FACULTAD DE CIENCIAS ECONÓMICAS Y DE ADMINISTRACIÓN

IECON INSTITUTO DE ECONOMÍA

UNIVERSIDAD DE LA REPÚBLICA URUGUAY

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INSTITUTO DE ECONOMÍA
Serie Documentos de Trabajo

Mayo, 2022
DT 08/2022

ISSN: 1510-9305 (en papel)
ISSN: 1688-5090 (en línea)

Forma de citación sugerida para este documento: De Rosa, M. (2022) “On Capital: an essay on inequality, capital and value theory”. Serie Documentos de Trabajo, DT 08/2022. Instituto de Economía, Facultad de Ciencias Económicas y Administración, Universidad de la República, Uruguay.

On Capital: an essay on inequality, capital and value theory

Mauricio De Rosa*

Abstract

Capital is back at the center of the empirical distributional research agenda. New estimates of wealth accumulation, distribution and inheritance, fully consistent with national accounts' definitions and deeply rooted in standard neoclassical growth models, are now available. This provides the new inequality literature with clear-cut insights and empirical firepower. But while the empirical flank is increasingly well protected, the theoretical one is exposed. I revisit the debates on the underlying theory of capital and document its drawbacks, highlighting that it is particularly ill-equipped for inequality analysis and that its central problem is the theory of value. Does this mean that we should to start anew? I argue on the contrary, showing that under a one-good model assumption, there is accounting correspondence with the labor theory of value, which gives room for reinterpretation of most available estimates. Moreover, it is possible to establish clear accounting links between famous drivers of the economic system such as r>g and Marx's falling rate of profits. However, even under this accounting correspondence, taking distance from the scarcity theory of value has relevant implications for the inequality narrative, insofar it forces us to abandon the merit-inheritance discussion to include the role of exploitation. Keywords: prenatal visits, weeks of gestation, premature births, vital statistics, household surveys, validation

Key words: capital theory, history of economic thought, wealth inequality, national accounts.

JEL Classification: B24, D46, D31, E01, E13

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Resumen

El capital vuelve a estar en el centro de la agenda de investigación distributiva. Nuevas estimaciones sobre acumulación, distribución de la riqueza y herencia, consistentes con las definiciones de las cuentas nacionales y profundamente arraigadas en los modelos de crecimiento neoclásicos se encuentran ahora disponibles. Esto le otorga a la nueva literatura sobre desigualdad interpretaciones claras y potencia de fuego empírica. Pero mientras el flanco empírico está crecientemente protegido, el teórico está descubierto. En este artículo, reviso los debates sobre la teoría del capital y documento sus inconvenientes, destacando que está particularmente mal equipada para el análisis de la desigualdad y que su problema central es la teoría del valor. ¿Significa esto que debemos empezar de nuevo? Argumento en contrario, mostrando que bajo el modelo de un solo bien, existe una correspondencia contable con la teoría del valor trabajo, lo que permite reinterpretar la mayoría de las estimaciones disponibles. Por otra parte, muestro que es posible establecer vínculos contables claros entre famosos determinantes del sistema económico, como r>g y la ley de la tendencia decreciente de la tasa de ganancia de Marx. Sin embargo, incluso bajo esta correspondencia contable, tomar distancia de la teoría de la escasez como fundamento del valor tiene implicaciones relevantes para la narrativa de la desigualdad, en la medida que nos obliga a abandonar la discusión sobre mérito-herencia para incluir el papel de la explotación.

Palabras clave: teoría del capital, historia del pensamiento económico, desigualdad de riqueza, cuentas nacionales

Código JEL: B24, D46, D31, E01, E13

1 Introduction

Over the past decade, *capital* made an astonishing comeback to the empirical distributional research agenda. Based on novel data and a variety of revised and new methodologies, the recent wealth accumulation, inheritance and inequality results are undoubtedly better estimated than ever before. Most of the new estimates of wealth accumulation and distribution are anchored to the System of National Accounts (SNA) framework, hence fully consistent with internationally accepted definitions. Thus, there is an increasing amount of evidence on wealth distribution (see for instance Saez and Zucman (2016), Alvaredo, Atkinson, and Morelli (2018), and Garbinti, Goupille-Lebret, and Piketty (2017)) and inheritance's flow and stock (Piketty, 2011; Atkinson, 2018), which is at the same time consistent with national wealth estimates (e.g. Piketty and Zucman (2014), Blanco, Bauluz, and Martınez-Toledano (2021), and Bauluz (2019)). Moreover, SNA-based wealth estimates are fully consistent by construction with national income and its distribution, hence providing a full depiction of income and wealth dynamics, both micro and macro-economically. This allows researchers to simultaneously account for variables such as growth, the capital share, the wealth to income ratio, inheritance flows, rate of return and wealth distribution under SNA's framework, endowing the empirical inequality literature with renewed firepower.

These variables and capital itself are not only estimated consistently with National Accounts, but can also be easily linked with standard neoclassical growth models, hence providing the theoretical foundation for the wealth accumulation dynamics (Piketty and Zucman, 2015). While there is no 'unified theory of inequality' (Atkinson and Bourguignon, 2000), for macro-distributional purposes standard neoclassical growth models are the main reference point. These models are admittedly limited, in particular the "one-good, perfect competition model is not a very satisfactory model, to say the least " (Piketty, 2015, p. 81). Nevertheless, they do provide important insights and intuitions on these macro variables and their likely evolution in the future (Piketty, 2014), and have been extensively debated (see e.g. Acemoglu and James Robinson (2015), Jones (2015), and Piketty (2015)). Even if only used as a general reference point, it is still *the* theoretical reference that the vast majority of the empirical literature uses, for better or worse. But is it adequately equipped for distributional analysis? In case it is not, can we do better? To explore these questions is the aim of this short epilogue, in which I ponder on the economic theory that underlies most macro-distributional empirical efforts.

Debates over these broad set of models are not new, as the standard neoclassical growth theory is rooted in extremely questioned assumptions and definitions. To begin with, the very definition of *capital* is problematic and has been subject to a significant amount of controversy (Hodgson, 2014). The last of the great controversies that raged during the 1950s until mid 1980s, i.e. the famous Cambridge Capital controversies, was primarily focused on how to

measure capital. Yet, that was only the corollary: the main issue at stake was the very essence of what capital was (Harcourt, 2014). Kick-started by the call to arms of Joan Robinson's attack on the existence of a production function (Joan Robinson, 1954), it was shown that neoclassical growth theory was unable to provide convincing explanations for the main driving variables of the capitalist system, especially for the rate of return r (Cohen and Harcourt, 2003). In particular, it was shown that once one leaves the one-commodity-model assumption, it is not longer possible to determine the rate of return to capital, hence turning it impossible to provide an explanation for the macro-distribution of income.

Although the exact significance of the overall conclusion was not settled (and nor will I try to do so in this short essay), it was indeed admitted even by the neoclassical side that standard growth models were unable to produce an adequate theory of factor prices, i.e. of income distribution. In the final paragraph of an overview of capital theory, Robert Solow claimed:

> "Very little has been said in this survey about income distribution (in other words, about the determination of factor prices). That is because there is no special connection between the neoclassical model of growth and the determination of factor prices. The usual practice is to appeal to the same view of factor pricing that characterizes static neoclassical equilibrium theory. If the working assumption that all markets clear were to be lifted, an alternative theory of factor prices would certainly be needed. Much else would change besides" (Solow, 2000, p. 378).

During the Cambridge Capital controversies, the debate focused on very specific details of the production function such as *reswitching* and *capital reversing*, which allegedly entailed the impossibility of the determination of r. Yet under the surface, the true underlying difficulty of the neoclassical theory lies in the theory of value on which is founded. The scarcity theory of value can be traced back to the irruption of the Marginalist revolution, which entailed a very different departure point than the previous economic thinkers. Following Cohen and Harcourt (2005), in the classical vision of political economy, the fundamental economic problem is the allocation of surplus output. Social class is the fundamental unit of analysis, and consumption is conceived as an indirect form of exchange for the purpose of satisfying the goal of production. The rate of profits is the rate of self-expansion of capital, the outcome of the accumulation process, and it arises from social relationships in production. In the canonical neoclassical models, the rate of return is in turn just the result of decreasing marginal productivity of capital and households maximizing inter-temporal utility. Lifetime utility-maximizing consumption decisions of individuals are the driving force of all economic activity, for which the allocation of scarce resources is the fundamental economic problem.

Under a one-good model world, these contradictory 'visions' of the fundamental functioning of the economic system may still provide similar conclusions, while true divergence emerge

only once one leaves such an assumption (Cohen and Harcourt, 2005). In fact, most of the neoclassical growth model distributional results are accounting identities, true by definition, either in all settings or in the steady state (Piketty and Zucman, 2015). Nevertheless, even if by assuming simple underlying one-good models the inequality literature manages to dodge the main criticisms to the capital theory, it still remains rooted in a scarcity approach to value and to a narrow view of the economic process. The question that I try to address is whether it is possible to use the classical political economy approach to account for the distributional and growth estimates, given the same one-good model assumption, and if so what can we learn.

There are a number of different alternatives to do this. Cohen (1989) shows that both the classical and neoclassical approaches are associated with "robust results that hold without exception" within one-good models such as Samuelson's surrogate production function (see 3.2) and the neo-Ricardian corn model. Tobón and Ríos (2020) show in turn that long run distributional results from one good-models such as the ones used by Piketty (2014) could be routed funded in post-keynesian models (i.e. with no production function), using as the key link the "Cambridge Equation" (Pasinetti, 1979). I will in turn focus on Marx, since it allows me to better discuss the theory of value implications in a more tractable way.

Marxian insights have been highlighted by many authors who were not themselves Marxists as Veblen, Schumpeter or post-Keynesians such as Sraffa (Bellofiore, 2008) or Robinson (Alves, 2022). Indeed, in her *essay on Marxian economics*, Robinson said that if "the orthodox notion of a definite supply price of capital thus disintegrates upon examination, we are left with nothing but Marx's notion that capital is accumulated and maintained because capitalists are forced to accumulate in order to survive" (Joan Robinson, [1942]1967, p. 61). Following this thread, I explore the accounting links –if any– between the neoclassical macro distributional theory and Marx's labour theory variables (Marx, [1867]1976), in an attempt to better understand their differences and the implications for empirical research. The aim is simply to understand the conditions under which there is *accounting correspondence* between these two approaches.

The starting point is the recognition of the conceptual gaps between similarly named variables. In particular, I contrast the definitions of the rate of return r and Marx's rate of profits P and, more importantly, I discuss the fact that –unlike neoclassical theory– Marx's capital is closer to a flow than to a stock. These two points alone help clarify much of the confusion which often result from these terms. Moreover, I show that under a one-good model assumption, accounting correspondence does exist between available estimates and Marx's labour theory of value, which can in turn be linked with main variables of standard neoclassical growth model and hence to the empirical inequality literature.

