# Behavior of Reactive Electronic Meters and Power Factor Meters Under Non-sinusoidal Waveforms and Unbalance Three Phase Systems

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***Abstract-*** **The behavior of electronic meters under distorted and unbalanced systems is studied. The computing algorithms implemented on them by the manufacturers for the calculation of reactive power, apparent power and power factor are deduced from external tests. It is shown that differences as large as 50% in the reactive power exist between different meters, running under field conditions.**

***Index terms*:** **Energy measurement, reactive power, power factor, distortion, harmonic, watthour meter.**

## I. INTRODUCTION

Historically, in power networks, the reactive energy has been measured using induction type meters and the power factor has been calculated from the computation of the active and reactive energies during the billing period. These systems, used under sinusoidal waveforms and balanced networks, have low measuring errors. However, in the present-day networks, large measuring errors appear due to distorted loads and asymmetries systems [1]. These situations cannot be solved by these kind of instrumentation.

On the other hand, in the last years new electronic energy meters have been developed with large computation power. Manufacturers (and in some models, also users) can implement on them different algorithms to compute reactive energy and power factor.

The main problem remains in the lack of an universal definition for those parameters, valid for all electrical situations. Even under sinusoidal waveforms, the apparent power has not a unique accepted definition if there are asymmetries in the three-phase network. Many definitions have been proposed, but no one proved to be useful under all situations. In this paper, it is focused the application to billing propose.

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Due to these facts, different manufacturers implement different algorithms to compute apparent power, reactive power and also power factor. Indeed, some models have contradictory algorithms implemented in themselves. This leads to large confusing situations on consumers and network administrators. In the next chapters different definitions are analyzed and the real behavior of electronic meters are tested.

## II. DEFINITIONS

The electric parameter that quantifies the increase of the losses in power networks is the power factor *PF* (defined as the ratio between the active power and the apparent power). Indeed, most electric companies limit in their regulations, not the reactive energy but the power factor, which should be greater to certain value to avoid additional billing.

Nowadays, several electronic measuring systems compute the *PF*, however historically it was only possible to calculate it starting from the measurement of the active and reactive energy, by means of two different meters. Even presently, most of the instruments used by electrical utilities are electro-mechanical meters of active and reactive energy. Billing systems are based on the computation of these two values. In utilities, an institutional culture exists on the relation between the active power $P$, reactive power $Q$, and the power factor given by the simple equation

$$PF = \frac{1}{\sqrt{1 + \frac{Q^2}{P^2}}} \quad (1)$$

The parameters of this equation are fully defined for sinusoidal waveforms in single-phase circuits, where the power factor coincides with the cosine of the angle between the current and the voltage. However, already in the 20´s decade of the last century, it was shown that with distorted waveforms of current or voltage, this equation leads to different concepts to those of reciprocating components of power [1]. A similar situation occurs in non balanced three-phase systems, even working under sinusoidal conditions. Another method for calculating Q is given by

$$Q = \sqrt{S^2 - P^2} \quad (2)$$

where $S$ is the apparent power. The value defined by (2), in non sinusoidal conditions, is generally called $N$ (non-active power). It takes into account reciprocating powers as well as distorted power [2]. There have been many attempts of decomposing the non-active power in several components [3]. Budeanu separated the non-active power in two components

$$N^2 = Q_B^2 + D_B^2 \quad (3)$$

being $Q_B$ the sum of the reactive powers produced by each harmonic component. Fryze proposed, in the domain of time, to split the non-sinusoidal current in two components $i_a$ and $i_b$. The first one is proportional to the voltage $v$ (equal waveform) and the second is the remaining difference.

$$i_a = \frac{P}{V_{rms}^2} v \quad (4)$$

$$i_b = i - i_a \quad (5)$$

Kimbark also divided the non-active power in two components. The first one is $Q_1$ (reactive power associated only to the fundamental harmonic components of voltage and current) and a second component with the remaining quadratic difference ($D_K$).

$$N^2 = (V_1 I_1 sin\varphi_1)^2 + D_K^2 \quad (6)$$

The main propose of most of these definitions is to get a decomposition that could separate different causes of low power factor, such as reciprocating powers and, on the other hand, problems caused by harmonics. The first one would be compensated by capacitors (in case of inductive behavior), but the second one needs more complicated electronic compensators. Unfortunately, from Budeanu's proposal up to the last ones, there are examples in which the separation in different causes of the phenomenon is not according to the author's proposals. These contradictions lead to the present situation in which no definition is accepted as the right one for all electrical applications. On the other hand, in this paper, only the application to billing systems is analyzed. The main purpose, in this case, is to detect measurement problems related to low power factors regardless of the causes of them. However, if (1) is used for calculating the power factor, as most utilities do, with the definition of $Q=Q_1$ proposed by Kimbark (implemented by electro-mechanical meters and many electronic ones), then the computed power factor does not take into account the harmonic effects. To get the right power factor, it is necessary to use (2) as the definition of $Q$. This definition coincides with that proposed by Fryze.

