## An experiment to address conceptual difficulties in slipping and rolling problems

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**Abstract.** A bicycle wheel that was initially spinning freely was placed in contact with a rough surface and a digital film was made of its motion. Using Tracker software for video analysis, we obtained the velocity vectors for several points on the wheel, in the frame of reference of the laboratory as well as in a relative frame of reference having as its origin the wheel's center of mass. The velocity of the wheel's point of contact with the floor was also determined obtaining then a complete picture of the kinematic state of the wheel in both frames of reference. An empirical approach of this sort to problems in mechanics can contribute to overcoming the considerable difficulties they entail. Introduction. Problems involving rolling without slipping are frequently included in mechanics courses at upper secondary and introductory university levels. Although the problems appear to be simple, they pose a number of difficulties for students because they require simultaneous application of static and dynamic concepts. Studies on rolling without slipping in the field of *Physics Education Research* confirm these difficulties. Some studies demonstrate, for instance, that students have difficulty in recognizing that the direction of the static frictional force on a body that is rolling without slipping does not necessarily oppose the direction of rolling [1]. They also have difficulty determining the direction of velocity at different points on the wheel [2].

Although such problems are frequently dealt with theoretically in textbooks [3], they are seldom studied experimentally. The absence of an empirical approach may have a negative effect on learning processes because students may be led to think that the theoretical model has no connection with real life, without actually having tested their preconceptions against experimental evidence. In terms of experimental approaches, it is worth mentioning the study of the velocity of the center of mass of a cylinder during rolling with and without slipping by means of video analysis [4] and proposal focused on the transition between slipping and rolling [5].

Here we describe an experiment to study the velocity distribution of the wheel viewed both from the laboratory frame of reference and from a relative frame of reference with its origin at the wheel's center of mass. The wheel initially is spinning freely, and is then put in contact with a rough surface, so that eventually it is rolling without slipping. This analysis enabled students to visualize and gain intuition into the kinematics of a rigid body that is rolling with and without slipping.

**Experimental setup.** The system consists of a bicycle wheel of radius as depicted in Fig. 1. The rim was marked at equal intervals in order to facilitate automatic tracking of the wheel's motion with Tracker software [6]. Motion was recorded at 30 *fps* using a digital video camera (Kodak PlaySport) mounted on a tripod in such a way that its optical axis was at right angles to the plane of movement of the wheel. In order to obtain the clearest possible image, we used spotlights to improve luminosity and increase the camera's shutter speed. We shall assume that the mass of the wheel is symmetrically distributed around its physical axis.

The wheel, initially spinning is then placed on a horizontal floor and the velocity of center of mass, initially equal to zero, increases with time, due to the force of kinetic friction acting in the direction of movement. This situation takes place over a very short time interval until the velocity of the point of contact with the floor is zero in the laboratory frame of reference. Then, the wheel begins to roll without slipping so that both the velocity of the center of mass and the angular velocity remain constant. As can readily be deduced from the equations of motion, when the wheel rolls without slipping, the resultant external force must be zero and therefore the static frictional force (the only force acting in the direction of the horizontal axis) must also be zero. In practice the wheel will not keep on rolling indefinitely because over longer timespans, the rolling resistance that we here chose to neglect, must be taken into account.



Figure 1. A wheel is made to spin at an angular velocity  $\omega_0$  and is then placed in contact with the floor. Once the wheel is released, the center of mass begins to accelerate due to the force of kinetic friction acting in the direction of movement.

Velocity distributions. The velocity at a given point on the rim can be thought, as shown in Fig. 2, as the addition of two vectors, one translational, in which all the points on the wheel have the same velocity as the center of mass, and one rotational, where the direction of the velocity of each point on the rim is tangential to the rim.



Figure 2. Diagram of the velocity vector of a point Q on the rim of the wheel represented as the addition of the vectors of rotation and translation. The point P in contact with the floor is also indicated.

In the laboratory frame of reference, while the wheel is slipping, the velocity of the centre of mass is increasing and the angular velocity is decreasing, the velocity of the point P in contact with the floor is opposite in direction to the wheel's motion and it decreases until it becomes zero when the wheel is rolling without slipping. On the other hand, when viewed in the frame of reference fixed to the wheel's center of mass, point P describes a circular motion that is uniformly decelerated until the wheel is rolling without slipping, when P begins to describe uniform circular motion.

**Experimental results** We used Tracker to analyse the changing velocities of six points on the rim from the time slipping started until rolling without slipping was established. We did this in the laboratory frame of reference and in a frame of reference

with its origin at the wheel's center of mass. Figure 3 shows a sequence of four images in which are represented the velocity vectors at different points on the rim, during the period when slipping is occurring and in the laboratory frame of reference. As expected, the direction of the velocity of the point of the wheel in contact with the floor is opposite to that of the center of mass, and its modulus decreases as time elapses.



Figure 3. Velocity vectors corresponding to several points of the wheel (drawn with different colours) in the laboratory frame of reference during slipping motion. Elapsed time is indicated in each panel.

Making the wheel's center of mass the origin of a coordinate system, the sequence of images in Fig. 4 shows how the profile of the velocities develops over time during the slipping phase in the frame of reference centred on the hub. In contrast to the previous figure, in all these images the velocity vectors are tangent to the rim and in each snapshot are approximately equal in magnitude. It can be also appreciated, in accordance with the theory, that the modulus of the velocity diminishes as time elapses.

To gain further insight, Fig. 5 shows the velocities of the same points, in the laboratory frame of reference (left) and in the frame of reference centred on the hub of the wheel (right), but this time when the wheel is rolling without slipping. Direct inspection of the left panel shows, as theory predicted, that the velocity of the point of contact with the floor is zero, while the velocity of the highest point is twice that of the centre of mass [3]. On the other hand, the right panel shows that the modulus of the velocity at the highest and lowest points are similar to each other and similar, also, to the modulus of the velocity of the centre of mass in the laboratory frame of reference.

**Conclusion.** We studied the changes in velocities over time of several points on the rim of the wheel with respect to the two frames of reference described above. By analysing these changes, the students are able to visualize and better comprehend what the directions of the velocity vectors are at different points of the wheel. They are able



Figure 4. Similar to Fig. 3 but the velocity vectors are measured with respect to the frame of reference fixed to the hub of the wheel.



Figure 5. Tracker screen snapshots showing in different colours the velocity vectors at different points on the wheel with respect to the laboratory frame of reference (left) and with respect to the frame of reference with its origin at the center of mass (right). Below each panel, the values of the modulus of the velocity of the centre of mass (A) and of the highest point (F) of the wheel are indicated.

to observe, for example, how when the wheel is slipping the velocity of the point of the wheel in contact with the floor is in the opposite direction to motion, and they could see visual evidence for the direction of action of the kinetic frictional force. At the same time, they are able to comprehend the nature of the motion of the wheel from the point of view of a frame of reference centered on the wheel hub (the center of mass). This reinforced the idea of thinking about the set of velocities of points in the laboratory frame of reference as the addition of two velocity vectors, one translational and one rotational. This concept is fundamental to the understanding of rolling without slipping. Given all of the above, the analysis of the changes over time of the set of velocities can be a very powerful tool to help students overcome certain conceptual difficulties associated with the kinematics of a wheel that is rotating with and without slipping.

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