After documenting the theoretical differences between the definition of capital and the rate of return (profits), I show that, under a one-good model and closed economy's assumption,

it is possible to interpret both labour theory of value and the new empirical literature in a simple unified accounting framework. Moreover, it is possible to establish the accounting links between famous drivers of the capitalist system such as $r > g$ and Marx's falling rate of profits. This is not surprising, since under a one-commodity model, many different theories find their place. Thus, "while heuristically valuable, the insights one-commodity models provide do not allow us to distinguish between competing theories that view interest or profits as payment for the marginal productivity of capital, or as exploitation of workers" (Cohen and Harcourt, 2005). What I show is that under such a model, $r > g$ and the falling rate of profits yield perfectly consistent results. Specifically, I show that with a stable rate of return r larger than the growth rate g, results in both an increase in the wealth to income ratio and the capital share, as well as in falling rate of profits P, offset at the beginning by the increase in the capital share but falling anyway in the long run.

Schumpeter ([1942]1950) famously discussed different 'Marxs' –the prophet, the sociologist, the economist and the teacher–, I argue that by also considering 'Marx the accountant'[1], there is room to re-interpret important empirical results in light of a theory that better allows to discuss distributional issues. Indeed, Marx's theory of value has at least the merit of being quite intuitive form a strictly accounting perspective (Sweezy, [1942]1970; Shaikh et al., 1997; Bryer, 1999). Labor theory of value is by no means exempt from criticisms, but the case can be made that "adverse judgment or even exact disproof, by its very failure to injure fatally, only serves to bring out the power of the structure" (Schumpeter, [1942]1950, p. 3). Moreover, it is nonetheless true that most of the criticism to the labour theory of value emerges in a multi-sector model, just as in the case of the neoclassical approach. In the best case scenario, this simple exercise provides a bridge with existing empirical literature and classical political economy, including Marx. The main takeaway is that empirical wealth and income inequality literature needs not to get corseted in a restrictive neoclassical framework. However, I argue that while at the one-commodity-model level there is overall accounting consistency between the different approaches, the shift from a scarcity theory of value to a the classical approach entails relevant implications. One immediate consequence is that to the statement that "in all societies, there are two main ways of accumulating wealth: through work or inheritance" (Piketty, 2014, p. 379), it would be necessary to be add exploitation of labour. And that completely changes the narrative.

This essay is organized as follows. Section 2 presents the System of National Accounts' wealth and capital definition and provides an overview of the main neoclassical growth model distributional results, focusing on the steady state determination of r and its gravitating effect in both wealth accumulation, the capital share and the personal wealth distribution. In Section

[1]Marx's accounting has been highlighted by the literature, see Bryer (1999) for a discussion, where he claims that Marx provides a general theory of accounting.

3, the literature on the definition of capital and the main caveats of the neoclassical growth model are briefly discussed, highlighting its distributional implications and tracing them back to the scarcity theory of value. As an alternative, classic political economy and Marx's labour theory of value are presented, showing how it also faces theoretical difficulties once the one-good model assumption is abandoned. The accounting equivalence between the models of sections 2 and 3 is discussed in Section 4. Section 5 concludes the essay and points at possible ways forward.

2 Wealth and capital theory

In this section, the national accounts' definition of wealth is presented, showing its link to the standard production function (2.1), followed by a discussion of the determinants of wealth accumulation and distribution, focusing on the rate of return r and the growth rate g (2.2). In section 2.3, the steady state determination of the key variable r in the standard neoclassical model is briefly summarized.

2.1 Wealth and capital from a National Account's perspective

The departure point of this analysis are the standard definitions of wealth and capital. The 'handbook definition' presents a powerful and coherent framework for its analysis, rooted in the general definitions of the System of National Accounts (SNA). Based on SNA balance sheet's definitions, private wealth W_t is defined as the net wealth (assets minus liabilities) owned by households (United Nations, 2008; WIL, 2021). These assets include "all the nonfinancial assets—land, buildings, machines, etc.—and financial assets—including life insurance and pensions funds—over which ownership rights can be enforced and that provide economic benefits to their owners" (Piketty and Zucman, 2015, p. 1309). Corporations are included in private wealth through the market value of equities and corporate bonds. Private wealth can be decomposed in housing assets, business assets (and other non-financial assets), financial assets and liabilities, while National wealth W_{nt} results from the addition of private and public wealth. It is also equivalent to to the sum of domestic capital and net foreign assets, as depicted in equation 1.

$$W_{nt} = W_t + W_{gt} = K_t + NFA_t \tag{1}$$

W_{nt}, W_{gt} and W_t represent net national, public and private wealth respectively, K_t domestic capital and NFA_t net foreign asset position. As for the second equivalence it is interesting to note that, intuitively, as all national financial assets and liabilities must cancel out (including the property of corporations), national wealth W_{nt} is equivalent to the sum of all non-financial

assets owned by household, corporate and government sectors, plus the net foreign asset position.

With Y_t being the national income, the private and national wealth to income ratios (β_t and β_{nt}) are hence defined as:

$$\beta_t = \frac{W_t}{Y_t}, \beta_{nt} = \frac{W_{nt}}{Y_t} \tag{2}$$

National wealth W_{nt} and domestic capital K_t have different magnitudes only if the net foreign asset position NFA_t is not zero; otherwise they are equivalent. In fact, in a closed economy or at the world level, where $NFA_t = 0$ by definition, aggregate wealth and capital coincide. In such a setting, capital and wealth are interchangeable, and not only $\beta_{nt} = \beta_t$, but also they are equivalent to the capital - output ratio K_t/Y_t. This is the case because $W_{nt} = K_t$, but also because output Y_{dt} is equivalent to net national income Y_t, given that $Y_t = Y_{dt} + r_t \cdot NFA_t$.

$$Y_t = Y_{dt} = F(K_t, L_t) \tag{3}$$

The capital - output ratio is important, since it can be directly be traced back to the production function, as in equation 3 (which assumes a closed system), where L_t is the aggregate labour input (Piketty and Zucman, 2015). This capital-output ratio has been one of the main concerns of growth theory, since the early attempts to model growth (Harrod, [1939]1972; Domar, 1946), and it is further discussed in the following section.

2.2 Growth, capital and inequality theory

When modelling the economic activity with a production function such as in equation 3, i.e. assuming a one-good model, some important steady-state conclusions may be easily drawn. First, in this one-good model setting, wealth growth is $W_{t+1} = W_t + S_t$, given S_t being the aggregate net savings rate. Considering equation 2 and noting that $s_t = S_t/Y_t$, we have:

$$\beta_{t+1} = \frac{\beta_t + s_t}{1 + g_t} \tag{4}$$

From equation 4, it is straightforward to show the steady state formula for β, with $g_t \to g$ and $s_t \to s$:

$$\beta_t \to \beta = \frac{s}{g} \tag{5}$$

This is a Harrod-Domar-Solow-Swan result (see section 2.3), which is an accounting definition that holds in the steady state of any micro-founded, one-good model of capital accu-

mulation, independently of the exact nature of saving motives (Piketty and Zucman, 2015).[2] Thus, independently from the long-run fundamentals of s and g, it is true by definition that β will increase the higher s and the lower g. In particular, in a low growth context, the amount of wealth (or capital) relative to income will be higher, which is the case for rich countries from 1970 onward (Piketty and Zucman, 2014).

The level of capital relative to output (or wealth to income) has a direct effect on the functional distribution of income. Again by construction, we have that the capital share of the economy α_t if given by:

$$\alpha_t = r_t \cdot \beta_t \tag{6}$$

where r_t is the average rate of return of the economy. In the standard neoclassical framework, r_t is equivalent by its marginal product (more below). Thus, considering a general and relatively flexible Constant Elasticity of Substitution (CES) production function, where $Y = F(K, L) = \left(a \cdot K^{\frac{\sigma-1}{\sigma}} + (1-a) \cdot L^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma-1}{\sigma}}$, then r_t is given by $r = F_K = a\beta^{-1/\sigma}$, being σ the capital-labour elasticity of substitution. As r_t inversely depends on β, which results from an inverse relation in the quantity of capital K_t and its price r_t the direction of changes in α_t will depend on the magnitude of the variations, which is given by σ.

$$\alpha = a \cdot \beta^{\frac{\sigma-1}{\sigma}} \tag{7}$$

Thus, if $\sigma > 1$ then an increase in β_t will result in a less than proportional decrease in r_t, hence pushing α_t upward. Naturally, the inverse is true when $\sigma < 1$, while $\sigma = 1$ results in the special case of the Cobb-Douglass production function, where changes in K_t (β_t) are exactly neutralized by opposite changes in the marginal product of capital (which equals r_t), hence leaving the capital share α_t unaffected.

Finally, it can be shown that these macro-results have relevant implications on micro (i.e. personal) distribution of wealth. As Piketty and Zucman (2015) show, in a wide variety of models, the concentration of wealth increases as the gap between r and g widens. Thus, as famously pointed out by Piketty (2014) and further discussed in Piketty (2015), if the inequality $r > g$ holds for sufficiently long periods, wealth concentration is likely to increase.

Piketty and Zucman (2015) show that all models with multiplicative random shocks in the wealth accumulation process give rise to distributions with Pareto upper tails.[3] Being z_{ti} the

[2] Authors show that these general conclusions hold in a variety of micro-funded general equilibrium models, where s is endogenous and depends on g Piketty and Zucman (2015, p. 1344-1347).

[3] They also present an example with closed form formulas, in which individual utilities are modeled as a Cobb-Douglas function, while the production function is a CES. The results represent a particular case of this general one.

normalized individual wealth[4] and ω_{ti} a i.i.d multiplicative shock with mean $Q = E(\omega_{ti}) < 1$ and ε_{ti} an additive shock, we have:

$$z_{t+1i} = \omega_{ti} \cdot z_{ti} + \varepsilon_{ti} \tag{8}$$

Aggregating individuals modelled as in equation 8 results in a distribution with a Pareto upper tail with a coefficient a, which must solve $E(\omega_{ti}^{a}) = 1$. Thus, for a given average $\omega < 1$, as the variance of the shock increase (goes to infinity), wealth concentration also does ($a \to 1$).

Regardless of the nature of the individual-level shocks, the shape of the upper tail (i.e. a) depends on $r - g$. Assuming a generation length H (e.g. 30 years) and each period of the model as lasting this long, ω can be written as (with r and g instantaneous rates)

$$\omega = s \cdot \frac{1+R}{1+G} = s \cdot \mathrm{e}^{(r-g)H} \tag{9}$$

with $1+R = \mathrm{e}^{rH}$ and $1+G = e^{gH}$ being the generational return and growth rates. Under binomial shocks, the inverted Pareto coefficient is $b = (\log(1/p))/(\log(1/\omega))$. This coefficient is thus extremely sensitive to variations in ω and therefore in $r - g$, which entail substantial concentration increases relative to minor $r - g$ upturns.

