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An analysis of the economic lot-sizing problem with return options focused on the remanufacturing plan

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Resumen

El Problema del Tamaño del Lote Económico (ELSP por sus siglas en inglés) puede ser definido como el problema de determinar los períodos y las cantidades a producir en cada período para satisfacer los requerimientos de demanda de un cierto artículo sobre un horizonte de planificación finito, minimizando la suma de todos los costos involucrados. Los valores de demanda se asumen conocidos y pueden ser diferentes para cada período, es decir, demanda determinística y dinámica. Hay costos fijos y variables por producir en cada período, y por unidad almacenada en inventario de un período a otro siguiente. El ELSP es un problema bien conocido en la literatura y varias extensiones han sido planteadas para atender de una mejor manera las necesidades de la industria. Una de las más recientes y relevantes extensiones del ELSP es cuando se incluye el retorno de artículos usados, los cuales pueden ser remanufacturados para satisfacer la demanda. También debe tenerse en cuenta que los artículos usados y retornados (o retornos simplemente) puedan ser descartados de una manera adecuada, como por ejemplo, cuando los mismos no satisfacen ciertos requerimientos mínimos para ser remanufacturados. La remanufacturación es un proceso de recuperación de artículos usados mediante la cual se puede asegurar que los productos remanufacturados ofrecen la misma calidad y funcionalidad que los artículos nuevos. Ejemplos de productos remanufacturados son: autopartes, motores, neumáticos, equipamiento de aviones, cámaras fotográficas, instrumentos médicos, muebles, cartuchos, fotocopiadoras, computadoras y equipo de telecomunicaciones. La remanufacturación ofrece beneficios para todas las partes involucradas. El consumidor puede obtener productos de la misma calidad a un precio generalmente inferior que el de uno nuevo. El fabricante se ve beneficiado ya que la remanufacturación necesita menos energía, menos materias primeras y menor trabajo. Por último, el medio ambiente también se ve beneficiado con un uso más eficiente de la energía y de las materias primas, y además la remanufacturación tiende a reducir el número total de artículos puestos en el mercado al extender la vida útil de los mismos.

En esta tesis consideramos la extensión del ELSP en la cual la demanda puede ser también satisfecha mediante la remanufacturación de artículos usados y retornados al origen, además de con artículos nuevos. Nos referiremos a este problema como el Problema del Tamaño del Lote Económico con Retornos (ELSR por sus siglas en inglés). Teniendo en cuenta que el ELSR es un problema NP-difícil en general e incluso para casos particulares de funciones de costos, decidimos analizar el ELSR desde el enfoque “divide y reinaras”, aplicado a la actividad de remanufacturación. Esta decisión está basada en el hecho de que la actividad de remanufacturación juega un rol fundamental en la resolución del ELSR ya que los planes de producción y de disposición final óptimos pueden ser determinados eficientemente y de forma independiente si el plan de remanufacturación es conocido, ya que ambos pueden ser formulados como problemas ELSP independientes. Por lo tanto en esta tesis nos enfocaremos en el problema de determinar la remanufacturación de una solución óptima del ELSR, el cual referiremos como el problema de obtener el Plan de Remanufacturación de Costo Perfecto. Se debe tener en cuenta que resolver este último problema es equivalente a resolver el ELSR, y por lo tanto es NP-difícil para los mismos casos. Considerando esta dificultad, analizaremos el problema de determinar las cantidades óptimas de remanufacturación suponiendo que el conjunto de períodos en donde la remanufacturación es posible ha sido definido con anterioridad, o en otras palabras el ESLR con Períodos Fijos para la

Remanufacturación. Este supuesto está soportado tanto por motivos académicos como de la vida real, como por ejemplo: razones operativas si los operarios y las máquinas son los mismos para la producción y la remanufacturación; disponibilidad de artículos usados solo en ciertos períodos; o razones económicas causadas por remanufacturación a bajo costo en ciertos períodos. Asumiendo que la cantidad a remanufacturar en los períodos permitidos es estrictamente positiva, y que los costos son no especulativos (es decir que es conveniente producir o remanufacturar lo más tarde posible) pudimos demostrar que la cantidad total de remanufacturación de una solución óptima puede ser obtenida de manera eficiente a través de un procedimiento de tiempo lineal en el número de períodos. Entre otras implicaciones, este resultado sirve como sustento teórico para una regla de remanufacturación simple pero efectiva, utilizada para resolver el ELSR con períodos fijos de remanufacturación. La regla establece que la cantidad a remanufacturar en un cierto período fijado debe ser el mínimo entre la cantidad de retornos disponibles en le período y la demanda acumulada desde el período en cuestión hasta el período inmediatamente anterior al próximo periodo fijado como de remanufacturación positiva. En esta tesis se muestra además que esta regla puede ser aplicada con muy buenos resultados para el caso del ELSR con Sustitución en Una Vía, es decir, los productos nuevos pueden utilizarse para satisfacer al demanda de artículos remanufacturados pero no viceversa. Esta clase de problemas ocurre en la práctica cuando hay segmentos de mercado diferentes para los productos nuevos y los remanufacturados. El análisis del ELSR centrado en la remanufacturación se completa demostrando que el problema de determinar las cantidades óptimas para cada uno de los períodos fijados es NP-difícil en general, aún en el caso en que el número de períodos que se han fijado como de remanufacturación positiva, es menor que el número total de períodos. Considerando este resultado para el caso general, se provee un algoritmo recursivo de orden pseudopolinomial que puede resultar efectivo en los casos en que la cantidad de períodos fijados es pequeña en relación al largo del horizonte de planificación, o la cantidad total de retornos es pequeña en comparación con la cantidad de demanda total. Finalmente, teniendo en cuenta los buenos resultados obtenidos para el ELSR bajo el supuesto de costos no especulativos, decidimos analizar la extensión del ELSP en la cual existen restricciones de capacidad en la producción (CLSP por sus siglas en inglés) bajo este supuesto en los costos. Para este problema fuimos capaces de mejorar el orden del reconocido algoritmo de Florian y Klein (1974) de $O(T^4)$ a $O(T^3)$ para el caso de costos cóncavos y no especulativos y capacidad de producción estacionaria. Hasta donde sabemos, este tipo de estructura de costos incluye casos de interés que no son abarcados por trabajos previos en la literatura.

Palabras Claves: Problema del Tamaño del Lote Económico, Remanufacturación, Control de Inventario, Optimización.

Abstract

The Economic Lot-Sizing Problem (ELSP) can be defined as the problem of determining the periods and the quantities to produce in order to meet the demand requirements of a single item for each one of the periods over a finite planning horizon on time, minimizing the sum of all the costs involved. The values of the demand are known in advance and can be different for each period, i.e. deterministic and dynamic demand. Set-up and unit costs are incurred by producing at each period, and unit costs for holding inventory from one period to the next. The ELSP is a well-known problem in the literature and several extensions have been considered in order to better attend practical and industrial needs. One of the recent and relevant extensions on the ELSP is to consider return options, i.e., used products returned to the origin that can be remanufactured to satisfy the demand requirements. On the other hand, used items (or returns) can be discarded properly, e.g. when they do not satisfy certain minimum requirements for remanufacturing. Among the recovery options, remanufacturing can be defined as the recovery process of returned products after which it is warranted that the remanufactured products offer the same quality and functionality that those newly manufactured. Products that are remanufactured include automotive parts, engines, tires, aviation equipment, cameras, medical instruments, furniture, toner cartridges, copiers, computers, and telecommunications equipment. Remanufacturing offers benefits for all of the parties involved. From the consumer's point-of-view, remanufactured products assume the same quality of new products and are sometimes offered at an inferior market price. For the manufacturer, remanufacturing provides cost savings in energy consumption, raw materials, and labor. Finally, the environment benefits from the more efficient use of energy and raw materials. In addition, remanufacturing tends to reduce the total number of products put in the market and later disposed, i.e. long-life products.

In this thesis we consider the ELSP extension for which the demand requirements can be also satisfied by remanufacturing used items returned to the origin. We refer to this problem as the Economic Lot-Sizing Problem with Remanufacturing and Final Disposal options (ELSR). Since the ELSR is an NP-hard problem, even for particular types of cost structures, we address the ELSR by means of a divide-and-conquer approach, focused on the remanufacturing activity. This approach is supported by the fact that the remanufacturing plan plays a key-role in the ELSR resolution since both the optimal production plan and the optimal final disposal plan can be determined separately and in an effective-time way if the remanufacturing plan is known, since both can be formulated as classical ELSP. Therefore, we focus on the problem of obtaining the remanufacturing plan of an optimal solution of the ELSR, which we refer as the remanufacturing plan of perfect cost. The problem of determining the remanufacturing plan of perfect cost can be considered equivalent to solve the ELSR, and then it is also NP-hard for the same cases. Considering this difficulty, we study the problem of obtaining the remanufacturing plan of perfect cost for which the periods where remanufacturing is allowed to be positive are known in advance, or in other words, the ELSR with fixed periods for remanufacturing. This assumption is supported by academic as well as practical reasons, e.g., operative reasons if the machinery and workers are the same for production and remanufacturing operations; availability of used items only in certain periods; or economic reasons due to periods with remanufacturing at low cost. Assuming strictly positive remanufacturing quantities in the periods fixed and a non-speculative motives on the costs (i.e., it is profitable to produce or remanufacture as late as possible) we prove that the total remanufacturing quantity of the remanufacturing plan of perfect cost can be obtained in linear time. Among further implications, this result serves as a theoretical support for a simple but effective rule proposed for obtaining the remanufacturing quantities of an ELSR with fixed periods for remanufacturing. This rule establishes that the remanufacturing quantity in a certain period is the minimum between the available returns and the accumulative demand from the current period until the period preceding the next period with positive remanufacturing. In this thesis we show that this rule can be effective employed for the case in which one-way substitution is allowed for the ELSR, i.e., manufactured items can satisfy the demand requirements of remanufactured items but not viceversa. This kind of situation occurs in practice when there are different market segments for new and remanufactured items. We complete the study of the remanufacturing plan showing that the problem of determining the optimal quantities for each one of the periods of an ELSR problem with fixed periods for remanufacturing is NP-hard in general, even in the case that the number of periods where remanufacturing is allowed is less than the total number of periods. In addition, we provide a recursive algorithm of pseudopolynomial time for solving the problem that can be time-effective if either the number of fixed periods for remanufacturing or the number of total returns is small. Taking into account the good results obtained for the ELSR with non-speculative motives on the costs, we decided to address the capacitated version of the economic lot-sizing problem (CLSP) under this last assumption. Thus, we are able to show that the well-known algorithm of Florian and Klein (1971) for the CLSP can be improved from $O(T^4)$ to $O(T^3)$ time for the case of concave cost functions with non-speculative motives and stationary capacities. This type of cost structure includes many situations of interest; including some particular cases that are not covered by algorithms proposed in previous works in the literature.

Keywords: Economic Lot-Sizing Problem, Remanufacturing, Inventory Control, Optimization.

List of included papers

- **Analysis of the quantities of the remanufacturing plan of perfect cost**, Pedro Piñeyro and Omar Viera. Revised version of the paper published in *Journal of Remanufacturing* 2012, 2:3, doi:10.1186/2210-4690-2-3. An initial version was published in Proceedings of ICoR: International Conference on Remanufacturing, 27-29 July 2011, University of Strathclyde, Glasgow, Scotland, ISBN 0947649816, 9780947649821.

- **The economic lot-sizing problem with return options and fixed periods for remanufacturing: complexity and algorithms**, Pedro Piñeyro and Omar Viera. Submitted to *Pesquisa Operativa*. An initial version was published in Annals of the Brazilian Symposium on Operations Research, 16th CLAIO: Congreso Latino-Iberoamericano de Investigación Operativa, 24-28 September 2012, Rio de Janeiro, Brazil, ISSN 1518-1731.

- **The economic lot-sizing problem with remanufacturing and one-way substitution**, Pedro Piñeyro and Omar Viera. Revised version of the paper published in *International Journal of Production Economics*, 124(2), 482–488, 2010.

- **An $O(T^3)$ algorithm for the capacitated economic lot sizing problem with stationary capacities and concave cost functions with non-speculative motives**, Pedro Piñeyro, Omar Viera and Héctor Cancela. Revised version of the paper published in *Lecture Notes in Management Science*, 5(1), 39-45, 2013, Proceedings of the 5th International Conference on Applied Operational Research (ICAOR), 2013, 29-31 July, Lisbon, Portugal.

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I dedicate this work to the memories of my father Pedro and my son Juan Martín. They are always in my mind and my heart.

