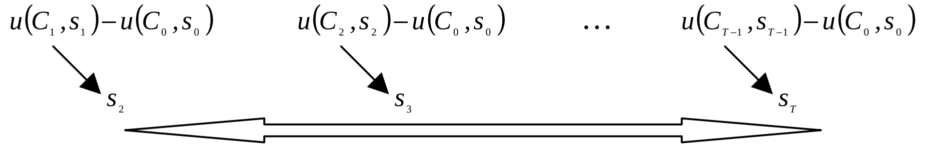
[10] It is assumed that $r(0) = 0$ and that $\exists s_{max} \; s.t. \, r(s) = 0, \forall s \geq s_{max}$

[11] This is a simplifier assumption, because the resilience of the environmental resources (like air, for instance) depends on its previous state, but in the present state of other environmental resources too.

[12] This means that the Consumption's trajectories do not intersect, mathematically:

$$\sim \exists s_g, s_{g'} \quad s.t. \quad C^1(s_g) > C^1(s_{g'}) \Rightarrow C^2(s_g) < C^2(s_{g'}), g \neq 0 .$$

[13] Because of the assumption of comonotonicity, the order can be established in an unique way for all the range of s_{g+1}



environmental stocks tomorrow (consequences' ranking)[14]

Then, the set of consequences is partitioned in three subsets in the following way[15]:

- set of non-familiar gains: $[s_1^+ ; s_i^+]$
- set of familiar gains and losses: $[s_{i+1}^+ ; s_j^-]$
- set of non-familiar losses: $[s_{j+1}^- ; s_T^-]$

It is assumed that the DM is pessimistic with respect to unfamiliar losses (the net utility is negative), neutral with respect to the familiar world and optimistic with respect to the unfamiliar gains (the net utility is positive). The capacity ν that will represent the uncertainty of the DM will be then concave in $[s_1^+ ; s_i^+]$, linear in $([s_{i+1}^+ ; s_j^-]$ and convex in $[s_{j+1}^- ; s_T^-]$.

The windfall gain is defined as $\bar{s} = max\{s : s \in [s_1^+ ; s_i^+]\}$ and the catastrophic loss as $\underline{s} = min\{s : s \in [s_{J+1}^- ; s_T^-]\}$. By the established ranking, $\bar{s} = s_1^+$ and $\underline{s} = s_T^-$.

Let's define ν as a simple capacity if $\nu(A) = \gamma . \pi(A), \forall A \in \Omega , \nu(\Omega) = 1$, where γ is a factor of distortion that corresponds to the degree of confidence that the DM assigns to the weights settled by π (an additive probability distribution function), $\gamma \in [0;1]$. By definition, then, ν (simple capacity) is convex.

Starting from ν, its dual capacity is defined: $\nu'(A) = 1 - \gamma . \pi(\Omega \setminus A), \forall A \in \Omega, \nu'(\Omega) = 1$

Then, formally, the capacity defined in 2^Ω is:

$$\begin{cases} \gamma . \pi(A), \forall A \subseteq \Omega^+ \\ \pi(A), \forall A \subseteq \Omega^{+-} \\ 1 - \gamma . \pi(A^c), \forall A \subseteq \Omega^+ \end{cases}$$

, where $\Omega = \{\Omega^+ ; \Omega^{+-} ; \Omega^-\}$ represents the partition in the space of the states that corresponds to the one established in the consequences space:

$$[s_1^+ ; s_i^+] \cup [s_{i+1}^+ ; s_j^-] \cup [s_{j+1}^= ; s_T^-]$$

It is proved that if the capacity ν exists and the partition above can be settled in the space of consequences, then the expectance "à la Choquet" that represents the hard uncertainty faced by the DM can be expressed as follows:

$$CEU = \gamma . \sum_{k=1}^{k=N} u(s_k)\pi(C_k) + (1-\gamma)[u(\bar{s}) + u(\underline{s})] = \gamma . \sum_{k=1}^{k=N} u(s_k)\pi(C_k) + (1-\gamma)(u_1 + u_T)$$

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[14] If consequences do not correspond exactly to the initial environmental stocks considered in Ω, due to the continuity of the range of consumption's values for instance, a discretization can be established to make them compatible with the states of nature defined previously.

[15] From now on the index on the environmental stocks correspond exactly to the order defined (the biggest stock will correspond to the lowest index and vice versa)

This is the *W* function that will be maximized in this proposed sustainability criterion. The function *W* will be maximized over all the possible paths of consumption.

In considering the axioms defined before, this criterion defines sustainable preferences. In fact, it is well defined: $W:\Omega\rightarrow\Re^{+}$ is continuous and monotonically non-decreasing:

- $u_g = u(s_g)$ is continuous by hypothesis, π is a probability distribution function of a continuous variable and the *min* and *max* are continuous too. Because the product and the composition of continuous function is a continuous function too, W is continuous.

- If the length of the sequence grows and a term bigger than the maximum is added, the sum corresponding of the first term does not decrease because all its terms are greater or equal to zero. As the second term is concerned, the longer is the considered stream, the bigger can be the *max* (and because $u_g = u(s_g)$ is neoclassical, it is not decreasing on its argument). If the term is lower than the minimum, using an analogous argument, the *min* will be the new value, and the corresponding value of *W* will be lower too. Then, the welfare function defined is montonically non-decreasing.

The non-dictatorial role neither of the present nor of the future can be shown considering $\forall i, 1\le i\le T$. There are three possibilities for the corresponding s_i:

- to belong to the familiar world, and so it is considered in the first term of the sum
- to belong to the unfamiliar losses, and so it is considered in computing the $\underline{s} = min\{s : s\in [s^{-}_{J+1}; s^{-}_{T}]\}$
- to belong to the unfamiliar gains, and so it is considered in computing the $\overline{s} = max\{s : s\in [s^{+}_{1}; s^{+}_{i}]\}$

The propriety of level independence is valid too. In fact, the marginal rate of substitution between the utilities of two generations g and g' will depend only on the identification of both generations and not on their utility levels. Then, we are facing preferences that are sustainable and level independent.

An extension: infinite time horizon ($T=\infty$)

The above results are valid if the time horizon is not finite anymore. In fact, assuming the set of the set χ to be compact (close and bounded), and that the utility function is continuous, there is a result[16] that proves that:

$$\int u\, d\nu = \gamma\int u\, d\pi + (1-\gamma)\min_{C_g\in\chi} u(C_g), \text{ if } \nu \text{ is a simple capacity and that}$$

$$\int u\, d\nu = \gamma\int u\, d\pi + (1-\gamma)\max_{C_g\in\chi} u(C_g), \text{ if } \nu \text{ is a dual simple capacity}$$

The continuity of *u* and the compactness of the consumption set[17] warrant that the minimum can be reached. The solely continuity of *u* is in the base of the existence of the integral.

[16] Eicherberg, J. and Kelsey. D, 1999

[17] In fact, $\forall g = 1,2\cdots T$ we have that $0\le C_g \le s(0)(1+r(s_0))^T$

Celina A. Gutiérrez
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