2.3 The steady-state and the role of r

The discussion of the steady-state is important since it allows us to distinguish the main forces at play under the neoclassical growth model. To be sure, it is a useful abstraction but does not necessarily reflect real world economies, especially in the short run (Piketty and Zucman, 2014; Piketty and Zucman, 2015). However, it may provide insights as to what may happen in the 21st century, provided there are no massive shocks as the ones of the previous one (Piketty, 2014).

The models briefly summarised in section 2.2 show that the gap $r - g$ is central to understanding the main macro and micro-distribution of capital, especially after a long-run cumulative process. In particular, it was shown that a low growth rate g result in high wealth to income (or capital-output in closed systems) ratio β. In that setting, given a capital-labour elasticity of substitution of $\sigma > 1$, the rate of return r decreases less than proportionally to the increase in β, resulting in a higher capital share α. Moreover, the gap $r - g$ produces thick-tailed wealth distributions under a wide variety of models.[5]

[4] The model assumes a stationary population $N_t = [0, 1]$, made of continuous agents, which results in equivalent aggregate and average wealth and income variables, i.e. $W_t = w_t$ and $Y_t = y_t$. From there, it z_{ti} is defined as $z_{ti} = {}^{w_{ti}}/_{w_t}$.

[5] This general distributional conclusions have been questioned by a number of scholars. For an interesting discussion see Jones (2015), and for a not-so interesting one see Acemoglu and James Robinson (2015).

The growth rate g is probably one of the most studied macro economic variables of all times, and has been modeled in a variety of ways.[6] In the basic Harrod-Domar model (Harrod, [1939]1972; Domar, 1946), output is equal to the sum of savings S_t and consumption C_t from an accounting perspective, so $Y_t = C_t + S_t$. It then follows that output may be expressed as the sum of consumption plus investment I_t, so $S_t = I_t$. As by definition total capital is the result of the interaction between investment and depreciation, we get $K_{t+1} = (1 - \delta)K_t + I_t$, and thus is the capital-output the one depicted in equation 5.[7] In this model, the savings rate is of paramount importance as it determines investment and growth, but is assumed to be exogenous, and if it is sufficiently high, it may lead to an extremely high level of β.[8]

The Solow-Swan version of the model (Solow, 1956; Swan, 1956) makes β endogenous, by introducing profit-maximizing firms facing a production function with constant returns to scale but diminishing returns to each factor (especially to capital), which "chokes-off per-capita growth in the steady-state" (Ray, 1998). As a result, in such a model long-run growth rate is the result of population growth n and a (exogenous) productivity growth component h so that $g = n+h$.[9] Moreover, in this simple model the rate of return, which is equivalent to the marginal product of capital, is equal to the growth rate of the economy. This is known as the *golden rule* of capital accumulation (Phelps, 1961). It essentially says that if equal weight is given to present and future generations, capital will be accumulated (and hence output increased) until $r = g$, so that each generation's consumption is the same and the highest possible for all t.

In the Harrod-Domar-Solow-Swan models, the savings rate s_t is exogenous, which is a major drawback. Following the seminal contributions by Ramsey (1928), in the Cass-Koopmans version of the neoclassical model (Koopmans, 1965; Cass, 1965), an infinite-horizon household who maximizes dynastic consumption is also introduced into the model, providing an endogenous savings rate. Firms will use capital until its marginal productivity equals r, which is in turn set by the interaction of this demand with the supply price of capital, resulting form a combination of the reward of waiting and the risk of lending capital. This is the essence of the *modified golden rule*, which results from the steady state equilibrium of profit-maximizing firms and inter-temporal consumption-maximizing households (Barro and Sala-i-Martin, 2004)[10]:

$$r = \theta + \gamma g \tag{10}$$

[6] See section 1.4 of Barro and Sala-i-Martin (2004) for a brief summary.

[7] In the standard model, it is expressed as ratio is $\beta = {}^{s}/_{g+\delta}$, therefore it is important to bare in mind that equation 5 is net of depreciation

[8] It is important to note, at this point and considering what follows, that the original Harrod-Domar's macro-dynamic model actually confronted with the dominating Marginalist mainstream (Pasinetti, 1983).

[9] If $L_t = N_t \cdot h_t$, with $N_t = N_0 \cdot (1+n)^t$ and $h_t = h_0 \cdot (1+h)^t$, then $1 + g = (1+n) \cdot (1+h)$, i.e., $g \approx n + h$ (Piketty and Zucman, 2015).

[10] The notation is not the same: it was adapted for consistency reasons.

where γ is the curvature of the utility function, and θ is the rate of time preference. The model requires that the transversality condition that (net of depreciation) return rate r is higher than the growth rate g, so that household can follow an optimal inter-temporal consumption path given their relative impatience θ. This is argued to be one possible explanation of the relative stability of the rate of return in the long run, which might be interpreted as an interval of psychologically plausible time preference parameters (Piketty, 2015). The return rate r in turn determines the level of capital, since it is accumulated until its marginal product equals the rate of return. The key to determine capital per-capita k is hence the diminishing return's assumption.

Thus, under this general neoclassical growth model, the inequality $r > g$ is a condition for inter-temporal efficiency: g and r are determined by population and productivity increases on the one hand, and the interaction of profit-maximizing firms with diminishing returns to capital and infinite-horizon consumption maximizing households on the other. However, the determination of the rate of return is possible only within a number of highly restrictive assumptions. Moreover, while these assumptions may work in a one-good-model setting, it has been extensively argued that they do not hold under slightly more realistic assumptions. Indeed, once we leave the one-good model, it becomes essentially impossible to theoretically determine r, hence substantially compromising the theoretical foundations of the neoclassical inequality predictions. This is discussed in the following section.

3 Capital, distribution and value theory

In this section, controversies over the definition of capital are briefly summarized (3.1), in order to introduce the Cambridge capital controversies and the impossibility of determining r outside the one-commodity model (3.2). Section 3.3 discusses to what extent this is the result of the underlying scarcity theory of value, as well as the classical political economy alternative; while 3.4 focuses on Marx's theory of value, presenting the basic outlay of his value theory, necessary for the discussion of the accounting equivalence of section 4.

3.1 A controversial definition

In section 2, I presented what appears to be the growing consensus on how to measure wealth (and capital) accumulation and distribution, which anchors the main definitions in the System of National Accounts, providing a number of important advantages discussed above. However, these definitions have been the object of an recurring debate (Hodgson, 2014; Cohen and Harcourt, 2003). Are capital and wealth really interchangeable? Is capital a sum of money or a bunch of stuff? Is it an accounting magnitude, a historically-specific process or the reflect of

investment's time-structure? The discussion goes far beyond the definition, as it is both a manifestation and a driver of our understanding of what is valuable, how that value is distributed, what is the underlying justification for such a distribution and the very nature of the economic system itself. The sheer number of heavy-weight scholars who dedicated time and effort in engaging in this two-and-a-half century long debate is telling of its scope and relevance. The sometimes confusing nature of the debate, especially in the twentieth century reveals that, as Solow bluntly said, "there is also an intrinsic reason for the controversial character of capital theory: it is very complicated and very difficult" (Solow, 1963, p. 11).

Wealth and Capital

As discussed in section 2.1, although capital and wealth are not equivalent in the standard definition, under relatively general assumptions (i.e. that the Net Foreign Asset Position is zero[11] or at the world level), they may well be considered interchangeable in the national balance sheet. However, there is room for differentiating capital and wealth from a SNAs perspective. Following WIL (2021), such distinction can be traced back to the sequence of non-financial accounts, since operating surplus and mixed income represent the flow of income accruing to capital stock-owners, while property income is received by owners of financial or non-produced assets. This separates assets used in production (capital) from other assets such as equity, bonds or land (rest of wealth). Under this perspective, wealth includes capital but also other non-produced assets.

Distinguishing capital and wealth based on its use in production is possible but potentially tricky, since many assets can be used for more than one purpose. For instance, some buildings may have residential uses, while others used as offices or storage facilities, and there are many similar examples, for which the all-encompassing accounting wealth definition may be less ambiguous and more pragmatical. Considering wealth and capital simply as the sum of all the (net) assets that may be bought or sold, presents the enormous advantage of dodging the need to discuss the nature of each of them (Piketty, 2014).

Nevertheless, it may be argued that it is important to keep capital and wealth as distinct concepts. While it is clear that wealth generates an income flow accrued by wealth holders, allowing consumption smoothing when income declines (Davies and Shorrocks, 2000), and more importantly contributing to shape income distribution, the notion of the power dimension of wealth is very relevant. As Atkinson pointed out, "wealth is important because it gives not only income (interests, dividends and rent) but also security, freedom of maneuver, and economic and political power" (Atkinson, 1973, p.239). The power dimension of wealth has a long-standing tradition in economics, which can be traced back to Adam Smith, who famously claimed that on top of being a source of a stream of income, wealth was a synonym of power, which was in

[11] Which is actually quite close to reality, at least for most developed countries (Piketty and Zucman, 2014).

turn closely linked with his theory of value.

> "Wealth, as Mr. Hobbes says, is power. (...) The power which that possession immediately and directly conveys to him, is the power of purchasing; a certain command over all the labour, or over all the produce of labour (...) the exchangeable value of everything must always be precisely equal to the extent of this power which it conveys to its owner" (Smith, 1776, p.51).

It is precisely the power dimension which may be behind the conceptual difference between wealth and capital, since again following Atkinson "wealth is now quite widely distributed, but much of the wealth that people own conveys little or no control over the productive activities of the economy beyond their own front door" (Atkinson, 2015, p. 95). In marxian terms, it is not only that capital is power, but it is a very specific type of power also linked to the theory of value, which in the case of labour theory of value is related with surplus labour (see 3.4). On Smith's notion, Marx says that "capital, therefore, is not only the command over labour, as Adam Smith thought. It is essentially the command over unpaid labour. All surplus-value, whatever particular form (profit, interest or rent) it may subsequently crystallize into, is in substance the materialization of unpaid labour-time. The secret of the self-valorization of capital resolves itself into the fact that it has at its disposal a definite quantity of the unpaid labour of other people" (Marx, [1867]1976, p. 672).