Contents

Part I

1 Introduction and motivation

The economic lot-sizing problem (ELSP) can be defined in its simplest form as the problem of determining the periods and the quantities to produce in order to meet the demand requirements of a single item on time, minimizing the sum of all the costs involved. There are set-up and unit costs for carrying out the production activity and unit costs for holding inventory from one period to the next. The demand and costs values are assumed known in advance and can vary for each period. In addition, capacity on production and storage is assumed unbounded and backlogging demand is not allowed. The ELSP is a well-known problem in the literature of Operations Research and Management Science (OR/MS) from the late 1950s as it can be considered a basic problem in production planning and inventory management (Karimi et al., 2003; Brahimi et al., 2006). Wagner and Whitin (1958) address the ELSP considering that the unit production cost is the same for every period. They provide an $O(T^2)$ time algorithm for the ELSP based on a dynamic programming approach, with T the planning horizon length. The algorithm is based on the today well-known zero-inventory property, introduced by the authors in their seminal paper: there is an optimal solution for which the production quantity in a certain period is positive if and only if the entering inventory level is zero. Later it was shown that the algorithm is also valid for the case of general concave cost functions (Zangwill, 1968). As an evidence of the impact of the Wagner and Whitin (1958) paper on ELSP, it was reproduced in January 2004 in the special issue of the journal Management Science celebrating the 50th volume of this leading journal in OR/MS. More recently, Federgruen and Tzur (1991), Wagelmans et al. (1992) and Aggarwal and Park (1993) propose faster algorithms for the ELSP of $O(T\log T)$ time and $O(T)$ time for the Wagner and Whitin case (constant unit production costs). In particular, Atamtürk and Küçükyavuz (2008) provide an $O(T^2)$ time algorithm for a more general formulation of the ELSP with inventory bounds and fixed costs for holding inventory.

From the seminal paper of Wagner and Whitin (1958), many extensions have been considered in the literature in order to better attend practical and industrial needs, e.g., more general cost functions (Zangwill, 1968; Shaw and Wagelmans, 1998), multi-item (Pochet and Wolsey, 1991; Tempelmeier and Helber, 1994), multi-level (Zangwill, 1969; van Hoesel et al., 2005), allowing shortage and/or lost sales (Pochet and Wolsey, 1988; Sandbothe and Thompson, 1990), capacity constraints on production (Florian and Klein, 1971; van den Heuvel and Wagelmans, 2006), limited storage (Love, 1973; Gutiérrez et al. 2001, Wolsey 2006; Atamtürk and Küçükyavuz 2008) and perishable items (Friedman and Hoch, 1978; Hsu 2000), some of them considering more than one of the extensions listed above in the same paper. We refer to Brahimi et al. (2006) for details about the ELSP and its extensions.

A recent and relevant extension on the ELSP is to consider return options, i.e., used products returned to the origin (supplier) that can be reconditioned to satisfy the demand requirements. Additionally, used items (or returns) can be discarded properly, e.g. when there is an overstock of used products or they do not satisfy certain minimum requirements for recovering. This kind of problem has been receiving an increasing academic attention in recent years as the industry has been involved with the recovery of used products (Gungor and Gupta 1999; Guide 2000; de Brito and Dekker, 2002, Hatcher et al. 2013). Environmental problems related to industrial activity from the middle of the last century and uncontrolled usage of natural resources puts in danger both the present levels of production and the development of future generations. These problems were caused in part by the production and massive consumption of short-life products. In view of this situation, governmental and social pressures as well as economic opportunities have motivated many firms to become involved with the return of used products for recovery. The management and activities related to the return flow from the consumer to the producer are known as Reverse Logistics, and they stand in contrast to the forward logistics from the producer to the consumer. Among the industrial options for recovering, the remanufacturing activity can be defined as the recovery process of returned products after which it is warranted that the remanufactured products offer the same quality and functionality that those newly manufactured (Ijomah, 2002; Matsumoto and Ijomah, 2013). Products that are remanufactured include automotive parts, engines, tires, aviation equipment, cameras, medical instruments, furniture, toner cartridges, copiers, computers, and telecommunications equipment. Remanufacturing offers benefits for all of the parties involved. From the consumer's point-of-view, remanufactured products assume the same quality of new products and are sometimes offered at an inferior market price. For the manufacturer, remanufacturing provides cost savings in energy consumption, raw materials, and labor. Finally, the environment benefits from the more efficient use of raw materials and energy employed during the production phase. In addition, remanufacturing tends to reduce the total number of products put in the market and later disposed, i.e. long-life products. For overviews about reverse logistics and remanufacturing benefits, the reader is referred to Gungor and Gupta (1999), Guide (2000), de Brito and Dekker (2002), Ijomah (2002), Hormozi (2003), Sundin (2004) and Gurler (2011). As an example of the today relevance of the remanufacturing activity in the industry, the European Division of the Automotive Parts of Remanufacturers Association (APRA) reports that the remanufacturing industry involves more than 400.000 directly or indirectly jobs in the world, and the annual production of remanufactured products in Europe grew from 10 millions of units in 1995 to 20 millions of units in 2005, with a potential of 30 millions of units in 2015. The U.S. International Trade Commission (USITC) report of October 2012 reports that during the period 2009-2011, U.S. production of remanufactured goods grew by 15 percent to at least $43 billion, supporting 180.000 full-time jobs, and exports of remanufactured goods totaled $11.7 billion in 2011.

In this thesis we consider the ELSP extension with return options for which the used items returned to the origin can be remanufactured for satisfying the demand requirements or possibly discarded in an adequate form for economic reasons. Costs are incurred for carrying on the activities (producing, remanufacturing or disposing) and for holding inventory of both used and serviceable items (produced or remanufactured items). The objective is to determine the quantities to produce, remanufacture and dispose at each period over a finite planning horizon in order to meet the demand requirements on time, minimizing the sum of all the involved costs. We refer to this problem as the Economic Lot-Sizing Problem with Remanufacturing and Final Disposal options (ELSR). The ELSR was shown to be NP-hard, even for the case of cost functions composed of both setup and variable costs for the activities (van den Heuvel, 2005; Retel-Helmrich et al., 2014). Considering this difficulty, several particular cases have been analyzed in the literature and heuristic procedures have been suggested (Richter and Weber, 2001; Golany et al., 2001; Yang et al., 2005;

Teunter et al., 2006; Retel-Helmrich et al., 2014; Li et al., 2014). One way to address the problem is by means of a divide-and-conquer approach, solving separately the activities of production, remanufacturing and final disposing. In Piñeyro and Viera (2009) we suggest and evaluate several inventory policies for the ELSR based on the divide-and-conquer approach (this paper is not included in this thesis as it was part of the MSc thesis by the same author of the present one; it was an important starting point for this work). We show that this approach is supported by the fact that the remanufacturing activity plays a key role in the resolution of the ELSR since the remanufacturing involves both serviceable and used items while the other activities involve only one of them. Thus, if the remanufacturing plan is known in advance (periods and quantities), the production and final disposal plans can be solved in an effective-time way as separate ELSP. Thus, the problem of solving the ELSR can be reduced to the problem of finding a remanufacturing plan of perfect cost in an independently way, i.e., the remanufacturing plan of an optimal solution of the ELSR. We note that this last problem is NP-hard, since it is equivalent to the ELSR. In Piñeyro and Viera (2009) we address the problem of finding a remanufacturing plan of perfect cost by means of analyzing the problem of determining a remanufacturing plan which maximizes the total remanufacturing quantity, thus considering both ecological as well as economical benefits simultaneously. In this sense, we introduce a simple but effective rule for determining the remanufacturing quantities for a given set of periods fixed as positive remanufacturing periods. This rule establishes that the remanufacturing quantity in a certain period fixed is the minimum between the available returns and the accumulative demand from the current period until the period preceding the next period with positive remanufacturing. In Piñeyro and Viera (2009) we use this rule in a Tabu Search-based procedure developed for solving the ELSR that we refer as Basic Tabu Search procedure (BTS). The procedure receives among other parameters, the periods where remanufacturing is allowed. In the numerical experiment carried out, the BTS procedure showed a very good behavior, finding in many instances the optimal solution. The success achieved by the BTS is supported in part by the key role that the remanufacturing plan plays in the ELSR resolution. In addition, we think that the simple rule for determining the remanufacturing quantities also plays a decisive role in the BTS success. One of the main objectives of the research carried out in this thesis is to confirm this hunch.

Considering the relevance of the remanufacturing activity in the ELSR resolution, we focus on the problem of determining the remanufacturing quantities plan of perfect cost under the assumption that the periods where remanufacturing is allowed to be positive are known in advance. This assumption is supported by academic as well as practical reasons, e.g., operative reasons if the machinery and workers are the same for production and remanufacturing operations; availability of used items only in certain periods; or economic reasons due to periods with remanufacturing at low cost. Assuming that the remanufacturing quantities in the periods fixed is strictly positive and a non-speculative motives on the costs (i.e., it is profitable to produce or remanufacture as late as possible) we are able to prove that the total remanufacturing quantity of the remanufacturing plan of perfect cost can be obtained in linear time. Among further implications, this result serves as a theoretical support for a simple but effective rule that we introduce in Piñeyro and Viera (2009) for obtaining the remanufacturing quantities of an ELSR with fixed periods for remanufacturing. We also show that the remanufacturing rule can be effectively employed for the ELSR extension in which one-way substitution is allowed, i.e., manufactured items can satisfy the demand requirements of remanufactured items but not viceversa. This kind of situation occurs in practice when there are different market segments for new and remanufactured items. We complete the study of the remanufacturing plan of perfect cost, showing that the problem of determining the optimal quantities for each one of the periods of an ELSR problem with fixed periods for remanufacturing is NP-hard, even assuming only that the costs related to the used items are at most equal to the costs of new items. Considering this difficulty, we suggest a recursive algorithm of polynomial time for the problem that can be time-effective if either the number of fixed periods for remanufacturing or the number of total returns is small. Considering the good results obtained for the ELSR with non-speculative motives on the costs, we decided to address the CLSP (the capacitated version of the ELSP) under this cost assumption. We show that the algorithm of Florian and Klein (1971) for the CLSP with concave cost functions and stationary capacities can be improved from $O(T^4)$ to $O(T^3)$ time for the case of non-speculative motives. This type of cost structure includes many cases of interest; in particular the cases where the production set-up costs are non-decreasing and non-linear cost functions, which according to our best knowledge, are not covered by the algorithms proposed in previous works in the literature of the CLSP. The improvement is achieved thanks to the fact that there is an optimal solution that is composed exclusively by a kind of sequences that we refer as ascending constrained capacity sequences: the only period with positive production below capacity, if it exists, is the first among all the positive periods of the sequence.

1.1 Outline

The thesis is structured in two parts. Part I introduces the problems and the results obtained during the research reported in this thesis. It is composed by the following sections. Section 1 presents the problem and the motivations for this thesis research. Section 2 begins with the definition and mathematical formulation for the ELSR, and the review of those papers closest to our work. Section 2.1 is about the relevance of the remanufacturing activity in the ELSR resolution and the motivation for the study of the ELSR with fixed periods for remanufacturing. In Section 2.2 we describe in detail the simple but effective rule for obtaining the remanufacturing quantities. Section 2.3 provides the results obtained for the ELSR with fixed periods for remanufacturing. We present first the results for the case of only one period fixed (Section 2.3.1) and then the results for the general case of more than one period (Section 2.3.2). In Section 2.4 we present the analysis and the results for the ELSR with one-way substitution. Section 3 is dedicated to the CLSP (the capacitated version of the ELSP) with non-speculative motives on the costs. We provide the formulation of the problem and a literature review focused in those papers considering non-speculative costs. The results for the CLSP are presented in Section 3.1. Part I ends in Section 4, with the conclusions and directions for future research. Part II contains the four papers that describe in details the problems and results obtained during the development of this thesis. Each one of them can be read independently. However, we note that the first two papers are close related and they were written in the order that they appear.

2 The economic lot-sizing problem with return options

We consider a finite, deterministic and dynamic lot-sizing problem of a single item for which the demand requirements can be satisfied either by producing new items or by remanufacturing used items returned to the origin. New and remanufactured items are called indistinctly serviceable items since they are identical from the customer's point-of-view. Additionally, used items can be disposed off in an adequate form. Figure 1 below shows a picture of the flows of items for the inventory system that represents the lot-sizing problem under consideration.

