# A WATTMETER BASED ON PHASE MEASUREMENTS WITH FEW µW/VA UNCERTAINTY

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# Abstract

The proposed wattmeter is based on separate measurements of voltage, current and power factor. This last parameter is calculated from the phase angle between voltage and current. To achieve uncertainties around 1 part in  $10^6$ , many error sources have been analyzed. New methods are used for scaling (120 V to 1 V and 10 A to 10 mA), and for reducing the effect of waveform distortion.

## **Introduction**

There are many proposed methods for high-accuracy power measurement. Some of them are based on adding devices and rms voltmeters [1], RC bridges [2], and digital sampling [3]-[4]. Although some methods have been proposed based on zero-crossing phase measurements, they do not reach high accuracy. Usually, the power factor is calculated as a derived quantity from the voltage, current and power, but the inverse relation is also possible. In this way, the power factor is directly measured, computing the phase angle between zero crossings of voltage and current. Although this is a well known method, it is not used in high precision instruments because of the distortion influence. Even a small distortion in the input waveforms produces large power factor errors. However, methods to reduce that influence have been proposed [5]. Improving them, the next sections show that very high accuracy can be obtained for power measurements at low frequencies (50 Hz to 60 Hz) with low distorted signals. The system measures the rms values of input voltage and current, and the power factor. Then, active, reactive and apparent powers are computed.

#### **Measuring system**

The proposed system, intended for low distorted conditions, separately measures the rms voltage, the rms current, the power factor, and multiples all these terms. For the last term, the system eliminates all the harmonic content of the input signals and measures the cosine between the fundamental components of current and voltage ( $\cos \phi_1$ ), the displacement factor. The errors generated by this truncation are very low, as it will be shown by the following equations. As usual, the relative power error ( $\varepsilon_p$ ) is defined referred to the apparent power

(*S*), as

$$\mathcal{E}_{\rm p} = \frac{P_{\rm m} - P_{\rm a}}{S},\tag{1}$$

where  $P_{\rm m}$  is the value measured by the instrument and  $P_{\rm a}$  the actual power. In our case, the value of this error is

$$\mathcal{E}_{\rm p} = \cos \phi_{\rm l} - \lambda \,, \tag{2}$$

where  $\lambda$  is the actual power factor. It can be expanded as

$$\lambda = \cos \phi_1 \frac{V_1 I_1}{V_{\rm rms} I_{\rm rms}} + \sum_{n=2}^{\infty} \frac{V_n I_n \cos \phi_n}{V_{\rm rms} I_{\rm rms}},$$
 (3)

being  $V_1$ ,  $I_1$  the fundamental components of voltage and current, and  $V_n$ ,  $I_n$  the remaining harmonic content.  $\phi_n$  is the angle between current and voltage of the harmonic number *n*. From (2) and (3), the power error becomes

$$\mathcal{E}_{\rm p} = \cos\phi_{\rm l} \left(1 - \frac{V_{\rm l}I_{\rm l}}{V_{\rm rms}I_{\rm rms}}\right) - \sum_{n=2}^{\infty} \frac{V_{\rm n}I_{\rm n}\cos\phi_{\rm n}}{V_{\rm rms}I_{\rm rms}} \,. \tag{4}$$

The parenthesis of the first term of the right-hand side of (4) can be expressed as a function of the distortion of voltage  $(D_v)$  and current  $(D_i)$ , approximately, as  $(D_v^2+D_i^2)/2$ . If both sources have distortion lower than 0.05%, this term produces errors smaller than  $3\times10^{-7}$ . Regarding the second term, it depends on the actual harmonic content, generally not known. Some statistic compensation will occur, but it depends on the sources. A worse case can be supposed, assuming that there is only one significant component *h*. In this case, also this second term can be expressed as a function of distortion as  $D_v D_i \cos \phi_h$ . With the previous assumption of THD=0.05% for both sources, a limit of its absolute value can be estimated in  $3\times10^{-7}$ .

