# HIGH PRECISION VOLTAGE RATIO MEASUREMENTS USING A 3458A MULTIMETER 

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#### Abstract

This paper shows that very high precision ratio measurements (uncertainty of 3 parts in $10^{6}$ ) can be performed, also at frequencies of 1 kHz , using the multimeter HP 3458A. It is proposed to slightly change known algorithms to control the instrument,


 and a new uncertainty calculation is presented.
## Introduction

Many systems need to measure voltages ratios, as impedance meters. The use of a commercial voltmeter is an interesting alternative solution than the use of conventional bridges [1]-[2]. However, these systems generally operate at 1 kHz and even best available ac voltmeters have high errors at that frequency. The specifications of the Hewlett Packard/Agilent 3458A digital multimeter (DMM) for ac measurements at 1 kHz show errors around 100 $\mu \mathrm{V} / \mathrm{V}$. Using the external control algorithm proposed by Swerlein [3], the uncertainty decrease to $10 \mu \mathrm{~V} / \mathrm{V}$ at 50 Hz , but at 1 kHz it raises up to $40 \mu \mathrm{~V} / \mathrm{V}$. All these values apply to single measurements. In case of ratio measurements, both voltages are strongly correlated, and many uncertainty contributions vanish. A recently proposed algorithm [4] was modified to adapt it to ratio measurements. Here we report the modifications and the new uncertainty calculation.

## Rms computation

The Swerlein algorithm is based on a sampling method. The rms value is calculated as

$$
\begin{equation*}
V=\sqrt{\frac{\sum_{1}^{n} v_{i}^{2}}{n}} \tag{1}
\end{equation*}
$$

where $v_{i}$ is the $i$ sample, and $n$ the total number of samples (corresponding to a integer number of cycles). The algorithm controls the analog to digital converter of the DMM, and performs all calculations. An external computer receives the rms value and calculates the uncertainty of the measurements. Although the method seems to be simple, there are
many sources of uncertainties [3] that increase at high frequencies. They will be discussed in the following paragraphs, for ratio measurements.

## A. DC voltage errors.

The DMM's DCV specifications show different accuracies depending on the range and period of calibration. To reduce this uncertainty source, we propose to use the same range for both measurements, up to $10: 1$ ratio. In this case, Transfer Accuracy/Linearity specifications apply because only the short term stability and the linearity have influence on the measurement. Scale error vanishes in ratio measurements. As an example, for the 10 V dc range, the accuracy specification is $0.05 \mu \mathrm{~V} / \mathrm{V}$ of the reading plus $0.05 \mu \mathrm{~V} / \mathrm{V}$ of the range. In a ratio measurement of 8 V to 0.8 V , that leads to an uncertainty contribution ( $\mathrm{k}=2$ ) of 0.8 parts in $10^{6}$ of the nominal ratio.

## B. Aperture time

When measuring variable signals, it seems to be important to use short aperture times $t_{\mathrm{a}}$ (time required by the analogue to digital converter (ADC) to perform one conversion). However, this criterion leads to large errors in the ADC as it will be discussed later. On the other hand, if large aperture times are selected, the signal will vary during that time. As the ADC of the analyzed DMM integrates the signal during the conversion, an error in the calculation of the rms value will appear. Although the value of this error $\left(e_{\mathrm{a}}\right)$ can be large, it can be calculated with low uncertainty [3], and then it can be corrected. The value of this error is

$$
\begin{equation*}
e_{\mathrm{a}}=\frac{\sin \left(\pi t_{\mathrm{a}} f\right)}{\pi t_{\mathrm{a}} f}-1 \tag{2}
\end{equation*}
$$

It depends on $t_{\mathrm{a}}$ and $f$ (signal frequency), and both depend on the time base of the DMM. However, permanent deviations of the time base do not affect the product $t_{\mathrm{a}} \times f$, because time errors have opposite relative values than frequency errors. Only shot term instabilities increase type A uncertainties.
We propose to use $t_{\mathrm{a}} \geq 200 \mu \mathrm{~s}$ ( 5 samples per cycle at 1 kHz ), because this is the lowest time stated in the

DMM specifications for Additional Gain Error and RMS Noise. For shorter values, there is no information on these errors. The original program was changed to allow that, because it did not permit this selection For $f=1 \mathrm{kHz}$, the value of this error is $e_{\mathrm{a}}=-64511 \times 10^{-6}$, which is corrected by the program. The uncertainty produced by this correction can be estimated from the jitter specification ( 50 ns ). A change of 50 ns in each sample produces a variation in $e_{\mathrm{a}}$ of $32 \times 10^{-6}$. Although it is a large value, it must be taken into account that the algorithm gets about 3000 samples per measurement, so that the estimated uncertainty is reduced to values lower than $1 \mu \mathrm{~V} / \mathrm{V}$ for each rms measurement.