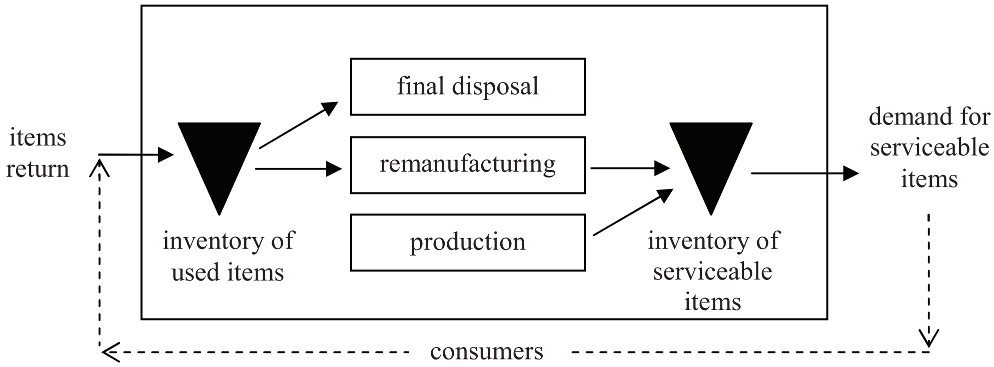

Figure 1. Flow of items in the ELSR.

The objective is determining the quantities to produce, remanufacture and dispose at each period in order to meet the demand requirements on time, minimizing the costs of carrying out the activities and for holding inventories from one period to the next. Formally, we consider a lot-sizing problem of T periods, with $0 < T < +\infty$, where D_t and R_t represent the demand and return values in periods $t = 1,...,T$, respectively; K_t^p, K_t^r, K_t^d, c_t^p, c_t^r and c_t^d represent the set-up and unit costs for production, remanufacturing and final disposing in periods $t = 1,...,T$, respectively; h_t^s and h_t^u, denote the unit cost of holding inventory for serviceable and used products in periods $t = 1,...,T$, respectively; We also define D_{ij} and R_{ij} as the accumulative demand and returns between periods i and j, with $1 \le i \le j \le T$. The quantities to produce, remanufacture and final disposing are denoted by p_t, r_t and d_t, respectively, for each period t in $t = 1,...,T$; finally, y_t^s and y_t^u, denote the inventory level during periods $t = 1,...,T$, for serviceable and used items respectively. We refer to this lot-sizing problem as the Economic Lot-Sizing Problem with Remanufacturing and Final Disposal options (ELSR). The ELSR can be modeled as the following Mixed Integer Linear Programming (MILP) problem:

$$\min \sum_{t=1}^{T} \left\{ K_t^p \delta_t^p + c_t^p p_t + K_t^r \delta_t^r + c_t^r r_t + K_t^d \delta_t^d + c_t^d d_t + h_t^s y_t^s + h_t^u y_t^u \right\} \quad (1)$$

subject to:

$$y_t^s = y_{t-1}^s + p_t + r_t - D_t \qquad \forall t = 1,2,...,T \quad (2)$$

$$y_t^u = y_{t-1}^u - r_t + R_t - d_t \qquad \forall t = 1,2,...,T \quad (3)$$

$$M\delta_t^p \geq p_t \qquad \forall t = 1,2,...,T \quad (4)$$

$$M\delta_t^r \geq r_t \qquad \forall t = 1,2,...,T \quad (5)$$

$$M\delta_t^d \geq d_t \qquad \forall t = 1,2,...,T \quad (6)$$

$$y_0^s = y_0^u = 0 \quad (7)$$

$$\delta_t^p, \delta_t^r, \delta_t^d \in \{0,1\} \;\; p_t, r_t, d_t, y_t^s, y_t^u \geq 0 \qquad \forall t = 1,2,...,T \quad (8)$$

Where M is a number at least as large as $\max\{D_{1T}, R_{1T}\}$, and δ_t^p, δ_t^r and δ_t^d, are binary variables equal to 1 if production, remanufacturing or disposing is carried out in periods $t = 1,...,T$, or 0 otherwise, respectively. Constraints (2) and (3) are the inventory balance equations for serviceable and used items. Constraints (4) to (6) indicate that a set-up is made whenever an activity is carried out in a period for a positive quantity. Constraint (7) states that the initial inventory-level for both serviceable and used items is zero. Finally, the set of possible values for each decision variable is specified by constraint (8).

After the seminal work of Simpson (1978) considering distinct inventories for serviceable and used items, papers regarding deterministic inventory problems with return options began to appear in the late 1990s. Richter and Sombrutzki (2000) and Richter and Weber (2001) consider the ELSR for the particular case when the number of returns in the first period is sufficient to satisfy the total demand over the planning horizon. Golany et al. (2001) suggest a Network Flow formulation for the ELSR and provide an exact algorithm of $O(T^3)$ time for the case of linear cost functions. They also show that the ELSR is NP-hard for the case of general concave cost functions. Yang et al. (2005) show the same result of complexity for the case of stationary concave cost functions and suggest a heuristic procedure of $O(T^4)$ time for the ELSR. van den Heuvel (2004) shows that ELSR is NP-hard for the case of set-up and unit costs for the activities and unit costs for holding inventory, even in the case that they are stationary, i.e. the same values for every period. Teunter et al. (2006) consider the ELSR with joint set-up costs for the production and remanufacturing activities, and suggest an $O(T^4)$ time algorithm based on a dynamic programming approach. Also several heuristic for the problem are provided. In Piñeyro and Viera (2009) we suggest and compare several inventory policies for the ELSR using a divide-and-conquer approach and a Tabu Search-based on procedure. We also show the key role that the remanufacturing plan plays in the ELSR resolution. Recently, Li et al. (2014) suggest a sophisticated Tabu Search based on procedure for the ELSR with stationary costs which outperforms other available algorithms in the literature. Nenes et al. (2010) analyze the ELSR taking into account the quality of the returns and Retel-Helmrich et al. (2014) provide and compare different mathematical formulations for the ELSR with separate and joint set-up costs for the activities.

2.1 The relevance of the remanufacturing activity for solving the ELSR

The remanufacturing plan plays a key-role in the ELSR resolution. While the production activity affects only the inventory level of serviceable items and the final disposal affects only the inventory of used items, remanufacturing affects both simultaneously. This means that for a given remanufacturing plan (periods and quantities for remanufacturing), the production and final-disposal subproblems can be tackled separately and solved efficiently as they can be considered as separate ELSP problems. The ELSP can be solved in $O(T^2)$ time if a Wagner and Whitin (1958) algorithm type is used, or in at most $O(T\log T)$ time if any of the faster algorithms of Federgruen and Tzur (1991), Wagelmans et al. (1992) or Aggarwal and Park (1993) is used. Thus, we can say that solving the ELSR is equivalent to the problem of finding the remanufacturing plan of an optimal solution of the ELSR. We refer to this problem as the remanufacturing plan of perfect cost.

The problem of obtaining the remanufacturing plan of perfect cost is NP-hard in general and for those cases where the ELSR is NP-hard, since they are equivalent problems. Therefore, it is unlikely that we can develop any efficient time procedure for obtaining the remanufacturing plan of perfect cost. Considering this difficulty, the way that we propose to address the problem is to assume that the periods where remanufacturing can be positive are fixed in advance. Thus, the original problem reduces to finding the remanufacturing quantities for each one of the periods fixed. In the following section we provide a rule for determining these quantities.

2.2 A simple but effective rule for determining the remanufacturing quantities

Consider an ELSR instance with a given set of periods $F \subseteq \{1,...,T\}$ with positive remanufacturing. The remanufacturing quantity for each period $i \in F$ is obtained by means of the following rule: the minimum between the number of available returns in period *i* and the accumulative demand from the current period *i* to the future period $(j-1)$, with $j \in F$ the next period fixed as positive remanufacturing period, if it exists, otherwise the accumulative demand from the current period *i* to the last period *T*. We define formally the rule by means of the following expression:

$$r_i = \min(y_{i-1}^u + R_i, D_{i(j-1)}), \quad r_i > 0,\ j = \text{npr}(i) \tag{9}$$

with function $\text{npr}(): F \to F$ defined as follows:

$$\text{npr}(i) = \begin{cases} j, & r_i, r_j > 0, r_t = 0\ \forall t : i < t < j \leq T \\ T, & r_i > 0, r_t = 0\ \forall t : i < t \leq T \end{cases}$$

We note that the remanufacturing rule of (9) establishes that the remanufacturing quantity in a certain period fixed is the maximum feasible quantity, since it is at most equal to the number of available returns ($r_i \leq y_{i-1}^u + R_i$). This seems to be the correct decision for those cases where the remanufacturing costs are at most equal to the production costs. We introduce the remanufacturing rule of (9) in Piñeyro and Viera

(2009). The rule is used in a Tabu Search based-on procedure, referred as a Basic Tabu-Search procedure (BTS), for solving the ELSR. The procedure explores different solutions by means of changing at each step the periods where remanufacturing can be positive, determining the remanufacturing quantity for each period fixed by means of the rule formulated in (9). The BTS procedure was tested for a wide range of return-demand relationships, cost settings, and planning horizon lengths. For all of them, the BTS showed a very good behavior (less than 2% of average gap between the cost of the solution obtained from BTS and the cost of the optimal solution), finding in many instances the optimal solution.

One of the main objectives of the analysis carried out in this thesis is to find the theoretical foundations for the success of the remanufacturing rule formulated in expression (9). We also show that this rule and the BTS procedure can be successfully applied to the ELSR extension for which the one-way substitution activity is allowed: different markets for new and remanufactured items where remanufactured items can be substituted by new ones but not viceversa. We note that there can be real situations for which it makes sense to restrict the periods where remanufacturing is positive, e.g., operative reasons if the machinery and workers are the same for production and remanufacturing operations; availability of used items only in certain periods; or economic reasons due to periods with remanufacturing at low cost.

2.3 Results for the ELSR with fixed periods for remanufacturing

In this section we provide the contributions obtained for the ELSR with fixed periods for remanufacturing. The results are about the conditions for which the remanufacturing rule of (9) is optimal as well as about the computational complexity of the problem of determining the remanufacturing quantities for each period. First we present the results for the case of only one period for remanufacturing (single-period case) and then for the general case of more than one period fixed as positive remanufacturing period (multi-period case). The following definition on the costs is assumed for all of the results obtained.

Definition 1. We say that the costs of the returns are at most equal to the costs of the new items when the expressions below are fulfilled by the cost components:

$$K_i^r \leq K_j^p, \tag{10.1}$$

$$c_i^r \leq c_j^p, \tag{10.2}$$

$$h_i^u \leq h_j^s, \tag{10.3}$$

for any couple of periods *i* and *j* in $1,...,T$.

Expressions (10.1) and (10.2) state the fact that the remanufacturing of used items is economically preferred to the production of new items. This can happen in practice due to the savings of energy and raw material of the remanufacturing activity. Expression (10.3) is fulfilled as it is assumed that value is added to the used items in order to make them serviceable. In addition we assume that the set-up costs are at least equal to the unit costs for each activity, i.e., $K_t^p \geq c_t^p \geq 0$, $K_t^r \geq c_t^r \geq 0$ and $K_t^d \geq c_t^d \geq 0$, for each period $t = 1,...,T$.

2.3.1 The single-period case

In this section we present the results for solving the ELSR with only one period fixed as a positive remanufacturing period. Assuming only that the conditions of Definition 1 are fulfilled, we show that either nothing or as much as possible of the demand of a certain future period *j* must be satisfied by remanufacturing in period *i*, where *i* is the only period fixed as positive remanufacturing period, with $1 \le i \le j \le T$. This means that the optimal remanufacturing quantity is either zero or the minimum between the number of available returns and the accumulative demand from period *i* to the last period *T*, i.e. the remanufacturing rule of (9). The optimal production and final disposal plans are obtained by solving two independent ELSP instances in at most $O(T^2)$ time, after we have calculated the remanufacturing quantity of period *i*. Thus, the ELSR with only one period fixed as positive remanufacturing period can be solved in $O(T^3)$ time if the algorithm of Wagner and Whitin (1958) is used for obtaining the optimal production and disposal plans or in $O(T^2 \log T)$ time if the faster algorithms mentioned in Section 2.1 are used.

We also note that the optimal single period for remanufacturing of an ELSR instance can be computed in $O(T^4)$, or $O(T^3 \log T)$ time if the faster algorithms are used for solving the ELSP subproblems, since we must consider each one of the *T* different periods for positive remanufacturing, with *T* the length of the planning horizon.

The following definition on the costs provides the necessary conditions for maximizing the remanufacturing quantity in a given period, and then for computing the remanufacturing quantity in the single period case in constant time.

Definition 2. Given a period *i* of an ELSR instance of *T* periods, such that $r_i > 0$ and $r_t = 0$ for all *t* in $1,...,T$ and $t \ne i$, we say that it is profitable to maximize the remanufacturing quantity of period *i* if the expression:

$$c_i^r + \sum_{t=i}^{j-1} h_t^s \le c_j^p \tag{11.1}$$

is fulfilled for each period *j* in $i,...,k$, with $1 \le i \le k \le T$, or

$$c_i^r + \sum_{t=i}^{j-1} h_t^s \le c_j^p + \sum_{t=i}^{T} h_t^u \tag{11.2}$$

in the case that the final disposal of used items is not considered.