Money and Stuff

Marshal, building on Adam Smith, presents a definition of aggregate capital that is closer to the one discussed in section 2. When considering capital from an aggregate perspective, he says that "this brings us to consider the use of the term capital from the point of view of inquiries into the material well-being of society as a whole. Adam Smith said that a person's capital is that part of his stock from which he expects to derive an income. And almost every use of the term capital, which is known to history, has corresponded more or less closely to a parallel use of the term Income: in almost every use, capital has been that part of a man's stock from which he expects to derive an income" (Marshall, [1890]2009, p. 53). He linked capital to income producing assets, excluding land, which is related to the fact that he, like the classics, is thinking of the tree main classes associated with capital, labour and land. It is interesting to note that Marshall clearly had in mind the link between capital and national income, which is behind National Accounts' definition (see section 3.1) and is the backbone of wealth distribution methodologies such as the capitalization method (Saez and Zucman, 2016).[12]

If Marshall represents a bridge between classical and neoclassical theories, it is with Fis-

[12]He makes it explicit by stating that "labour together with capital and land thus defined are the sources of all that income of which account is commonly taken in reckoning up the National Income" (Marshall, [1890]2009, p. 54-55).

cher that capital is really placed within neoclassical walls and acquires the features that we now attribute to the canonical capital theory. In probably one of the most widely read economics handbooks of the twentieth century, Samuelson and Nordhouse (2009) argue that the modern idea of the capital stock (and interest rate) being determined by the the interplay between psychological preferences and diminishing marginal returns presented in section 2.3 is attributable to Fischer (1906) and Fischer (1907). This locked capital theory well within the roam of scarcity theory of value (see section 3.3), hence away from any association between capital, power and class. The controversies, however, did not stop and returned over and over again from the early XXth century to mid 1980s: Fischer himself was a part of a major controversy with Böhm-Bawerk, Clark and Veblen at the turn of the 1900s, followed by Hayek and Knight, who battled on during the 1930s, ending up with the train's collision of the Cambridge Capital Controversies, which went on for a solid 30 year period which started in the 1950s. I will turn to some of the issues raised in the reminder of this section and the following ones.

Following Cohen and Harcourt (2005), most of capital controversies originate in the dual nature of capital. i.e. whether it is a set of items used in production or an sum of money. The classics, beginning with The Wealth of Nations (Smith, 1776, ch. 9), talked about the 'stock' when referring to capital. Hodgson (2014) complains that it was with Smith that the term capital was corrupted forever, abandoning the use given for centuries by businessmen, for whom capital meant a collateral, and replaced it for 'stuff'. However, in the classics the idea of capital is also associated with a 'fund' that the capitalists advance to begin production and which also includes wages (Dobb, [1937]1961).

For Marx, the discussion of what capital is is inseparable from the theory of value. It is related to the process of creation and appropriation of surplus value, and as we will discuss below and has been extensively argued, it is a process, not a thing. As such, it can be either money or commodities, depending on the purpose of its use –as opposed to its nature– and on which stage of the Money-Commodity-Money cycle one is located (see 3.4).

> "If we pin down the specific forms of appearance assumed in turn by self valorizing value in the course of its life, we reach the following elucidation: capital is money, capital is commodities. In truth, however, value is here the subject of a process in which, while constantly assuming the form in turn of money and commodities, it changes its own magnitude, throws off surplus-value from itself considered as original value, and thus valorizes itself independently. (...) By virtue of being value, it has acquired the occult ability to add value to itself. It brings forth living offspring, or at least lays golden eggs" (Marx, [1867]1976, p. 255).

It is interesting to note that, while capital may well be a sum of money in Marx's view, or even the net worth of a firm's balance sheet, a pure accounting definition –i.e. the sum of owned net assets– is not enough to define capital.[13] However, as pointed out by Paul Sweezy,

[13] "If I state, like for example Say, that capital is a sum of values, then I state nothing more than that capital

capital is for Marx essential one-dimensional, i.e. "'capital' is not simply another name for means of production; it is means of production reduced to a qualitatively homogeneous and quantitatively measurable fund of value. The concern of the capitalist is not with the means of production as such, but with capital, and this necessarily means capital regarded as a quantity, for capital has only one dimension, the dimension of magnitude" (Sweezy, [1942]1970, p. 338).

In his *Essentials of Economic Theory*, Clark ([1907]2007) distinguished 'pure capital' and 'capital goods', the first referring to the value of all capital and the second to the actual equipment. Veblen (1908), debating Clark but also Fischer, questioned them for their use of 'capital value' and 'capital'[14], which are "conceptually distinct, tho substantially identical". He stresses their contradictions (e.g. when discussing the transfer of capital, which makes sense as value but not necessarily as goods), but also the fact that they do not include 'intangible assets' or 'immaterial wealth', which are a part of pecuniary definitions of capital. He has the classic's and Marx's notion of power in mind, when he claims that owners are able to "corner the wisdom of the ancients and the accumulated experience of the race" (Veblen, 1908, p. 154), and that this explains why any "natural law" of the share of capital in product is misleading.

> The returns actually accruing to him under competitive conditions would be a measure of the differential advantage held by him by virtue of his having become legally seize of the material contrivances by which the technological achievements of the community are put into effect. (Veblen, 1908, p. 167)

A few years later, the debate re-emerged, this time with Knight defending the neo-classical position and Hayek attacking it from the Austrian perspective who, unlike his predecessor Böhm-Bawerk, did not considered capital as heterogeneous items but as a fund of value that is 'malleable and perpetual' (Hodgson, 2014).[15] But it was in the 1950 where these long-standing controversy took-off and acquired epic proportions. Joan Robinson wrote an incendiary article which questioned the definition of capital behind which neoclassical economists had been barricaded. She famously went after the neoclassical production function, such as the one presented in equation 3 of section 3.1, arguing at the very start of her piece that:

> "The production function has been a powerful instrument of miseducation. The student of economic theory is taught to write $O = f(L, C)$ (...). He is instructed to assume all workers alike, and to measure L in man-hours of labour; he is told something about the index-number problem involved in choosing a unit of output; and then he is hurried on to the next question, in the hope that he will forget to ask in what units C is measured. Before ever he does ask, he has become a professor, and so sloppy habits of thought are handed on from one generation to the next" Joan Robinson (1954, p. 81).

= exchange value. Every sum of values is an exchange value, and every exchange value is a sum of values. I cannot get from exchange value to capital by means of mere addition. In the pure accumulation of money, as we have seen, the relation of capitalizing [Kapitalisieren] is not yet posited"(Marx, [1857-58]1997).

[14]These are the terms used by Fischer, in Clark's writings they are capital and capital-goods.

[15]For a survey of the Hayek-Knight debate, see Cohen (2003).

This triggered a sequence of offensives and counter-offensives with no evident victor (Cohen and Harcourt, 2003), but with potentially important consequences anyway, which will be discussed in section 3.2.

Time and History

To discuss the theoretical implications of the above-mentioned heterogeneity of capital goods, it is necessary to understand the time-dimension of capital. Capital is intrinsically interlinked with time since (i) capital is invested in a production process that takes time; (ii) capital is consumed within one or several periods and (iii) the income stream resulting from capital investments are spread through several periods. This time-dimension is at the base of the *Wicksell effects*, which when interacted wit heterogeneous capital goods creates serious problems for the theoretical determination of the rate of return r, as I will discuss in section 3.2. The dimension of time, and the related but conceptually different of change throughout time and more generally of history, where an important part of the capital controversies.

In the classical tradition, the only important distinction in capital is precisely the one between circulating and fixed capital, i.e. the part of capital that lasts more than one period and the one that is consumed within the production process. In Mill's words, the former is the one "which fulfills the whole of its office in the production in which it is engaged, by a single use", and the latter "exists in a durable shape and the return to which is spread over a period of corresponding duration" (Mill, [1848]1900). In the second book of Capital, Marx argues against this distinction, since for him conceals the true origin of value, but also discusses capital's rotation period, i.e. the time it takes for an investment produce surplus value and how it affects the rates of profit (more on this in 3.4).

It was the Austrians who used the concept of time in the most radical way, merging it with capital in such a way that capital was time, reaching its most developed form in Hayek ([1941]2009). The idea was that profits were a product of time itself and the different investment's time-structure was central for the Austrians, although heavily criticized. Solow for one claimed that "I think the Austrian school overdid the identification of capital itself (and capital theory) with time –it was an inspired simplification that didn't work– but the need for a theory of capital does arise only when we try to take account of production processes which involve time in some essential way (Solow, 1963, p. 11)."[16]

But the dynamic effect was also big part of the assault to the neoclassical definition of capital, especially during the more advanced stages of the Cambridge Capital Controversy. Post-Keynesians such as Joan Robinson criticized capital and growth theory in general from the methodological standpoint, arguing against the very idea of an equilibrium and, more

[16]On Hayek's Pure Theory of Capital, Mirowsky said that it was "a deeply flawed attempt to explicate the Austrian theory of capital using simple deterministic geometric models" (Mirowski, 2002).

importantly, that a series of equilibrium can adequately capture the path between them. She set a distinction between logical time and historical time, which was rooted in her readings of Marx (Alves, 2022). In Robinson's words, "the problem of the 'measurement of capital' is a minor element in the criticism of the neo-classical doctrines. The major point is that what they pretend to offer as a alternative or rival of the post-Keynesian theory of accumulation is nothing but an error in methodology—a confusion between comparisons of imagined equilibrium positions and a process of accumulation going through history" (Joan Robinson, 1974, p. 213).

Finally, the historical specificity of capital is another dividing line between competing theories. Marx is one of the main advocates of this view, arguing that capital is not only "the all-dominating economic power of bourgeois society" (Marx, [1857-58]1997), but also that is specific to it. Thus, there is no capital outside capitalism –neither theory of value (Sweezy, [1942]1970). This notion of capital inextricably linked with the theory of value and hence specific to the capitalist mode of production is quite different from the current consensus, which focus on things that can be owned and sold rather than in the social relations of production. Thus, both financial assets and slaves can be considered as part of the evolution of aggregate capital and compared through time (Piketty and Zucman, 2014). However, this does not seem to represent a rupture with the classical political economy prior to Marx, which also considered capital as things that were useful for production regardless the historical phase. Capital hence existed in all places and times, since Smith's "rude state of society" (Smith, 1776, ch. 6). As Ricardo made explicit, "even in that early state to which Adam Smith refers, some capital, though possibly made and accumulated by the hunter himself, would be necessary to enable him to kill his game" (Ricardo, [1817] 2001, p. 17). The recourse to the rude state of society is typical of classical political economy but survived it, very clearly so for instance since the early chapters of Clark ([1907]2007), but has been repeatedly criticized. For instance, when analyzing Clark's work, Veblen says, emphasizing by the way the role of immaterial assets:

> "The best excuse that can be offered for these excursions into 'primitive life' is that they have substantially nothing to do with the main argument of the book, being of the nature of harmless and graceful misinformation. (...) The 'capital' possessed by such a community-as, e.g., a band of California 'Digger' Indians-was a negligible quantity, more valuable to a collector of curios than to any one else, and the loss of which to the 'Digger' squaws would mean very little. What was of 'vital concern' to them, indeed, what the life of the group depended on absolutely, was the accumulated wisdom of the squaws, the technology of their economic situation (...) growth contemplated by Mr. Clark. The 'natural' system of free competition, or, as it was once called, 'the simple and obvious system of natural liberty', is accordingly a phase of the development of the institution of capital; and its claim to immutable dominion is evidently as good as the like claim of any other phase of cultural growth" (Veblen, 1908, p. 152-154).