## C. ADC gain errors

The ADC increases it gain error as the aperture time decreases. According to the specifications, an additional error of $17 \mu \mathrm{~V} / \mathrm{V}$ exists for $t_{\mathrm{a}}=200 \mu \mathrm{~s}$ and $\mathrm{f}=1 \mathrm{kHz}$. However, in ratio measurements the same error affects both measurements in the same way. Then, this problem has no influence on the results.

## D. Voltage noise

As the aperture time decreases, the noise of the samples increases. For the 10 V range, at $10 \%$ of full scale and $t_{\mathrm{a}}=200 \mu \mathrm{~s}$, the noise can be as large as 25 $\mu \mathrm{V} / \mathrm{V}$. However, as 3000 samples are taken by each measurement, the effect of it is reduced under 1 $\mu \mathrm{V} / \mathrm{V}$. For 1 V and 100 V ranges, the noise duplicates that value, and it is 20 times greater for the 0.1 V range.

## E. Sampling period

If the time between samples $T_{\mathrm{s}}$ multiplied by the number of samples $n$ is not an integer number of the signal period, an error will appear. This error can be reduced increasing $n$, but a limit of $n=4000$ applies for this DMM (internal memory). The value of this error can reach $100 \mu \mathrm{~V} / \mathrm{V}$ [3] for $f=1 \mathrm{kHz}$ and $T_{\mathrm{s}}=200$ $\mu \mathrm{s}$. To reduce this influence, the algorithm generates multiples trains of samples with a phase shift between them. It is shown that six trains eliminate this error for the fundamental component of the signal, and also for the second harmonic. For higher harmonics some error remains, but with low distorted signals (THD $=0.1 \%$ ), this source of uncertainty is around $1 \mu \mathrm{~V} / \mathrm{V}$.
affects absolute measurements but there is no influence in ratio measurements because both voltages have the same relative error.

## G. Dissipation factor

The dissipation factor of the straight input capacitance generates a frequency dependant error. Anyway, no influence exists for ratio measurements, as the same error affects both measurements.

## H. Combined uncertainties

The proposed control program calculates all uncertainties sources and combines them according to the GUM [5], for each measuring ratio. As an example, the type B uncertainty for the ratio 8 V to $0.8 \mathrm{~V}, 1 \mathrm{kHz}$, is $1.3 \times 10^{-6}$ of the nominal ratio $(\mathrm{k}=1)$.

## Experimental evaluation

An inductive voltage divider (ratio $32 / 3$ ) was used as ratio standard for testing two DMMs. Its ratio uncertainty is $0.5 \times 10^{-6}(\mathrm{k}=2)$. The voltages used were 8 V and $0.75 \mathrm{~V}, 1 \mathrm{kHz}$. The standard deviation of each ratio measurement was around $10 \times 10^{-6}$, similar for both DMMs. Then, to reduce the type A uncertainty to $2 \times 10^{-6}, 25$ measurements were done. For each voltage, the average value of five single measurements was computed, and 5 complete ratio cycles were observed. The ratio deviations were $0.7 \times 10^{-6}$ for one DMM, and $-1.1 \times 10^{-6}$ for the other, with a total expanded uncertainty $(\mathrm{k}=2)$ of $3.3 \times 10^{-6}$. These results confirm the theoretical conclusions.

## References

[1] B. C. Waltrip, N. M. Oldham, "Digital impedance bridge," IEEE Trans. Instrum. Meas., vol. 44, no. 2. pp. 436-439. Apr. 1995.
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[3] R. L. Swerlein, "A 10 ppm accurate digital AC measurement algorithm," Proc. NCSL Workshop Symp., pp. 17-36, 1991
[4] G. A. Kyriazis. R. Swerlein, "Evaluation of Uncertainty in AC Voltage Measurement Using a Digital Voltmeter and Swerlein's Algorithm." Conference on Precision Electromagnetic Measurements 2002, CPEM Digest, pp. 2425, 2002.
[5] ISO Guide to the Expression of Uncertainty in Measurement. 1995.

## F. Bandwidth

The DMM has a limited bandwidth $(150 \mathrm{kHz}$ in 1 V and 10 V ranges, and lower frequencies for the others). It