For the cases in which Definition 2 is fulfilled for any couple of periods (i,k) within the planning horizon with $1 \le i \le k \le T$ and $r_i > 0$, we can assure that it is optimal to remanufacture as much as possible in the period fixed for remanufacturing, i.e., the remanufacturing rule of (9) is the optimal choice for the period under consideration. In fact, it is sufficient that Definition 2 is fulfilled between the period fixed as positive-remanufacturing period and the last one for which at least a portion of its demand is attainable by remanufacturing in the period fixed. The last attainable period can be determined as the earliest period *t* for which $D_{it} > y_{i-1}^u + R_{it}$ with *i* the single remanufacturing period. Then, assuming that Definition 2 is fulfilled, the optimal solution of an ELSR with only one period fixed as positive remanufacturing period can be solved in $O(T^2)$ time if the algorithm of Wagner and Whitin (1958) is used or in $O(T \log T)$ time if the faster algorithms mentioned in Section 2.1 are used. Real situations where Definition 2 is fulfilled include the cases for which the holding costs of both used and serviceable items are similar or negligible, very low remanufacturing costs as well as instances with few periods.

For details about the analysis and the results presented in this section, we refer to the papers "Analysis of the quantities of the remanufacturing plan of perfect cost" and "The economic lot-sizing problem with return options and fixed periods for remanufacturing: formulation, algorithm and complexity" included in Part II of this document.

2.3.2 The multi-period case

In this section we present the results obtained for the ELSR with at least two periods fixed as positive remanufacturing periods. The first result that we present below is about the form of the remanufacturing plan under particular assumptions in the number of returns.

Let us consider an ELSR instance with a given set of periods $F \subseteq \{1,...,T\}$ with positive remanufacturing. If the number of available returns in a certain period i fixed as a positive-remanufacturing period is sufficient to fully cover the demand until the end of the planning horizon, i.e., $R_i + y_{i-1}^u \geq D_{iT}$, $r_i > 0$, with $1 \leq i \leq T$, we are able to prove that there is at least one optimal solution of the ELSR for which the total remaining demand from period i is satisfied only by remanufacturing from period i onwards, i.e., $r_{iT} = D_{iT}$, with $r_{ij} = \sum_{t=i}^{j} r_t$, $1 \leq i \leq j \leq T$.

This result helps us to identify the form of a remanufacturing plan of perfect cost for the ELSR in the particular case that the number of available returns in a period fixed as positive-remanufacturing period is sufficient to meet all the remaining demand until the end of the planning horizon. The results for the cases for which no relationship is assumed between the demand and returns quantities, is based on the following definitions.

Definition 3. We say that the remanufacturing costs are non-speculative with respect to the transfer when they satisfy the following expressions:

$$K_i^r + c_i^r + \sum_{t=i}^{j-1} h_t^s \geq K_j^r + c_j^r + \sum_{t=i}^{j-1} h_t^u \qquad (12.1)$$

$$c_i^r + \sum_{t=i}^{j-1} h_t^s \geq c_j^r + \sum_{t=i}^{j-1} h_t^u \qquad (12.2)$$

for any couple of period i and j in $1,...,T$.

Expression (12.1) states that it is profitable to transfer the entire remanufacturing quantity from certain period to other future period that was inactive, while (12.2) states that it is profitable to transfer forward at least one unit between two periods with positive remanufacturing. We note that expressions given in (12) are fulfilled in different settings of practical interest, e.g., when all the costs involved are stationary or they do not increase over time. Non-speculative motives on the costs have been used successfully in the literature about lot-sizing problems, e.g. in the seminal paper of Wagner and Whitin (1958) costs are assumed stationary and then they are non-speculative. The faster algorithm of Federgruen and Tzur (1991) of $O(T\log T)$ time for the ELSP runs in $O(T)$ when there is non-speculative motives. Wolsey (2006) provides new results for the lot-sizing problem with delivery time windows for the case of non-speculative costs. In Section 3 we will see also the relevance of non-speculative motives for the capacitated extension of the ELSP.

Definition 4. Given an ELSR instance with a set of periods fixed as positive remanufacturing periods and a feasible remanufacturing plan *r*, we define the *upper bound of remanufacturing* of a certain period *i* to the quantity $u_i = 0$ if $r_i = 0$ and $u_i = \min(R_i + y_i^u, D_{i(j-1)})$ if $r_i > 0$, where *j* is either the next positive-remanufacturing period within the planning horizon, or $(T+1)$ if *i* is the last positive-remanufacturing period, i.e., $r_t = 0$ for all periods *t* in $(i+1),...,T$.

Considering that the costs are non-speculative according to Definition 3, we first are able to prove that there is at least one optimal solution of the ELSR with fixed periods for remanufacturing for which the remanufacturing quantity of each period is at most equal to its upper bound of remanufacturing, i.e., $0 \le r_t \le u_t$, for all periods $t = 1,...,T$. Then, we prove that there is an optimal solution for which the total remanufacturing quantity is equal to the sum of the upper bounds. This last result implies that in order to determine a remanufacturing plan of perfect cost for an ELSR instance with certain periods fixed as positive-remanufacturing periods, we only need to explore those remanufacturing plans for which the total remanufacturing quantity is equal to the sum of the upper bounds of remanufacturing. These values can be determined in linear time by means of applying Definition 4 period by period, beginning with the first period fixed as positive-remanufacturing period. For example, consider an ELSR instance with $T = 5$, a demand vector $D = (5,3,6,4,5)$ and a returns vector $R = (3,2,2,2,3)$, where the periods 2, 4 and 5 are fixed as positive remanufacturing periods. The cost values are as follows: $K_t^p = 200$, $c_t^p = 20$, $K_t^r = 150$, $c_t^r = 15$, $K_t^d = 100$, $c_t^d = 10$, $h_t^s = 5$ and $h_t^u = 2$, with $1 \le t \le 5$. Note that maximizing the remanufacturing quantity is profitable according to Definition 2 for all the meaningful pair of periods, i.e., (2,3), (4,4) and (5,5). Applying Definition 4 we have that the total remanufacturing quantity is 12, since the upper bounds of remanufacturing obtained sequentially are $u = (0,5,0,4,3)$. Table 1 below provides the candidate remanufacturing plans that we must consider in order to determine the remanufacturing plan of perfect cost for the ELSR instance under consideration.

t	D	R	r					
1	5	3	0	0	0	0	0	**0**
2	3	2	5	5	5	4	4	**3**
3	6	2	0	0	0	0	0	**0**
4	4	2	4	3	2	4	3	**4**
5	5	3	3	4	5	4	5	**5**

Table 1. Candidate remanufacturing plans.

These candidate plans were obtained by assigning to each period the maximum quantity according to its upper bound, and then transferring unit by unit from period 4 to period 5, and from period 2 to period 4. The last column of Table 1 in bold corresponds to the remanufacturing plan of perfect cost. The corresponding production and final dispose plans of the optimal solution are $p = (11,0,0,0)$ and $d = (0,0,0,0)$, respectively.

At this point we want to note that the upper bound of remanufacturing given in Definition 4 is the remanufacturing rule given in expression (9) for determining the remanufacturing quantities in the periods fixed. Thus, by means of the rule given in expression (9) we can obtain the total remanufacturing quantity of an optimal solution of an ELSR with a given set of periods for positive remanufacturing and this quantity can be computed in linear time as we showed in the numeric example above.

The result about the total remanufacturing quantity of an optimal solution from the remanufacturing rule of (9) under non-speculative motives serves to explain in part the good performance of the BTS procedure of Piñeyro and Viera (2009) for the ELSR. In addition, these theoretical contributions can be used for evaluating the solutions obtained for any algorithm developed for the ELSR under the assumptions on the costs of Definitions 1 to 3, i.e. 1) remanufacturing costs at most equal to the production costs, 2) conditions on the costs for maximizing the remanufacturing quantity in the periods fixed, and 3) non-speculative motives on the remanufacturing costs. We also note that considering the good performance of the BTS procedure reported in Piñeyro and Viera (2009), the upper bounds are tight to the optimal values for those ELSR instances where conditions of Definitions 1 to 3 are fulfilled.

2.3.3 Computational complexity and an exact algorithm for the ELSR-F

We describe below the results obtained about the computational complexity of the ELSR-F in the general case of number of periods fixed as positive remanufacturing periods and provide a recursive algorithm for solving the ELSR-F focused on the remanufacturing activity.

Since the ELSR-F can be considered an extension of the ELSR, it is easy to see that the ELSR-F is NP-hard in the case that $F = \{1,...,T\}$. However, we show that the ELSR-F is also NP-hard in the case that $F \subset \{1,...,T\}$. In order to demonstrate this last result we were able to construct a particular ELSR-F instance that is equivalent to a traditional ELSR for which the number of periods is $|F|$.

We also suggest a recursive algorithm of pseudopolynomial time for solving the ELSR-F. The algorithm is based on the key role that the remanufacturing plays in the ELSR resolution. The algorithm evaluates all the feasible remanufacturing plans and for each one of them, the optimal production plan and the optimal final disposal plan are obtained by means of a time-effective algorithm for the ELSP. We note that the algorithm can be efficient in practice if either the number of fixed periods for remanufacturing or the number of total returns is small.

For details about the analysis and the results presented in Section 2.3.1 to Section 2.3.3, we refer to the papers “Analysis of the quantities of the remanufacturing plan of perfect cost” and “The economic lot-sizing problem with return options and fixed periods for remanufacturing: complexity and algorithms” included in Part II of this document.

2.4 Results for the ELSR with one-way substitution

In this section we report the results obtained for the extension of the ELSR in which there are different demand requirements for new and remanufactured items, and the requirements for remanufactured items can be satisfied by new items if it necessary, but not viceversa, i.e., one-way substitution.

Different demand segments for new and remanufactured items arise when they are not identical from the consumer’s viewpoint. Possible downgrading in the remanufactured products may cause that they are offered at an inferior market price than the new ones. Gutowski et al. (2011) show that there are products for which remanufactured items consume more energy than new ones, if we take into account the use phase of the products. The reason is that the new products are composed by devices more efficient from an energy point of view. Therefore, there may be cases for which remanufactured products do not offer the same characteristics than the new products. Industrial applications where segmented market for manufactured and remanufactured occurs include photocopiers, tires and personal computers (Ayres et al., 1997; Ferrer, 1997; Maslennikova and Foley, 2000; Inderfurth, 2004). When the number of available returns is not sufficient to meet the demand requirements for remanufactured products on time, a manufacturer’s market strategy is to allow substitution of remanufactured products by new ones, possibly maintaining the selling price of the remanufactured products in order to avoid losing potential customers (Bayindir et al., 2007; Inderfurth, 2004). Thus, we can consider the substitution necessary rather than desirable.

We investigate the single-item economic lot-sizing problem with remanufacturing and final disposal options and different demand streams for new and remanufactured products, where in addition the requirements for the remanufactured items can be also satisfied by new items, but not vice versa (i.e. one-way substitution). We refer to this problem as the Economic Lot-Sizing Problem with Remanufacturing and Final Disposal options and one-way Substitution (ELSR-S). Figure below shows a picture of the flows of items for the ELSR-S.

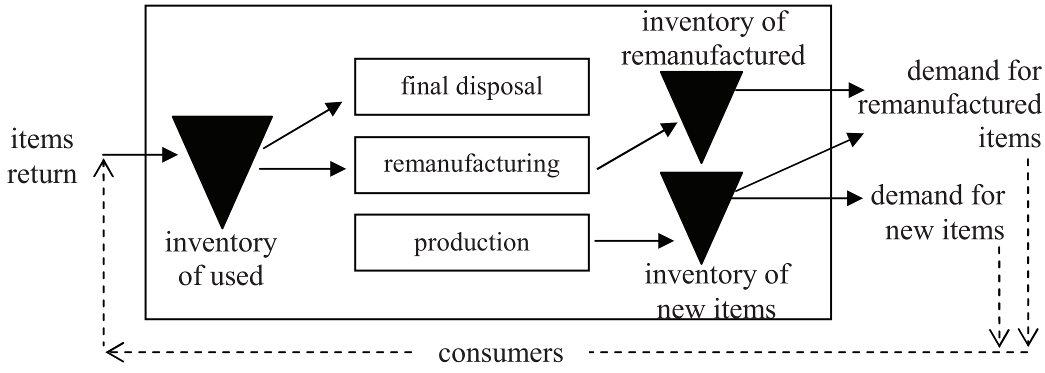

Figure 2. Flow of items in the ELSR-S.