The unbalance of three–phase systems is another factor that increases the losses, and wastes network-capabilities. Even under sinusoidal waveforms, unbalanced systems affect the efficiency of distribution networks. A non-symmetric system overloads some phases and wastes installed capacity in others. In the example shown in Fig. 1 ($V$: rms phase voltage), the source is loaded only by one resistor ($R$). Through phases $a$ and $b$ the current is $I_d$ and, on the other hand, there is null current through phase $c$. The value of $I_d$ is

$$I_d = \frac{\sqrt{3}V}{R} \quad (7)$$

The active power $P_d$ is

$$P_d = \sqrt{3} V I_d \quad (8)$$

and the value of the network losses ($P_{Ld}$) is

$$P_{Ld} = 2 R_c I_d^2 \quad (9)$$

being $R_c$ the resistance of each phase. On the other hand, with a symmetrical resistive load the active power $P_s$ would be

$$P_s = 3 V I_s \quad (10)$$

where $I_s$ is the rms current corresponding to the symmetrical load. In this case, the value of the network losses ($P_{Ls}$) would be

$$P_{Ls} = 3 R_c I_s^2 \quad (11)$$

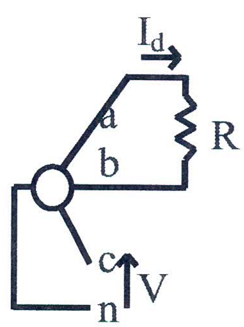


Fig. 1. Three-phase network with asymmetric resistive load

If we impose $P_{Ls}=P_{Ld}$, it is concluded that

$$I_d = \sqrt{\frac{3}{2}} I_s \qquad (12)$$

From (8), (10) and (12) it is concluded that the relation between active powers is $P_s/P_d = \sqrt{2}$. This means that with the same losses it could be possible to feed a resistive symmetrical load with a power $\sqrt{2}$ times larger than the corresponding to the asymmetric example. To penalize this consumer appropriately, according to the losses criterion, a power factor of $1/\sqrt{2}$ must be associated to this load. This can be achieved using an appropriate definition of the apparent power in (2). However, in most cases, electrical utilities compute the power factor from active and reactive meters. Induction type reactive meters and many electronic ones implement the following computation for the reactive power

$$Q = \frac{-1}{\sqrt{3}T}\int_0^T \left[(v_a - v_b)i_c + (v_b - v_c)i_a + (v_c - v_a)i_b\right]dt \qquad (13)$$

where $v_i$ and $i_i$ are instantaneous values of voltage and current. In the example of Fig. 1, (13) leads to

$$Q = -VI_d(\cos 2\pi/3 + \cos \pi/3) \qquad (14)$$

Then, $Q$=0; and using (1), the computed power factor would be $PF$=1. This shows that this definition of reactive power, jointly with the calculating method of the power factor used by most of electric utilities for billing systems, is insensitive to asymmetries in the load.

Many different definitions of apparent power have been proposed, leading to very different results. One of the simplest definitions for apparent power, that only adds the apparent powers of each phase, neither contemplates the described problem.

$$S_{su} = \sum_{i=1}^{3} V_i I_i \qquad (15)$$

Applying this definition to the previous examples

$$S_{su} = 2VI_d \qquad (16)$$

So that, the value of the power factor calculated from (1), (2) and (16) is $\sqrt{3}/2$, larger than $1/\sqrt{2}$, as expected. The following definition of apparent power is correct from the losses point of view [4].

$$S_e = 3\sqrt{\frac{V_a^2 + V_b^2 + V_c^2}{3}}\sqrt{\frac{I_a^2 + I_b^2 + I_c^2}{3}} \qquad (17)$$