3.2 The Cambridge capital controversy and the determination of r

In the previous section, a number of issues concerning the definition of capital were discussed. Two of them, i.e. the fact that capital is actually composed of very heterogeneous items, and the fact that it operates and that 'produces' income throughout time, creates theoretical problems for its valuation, and (perhaps more importantly) for the determination of r. This was only one of the vast and sometimes confusing number of issues debated during the Cambridge Capital Controversies, but since it is the key for the present discussion, I will focus on it. However, it is worth stressing that the topics covered were enormous and have been extensively surveyed. The participants included scholars such as Joan Robinson, Luigi Pasinetti, Pierangelo Garegnani and Piero Sraffa on one side, versus Paul Samuelson, Robert Solow, Christopher Bliss and Frank Hahn on the other, in a debate that, for all practical purposes, seems to have been the last time the classical political economy tradition offered and organized defense.

The problem is best explained recurring to 'Samuelson's parables'. While still arguing that capital theory could actually be developed assuming capital is composed of heterogeneous items, i.e. without recurring to the "Clark-like concept of aggregate 'capital' " (see section 3.1), Samuelson (1962) offered a "simple neoclassical capital models in a rigorous and specifiable sense can be regarded as the stylized version of a certain quasi-realistic MIT model of diverse heterogeneous capital goods' processes" (Samuelson, 1962, p. 201-202). He used surrogate production functions or *as if* productions functions, which predict how complex heterogeneous capital models can work as if they had come from a simple generating production function.[17] Based on this approach, he produced the famous three parables of the neoclassical production function[18], which are: (i) the real return on capital is determined by diminishing marginal productivity of capital; (ii) a greater quantity of capital leads to a lower marginal product of additional capital and thus to a lower rate of interest, and the same inverse, monotonic relation with the rate of interest also holds for the capital/output ratio; (iii) the distribution of income between laborers and capitalists is explained by relative factor scarcities and marginal products.

These parables are true under the surrogate production function or, what is by all practical purposes the same, under a one-commodity model. However, they do not hold once the one-good assumption is lifted. The basic problem is that once one considers heterogeneous capital goods, they need to be valuated in order to be aggregated and incorporated in the model. Valuation, in turn, is interlinked with time as discussed in section 3.1, since whether value is measured as cost of production or as present value of future output, time is involved. Thus, one needs the interest rate to perform the valuation, but the interest rate is itself endogenous, i.e. is the result of the quantity of 'capital'. There is an inherent circularity, by which one

[17] As Samuelson says, Nicholas Kaldor –making fun of his own terminology– wrote to him that he was trying to pretend Clark could be defended as a 'stylized Samuelson'.

[18] It is interesting that Samuelson claims that, at this point, they should be called 'neo-neo-classical'.

needs the interest rate to determine the interest rate, or the rate of return r for that matter. This problem is behind *reswitching* and *capital reversing*, which were two of the main technical objections to the neoclassical production function and capital theory in general.

Reswitching occurs when the same physical capital/labour ratio is preferred at two or more interest rates, while a different one is preferred at intermediate ones, therefore switching to a different technique (capital/labour) and then reswitching back. Capital-reversing implies that a lower capital/labour ratio is associated with a lower interest rate, which is equivalent to having lower interest rates when capital is more scarce, and thus an upward slope demand curve for capital. Reswitching thus violates parables 1 and 2, and capital-reversing violates parables 2 and 3. The main takeaway is that changing quantities are no longer associated with unambiguously signed price effects.

Samuelson (1966) provides a toy example, which in his words "tells more simply the full story of the twenty-fifth and eighth degree polynomials of the Sraffa-Pasinetti example of reswitching", in which Champagne is produced using only labour and time. When different amounts of labour are organized in different time arrangements (in the example, 7 labour units in $t-2$ versus 2 labour units in $t-3$ and 6 in $t-1$) to produce the same output, the cost-minimizing technique in not unambiguously determined. At either very high or very low interest rates, the first technique is chosen, while for intermediate ones the second one is preferred. This problem crates the theoretical possibility for multiple equilibria, which make the one-way scarcity principle useless to explain r and thus the income distribution. Moreover, reswitching "leads to a pattern of capital/output ratio that fails to move in on direction". This is very important in the context of the our discussion, since these patterns are precisely the foundation of the model that determines r, which is turn critical to explaining macro and micro income distributions, as discussed in section 2.3.

Naturally, there were a number of replies and counterattacks, which were inconclusive (Cohen and Harcourt, 2003) and are beyond the scope of this essay. The key point is that a relevant and substantial critique was delivered to the neoclassical model, which especially affects its ability to unambiguously determine key distributional variables such as r, α and β presented in 2.2. More importantly, these points were taken by the neoclassical side, with unequivocal clarity in most cases (see e.g. Solow (2000)). As Samuelson himself pointed out in his 'summing up', "if all this causes headaches for those nostalgic for the old time parables of neoclassical writing, we must remind ourselves that scholars are not born to live an easy existence. We must respect, and appraise, the facts of life" (Samuelson, 1966, p. 583).

While important at the moment and for my purpose in this essay, these technical issues were only at the surface of the debate, which in reality was about deeper concerns by the critiques of the neoclassical school. In the end, for people such as Joan Robinson, it mattered more what capital was than its corollary, i.e. how to measure it (Harcourt, 2014). To dive into

that level, one needs to address the underlying problem of the theory of value, which is behind the theory of capital and of our whole understanding of the functioning of the economy.

3.3 Capital and the theory of value

To understand the effect of the theory of value in neoclassical capital theory, it is necessary to start by explicitly establishing the aim and requisites for a theory of value. This is not easy, since the focus of mainstream economics is not on value, but on prices. Price theory establishes how production, consumption, distribution and exchange interact to determine prices under a general equilibrium framework (Cohen and Harcourt, 2005). They may depend on preferences, endowments and technology as in the neoclassical case, or in costs and production as in the classical. Value theory, on the other hand, aims at accounting for the underlying forces at stake, and therefore of the ultimate determinant of prices, as opposed to its mutual interdependence.

Following Dobb ([1937]1961), a theory of value needs to solve the problem of value of commodities and the distribution of income at the same time, based on some external variable that is itself not a value (in a general long-run equilibrium). It should be sufficiently obvious that the value of something cannot be explained (i.e. determined in a causal way) by other values, which are at the same time determined by the former. In partial short-run equilibrium models, this may not be difficult, since a number of elements can be considered as unalterable, such as wages or the amount of capital. But in the long run, all these elements of the system may change. What can be considered as such an external variable? The classical political economy found this independent variable in an objective element of the production process, i.e. labour, while the neoclassical in a subjective factor underlying consumption and demand; the former resulting in the labour theory of value, and the latter in the scarcity theory of value.[19]

The father of the labour theory of value was Ricardo, after an insightful but unclear early discussion in The Wealth of Nations (Smith, 1776). For his discussion of value, Ricardo focused his attention on those commodities that could be produced by human effort, explicitly leaving aside the relatively unimportant –both numerically and conceptually– scarce goods: "in speaking then of commodities, of their exchangeable value, and of the laws which regulate their relative prices, we mean always such commodities only as can be increased in quantity by the exertion of human industry, and on the production of which competition operates without restraint" (Ricardo, [1817] 2001, p. 9). Neoclassic economists, in turn, focused their attention precisely on those goods which were taken out of the analysis by the classics. i.e. scarce goods.

Pasinetti (1983) claims that this distinction can be traced back to the trade and industry phases: one static, focused on one time-gains and subject to maximization; the other dynamic,

[19]The scarcity theory of value is also called *subjective* theory, but in this point I will follow the names given by Cohen and Harcourt, 2005, since they provide greater clarity to the present discussion.

focused on learning and production. This opinion, regardless of its validity, does show what are the main dividing lines between the two: one focused on maximization of utility given an endowments restriction, and the other on production and the conditions for expanding it. These two approaches were in fact so opposed to each other that manifested into different definitions of economics: one that defines the object of economics as the study of efficient allocation of scarce goods, popularized by Robbins ([1932]2007), which was at odds with the classical view of political economy as the study of the economic process (see e.g. Mill (1836)).

Following Pasinetti (1983), it is interesting to note that the classics would probably not have objected the neoclassical approach, and would not have denied that there exists a rational problem of allocation of scarce goods. But they would not have made it more than a minor problem, and an easy one to solve . However, neoclassic economists went too far and began to "advance a disproportionate claim", i.e. to apply this to all branches of the economic analysis. In the 1890s, the theory of marginal productivity emerged as an extension of the field of optimum allocation of scarce goods. In particular, they end up considering capital as equivalent to land (or labour), i.e. a scarce good: they were all factors of production now.

Under this approach, capital is considered as a factor of production and its price is determined by the relative scarcity of capital. Moreover, "the scarcity theory of value entails, *ceteris paribus*, a unique inverse relationship between a commodity's quantity and its price" (Cohen and Harcourt, 2005). Thus, by extending the scarcity theory to the production, factorial income distribution becomes a subset of the general theory of price determination. As I discussed in section 3.2, the Cambridge controversies showed that there is no unequivocal one-direction relation between scarcity of capital and its price, i.e. the interest rate. It ends up in a circular argument under relatively more general models than the one-good model. Thus, the scarcity theory of value cease to be effective as an explanation of value *and* of price. Prices can be determined by solving a number of simultaneous equations, but this is not enough to *explain* them.

Thus, the extension of the scarcity theory of value to all areas of the economic analysis and, in particular, to the capital theory, entails major difficulties for our understanding of capital and its relation to income distribution. The irruption of such a theory, and its blitzkrieg-like success is in itself an study object (Mirowski, 2002), but it is hard not to associate it with the almost simultaneous challenge proposed by Marx's *Capital* (Pasinetti, 1983; Dobb, [1937]1961). Whether Marx was a classical economist or not is debated[20], yet it seems evident that it had an impact on economic theory, both as a result of his studies but probably more so because of the reaction it generated, since:

[20]In one view, "objectively, Karl Marx was a Classical economist in the full sense of the word", yet "subjectively [...] he used classical theory for purposes which were diametrically opposite to those of the classical economist." (Pasinetti, 1983, p. 12).

> "Marx's overall arguments were not easy to challenge. The obvious procedure to follow would have been to question the premises. But this is precisely what was so difficult. Marx's premises were exactly the same as those of Smith and Ricardo, i.e. of all established economists" (Pasinetti, 1983, p. 13).

Certainly, a theory of value which could make production virtually invisible could do the trick. By introducing profit-maximization behavior to firms, the neoclassical model defines a correspondence between demand of commodities and demand of (scarce) production factors. Everything in between is downplayed, and "for all essential purposes, the model had, so to speak, eliminated the process of production from the analysis" (Pasinetti, 1983, p. 16). In the next section, I explore Marx's labour theory of value and some of its main caveats, in order to establish the background for the accounting correspondence discussion of section 4.