The literature about the ELSR-S is very scarce. Inderfurth (2004) suggests and analyzes a profit model for the single-period hybrid manufacturing/remanufacturing system with product substitution. Bayindir et al. (2005) and Bayindir et al. (2007) propose profit models in order to investigate the effect of substitution on the optimal utilization of remanufacturing options under capacity constraints. Several observations and managerial insights are derived for the numerical experiment carried out by the authors. A multi-product version of the problem is studied by Li et al. (2006), without considering the final disposal of used items and without distinction among produced and remanufactured items.

The contributions presented in this thesis about the ELSR-S are as follows. We provide a mathematical model for the problem and show it is NP-hard, even under stationary cost parameters. By means of a numerical example, we show that allowing substitution can result in cost savings, even when the returns are sufficient to fulfill the requirements of remanufactured products in any period and the remanufacturing costs are favorable. As with the ELSR, we show that in order to solve the ELSR-S, we can apply a divide-and-conquer approach determining first the remanufacturing and substitution quantities, i.e, the ELSR-S reduces to the problem of finding a remanufacturing plan and a substitution plan of perfect cost. Considering this last result, we suggest a Tabu Search-based procedure for solving the ELSR-S, exploiting the key role that the remanufacturing plays in the problem resolution as in the traditional ELSR. The procedure receives the periods where remanufacturing is allowed and explores different remanufacturing plans, guided by the rule of maximizing the useful remanufacturing quantity for each period fixed as positive remanufacturing-period. The substitution quantities at each period are determined as the portion of the demand for remanufactured products that cannot be fulfilled by remanufacturing. After that the remanufacturing and substitution quantities are determined, the corresponding optimal production and final disposal plans are obtained by means of the Wagner and Whitin (1958) algorithm. The experiment conducted shows that the suggested procedure is cost-effective for a wide-range of problem instances. For all cases the average gap is less than one percent and the optimal value was attained for 153 cases; this is 28.33% of the total tested cases. In 58.89% cases the gap among the cost of the solution obtained and the optimal solution was positive and less than 1%. Only in 13% of the total tested cases the gap was superior to 1% and always less than 5%. Referring to the procedure time efficiency we note that the solution was found in the first 20 iterations for all the tested instances, and the running time was less than 250 milliseconds. To the best of our knowledge, this is the first time that a metaheuristic, and in particular the Tabu Search, is used for solving this kind of problem.

For details about the analysis and the results presented in this section, we refer to the paper "The economic lot-sizing problem with remanufacturing and one-way substitution" included in Part II of this document.

3 The CLSP with stationary capacities and non-speculative concave costs

This section present the result obtained for the Capacitated Economic Lot-Sizing Problem (CLSP), i.e., the ELSP with capacity constraints on the production activity. We consider the problem with stationary capacities (i.e., equal capacity upper bounds for each period) and concave cost functions with non-speculative motives (i.e. it is profitable to produce as late as possible). The CLSP in the general case can be formulated as follows:

$$\min \sum_{t=1}^{T} \{ f_t(x_t) + h_t(y_t) \} \qquad \text{(P)}$$

subject to:

$$y_t = y_{t-1} + x_t - D_t, \quad \forall t = 1,...,T \qquad (1)$$

$$y_0 = 0 \qquad (2)$$

$$x_t \le C_t, \quad \forall t = 1,...,T \qquad (3)$$

$$x_t, y_t \ge 0, \quad \forall t = 1,...,T \qquad (4)$$

The objective function (P) is the sum of the production cost $f_t(.)$ and the holding inventory cost $h_t(.)$ for each period $t = 1,...,T$. Constraint (1) states the well-known inventory balance equations. Constraint (2) establishes that the initial inventory quantity must be zero. The capacity constraints are given in (3), and constraints (4) state the set of possible values for the decision variables.

The CLSP is a well-known problem in the literature. Florian and Klein (1971) show that the optimal solutions of the CLSP are composed by a particular kind of subplans called capacity constrained sequences, for which the production quantities of the periods involved are either zero or equal to the capacity, except in at most one period, which is called the fractional period. Based on this property, they provide an $O(T^4)$ time algorithm for solving the CLSP with stationary capacity-pattern and general concave cost functions. More recently, faster algorithms of $O(T^3)$ and $O(T^2 \log T)$ times have been suggested by van Hoesel and Wagelmans (1996) for the case of linear inventory holding costs and by van Vyve (2007) for the case of linear costs with non-speculative motives, respectively. Bitran and Yanasse (1982) introduce the notation $\alpha / \beta / \gamma / \delta$ in order to represent different capacitated lot-sizing problem settings, where $\alpha, \beta, \gamma, \delta$ specify the set-up costs, the holding costs, the unit production costs, and the capacity pattern, respectively. Letters G, C, ND, NI, Z are used to indicate arbitrary pattern, constant, non-decreasing, non-increasing and zero, respectively. They suggest polynomial time algorithms for the cases NI/G/NI/ND, NI/G/NI/C, C/Z/ND/NI, and ND/Z/ND/NI of the CLSP. For the case NI/G/NI/ND of the CLSP, Chun and Lin (1988) provide an algorithm of $O(T^2)$ time. van den Heuvel and Wagelmans (2006) also consider the NI/G/NI/ND case, providing other $O(T^2)$ time algorithm which may run faster in practice. Chen et al. (2008) provide a pseudo-polynomial time algorithm for the same CLSP case but with more general cost functions. For surveys on the CLSP, we refer the readers to Brahimi et al. (2006) and Karimi et al. (2010).

3.1 An improvement algorithm for the CLSP

Taking into account the good results presented in Section 2 for the ELSR with non-speculative motives, we analyze the CLSP under the same assumption on the costs. We note that previous works in the literature have considered non-speculative motives but for more restrictive cost functions (Hoesel and Wagelmans, 1996; Wolsey, 2006; Van Vyve, 2007). In particular, we note that our assumptions on the costs may include the case of fixed costs for holding inventory, which arise in the pharmaceutical and software industries (Atamtürk and Küçükyavuz, 2008).

We show that the subplans composing an optimal solution of the CLSP with stationary capacities and concave cost functions with non-speculative motives have a particular structure and can be obtained by means of a linear time procedure. We refer to these particular subplans as *ascending capacity constrained sequences* since the production level in these subplans is increasing over time: the only period below capacity, if it exists, is the first among all the positive periods in the sequence. This last property of the optimal solutions is because the non-speculative motives on the costs indicate to transfer the production to the later periods as much as it is possible. By means of this result about the structure of the optimal solutions we can improve the running time of the well-known algorithm of Florian and Klein (1971) for the CLSP from $O(T^4)$ time to $O(T^3)$ time for the case of non-speculative motives on the costs. According to our best knowledge, our approach can be applied over situations that are not covered by previous related works in the literature. In addition, we note that our approach is simpler than the approach of Van Vyve (2007).

For details about the analysis and the results presented in this section, we refer to the paper "An $O(T^3)$ algorithm for the CLSP with stationary capacities and concave cost functions with non-speculative motives" included in Part II of this document.

4 Conclusions and future research

In this thesis we present different results for the economic lot-sizing problem with remanufacturing (ELSR) assuming that the periods where remanufacturing can occur are fixed in advance. This assumption is supported by academic as well as practical reasons. For the particular case of only one fixed period, we show that the optimal remanufacturing quantity can be determined in at most $O(T^3)$ time. For the general case of more than one fixed period, we show that the problem of obtaining the optimal remanufacturing quantity for every period is NP-hard. However, we are able to show that under the assumption of non-speculative motives on the costs, the total remanufacturing quantity of an optimal solution can be determined in linear time by means of the sum of certain quantities that we refer as the upper bounds of remanufacturing. Among further implications, this theoretical result serves as support for the simple but effective rule of remanufacturing that we introduce in Piñeyro and Viera (2009): the minimum between the available returns and the accumulative demand from the current period and the period preceding the next period fixed as positive-remanufacturing period. This theoretical result can be used for improving available algorithms in the literature. For example, the recent approach of Li et al. (2014) can be improved by considering the result about the total remanufacturing quantity in the Linear Programming formulation that they suggest for the ELSR under certain particular circumstances. In addition, we suggest a recursive algorithm for the ELSR that can be effective in time if either the number of fixed periods for remanufacturing or the number of total returns is small. We consider then the ELSR with one-way substitution, i.e. remanufactured products can be substituted by new ones but not viceversa. This kind of situation occurs in practice when there are different market segments for new and remanufactured items. For this ELSR extension we develop and evaluate a Tabu Search-based on procedure, guided by the rule of remanufacturing described above. The numerical experiment conducted shows that the suggested procedure will be cost-effective for a wide-range of problem instances. We note that the optimal solution was found for nearly one third of tested cases and for most cases the gap with the optimal solution was less than 1%. Finally, we consider the CLSP with non-speculative motives on the costs. For this problem we are able to improve the well-known algorithm of Florian and Klein (1971) from $O(T^4)$ time to $O(T^3)$ time for the case of stationary capacities and non-speculative motives on the costs. According to our best knowledge, our approach can be applied over situations that are not covered by previous related works in the literature.

Future research on the problem of determining the quantities of the remanufacturing plan of perfect cost in an independent way may be done relaxing some of the assumptions imposed in this thesis or considering more realistic assumptions such as the quality of the returns (Nenes et al., 2010; Jin et al., 2011), limited times for remanufacturing of the products (El Saadany et al., 2013) or different firms for producing and remanufacturing (Xiong et al., 2013; Georgiadis and Athanasiou 2013, Jonrinaldi and Zhang; 2014). In addition, the problem of determining the periods with positive remanufacturing should be tackled. In this direction, we can resort to the Useful Remanufacturing Problem (URP) defined in Piñeyro and Viera (2009). The URP refers to the problem of determining the remanufacturing plan that minimizes the involved costs and maximizes the use of the returns. Then, it would be interesting to investigate the relationship between the positive periods of an URP solution and the positive periods of a remanufacturing plan of perfect cost. Another interesting direction for future research for the ELSR is to study the situation which arises when imposing a minimum quantity for remanufacturing, e.g., remanufacturing at least fifty percent of the total quantity of available returns within the planning horizon.

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Part II

6 Analysis of the quantities of the remanufacturing plan of perfect cost

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Abstract. The remanufacturing plan of perfect cost makes reference to the remanufacturing plan of an optimal solution of the economic lot-sizing problem with remanufacturing (ELSR). In this paper we address the problem of determining the quantities of the remanufacturing plan of perfect cost in an independent way. Assuming that the periods where the remanufacturing is strictly positive are known in advance and certain other assumptions on the costs, we can show that the total remanufacturing quantity of a remanufacturing plan of perfect cost can be determined separately and in a time-effective way. We consider that the theoretical results obtained in this paper contribute to a deeper knowledge of the characteristics of the ELSR optimal solutions. Thus, the results obtained can be used to develop an effective algorithm for solving the ELSR.

Keywords: Remanufacturing, Economic Lot-Sizing Problem, Inventory Control, Reverse Logistics.

Introduction

We consider a single item economic lot-sizing problem where the demand can also be satisfied by remanufacturing used items backed to the origin. This problem is commonly known in the literature as the economic lot-sizing problem with remanufacturing (ELSR) and refers to the problem of determining the quantities to produce, remanufacture, and dispose in each period over a finite planning horizon in order to meet the demand requirements of a single item on time, minimizing the involved costs. Used products returned by the customers are available at each period for remanufacturing. In addition, we consider that the returns can be disposed off, e.g., when there is an overstock of used products. This kind of problem has been receiving an increasing academic attention in recent years as the industry has been involved with the recovery of used products. This has been the result of governmental and social pressures as well as economic opportunities. Remanufacturing can be defined as the recovery process of returned products after which it is warranted that the remanufactured products offer the same quality and functionality that those newly manufactured (Ijomah [7]). Remanufacturing tasks often involve disassembly, cleaning, testing, part replacement and reassembling operations. Products that are remanufactured include automotive parts, engines, tires, aviation equipment, cameras, medical instruments, furniture, toner cartridges, copiers, computers, and telecommunications equipment. Among the recovery options, the remanufacturing offers benefits for all of the parties involved. We refer the readers to de Brito and Dekker [1], Guide [3], Gungor and Gupta [4], and Hormozi [6] for details descriptions about the remanufacturing benefits.