Obviously, for a complete uncertainty calculation, it is necessary to add the uncertainty of the measurement of  $V_{\rm rms}$ ,  $I_{\rm rms}$  and  $\cos \phi_{\rm l}$ . It will be done in the next sections.

#### **Displacement factor meter**

There is a module in the system to compute  $\cos \phi_1$ . For that,

low-pass filters are applied to both signals, and zerocrossing techniques are used for detecting the angle between them. The error of this technique depends on the signal harmonic content. Even a small THD produces high errors. An error limit can be calculated [6] as

$$\left|\delta\right| \le \sum_{n=2}^{\infty} (I_n / I_1) + \sum_{n=2}^{\infty} (V_n / V_1)$$
 (5)

where  $\delta$  is the angular difference between the angle measured by the instrument and the angle between the fundamental components. Its influence factor in the power error, according to (2), is  $-\delta \sin \phi_1$ . That is, at unit power factor, this uncertainty contribution can be neglected, but at zero power factor, it directly affects the results. From (5), it is concluded that THD in the order of  $10^{-7}$  is required to reach the total uncertainty goals. For that, the filters must have an attenuation of 75 dB, to reduce the harmonic content of the sources to that level. An inverter switch is used to measure the phase angle between both, up and down crosses. Averaging these values, the influence of differences in signal amplitudes, remaining DC, and even harmonic components are canceled [6], being the third harmonic the smallest one to take into account. The attenuation factor at that frequency of the filter used in the prototype is 80 dB; and the worst case is that all the harmonic content is only due to that harmonic component. Calculating the maximum error  $\varepsilon_{\rm p}$  from (2), a value of 10<sup>-7</sup> is obtained at zero power factor. At unit power facto, this error is negligible.

# **Rms measurements**

For the rms voltage and current measurement, a planar multijunction thermal converter (PMJTC) [7] is used. Its nominal input voltage is 1 V, and the ac-dc transfer error, at power frequencies, is smaller than 0.3  $\mu$ V/V. The system automatically scans the input ac current and ac voltage proportional signals, and a standard 1 V dc source. Note that at nominal voltage and current inputs, the signals always have 1 V, whatever is the power factor. In this way, the PMJTC always operates in a fixed point increasing the accuracy and reducing the time required for each measuring cycle. Regarding the short term stability of the dc source, it is around 0.2  $\mu$ V/V.

# **Scaling**

For the voltage input, a binary inductive voltage divider [8] is used. Its error uncertainties are in the order of  $5 \times 10^{-7}$  in ratio and in phase. This device permits to scale from input voltages up to 240 V, to 1 V, which is the level required by the rms converter and the phase displacement measuring module.

For the current input, a current transformer form 1 A to 10 A in the primary, to 10 mA in the secondary is used [9]. Its output is connected to a resistor of 100  $\Omega$ . The standard uncertainty contribution of this transformer is lower than  $5 \times 10^{-7}$  in ratio and in phase. To reach these low values, it is calibrated against a self-calibrating current transformer that allows ratios from 100:1 to 1:1. The last one permits the self-calibrating procedure. It uses the same principle presented in [9] to eliminate the influence of internal capacitances. Regarding the 100  $\Omega$  resistor, a set of bulk metal foil resistors are used. Its dc short time stability is in the order of 0.2  $\mu\Omega/\Omega$ , and for the ac performance, it is calibrated against a calculable resistor [10]. The ac-dc transfer uncertainty is 0.1  $\mu$ rad in phase angle and negligible in module at power frequency.

## **Conclusions**

The proposed system is based on the computation of the displacement factor with uncertainties around  $10^{-7}$ . Other sources also produce uncertainties under 1  $\mu$ W/VA. The uncertainties of the calibration of the 1 V dc source and the 100  $\Omega$  resistor (in dc conditions) can be few parts in  $10^7$  if quantum standards are used. Otherwise, these will be the mayor uncertainty sources. More details on the devices and a combined uncertainty budget will be presented at the conference.

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