3.4 Marx the accountant

In this section, I will go over some of the main definition of Marx's theory of value, which is explicitly intertwined with his understanding of capital. Only a brief account of main variables is presented, not an in depth explanation (which would be impossible), nor a discussion of the validity of the assumptions or predictions. Out of the vast marxian contributions, only the definitions and issues that are directly related to the main point of this essay will be discussed. Given the multitude of often contradictory interpretations of Marx's work, I rely mostly on Marx's Capital, making the strong assumption suggested by Sweezy ([1942]1970) that he actually meant what he said.

In the foreword of her 1941 edition of the *essay on Marxian economics*, Joan Robinson described the spirit of this section, when she wrote:

> "the purpose of this essay is to compare the economic analysis of Marx's *Capital* with current academic teaching. The comparison is, in one sense, a violent anachronism, for the development of Marx's thought was influenced by controversy with his own contemporaries, not with mine. But if we are interested, not in the historical evolution of economic theory, bit in its possible future progress, this the relevant comparison to make" (Joan Robinson, [1942]1967).

On top of the 'violent anachronism', I focus on Marx's accountancy of value and capital, stripping it out of most of his most interesting insights. The starting point to understand Marx's accounting is the fact that capital necessarily describes a cycle of Money-Commodity-Money, or M-C-M for short, in which the capitalist has money, buys inputs and hires labour, to sell the product in exchange for money again. As discussed in section 3.1, capital is not money nor commodities, but this movement: "money which describes the latter course in its movement is transformed into capital, becomes capital, and, from the point of view of its function, already is capital."(Marx, [1867]1976, p. 248). This circulation is not for consumption, as would be

the case in the cycle C-M-C, where the entry point is a use-value[21] and exit with a different one. In the capital movement, the beginning and ending of the cycle are qualitatively the same (money), so they can only differ in terms of their exchange value. Only a quantitative difference justifies such an operation, hence capital describes the M-C-M', where $M' = M + \Delta M$, being ΔM the *surplus value*.

> "In simple circulation, the value of commodities attained at the most a form independent of their use-values, i.e. the form of money. But now, in the circulation M-C-M, value suddenly presents itself as a self-moving substance which passes through a process of its own, and for which commodities and money are both mere forms. But there is more to come: instead of simply representing the relations of commodities, it now enters into a private relationship with itself, as it were. It differentiates itself as original value from itself as surplus-value, just as God the Father differentiates himself from himself as God the Son, although both are of the same age and form, in fact one single person; for only by the surplus-value of £10 does the £100 originally advanced become capital, and as soon as this has happened, as soon as the son has been created and, through the son, the father, their difference vanishes again, and both become one, £ 110" (Marx, [1867]1976, p .256).

In this sense, Marx establishes a clear difference between the concepts of capital and what we now call wealth. A sum of money, not invested to result in more money, is wealth but not capital in the marxian sense. This is important, because it is not even related to the *power* over the production process as in Atkinson (2015). It is related to the movement and its aim, since "if the £110 is now spent as money, it ceases to play its part. It is no longer capital. Withdrawn from circulation, it is petrified into a board, and it could remain in that position until the Last Judgment without a single farthing accruing to it (Marx, [1867]1976, p .252)".

Capital is intertwined with value, insofar as to explain M-C-M' one needs to explain the origin of ΔM. As discussed in section 3.3, in order to explain exchange values, a theory of value needs an external element that is not itself a value. For Marx, as a 'pupil of Ricardo' (Schumpeter, [1942]1950), this element is labour. The intuition is that value cannot emerge from circulation alone (i.e. from buying and selling more dearly), since anyone's gains cancel-out with someone else's loss. Value hence emerges while combining commodities, i.e. in the production process, as a part of M-C-M'. "Capital cannot therefore arise from circulation, and it is equally impossible for it to arise apart from circulation. It must have its origin both in circulation and not in circulation (Marx, [1867]1976, p. 268)". Classical political economy, especially Ricardo, found a circularity when trying to explain value as labour, which lead them to slide to the more general concept of 'cost of production'. In an introduction to Marx's 'Wage Labour and Capital', Engels explains that "as soon as the economists applied this determination of value by labour to the commodity 'labour', they fell from one contradiction into another.

[21] In Marx, commodities have use-values insomuch they satisfy any human need or want, while exchange value –or plain value– is an exchange relation with other commodities.

How is the value of 'labour' determined? By the necessary labour embodied in it. But how much labour is embodied in the labour of a labourer of a day a week, a month, a year. If labour is the measure of all values, we can express the "value of labour" only in labour" (Engels, 1891). Marx's solution to this problem, was to notice that the capitalist does not hire labour, but labour-power, which, like every other commodity has a price.

> "The rock upon which the best economists were stranded, as long as they started out from the value of labour, vanishes as soon as we make our starting-point the value of labour-power. Labour-power is, in our present-day capitalist society, a commodity like every other commodity, but yet a very peculiar commodity. It has, namely, the peculiarity of being a value-creating force, the source of value, and, moreover, when properly treated, the source of more value than it possesses itself" (Engels, 1891).

Thus, if the exchange value of every commodity is its embedded labour, then labour-power's value also is: the labour needed to reproduce the labour-power, i.e. the worker herself. Since the value of the commodities needed to maintain and reproduce workers is lower than what they actually produce during a working day, a surplus value emerges. This surplus value is a product of surplus labour, which results from the part of the day in which the worker is producing commodities that surpass the value of her labour-power. Labour-power, as every other commodity, is paid at its value[22], but is the only commodity that has the ability to produce more than it costs.

Marx thus defines two components of capital: constant capital C and variable capital V. Constant capital includes all the equipment and inputs used in production, while variable capital is the hired labour-power. Constant capital does not generate value, only transfers it in its interaction with labour. "Here, the dead labor that is present alongside living labor in the production process appears as an alien and hostile power — as capital. (...) Capital is now more than a claim on surplus; it has become a tangible force that drains the worker of all energy and cripples all his talents." (Elster, 1986, p. 55). Variable capital, in turn, "does undergo an alteration of value in the process of production. It both reproduces the equivalent of its own value and produces an excess, a surplus-value" (Marx, [1867]1976, p. 317). Labour, by producing more value than it costs to the capitalist, generates a surplus value S. Total value is therefore the sum of C, V and S.

From this components of value, three key relations are drawn. The first is the rate of surplus value $S' = {}^{S}/_{V}$, which is equivalent to the rate of exploitation, in the sense that it provides an account of how much the capitalist keeps of the value produced by workers.[23]. If the working day is eight hours long and the necessary work is 4 hours, then $S' = {}^{S}/_{V} = {}^{4}/_{4} = 100\%$.

[22] This is the key assumption of the first volume of Capital.

[23] It is interesting to note that Joan Robinson fully incorporated the idea of exploitation in her Joan Robinson ([1942]1967), without ever accepting the labour theory of value as such (Alves, 2022)

A second important relation is the organic composition of capital $Q = {}^{C}/_{V}$[24], which is a measure of the extent of inputs and equipment used in relation to labour-power. Finally, Marx's rate of profit $P = {}^{S}/_{C+V}$, which is the surplus value the capitalist keeps in relation to the totality of his investment, in both labour-power and inputs/equipment.

Noting that the rate of profit can also be written as $P = {}^{S'}/_{1+Q}$, is straightforward to visualize Marx's law of the falling tendency of the rate of profit. If one assumes a constant rate of surplus S', then P varies inversely with the organic composition Q. Assuming that capitalist development entails an upward tendency of Q, given by growing need for equipment and inputs in relation to labour, then it results that P falls. Marx enumerates a number of 'counteracting forces' to this law, from which one of the most important ones for my purposes is that the rate of surplus S' may increase, i.e. that workers may be more exploited. The assumption of constant rate of surplus is very demanding, since there are a number of reasons why it may increase, even as part of capitalist development. In fact, Marx stresses over and over again that the very "same causes that bring about a fall in the general rate of profit provoke counter-effects that inhibit this fall, delay it and even paralyze it (Marx, [1894]1981, p. 346). In this sense, Sweezy ([1942]1970) is right in saying that he should have included these counteracting factors as part of the law itself, given their importance. In any case, it is worth pointing out that Marx is evidently thinking of a tendential law, in the sense of the classical methodological approach (Mill, 1836) and also the ones that followed, typically Marshall ([1890]2009).

One final important remark is that, just as neoclassical economists, Marx's theory of value is challenged when abandoning the one-commodity model. In fact, the problem emerges as soon as one adds a multi-sector production (or departments, as Marx would call them). Again, if S' is the same across sectors, since workers would move from one industries were they are less exploited in the sense described above, then if Q varies across sectors, P should be different too. But this is a contradiction insofar capitalist would move to those sectors with the higher rate of profit. The theory would require, as in Ricardo ([1817] 2001), the assumption that the organic composition is the same across sectors, which is clearly untrue.

In the second part of the third volume, Marx provides an explanation based on 'production prices', which are the results of the *average* rate of profit of the system to the capital invested $(C + V)$. In this way, the assumption that goods are sold at their values is lifted and now commodities are exchanged at their prices. Thus, capitalist with higher organic compositions Q, would be compensated by appropriating not only the surplus value produced in their firms, but also that of others, who will be the losing end of the thread. This procedure entails that in the end each capitalist will obtain a share of the overall surplus value produced by the system according to his own total capital investment. This 'solution' has been criticized and

[24]Some authors express this relation as $Q = {}^{C}/_{V}$ (Sweezy, [1942]1970), which does not change this results, nor the ones presented in 4.

several alternatives were proposed (Sweezy, [1942]1970), but I will not go beyond this succinct presentation.

It is worth mentioning, following Dobb ([1937]1961), that it is not that Ricardo or Marx were not aware of the potential problems of different capital compositions (organic compositions) generated, it is more likely (and more relevant to us here) that they considered them relatively unimportant given their objectives. They were not interested in determining the prices of specific commodities, but rather on "macroscopic problems" (today we would say macroeconomic), i.e. determining rents, profits and wages and values of commodities in relation to those. It is at this high aggregation level that I discuss the accounting equivalence in section 4.

4 One-commodity-model accounting equivalence

4.1 The one commodity model

I will now focus on establishing the accounting equivalence between the macro distributional model presented in section 2 and the Marx's value categories of section 3.4. It is worth stressing that these are by no means conceptual equivalences, in the sense that they do refer to different (and often opposite) ways of understanding the functioning of the economic system, the underlying theories of value and the very definition of what *economics* is. The aim is more modest, and is simply to understand the conditions under which there is accounting equivalence between these two approaches.

The setting under which some degree of equivalence holds is the one-commodity model. This is not a novelty, insofar "by simplifying so extensively, one-commodity models allow many competing theories to demonstrate their results" (Cohen and Harcourt, 2005, p. 44).[25] However, clearly comparing the two may help understand their conceptual differences while at the same time establishing if some of their predicted outcomes are incompatible or not.