This paper is focused on the analysis of the quantities of the remanufacturing plan of an optimal solution of the ELSR that we refer as the remanufacturing plan of perfect cost. The remanufacturing plan plays a key-role in the ELSR resolution since both the optimal production plan and the optimal final disposal plan can be determined separately and in an effective-time way if the remanufacturing plan is known (Piñeyro and Viera [9]). Thus, we can say that solving the ELSR reduce to the problem of finding the remanufacturing plan of perfect cost, i.e., the remanufacturing plan of an optimal solution of the ELSR. We note that the problem of finding a remanufacturing plan of perfect cost is a NP-hard problem, since it is equivalent to the ELSR, which is a known NP-hard problem even under stationary cost structures (Golany et al. [2], Yang et al. [16], van den Heuvel [14]). Considering this difficulty, we tackle the problem of determining the quantities of a remanufacturing plan of perfect cost under the assumption that the periods where the remanufacturing is strictly positive are known in advance. This can occur in practice if cores, parts, machinery or workers are only available in certain periods within the planning horizon. We also assume certain constrains on the costs that can be fulfilled in the real life, such as non-speculative motives or that the costs related to used items are at most equal to those related to new items. In addition, we provide a constraint on the costs which makes it more profitable to maximize the remanufacturing quantity in those periods where remanufacturing is carried out. This can be fulfilled in practice if the unit cost of producing is much greater than other unit costs of the problem, or in those cases for which the inventory holding costs can be neglected.

The rest of the paper is organized as follows. In Section 2 a short literature review is provided. The problem formulation is given in Section 3. Section 4 is devoted to the analysis of the quantities of the remanufacturing plan of perfect cost with fixed periods for positive remanufacturing. Section 5 concludes the paper with possible directions for future research.

Literature review

According to our best knowledge Richter and Sombrutzki [11] and Richter and Weber [12] are the first to analyze the ELSR. They consider the particular case that the returns in the first period are sufficient to satisfy the total demand over the planning horizon. Golany et al. [2] suggest a Network Flow formulation for the ELSR and provide an exact algorithm of $O(T^3)$ time for the case of linear cost functions. They also show that the ELSR is a NP-hard problem for the case of general concave cost functions. Yang et al. [16] show the same result of complexity for the case of stationary concave cost functions and provide a heuristic procedure of $O(T^4)$ time for the ELSR. van den Heuvel [14] show that ELSR is NP-hard for the case of set-up and unit costs for the activities and unit costs for holding inventory, even in the case that they are stationary. Teunter et al. [13] consider the ELSR with joint set-up costs for the production and remanufacturing activities, and suggest an $O(T^4)$ time algorithm based on a dynamic programming approach. Also several heuristic for the problem are provided. Piñeyro and Viera [9] suggest and compare several inventory policies for the ELSR using a divide-and-conquer approach and a Tabu Search based on procedure. They also show the key role that the remanufacturing plan plays in the ELSR resolution and introduce the concept of the remanufacturing plan of perfect cost. Piñeyro and Viera [10] consider the ELSR with different demand streams for new and remanufactured items where in addition substitution is allowed for remanufactured items but not viceversa. Recently, Nenes et al. [8] provide an analysis of the ELSR taking into account the quality of the returns and Helmrich et al. [5] provide and compare different mathematical formulations for the ELSR with separate and joint set-up costs for the activities.

Problem formulation

Figure 1 below shows a sketch of the flow of items for the inventory system that represents the lot-sizing problem that we are facing.

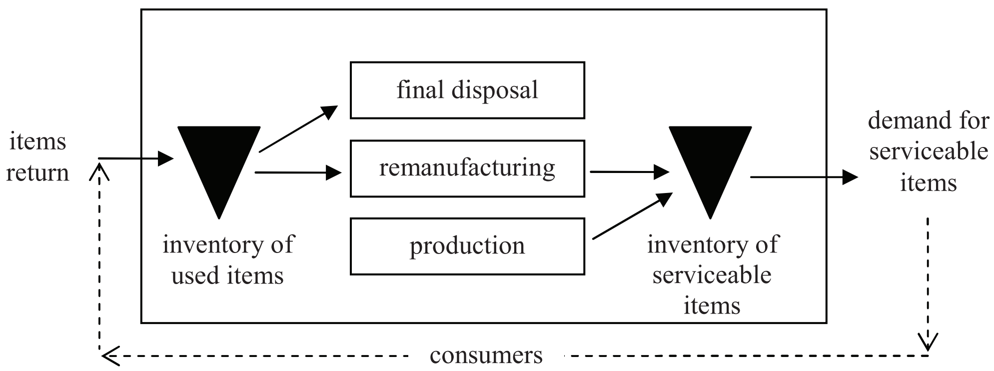

Figure 1. Flow of items in the system.

We consider a lot-sizing problem for which the demand and return values are known in advance for each period over the finite planning horizon. The demand for serviceable items must be satisfied on time, i.e., backlogging demand is not allowed. Infinite capacity for producing, remanufacturing and disposing is assumed with zero lead-times. The inventory level of both used and serviceable items is determined after all activities were carried out. Set-up and unit costs are incurred for producing, remanufacturing or disposing, and unit costs for carrying ending positive inventory from one period to the next. Finally, we assume that the initial inventory levels of both used and serviceable items are zero and the demand is positive for each period in the planning horizon. The objective is to determine the amounts to produce, remanufacture, and dispose for each one of the periods in the planning horizon such that all demand requirements are satisfied on time and the sum of all the involved costs is minimized. We refer to this problem as the Economic Lot-sizing Problem with Remanufacturing (ELSR) and it can be modeled as the following Mixed Integer Linear Programming (MILP) problem:

$$\min \sum_{t=1}^{T} \left\{ K_t^p \delta_t^p + c_t^p p_t + K_t^r \delta_t^r + c_t^r r_t + K_t^d \delta_t^d + c_t^d d_t + h_t^s y_t^s + h_t^u y_t^u \right\} \quad (1)$$

subject to:

$$y_t^s = y_{t-1}^s + p_t + r_t - D_t \qquad \forall t = 1,2,...,T \quad (2)$$

$$y_t^u = y_{t-1}^u - r_t + R_t - d_t \qquad \forall t = 1,2,...,T \quad (3)$$

$$M\delta_t^p \geq p_t \qquad \forall t = 1,2,...,T \quad (4)$$

$$M\delta_t^r \geq r_t \qquad \forall t = 1,2,...,T \quad (5)$$

$$M\delta_t^d \geq d_t \qquad \forall t = 1,2,...,T \quad (6)$$

$$y_0^s = y_0^u = 0 \quad (7)$$

$$\delta_t^p, \delta_t^r, \delta_t^d \in \{0,1\} \quad p_t, r_t, d_t, y_t^s, y_t^u \geq 0 \qquad \forall t = 1,2,...,T \quad (8)$$

In model (1) – (8) the parameters T, D_t and R_t denote the length of the planning horizon, demand and returns values in periods $t = 1,...,T$, respectively; K_t^p, K_t^r, K_t^d, c_t^p, c_t^r and c_t^d denote the set-up and unit costs for production, remanufacturing and final disposing in periods $t = 1,...,T$, respectively; h_t^s and h_t^u, denote the unit cost of holding inventory for serviceable and used products in periods $t = 1,...,T$, respectively; M is a number at least as large as $\max\{D_{1T}, R_{1T}\}$, where D_{ij} and R_{ij} are the accumulative demand and returns between periods i and j, with $1 \leq i \leq j \leq T$. The decision variables p_t, r_t and d_t, denote the number of units produced, remanufactured and disposed in periods $t = 1,...,T$, respectively; δ_t^p, δ_t^r and δ_t^d, are binary variables equal to 1 if production, remanufacturing or disposing is carried out in periods $t = 1,...,T$, or 0 otherwise, respectively; y_t^s and y_t^u, denote the inventory level during periods $t = 1,...,T$, for serviceable and used items respectively.

Constraints (2) and (3) are the inventory equilibrium equations for serviceable and used items, respectively. Constraints (4) to (6) indicate that a set-up is made whenever an activity is carried out in a period for a positive quantity. Constraint (7) states that the initial inventory-level for both serviceable and used items is zero. Finally, the set of possible values for each decision variable is specified by constraint (8).

The ELSR as modeled above is a NP-hard problem (van den Heuvel [14]). As we mentioned earlier, solving the ELSR is equivalent to find a remanufacturing plan of perfect cost, i.e., the remanufacturing plan of an optimal solution of the ELSR. In the following section we analyze this last problem assuming that the periods for which the quantity of remanufacturing is positive are known in advance.

Fixed periods for remanufacturing

In this section we tackle the problem of determining the quantities of the remanufacturing plan of perfect cost under the assumption that the periods with strictly positive remanufacturing (periods for which the quantity of remanufacturing is greater than zero) are known in advance. This means that the remanufacturing setup is already taken into account and hence only the remanufacturing variable cost must be considered in order to determine the optimal quantities. We begin considering the particular case of only one positive-remanufacturing period, and then we consider the case of more than one period. To conduct the analysis we resort to certain assumptions on the costs as well as on the number of the available returns in the periods fixed. The first assumption that we introduce below is about the costs related to the used items.

Definition 1. We say that the costs of the returns are at most equal to the costs of the new items when the expressions below are fulfilled by the cost components:

$$K_i^r \leq K_j^p, \tag{9.1}$$

$$c_i^r \leq c_j^p, \tag{9.2}$$

$$h_i^u \leq h_j^s, \tag{9.3}$$

for any couple of periods *i* and *j* in $1,...,T$.

Expressions (9.1) and (9.2) state the fact that the remanufacturing of used items is economically preferred to the production of new items. This can happen in practice due to the savings of energy and raw material of the remanufacturing activity. Expression (9.3) is fulfilled as it is assumed that value is added to the used items in order to make them serviceable. In addition we assume that the set-up costs are at least equal to the unit costs for each activity, i.e., $K_t^p \geq c_t^p \geq 0$, $K_t^r \geq c_t^r \geq 0$ and $K_t^d \geq c_t^d \geq 0$, for each period $t = 1,...,T$.

The single-period case

Consider an ELSR instance of *T* periods with only one period *i* fixed as positive-remanufacturing period, i.e., $r_i > 0$, with $1 \leq i \leq T$, and $r_t = 0$ for all *t* with $1 \leq t \leq T$ and $t \neq i$. The objective is to determine the optimal remanufacturing quantity Q_i^r of the period *i*, with $0 < Q_i^r \leq y_{i-1}^u + R_i$, assuming that $y_{i-1}^u + R_i > 0$. Note that case for which the remanufacturing quantity is zero is not considered since we are assuming only a strictly positive remanufacturing quantity in the fixed period.

First consider the case that the number of available returns in period i are at most equal to the demand of the period, i.e., $y_{i-1}^u + R_i \le D_i$. Then, by (9), the optimal remanufacturing quantity must be equal to all of the available returns, i.e., $Q_i^r = y_{i-1}^u + R_i > 0$. On the other hand, for the case that $y_{i-1}^u + R_i > D_i$, we must determine the last period j within the planning horizon for which it is more profitable to meet at least one unit of its demand by remanufacturing in period i. Assume first that the number of available returns is sufficient to exactly meet the accumulative demand from the current period i to certain future period k, i.e., $y_{i-1}^u + R_i = D_{ik}$, with $1 \le i \le k \le T$. Then, the optimal remanufacturing quantity of period i is $Q_i^r = D_{ij}$, with j the last period for which $c_i^r D_j + \sum_{t=i}^{j-1} h_t^s D_j \le K_j^p + c_j^p D_j + \sum_{t=i}^{T} h_t^u D_j$ is fulfilled, with $1 \le i \le j \le k \le T$ and $D_j > 0$. However, we note that in general the number of available returns in period i is sufficient to meet only a portion of the demand of certain future period k, i.e., $y_{i-1}^u + R_i = D_{i(k-1)} + \alpha < D_{ik}$, with $1 \le i \le k \le T$ and $\alpha \ge 1$. Without loss of generality let us assume that it is profitable to remanufacture in period i at least the needed quantity to cover the demand requirements from i to $(k-1)$, i.e., $D_{i(k-1)} \le Q_i^r < D_{ik}$ .This means that at least one unit of the demand requirement of period k is satisfied by means of the production of new items in certain period t with, $1 \le t \le k$. If $1 \le t \le i$, then there must be that $Q_i^r = D_{i(k-1)} + \alpha$, since $c_i^r \le c_t^p + \sum_{\tau=t}^{i-1} h_\tau^s$ is true by (9). Therefore, in the case of $i < t \le k$, we have that $Q_i^r = D_{i(k-1)} + \alpha$ only if the condition $c_i^r + \sum_{t=i}^{t-1} h_t^s \le c_t^p$ is fulfilled, otherwise $Q_i^r = D_{i(k-1)}$. This last condition can be relaxed by $c_i^r + \sum_{t=i}^{j-1} h_t^s \le c_j^p + \sum_{t=i}^{T} h_t^u$ in the case that final disposal of used items is not considered, which is supported by economic as well as ecological reasons. Teunter et al. [13] point out that disposing option "does not lead to a considerable cost reduction unless the remanufacturable return rate as a percentage of the demand rate is unrealistically high (above 90%) and the demand rate is very small (less than 10 per year)". We resume the reasoning above by means of the following assumption about the profitability of maximizing the remanufacturing quantity in a certain period.