The application of this definition to the previous example leads to $S_e = \sqrt{3}\sqrt{2}VI_d$, and $PF = 1/\sqrt{2}$. This definition takes into account the network losses, however it does not include the total effects of waste of network capability. In single-phase cases, the percentage of the increment of the losses always coincides with the reduction of active power capability. However, in three-phase systems, the total capability is limited by the most loaded phase. Then, it is possible to reach the maximum rating of cables, transformers and other components, although some phases were only partially loaded. In the discussed example, from (8) and (10) it is concluded that the maximum power that can be delivered is $1/\sqrt{3}$ of the value corresponding to a symmetrical load, supposing that the current is the same in both cases. Therefore, this consumer must be penalized associating to the billing a lower power factor. For this, a new apparent power definition is proposed ($S_p$), where the last factor (related to the currents) in the equation (17) is substituted for the maximum rms current of all phases ($I_{max}$). This current limits the capability of the system. Then

$$S_p = 3\sqrt{\frac{V_a^2 + V_b^2 + V_c^2}{3}} I_{max} \qquad (18)$$

In the analyzed example, $S_p = 3VI_d$ and $PF = 1/\sqrt{3}$ in accordance with the proposal.

Using the first analyzed definition of reactive power (13) with the definition of the power factor shown in (1), no unbalance in the currents is detected. This is the present situation for most of electrical utilities. Using the definition of apparent power shown in (15) with the usual definition of the power factor, it undervalues many of the problems like network losses. The third definition of apparent power (17) is right from the losses point of view, but still undervalues the reduction of installed power. Finally, the proposed definition (18), which always leads to smaller power factors than the other ones, may overvalue the effect on the losses; but it is the only one that takes fully into account the reduction of installed capability of the network.

## III. ENERGY METERS

Modern solid-state meters, based on diverse operation principles, can compute several parameters as power, active, reactive and apparent energy, power factor, current, voltage, and harmonic distortion. Generally, the internal algorithms for these computations are not known in detail by the users, so that it is not easy to predict their behaviors under non standard conditions.

Four electronic three-phase meters of different manufacturers, identified as A, B, C and D have been tested with the objective of determining the algorithms implemented in themselves. The first three are of class 0.2S and the fourth is class 1, according to IEC 60687, 61036, and class 2 according to IEC 61268 [5, 6, 7]. They were tested under normalized conditions, as established by the standard. All of them were according to the error limits. Complementarily, additional tests were done. Meters A and D were tested with non-linear single-phase current and voltage. In this case, serial-parallel connection was used. The waveforms shown in Fig. 2 and Fig. 3 reproduce real data collected from a water pump plant. The voltage waveform (Fig. 2) has the following values: Vrms =100.0 V, frequency=50 Hz, THD=2.9%, phase of the fundamental component = 0°. The total harmonic distortion is referred to the fundamental component.

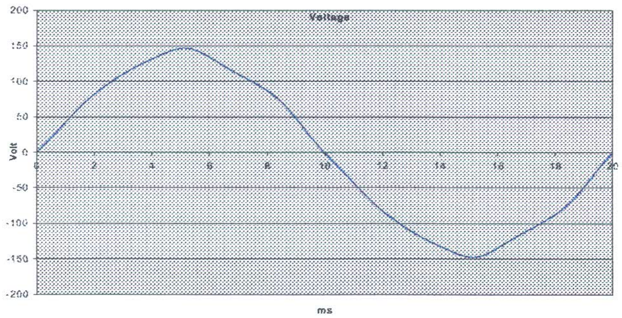


Fig. 2. Voltage waveform of a field measuring point.

The current waveform, shown in Fig. 3, has the following values: Irms=0.76 A, $I_1$=0.72 A, frequency=50 Hz, THD=35.5%, phase of the fundamental component=-12°.

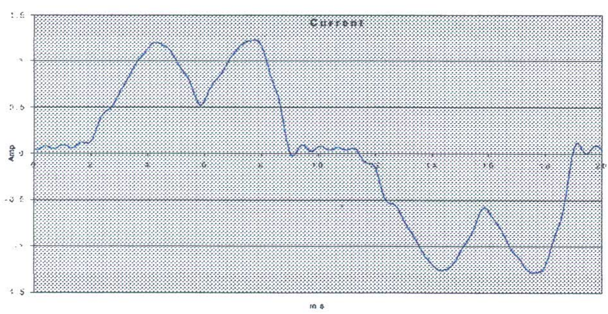


Fig. 3. Current waveform of a field measuring point.