More importantly, the one-commodity model is very important in particular for the marxian analysis, since it is not just a simplifying assumption as may be in the neoclassical model. The one-commodity model is a synonym in this case of considering the system as whole, and it is at the system level where the rate of profit P is determined, independently of what individual rate of profit each capitalists would face if commodities were to be sold at their values. At this level, profits and surplus value coincide, and so does aggregate production measured with prices or values. The one-commodity model is the key link between value and price, and therefore not a working assumption to be lifted and abandoned at later stages, but a central feature for understanding of the economic process. Thus, the one-commodity-model has tremendous

[25] In fact, Cohen, 1989 shows this for the standard neoclassical model and Sraffian model.

conceptual importance in Marx: it is were the theory of value manifests directly. Indeed, the point could be made that regardless of the way in which each class or even each individual ends up getting their share of aggregate net income and of how large is such a share, at the system level the theory of value holds true and best captures some basic characteristics of the functioning of a modern economy, as noted by Sweezy.

> "The entire social output is the product of human labor. Under capitalist conditions, a part of this social output is appropriated by that group in the community which owns the means of production. This is not an ethical judgment, but a method of describing the really basic economic relation between social groups. It finds its most clearcut theoretical formulation in the theory of surplus value. As long as we retain value calculation, there can be no obscuring of the origin and nature of profits as a deduction from the product of total social labor. The translation of pecuniary categories into social categories is greatly facilitated. In short, value calculation makes it possible to look beneath the surface phenomena of money and commodities to the underlying relations between people and classes (Sweezy, [1942]1970, p. 129)".

At this point, it is worth stressing two important conceptual differences between some of Marx's categories of section 3.4 and the variables discussed in Section 2, which will be key in understanding the accounting equivalence of the reminder of this section. First, besides the obvious fact that capital in Marx includes wages (as in most classical political economy) while the neoclassical theory does not of, it is important to note that Marx's capital is more a flow than a stock. This is not only because for Marx capital describes a certain 'movement' M-C-M', but also because being the sum of $C + V$ is literally closer in SNAs terms to output than to net worth. Marx's capital in its money form accounts for wages paid to workers, inputs of production and the wearing out of equipment in the period of production. These are all flow-like variables. The stock of equipment is behind the depreciation of the equipment, but does not enter in calculation as a stock. In Marx's words:

> "The value component c, which represents the constant capital *consumed* in the course of production, is not the same thing as the value of the constant capital *applied* in production. The materials of production are certainly completely consumed, and their value is therefore entirely transferred to the product. But only a part of the *fixed* capital is entirely consumed, its value thereby being transferred to the product. Another part of the fixed capital in machines, buildings, etc. continues to exist and to function just as before, even if its value is diminished by the annual wear and tear. This part of the fixed capital that continues to function does not exist for us when we consider the value of the product. It forms a part of the capital value that is independent of this newly produced commodity value and is present alongside it." (Marx, [1885]1978, p. 472, emphasis in original).

The preceding comment results in the second point, which is that the neoclassic rate of return r is a return on the *stock of capital* which does not include wages, while for Marx is a return on the invested flow. Theses two variables are, although similarly named, quite different

from a conceptual view point. Moreover, form a quantitative standpoint it is not evident at first sight which should be higher. Since r is expressed in terms of a stock, one could think that it should be lower than P. However, although P is the surplus value in relation to a flow, this denominator is very large insofar it includes all the economy's wage bill, plus depreciation plus all inputs. In the reminder of this section, I compare them more systematically in order to understand their role in their respective theories.

4.2 Equivalent aggregate variables

Before establishing the equivalence, I lay out the main assumptions and recall the variables to be compared from each approach.

Assumptions. We assume that there is no increase in population or hours worked and a one good model. Moreover, we assume that there are no inputs, i.e. there is only fixed capital in constant capital. The later assumption can be lifted with no relevant consequences, as shown in Appendix A. Moreover, it makes sense to exclude it since inputs (or circulating capital) are not part of the neoclassical production function, which considers only new output or added value.

The neoclassical variables. We begin by considering a generic production function.

$$Y = F(K, L)$$

F can be any production function. From an income perspective, considering Y_g gross income (eq. 11) and Y_n net income (eq. 12), we get:

$$Y_g = wL + rK + \delta K \tag{11}$$

where L and K are labour and capital, w and r are the wage and interest rate, while δ is the depreciation rate in terms of aggregate capital.

$$Y_n = wL + rK \tag{12}$$

Considering a depreciation rate d, which is very similar to the SNAs concept of consumption of fixed capital[26], expressed in terms of Y_g, i.e $d = CFK/Y_g$, we get:

$$Y_g(1 - d) = Y_n$$

Normalizing equation 11 in Y_g, we get:

[26]There are some differences which are unimportant in this context, see WIL (2021).

$$\frac{Y_g}{Y_g} = 1 = \frac{wL}{Y_g} + \frac{rK}{Y_g} + \frac{\delta K}{Y_g}$$

Hence

$$\frac{wL}{Y_g} + \frac{rK}{Y_g} + \frac{\delta K}{Y_g} = \frac{wL}{\frac{Y_n}{(1-d)}} + \frac{rK}{\frac{Y_n}{(1-d)}} + \frac{\delta K}{\frac{Y_n}{(1-d)}} = 1$$

Re-arranging:

$$(1-\alpha)(1-d) + \alpha(1-d) + \delta\beta(1-d) = 1 \tag{13}$$

With α being the capital share of net income, i.e. $\alpha = \frac{rK}{Y_n}$, and β being the capital to income ratio $\beta = \frac{K}{Y_n}$.

The labour theory of value variables.

Aggregate value W_{agg} is:

$$W_{agg} = S_{agg} + V_{agg} + C_{agg}$$

These values may be expressed in monetary terms to facilitate exposition, but actually they refers to embedded labour, so their magnitudes are unchanged at any level of production. This means that there is no Y_g or Y_n in Marx (as in Ricardo ([1817] 2001, ch. 20)), only value. Normalizing,

$$\frac{W_{agg}}{W_{agg}} = 1 = \frac{S_{agg} + V_{agg} + C_{agg}}{W_{agg}} = S + V + C$$

we get a macro-distributional definition, in which S, V, and C are shares of aggregate value. Expressing the different components or value as shares and not embedded labour does not affect computations, since we are always expressing these quantities in relation to others. Moreover, by presenting them as shares the distributional implications of the analysis are best captured.

The accounting correspondence

It has been long acknowledged that there is room for mapping Marxian categories to standard economic variables, at least at high level of abstraction. Following Sweezy, it is clear that "Marx's theory of value has the great merit, unlike some other value theories, of close correspondence to the actual accounting categories of capitalistic business enterprise" (Sweezy, [1942]1970, p. 63), as $V + S$ corresponds to net national income, which "includes payments to individuals plus business savings".

This holds no only overall, but also in V and S separately. "If the labour-power has performed its function, then the capital no longer consists of labour-power on the one hand and means of production on the other. The capital value that was laid out on labour-power is now value which has been added to the product (together with surplus-value)" (Marx, [1885]1978, p. 300). For Marx, $V + S$ represent the value equivalent of net national income, since C is the equivalent of consumption of fixed capital and inputs. This is discussed extensively in Shaikh et al. (1997), which represents the most thorough analysis of correspondence between National Accounts and Marxian variables. Although his aim goes far beyond the scope of this analysis[27], he establishes a direct link between key value components in the two approaches under the assumptions of "production sectors alone", which is close to the one-good model assumed here. In this setting, it is hence straightforward that:

$$\frac{wL}{Y_g} = V = \frac{wL}{\frac{Y_n}{(1-d)}} = \frac{wL(1-d)}{Y_n} \rightarrow \boxed{(1-\alpha)(1-d) = V} \tag{14}$$

$$\frac{rK}{Y_g} = S = \frac{rK}{\frac{Y_n}{(1-d)}} = \frac{rK(1-d)}{Y_n} \rightarrow \boxed{\alpha(1-d) = S} \tag{15}$$

$$\frac{\delta K}{Y_g} = C = \frac{\delta K}{\frac{Y_g}{(1-d)}} = \frac{\delta K(1-d)}{Y_n} \rightarrow \boxed{\delta\beta(1-d) = C} \tag{16}$$

4.3 The rate of return r and the rate of profit P

From the preceding accounting correspondences, it is possible to establish a comparison of the main driving forces of the system, i.e. $r > g$ in the neoclassical macro-distributional models and the falling tendency of the rate of profit P in the Marxian framework.

We start by deriving the organic composition of capital Q and the rate of surplus S'. First, we have that:

$$Q = \frac{C}{V} = \frac{\delta\beta(1-d)}{(1-\alpha)(1-d)} = \frac{\delta\beta}{(1-\alpha)} \rightarrow Q = Q(\overset{+}{\beta}, \overset{+}{\alpha}) \tag{17}$$

On the other hand:

$$S' = \frac{S}{V} = \frac{\alpha(1-d)}{(1-\alpha)(1-d)} = \frac{\alpha}{(1-\alpha)} \rightarrow S' = S'(\overset{+}{\alpha}) \tag{18}$$

Combining equations 17 and 18, we get that the system's profit rate P is:

[27] His objective is to "provide an alternate foundation for the measurement of the production of nations." This is done by mapping Marxian and national account's variables, and –more importantly– by systematically including the classical distinction of production and consumption activities, which excludes sectors such as the military or the police from production sector.

$$P = \frac{S}{C+V} = \frac{S'}{1+Q} = \frac{\alpha}{(1-\alpha)+\delta\beta} \rightarrow P = P(\overset{-}{\beta}, \overset{+}{\alpha}) \tag{19}$$

The result from equation 17 is straightforward. The organic composition of capital $^{C}/_{V}$ increases both when β or α grow, since in the first case it entails a larger C[28] through the effect of depreciation, while a larger α entails a higher capital share, i.e. lower V. The same idea applies to equation 18, since higher α is mechanically associated with increasing rate of exploitation in this setting. In the case of the system's rate of profit P, equation 19 shows that it is consistent with Marxs insights of the tendency of the rate of profit to fall. When β increases, it pushes Q up and hence P down, but the rise in α increases Q too (and also S'), and thereby pressing P upwards.[29] Note that this is one of the main counteracting forces of the tendency of the rate of profits discussed in Marx ([1894]1981): the increase in exploitation S' offsets the tendency of the profit rate to fall. The net effect is hence unknown.

Three things are worth noting. First, under the Cobb-Douglas assumption, the increase in β has no effect in α insofar it pushes r downwards in the exact proportion so as to keep α unaffected (sec. 2.2), thus Marx's downward trend for P prediction holds. In the end, the Cobb-Douglas production function is the equivalent of a constant exploitation rate s', which resonates in Marx's assumption throughout Capital. However, in the more flexible CES production function assumption, the increase in α may push P upwards, given the elasticity of substitution of capital σ is larger than 1. Naturally, α has an upper limit (probably even before its absolute maximum of 1), hence Marx's point re-emerges, as eq. 20 shows:

$$\lim_{\alpha \to 1} P = \lim_{\alpha \to 1} \frac{\alpha}{(1-\alpha)+\delta\beta} = \frac{1}{\delta\beta} \tag{20}$$

Second, if an increase in β ends up forcing P downwards, it is interesting to note that this can happen with a constant r, provided it is larger than g. In Marx's terms, this is through its effect on $C = \delta\beta(1-d)$, i.e. the part of the stock of physical capital and inputs that goes into the flow of value C. This point is entirely missed if one focuses on neoclassical rate of return r instead of Marx's profit rate P. What is missed in Marx, in turn, is the fact that β (hence C, given d and constant or increasing δ) tends to grow mechanically when $r > g$, as discussed in section 2.3. This is absent in his analysis, because despite acknowledging that there is a stock of capital, it is not in his accounting framework (only the part that passes on to value C) and –even worse– he is not interested in accounting for changes in aggregate production (which he assumes as a feature of the capitalist system) and therefore of g.