Definition 2. Given two periods i and k of an ELSR instance of T periods, with $1 \le i \le k \le T$, such that $r_i > 0$, we say that it is profitable to maximize the remanufacturing quantity of period i if the expression:

$$c_i^r + \sum_{t=i}^{j-1} h_t^s \le c_j^p \tag{10.1}$$

is fulfilled for each period j, with $1 \le i \le j \le k \le T$, or

$$c_i^r + \sum_{t=i}^{j-1} h_t^s \le c_j^p + \sum_{t=i}^{T} h_t^u \tag{10.2}$$

in the case that the final disposal of used items is not considered.

Thus, if Definition 2 is fulfilled for any couple of periods *i* and *j* in $1,...,T$, with $i \leq j$, we can assure that the strictly positive optimal remanufacturing quantity Q_i^r of a single period *i* fixed as positive-remanufacturing period is the minimum between the amount of available returns and the accumulative demand from the current period and the end of the planning horizon, i.e., to remanufacture as much as possible. We note that for a given instance it is sufficient that Definition 2 is fulfilled between the period fixed as positive-remanufacturing period and the last one for which at least a portion of its demand is attainable by remanufacturing in the period fixed. On the other hand, if Definition 2 is not fulfilled it is unlikely that we can determine the optimal remanufacturing quantity of a certain period without knowing the periods where production is carried out, since in the case that the available returns in period *i* are only sufficient for partially meet the accumulative demand to certain future period *k*, we need to know if the rest of the demand of period *k* is produced either in the same period or in a previous one.

Real situations where Definition 2 is fulfilled include cases where holding costs of both used and serviceable items are similar or negligible, very low remanufacturing costs as well as instances with few periods. We also note that the problem of finding the optimal positive-remanufacturing period for an ELSR instance for which it is profitable to remanufacture as much as possible at any period, can be solved in $O(T^3)$ time, since we must consider *T* different periods and the corresponding optimal production and final dispose plans can be obtained in $O(T^2)$ by means of a Wagner-Whitin algorithm type (Wagner and Whitin [15]).

We summarize the results obtained above for the single-period case in the following proposition.

Proposition 1. Consider an ELSR instance with only one period *i* fixed as strictly positive remanufacturing period with $y_{i-1}^u + R_i > 0$. Let us assume that Definition 2 is fulfilled for the pairs of periods (*i*, *j*), for any *j* in $i \leq j \leq \ell$, with ℓ the last period within the planning horizon for which at least a portion of its demand is attainable by remanufacturing in the period *i*. Then the optimal remanufacturing quantity Q_i^r of period *i* is equal to the minimum between the number of available returns in the period and the accumulative demand from period *i* to period ℓ, i.e. $Q_i^r = \min(y_{i-1}^u + R_i, D_{i\ell})$.

Proof. The proof is straightforward from Definition 2 applied to the pair of periods (i,ℓ).

■

The multi-period case

We now consider the problem of finding the remanufacturing quantities of a remanufacturing plan of perfect cost with at least two periods fixed as positive-remanufacturing periods. We first note that the amount to be remanufactured in a certain period depends in part of the remanufactured quantity in previous periods as well as affects the amount to be remanufactured in future periods. Then, it may not be possible to determine efficiently the optimal remanufacturing quantity for each period, even under the assumptions introduced in the previous section. In view of this difficulty, we focus on the problem of determining the total quantity of a remanufacturing plan of perfect cost assuming that the periods with strictly positive remanufacturing quantity are known in advance. Before tackle this problem, we provide a result about the form of the remanufacturing plan of perfect cost for a particular case.

Proposition 2. Consider an ELSR instance for which the number of available returns in a certain period i fixed as a positive-remanufacturing period is sufficient to fully cover the demand until the end of the planning horizon, i.e., $R_i + y_{i-1}^u \geq D_{iT}$, $r_i > 0$, with $1 \leq i \leq T$. If the optimal solution set is not empty, there is at least one optimal solution for which the total remaining demand from period i is satisfied only by remanufacturing from period i onwards, i.e., $r_{iT} = D_{iT}$, with $r_{ij} = \sum_{t=i}^{j} r_t$, $1 \leq i \leq j \leq T$.

Proof. Let us consider an optimal solution of the ELSR with $r_i > 0$, $R_i + y_{i-1}^u \geq D_{iT}$, and $r_{iT} < D_{iT}$. Then, the quantity $(D_{iT} - r_{iT}) > 0$ is satisfied by means of the production of new items. We can determine a new solution with $r_{iT} = D_{iT}$ from the current solution as follows. First, for each period t with $i \leq t \leq T$ and $p_t > 0$, we replace the entire production in t by remanufacturing, i.e., $r_t \leftarrow p_t$, $p_t \leftarrow 0$. Note that the replacement operation is possible as we are assuming the returns are sufficient. Secondly, while $r_{iT} < D_{iT}$, take the last period t with $p_t > 0$ and $1 \leq t < i$, and transfer units of the production of period t to the remanufacturing of period i, until $r_{iT} = D_{iT}$ or $p_t = 0$. By (9) the cost of the new solution is at most equal to the cost of the original. Therefore, there must be an optimal solution of the ELSR for which $r_{iT} = D_{iT}$, if $r_i > 0$ and $R_i + y_{i-1}^u \geq D_{iT}$ is complied.

■

Proposition 2 helps us to identify the form of a remanufacturing plan of perfect cost for the ELSR in the particular case that the number of available returns in a period fixed as positive-remanufacturing period is sufficient to meet all the remaining demand until the end of the planning horizon. We must note that if the amount of available returns in a certain period is sufficient to meet all the remaining demand but the period is not fixed as a positive-remanufacturing period, we cannot ensure the result above unless the period under consideration is the first one (see Richter and Sombrutzki [11]).

We consider now the problem in general sense, i.e., no kind of relationship is assumed between the returns and the demand values. First, we provide the following definitions about the costs and the quantities of remanufacturing.

Definition 3. We say that the remanufacturing costs are non-speculative with respect to the transfer when they satisfy the following expressions:

$$K_i^r + c_i^r + \sum_{t=i}^{j-1} h_t^s \geq K_j^r + c_j^r + \sum_{t=i}^{j-1} h_t^u \quad (11.1)$$

$$c_i^r + \sum_{t=i}^{j-1} h_t^s \geq c_j^r + \sum_{t=i}^{j-1} h_t^u \quad (11.2)$$

for any couple of period i and j in $1,...,T$.

Expression (11.1) states that it is profitable to transfer the entire remanufacturing quantity from certain period to other future period that was inactive, while (11.2) states that it is profitable transfer forward at least one unit between two periods with positive remanufacturing. We note that expressions given in (11) are fulfilled in different settings of practical interest, e.g., when all the costs involved are stationary or they do not increase over time.

Definition 4. Given an ELSR instance with a set of periods fixed as positive remanufacturing periods and a feasible remanufacturing plan *r*, we define the *upper bound of remanufacturing* of a certain period *i* to the quantity $u_i = 0$ if $r_i = 0$ and $u_i = \min(R_i + y_{(i-1)}^u, D_{i(j-1)})$ if $r_i > 0$, where j is either the next positive-remanufacturing period within the planning horizon, or $(T+1)$ if *i* is the last positive-remanufacturing period, i.e., $r_t = 0$ for all periods *t* in $(i+1),...,T$.

Proposition 3. Given an ELSR instance, there is at least one optimal solution for which the remanufacturing quantity of each period is at most equal to its upper bound of remanufacturing, i.e., $0 \leq r_t \leq u_t$, for all periods $t = 1,...,T$.

Proof. Without loss of generality, consider an optimal solution of an ELSR instance with only one period i for which $r_i > u_i = \min(R_i + y_i^u, D_{i(j-1)})$ and $r_j > 0$ with $1 \leq i \leq j \leq T$. First we note that the case $u_i = R_i + y_i^u$ is not feasible since the remanufacturing quantity is greater than the amount of available returns. Now consider the case that $u_i = D_{i(j-1)}$. Then, by (11) we can obtain a new solution with at most the same cost than the original by transferring remanufactured units from period i to the consecutive period j with $r_j > 0$, until $r_i = D_{i(j-1)}$ in the new solution. Therefore, an optimal solution for the same ELSR instance for which $r_t \leq u_t$ can be obtained, for all periods $t = 1,...,T$.

■

Proposition 3 states that the remanufacturing quantity of a certain period is upper bounded by the minimum between the number of available returns and the accumulative demand until the period preceding the next period with positive remanufacturing. We note that the upper bound value of certain period depends on the remanufacturing quantities of the previous periods. In addition, it may not be possible to determine how close or how far to its upper bound is the remanufacturing quantity of a certain period in an optimal solution of the ELSR. Despite these facts, the upper bound of remanufacturing allows us to determine the total remanufacturing quantity of a remanufacturing plan of perfect cost, as we show in the following proposition.

Proposition 4. Consider an ELSR instance with a set of periods F fixed as positive-remanufacturing periods such as for any pair of consecutive periods i and j of F, the Definition 2 is fulfilled for any pair of meaningful periods, i.e., pairs (i,t) with $i \in F$ and t the last period before j for which at least a portion of its demand is attainable by remanufacturing in i with $i \leq t < j$. Then consider the remanufacturing plan $\overline{r}$ obtained by remanufacturing in each period the amount given by the upper bound of remanufacturing applied in ascending order, i.e., $\overline{r_t} = u_t$, assuming that $\overline{r_1} = u_1, \overline{r_2} = u_2, ..., \overline{r_{(t-1)}} = u_{(t-1)}$, for all periods $t = 1,...,T$. Then, there is an optimal solution with a remanufacturing plan r^* for which $r^*_{1T} = \overline{r_{1T}}$, where $r^*_{ij} = \sum_{t=i}^{j} r^*_t$ and $\overline{r_{ij}} = \sum_{t=i}^{j} \overline{r_t}$, with $1 \leq i \leq j \leq T$.

Proof. We note that by Proposition 3 and Definition 4, there must be that $r^*_{1T} \leq \overline{r_{1T}}$. Without loss of generality, let us assume that $r^*_{1T} = \overline{r_{1T}} - 1$. Then, there exists a period i, with $1 \leq i \leq T$, for which $0 < r^*_i = \overline{r_i} - 1$, $r^*_{1(i-1)} = \overline{r_{1(i-1)}}$ and $y^{*u}_{(i-1)} = \overline{y^u_{(i-1)}}$. This means that the upper bound of remanufacturing of period i is the same for both remanufacturing plans under consideration, with $0 < r^*_i < u_i = \overline{r_i}$. We also note that $y^{*u}_t \geq 1$ is fulfilled for all periods $t = i,...,T$. Therefore, we can obtain a new feasible solution for the same ELSR instance with at most the same cost by increasing the remanufacturing in period i in one unit, i.e. $r^*_i \leftarrow r^*_i + 1 = \overline{r_i}$, without affecting the remanufacturing of the future periods and in the meantime by reducing the production of a certain period j in $1,...,T$. This new solution fulfills that $r^*_{1T} = \overline{r_{1T}}$ and its cost is at most the same than the cost of the original optimal solution as we are assuming that to maximize the remanufacturing quantity of the periods with positive remanufacturing is profitable according to Definition 2.

■

Proposition 4 states that in order to determine a remanufacturing plan of perfect cost for an ELSR instance with certain periods fixed as positive-remanufacturing periods, we only need to explore those remanufacturing plans for which the total remanufacturing quantity is equal to the sum of the upper bounds of remanufacturing. These values can be determined efficiently (linear time) by applying Definition 4 period by period, beginning with the first period fixed as positive-remanufacturing period. We show the usefulness of Proposition 4 through the following numerical example.

A numerical example

Consider an ELSR instance with $T = 5$, a demand vector $D = (5,3,6,4,5)$ and a returns vector $R = (3,2,2,2,3)$, where the periods 2, 4 and 5 are fixed as positive remanufacturing periods. The cost values are as follows: $K^p_t = 200$, $c^p_t = 20$, $K^r_t = 150$, $c^r_t = 15$, $K^d_t = 100$, $c^d_t = 10$, $h^s_t = 5$ and $h^u_t = 2$, with $1 \leq t \leq 5$. Note that the remanufacturing is profitable according to Definition 2 for all the meaningful pair of periods, i.e., (2,3), (4,4) and (5,5). Applying Definition 4 we have that the total remanufacturing quantity is 12, since the upper bounds of remanufacturing obtained sequentially are $u = (0,5,0,4,3)$. Table 1 below provides the candidate remanufacturing plans that we must consider in order to determine the remanufacturing plan of perfect cost for the ELSR instance.

t	*D*	*R*	*r*					
1	5	3	0	0	0	0	0	**0**
2	3	2	5	5	5	4	4	**3**
3	6	2	0	0	0	0	0	**0**
4	4	2	4	3	2	4	3	**4**
5	5	3	3	4	5	4	5	**5**

Table 1. Candidate remanufacturing plans.