Both meters were energized at the same time with the same values and waveforms. After 48 hours, the values of reactive energy for each meter were obtained. For the purpose to compare both behaviors, values of reactive power are deduced from the measured reactive energies. The obtained results (per phase) were:

meter A: Q = 15 var
meter D: Q = 30 var

It is remarkable that meter D shows double reactive power than meter A. If the internal computing algorithm of Q is based on Kimbark proposal, then

$$Q_K = V_1 I_1 sin\varphi_1 = 14.9\, var \qquad (19)$$

This shows that this must be the algorithm used by meter A. On the contrary, if the algorithm is based on the Fryze proposal (2), the reactive power value would be

$$Q_F = \sqrt{(V_{rms} I_{rms})^2 - \left(\sum V_n I_n \cos\varphi_n\right)^2} = 29.5\, var \qquad (20)$$

This is the case for meter D. For the power factor calculation, this meter will lead to the real value. Under this point of view, meter A has a huge error of -49%.

The power factor calculated using (1) leads to: $PF_K$=0.98 and $PF_F$=0.92. The real power factor, measured by a Fluke 41 analyzer was 0.92 coinciding with meter D.

Meter A displayed the rms value of the current correctly, that means that it considers all the harmonic components. From this value and the value of the voltage, it would be possible to calculate the apparent power (the result is 76 VA). However, the value of apparent power displayed in the screen is 72 VA. This value uses only the fundamental component of the current, discarding all harmonic components.

The displayed power factor is 0.97 which practically coincides with the cosine between the fundamental components of voltage and current.

That is, although the meter measures all harmonic components appropriately, the manufacturer decided to use algorithms that simulate the behavior of electro-mechanical meters, which are insensitive to harmonics. There is an inconsistency between the displayed apparent power and the calculated one obtained from the multiplication of the rms values of voltage and current, also displayed by the same meter. Other inconsistency comes from the displayed power factor and the calculated one from the ratio between the displayed active power and the product between voltage and current.

Meter D displayed a value of reactive power using (2). In case of distorted waveform, meters A and D show a big difference with the reactive energy storage values and this inconsistency create a big economic problem in the billing.

Additionally, A, B and C meters were tested with a resistive single-phase load connected as Fig. 1. Under asymmetric resistive load, models A and C showed values of $Q$=0, $PF$=1 and apparent power equal to the active power. This coincides with algorithms that implement the behavior of induction type meters, in agreement with (13) and (1). It is interesting to note that these meters have the possibility to compute other definitions of apparent power, since they compute the rms value of each one of the currents and voltages. But, these manufacturers choose to implement the same algorithm that is used with the traditional set of active and reactive induction meters.

On the other hand, model B computes the power factor using an algorithm that coincides with the computation of the apparent power according to (15). In case of single-phase resistive load, it displays the value 0.87 being the same value that was calculated in chapter II. However, in the same meter and under the same load, the displayed values of reactive power and energy are zero. Therefore, for the reactive power it uses an algorithm that coincides with (13). These ambiguous criteria can cause a great confusion in the evaluation of the power factor by the consumers. On one hand, it is concluded that the value of the power factor is 0.87 and, on the other hand, that the value is 1.

This analysis shows the disparity of algorithms that different manufacturers implement in their meters, which is the result of the present situation of lack of international agreement on the definitions related to non-balanced or distorted systems.

## IV. CONCLUSIONS

Different definitions of apparent and reactive power have been analyzed from the point of view of the billing system between electric utilities and consumers. Electronic meters, to implement these definitions, already exist; but the problem is to agree on the use of one of them as a standard definition. When Budeanu proposed his theory there were not meters to compute his power definitions. Nowadays, we can measure whatever we like, but we do not agree yet on what we really want. Considering this situation, meter's manufacturers implement different algorithms to compute the reactive power and power factor that create a large confusion in users.

After 70 years of discussion, is time to accept that there is no definition that works well for all application fields. It is proposed in this paper a definition of reactive power, shown in (18), only for using in the billing field, that maintain the present method of power factor calculation shown in (1). This condition is essential for the utility acceptability of the proposal. It takes into account not only the extra losses causes by distortion and non symmetries, but also the effects of waste of the network capability.

Additionally, it is necessary to decide what to do regarding to the large quantity of points of measurements that the companies already have, based on classic meters. These will not be able to implement any new definition; it will be necessary a change of instruments. Given the high number of these meters, the change of them implies a great investment. However, it has been shown that the power factor computed from these meters is greater than anyone of the other $PF$ proposals. This implies a benefit for the consumer and a damage for the electrical utility. From this point of view, it is natural to propose that the utility decides about the convenience or not of the change of instruments in each measuring point.

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