[28] If depreciation is stable or increasing. In the case of decreasing depreciation, it will depend on the rate of increase of β and δ.

[29] This happens because the increase in α has more effect in S' than in Q, as in both the effect on the denominator is the same, but in the case of S' it also increases the numerator.

This means that if $r > g$, and as a result both β and α increase (given the right elasticity of substitution), then in the long run the falling rate of profits P persists, even after accounting for the effect of the counteracting force of increasing α. This is clear when comparing r and P. Recalling that $\alpha = r \cdot \beta$ and equation 19, we have that:

$$\frac{r}{P} = \frac{\frac{\alpha}{\beta}}{\frac{\alpha}{(1-\alpha)+\delta\beta}} = \frac{\beta}{(1-\alpha)+\delta\beta} = \frac{\beta}{(1-\alpha)} + \frac{1}{\delta}$$

Except for extremely high values of α and implausibly low values of β (lower than one), $r > P$. Moreover, the gap between the two widens with increasing β and/or α. Thus, the world envisioned by Piketty (2014), with low growth and a constant r but $r > g$, entails both his result of growing β and α, but also the falling rate of profit P.

Third, if β varies inversely with the growth rate g (section 2.3), then low growth societies will mechanically amplify the downward pressure on the system's rate of profit P. Moreover, this is also true if we think of a higher obsolescence rate, which will increase the depreciating of capital δ, likely to happen in the next decades, and also if production becomes more inputs-intensive, which is the trivial yet important result of the generalization of the formula in Appendix A.

Once the differences between the rate of return and the rate of profits are presented, it is worth commenting on some implications for empirical distributional analysis. In particular, many studies use the (inverse of) the rates of return on different assets to compute the distribution of those assets based on observed capital incomes, in what is called the 'capitalization method' (see e.g. Saez and Zucman (2016), Garbinti, Goupille-Lebret, and Piketty (2017), Martınez-Toledano (2017), and De Rosa (2022)). From an accounting perspective, these rates of return do in fact represent the stream of incomes accruing the owners of the assets in the Smith-Marshall sense (see 3.1), and therefore the estimates do provide important insights on the distribution of wealth and capital. What changes once one assumes that the labour theory of value is operating behind the scenes, is the way in which we understand it, i.e. not as the remuneration of the factor capital, but the appropriation of surplus value. This has important consequences from the view point of the justification for this remuneration. In Robinson's words, "with the notion of the supply price of capital, the moral justification of profit as a necessary cost of production disappears, and the whole structure of the orthodox apology falls to the ground." Joan Robinson ([1942]1967, p. 62).

This idea is discussed in length by Marx at the beginning of the third volume of capital, in what he describes as describes the 'mystification of capital', which refers to the fact that even the capitalist himself misunderstands the origin of his profit as the result of his overall investment. He does not see that this profit results from surplus value, because he is not seeing variable capital as such, only a mass of investment which results in a flow of income. For Marx,

this was also the case for classical economists such as Smith or Ricardo, who by considering fixed and circulating capital instead of constant a variable capital (see section 3.1), were unable to see the true origin of value. More generally, this results from the twin ideas of alienation and fetishism, which applied to capital result in the notion that capital produces value by itself. As Elster summarized:

> "Alienation-as-subjection, though closely linked to exploitation, is not equivalent to it. Alienation adds to exploitation a belief on the part of the workers that the capitalist has a legitimate claim on the surplus, by virtue of his legitimate ownership of the means of production. (...) Capital fetishism is the belief that capital's power to produce is a faculty inherent in it, not one it owes to the labor process." (Elster, 1986, p. 56-57)

The phenomenon capital fetishism captures the main point of this essay. Under a one-commodity assumption there is consistency in some macro-distributive Marxian and neoclassical results as the one best described by Piketty and Zucman (2015). Yet while by using Marx's variables one is able to grasp the notion of where value is actually created and by whom, in the neoclassical setting this is obscured. The debate over the validity of Marx's theory of value is way beyond the scope of this short essay, but I do believe that allowing ourselves to look at empirical results on β, α, r and g through these lenses has the potential to open a fruitful conversation.

5 Concluding remarks

In this essay, I have tried to show that while our empirical distributional estimates are far better than they used to be, they are still anchored to a very questioned theory. It is a theory which, however useful for many purposes, is especially poorly equipped to deal with long-run macro distributional issues. This hinders our ability to understand past trends and look into the future.

It is true that it is not mandatory to buy the full neoclassical story to use it. Some general models, like the one-commodity long run growth model is useful as a tool for thinking about some of the main economic variables and contribute to a certain data-based historical narrative. We can use these models as subsidiary to our empirical efforts –a sort of general interpretative device– without necessarily trusting them blindly. It is a very productive way forward, in which a lot of progress is still to be done.

But theories are important. We need them to interpret, understand and predict whenever possible. And however problematic they may be, they do need to grasp the main features of the economic system. The scarcity approach to value, which is behind theoretical models and especially the conceptualization of capital, is useful for many purposes and does refer to important economic questions. Yet it hardly accounts for some very basic facts about

which are the drivers of the economic system. The functioning of modern economies cannot be modeled in the traditional marginalist fashion based on ahistorical atomistic individuals with perfect information maximizing their infinite-horizon utility, for whatever that is. But, as Pasinetti would say, "marginialism had the advantage of synthesis. For they have always clearly presented their arguments around a unifying problem (optimum allocation of scarce resources) and a unifying principle (the rational process of maximisation under constraints)" (Pasinetti, 1983, p.19).

Classical political economy, although outdated and flawed in many ways, still offers a much more simple, reasonable and credible starting point: that the economic system is driven by production decisions, heavily determined by profits and by the never-ending need to grow or die of capitalists, molded by the conflicting interaction of more or less stable sets of individuals who share certain roles in such a production. This was obvious for Shcumpeter, Robinson, Veblen, for most pre-marginalist economist of the 19th century and most importantly for Marx. Even though this essay was not about discussing Marx's theory, its potential or its crippling limitations, I do want to stand by the idea that –as Robinson would say when comparing him with neoclassical school– Marx's "intellectual tools are far cruder, but his sense of reality is far stronger, and his argument towers above their intricate constructions in rough and gloomy grandeur "(Joan Robinson, [1942]1967, p. 2). In the end, it just seams more reasonable to assume that is human activity which is behind all human production and exchange –and therefore it determines both value of things and social relations–, instead of isolated individuals harmonically interacting with stuff in a conceptual vacuum.

The only point I wish to make is that there is room for thinking in political economy terms with the estimates we already have. That, for instance, if we observe an $r > g$ which results in an increasing β, that probably means that the rate of profit P is falling, and that that can be offset by an increase in α but only for so long. And this opens a whole new conversation. More importantly, if we believe that the labour theory of value better explains these estimates than the scarcity theory, then it is no longer true that wealth can be accumulated through work and inheritance alone, insomuch the main (though not the only) way of accumulating wealth would be exploitation. Not exploitation as an ethical judgment, but as an objective category of analysis, and this indeed changes our narrative. In Solow's quote in the introduction, he warned us that if we abandoned the ad-hoc assumption that factors were priced at their marginal productivity "much else would change besides", and he was right.

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A Appendix: accounting correspondence with intermediate consumption

As above, aggregate value W_{agg} is:

$$W_{agg} = S_{agg} + V_{agg} + C_{agg}$$

Normalizing,

$$\frac{W_{agg}}{W_{agg}} = 1 = \frac{S_{agg} + V_{agg} + C_{agg}}{W_{agg}} = S + V + C$$

We assume that there is both fixed and circulating capital in C, and therefore we should also include intermediate consumption in the accounting framework. Thus, being I the intermediate consumption of the economy, i.e. the inputs of production, we consider output Y_o:

$$Y_o = Y_g + I = Y_n + \delta K + I$$

We proceed again by establishing that:

$$Y_o = Y_n(1 - d' - i)$$

being i the share of intermediate consumption in total output ($i = \frac{I}{Y_o}$), and d' as the depreciation rate in terms of output[30], i.e. $d' = \frac{\delta K}{Y_o}$.

In this setting, we get:

$$\frac{wL}{Y_o} = V = \frac{wL}{\frac{Y_n}{(1-d'-i)}} = \frac{wL(1-d'-i)}{Y_n} \rightarrow \boxed{(1-\alpha)(1-d'-i) = V} \tag{A.1}$$

$$\frac{rK}{Y_o} = S = \frac{rK}{\frac{Y_n}{(1-d'-i)}} = \frac{rK(1-d'-i)}{Y_n} \rightarrow \boxed{\alpha(1-d'-i) = S} \tag{A.2}$$

$$\frac{\delta K + I}{Y_o} = C = \frac{\delta K + I}{\frac{Y_n}{(1-d'-i)}} = \frac{(\delta K + I)(1-d'-i)}{Y_n} \rightarrow \boxed{\delta\beta(1-d'-i) + \frac{I(1-d'-i)}{Y_n} = C} \tag{A.3}$$

[30] Note that its different from d defined above, since this is defined in terms of output and not of gross income Y_g.

Under these conditions, note that S' does not change, but Q does.

$$S' = \frac{S}{V} = \frac{\alpha(1-d'-i)}{(1-\alpha)(1-d'-i)} = \frac{\alpha}{(1-\alpha)} \tag{A.4}$$

$$Q = \frac{C}{V} = \frac{\delta\beta(1-d'-i) + \frac{I(1-d'-i)}{Y_n}}{1-\alpha)(1-d'-i)} = \frac{\delta\beta + i'}{1-\alpha} \tag{A.5}$$

Being $i' = \frac{I}{Y_n}$, which is very similar to i, but the denominator is Y_n instead of Y_o. Thus, considering equations A.4 and A.5, we get:

$$P = \frac{S}{C+V} = \frac{S'}{1+Q} = \frac{\frac{\alpha}{(1-\alpha)}}{\frac{\delta\beta+i'}{(1-\alpha)}} = \frac{\alpha}{\delta\beta + i'} \tag{A.6}$$

Note that equation A.6 is very similar to equation 19, with the addition of i' in the denominator. This entails the trivial result that a larger share of inputs in the production process will mechanically reduce the rate of profit, and leaves the main conclusions of the evolution of G with respect to β and α.