These candidate plans were obtained by assigning to each period the maximum quantity according to its upper bound, and then transferring unit by unit from period 4 to period 5, and from period 2 to period 4. The last column of Table 1 in bold corresponds to the remanufacturing plan of perfect cost. The corresponding production and final dispose plans of the optimal solution are $p = (11,0,0,0)$ and $d = (0,0,0,0)$, respectively.

Effectiveness of the upper bound of remanufacturing

In Piñeyro and Viera [9] a basic Tabu Search based-on procedure (BTS) was suggested and evaluated for the ELSR. The procedure receives among other parameters, an initial (0,1) *T*-tuple, where a value of 1 in position *t* indicates that remanufacturing is allowed to be positive in period *t*, otherwise it must be zero. The procedure explores different remanufacturing plans by means of swapping the periods where remanufacturing can be positive. The remanufacturing quantity of each period *i* fixed as positive-remanufacturing period is equal to the minimum between the number of available returns in *i* and the accumulative demand from *i* to the period preceding the next period *j* with positive remanufacturing, i.e., the upper bound of remanufacturing of Definition 4. The BTS procedure was tested for a wide range of return-demand relationships, cost settings, and planning horizon lengths of 5, 10 and 15 periods. For all of the tested cases the BTS showed a very good behavior (less than 2% of average gap between the cost of the solution obtained from BTS and the cost of the optimal solution), finding in many instances the optimal solution.

The good performance observed for the BTS procedure can be explained in part by the theoretical results provided in this paper about the quantities of the remanufacturing plan of perfect, at least for those cases where the conditions of Definitions 1 to 3 are fulfilled. In this sense we note that for the numeric experiments of the BTS procedure of Piñeyro and Viera [9] it is assumed that the costs of the returns are at most equal to the costs of the new items according to expression (9) of Definition 1. In addition, horizon planning lengths of 5, 10 and 15 are used, thus it can be assumed that the conditions of Definition 2 and Definition 3 are fulfilled in many of the tested instances. On the other hand, for those cases where the conditions are not fulfilled, may be that the upper bound of remanufacturing is not a good option which in turn explains why the BTS procedure is not able to achieve high quality solutions for some of the tested instances, e.g. when the positive-remanufacturing periods are widely separated or the holding costs of serviceable items are relatively greater.

Conclusions and future research

In this paper we have addressed the problem of determining the quantities of the remanufacturing plan of perfect cost for the economic lot-sizing problem with remanufacturing (ELSR) assuming that the periods where remanufacturing is strictly positive are known in advance and that it is profitable to remanufacture as much as possible in a period fixed as positive remanufacturing period. Thus, we are able to determine the optimal remanufacturing quantity for the particular case of only one period fixed as positive-remanufacturing period. We also note that the problem of finding the optimal period for remanufacturing can be solved in $O(T^3)$ time. For the general case of more than one period fixed as positive-remanufacturing period, we note that it may not be possible to determine the optimal remanufacturing quantity for each one of them in an effective-time way. Nevertheless, we show that the total remanufacturing quantity of an optimal solution can be determined as the sum of the upper bounds of remanufacturing, assuming also that the remanufacturing costs are non-speculative respect to the transfer, i.e., remanufacturing occurs as late as possible. The upper bounds of remanufacturing can be computed period by period in a linear time way as the minimum between the number of available returns and the accumulative demand from the current period to the period preceding the next period with positive remanufacturing. The theoretical results obtained about the quantities of a remanufacturing plan of perfect cost serve to explain the effectiveness of the Tabu Search based-on procedure suggested in Piñeyro and Viera [9] for the ELSR.

More attention should be placed in the future on the problem of determining the quantities of the plan of perfect cost in an independent way by relaxing some of the assumptions imposed in this paper. More specifically, on identifying situations in which it is desirable to maximize the remanufacturing, even if the condition of Definition 2 is not fulfilled. In addition, the problem of determining the periods with positive remanufacturing should be tackled. In this sense, we can resort to the Useful Remanufacturing Problem (URP) introduced in Piñeyro and Viera [9]. The URP refers to the problem of determining the useful remanufacturing plan that minimizes the involved costs and maximizes the use of the returns. Then, we can assume that the positive periods of a useful remanufacturing plan are close to the positive periods of a remanufacturing plan of perfect cost. We may include also different demand streams for new and remanufactured items, as in Piñeyro and Viera [10].

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7 The economic lot-sizing problem with return options and fixed periods for remanufacturing: complexity and algorithms

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Abstract. In this paper we analyze the economic lot-sizing problem with return options assuming that the periods where remanufacturing is allowed to be positive have been fixed in advance. We begin considering the case of only one period fixed and then we tackle the general case of more than one period fixed. For the single-period case, we are able to derive an efficient time procedure for obtaining the optimal remanufacturing quantity under certain assumptions on the costs. For the multi-period case we show that the problem is NP-hard and suggest a recursive algorithm of pseudopolynomial time for solving the problem.

Keywords: Remanufacturing, Economic Lot-Sizing Problem, Inventory Control.

1. Introduction

We consider an economic lot-sizing problem (ELSP) with return options for which the demand requests of the periods can be satisfied either by producing new items or by remanufacturing used items backed to the origin. More specifically, the economic lot-sizing problem with remanufacturing (ELSR) refers to the problem of determining the quantities to produce, remanufacture, and dispose in each period over a finite planning horizon in order to meet the demand requirements of a single item on time, minimizing the sum of the involved costs. Used products returned by the customers are available at each period for remanufacturing. In addition, the returns can be disposed off, e.g., when there is an overstock of used products. The ELSR has been receiving an increasing academic attention from late 90s as the industry has been involved with the recovery of used products due to governmental and social pressures as well as economic opportunities. Remanufacturing can be defined as the recovery process of returned products after which it is warranted that the remanufactured products offer the same quality and functionality that those newly manufactured (Ijomah, 2002). Remanufacturing tasks often involve disassembly, cleaning, testing, part replacement and reassembling operations. Products that are remanufactured include automotive parts, engines, tires, aviation equipment, cameras, medical instruments, furniture, toner cartridges, copiers, computers, and telecommunications equipment. Remanufacturing offers benefits for all of the parties involved. We refer the readers to de Brito and Dekker (2002), Guide (2000), Gungor and Gupta (1999), and Hormozi (2003) for detail descriptions about the remanufacturing benefits.

The ELSP and extensions are well-studied problems in the literature (Karimi et al., 2003; Brahimi et al., 2006; Toledo et al., 2007; Atamtürk and Küçükyavuz, 2008; Toso et al., 2008). According to our best knowledge, Richter and Sombrutzki (2000) and Richter and Weber (2001) are the first to consider the ELSP extension with return options, analyzing the particular case for which the number of returns in the first period are sufficient to satisfy the total demand over the planning horizon. Golany et al. (2001) suggest a Network Flow formulation for the ELSR and provide an exact algorithm of $O(T^3)$ time for the case of linear cost functions. They also show that the ELSR is a NP-hard problem for the case of general concave cost functions. Yang et al. (2005) and van den Heuvel (2004) extend this last result about complexity for the cases of stationary concave cost functions and set-up and unit costs for the activities and for holding inventory, respectively. Teunter et al. (2006) consider the ELSR with joint set-up costs for the production and remanufacturing activities, and suggest an $O(T^4)$ time algorithm based on a dynamic programming approach. Piñeyro and Viera (2009) suggest and compare several inventory policies for the ELSR using a divide-and-conquer approach and a Tabu Search-based procedure. Piñeyro and Viera (2010) consider an ELSR extension with different demand streams for new and remanufactured items where in addition substitution is allowed for remanufactured items but not viceversa. Nenes et al. (2010) provide an analysis of the ELSR taking into account the quality of the returns and Retel-Helmrich et al. (2014) provide and compare different mathematical formulations for the ELSR with separate and joint set-up costs for the activities. They show that the ELSR with joint set-up costs is also NP-hard.

The analysis presented in this paper is motivated by the following facts. The remanufacturing plan plays a key-role in the ELSR resolution, as it was noted in

Piñeyro and Viera (2009). If the remanufacturing plan is known, the optimal production and final disposing plans can be obtained by solving independent ELSP problems, which can be solved in at most $O(T^2)$ time (Wagner and Whitin, 1958; Zangwill, 1968). Thus, the ELSR can be reduced to the problem of determining the remanufacturing plan of an optimal solution, which is referred as the problem of determining the remanufacturing plan of perfect cost. Piñeyro and Viera (2012) consider the problem of determining the quantities of the remanufacturing plan of perfect cost assuming that the periods with strictly positive remanufacturing are known in advance and that it is profitable to maximize the remanufacturing quantity in the periods fixed. Thus, they show that the total remanufacturing quantity of an optimal solution of the ELSR with fixed periods for positive-remanufacturing can be obtained in linear time and claim that is unlikely that we can determine the exact amount for each one of the periods fixed as positive-remanufacturing period by means of an efficient-time procedure. Here, we analyze this last problem relaxing some of the assumptions of this previous work. Besides the academic motivation exposed above, we note that there can be real situations for which it makes sense to restrict the periods where remanufacturing can be carried out, e.g., operative reasons if the machinery and workers are the same for production and remanufacturing operations; availability of used items only in certain periods; or economic reasons due to periods with remanufacturing at low cost.

In this paper we consider the ELSR with fixed periods for remanufacturing in a more broad sense that in Piñeyro and Viera (2012) of Chapter 6. First, the remanufacturing quantity in the periods fixed can be either zero or positive in contrast to Piñeyro and Viera (2012), in which the remanufacturing quantity in the periods fixed is assumed strictly positive. Second, some of the assumptions on the costs are relaxed in this paper. We provide an efficient-time algorithm for the case of only one period fixed and we show that the multi-period case of the problem is NP-hard. In addition, we suggest a recursive algorithm for the general case of the ELSR-F of pseudopolynomial time which can be time-effective in practice if either the number of periods where remanufacturing is allowed or the number of total returns is small.

The remainder of the paper is organized as follows. Section 2 introduces the notation used through the paper and the mathematical formulation for the problem. In Section 3 we analyze the single-period case of the problem and provide an efficient-time algorithm for determining the optimal remanufacturing quantity of the fixed period. In Section 4 we present the NP-hard result for the multi-period case and we suggest a pseudopolynomial time algorithm for solving the problem in the general case. Section 5 concludes the paper.

2. Notation and mathematical formulation

We consider the ELSR with T periods, with $0 < T < \infty$. Demand and return values are denoted by D_t and R_t for each period $t = 1,...,T$, respectively; K_t^p, K_t^r, K_t^d, c_t^p, c_t^r and c_t^d denote the set-up and unit costs for production, remanufacturing and final disposing in periods $t = 1,...,T$, respectively; h_t^s and h_t^u, denote the unit cost of holding inventory for serviceable and used products in periods $t = 1,...,T$, respectively. In addition, $F \in 2^T$ denote the set of periods for which the remanufacturing is allowed to be positive, i.e., $r_t \geq 0$ if and only if $t \in F$, $r_t = 0$ otherwise. We also denote by D_{ij}, R_{ij}, p_{ij}, r_{ij}, and d_{ij} the accumulative demand, returns, production, remanufacturing and disposing quantities between periods *i* and *j*, with $1 \leq i \leq j \leq T$, respectively. The objective is to determine the values for the decision variables p_t, r_t and d_t of producing, remanufacturing and final disposing at each period $t = 1,...,T$, respectively, and for holding inventory of serviceable and used items y_t^s and y_t^u, respectively, minimizing the sum of all the involved costs. We refer to this problem as the ELSR with Fixed periods for remanufacturing (ELSR-F).

The ELSR-F can be modeled as a Mixed Integer Linear Programming (MILP) problem. The model (1) – (9) below is similar to that given in Golany et al. (2001), Yang et al. (2005) and Piñeyro and Viera (2009) for the ELSR, except by constraint (8) which is introduced in order to indicate the periods where remanufacturing is not allowed to be positive.

$$\min \sum_{t=1}^{T} \left\{ K_t^p \delta_t^p + c_t^p p_t + K_t^r \delta_t^r + c_t^r r_t + K_t^d \delta_t^d + c_t^d d_t + h_t^s y_t^s + h_t^u y_t^u \right\} \tag{1}$$

subject to :

$$y_t^s = y_{t-1}^s + p_t + r_t - D_t \qquad \forall t = 1,2,...,T \tag{2}$$

$$y_t^u = y_{t-1}^u - r_t + R_t - d_t \qquad \forall t = 1,2,...,T \tag{3}$$

$$M\delta_t^p \geq p_t \qquad \forall t = 1,2,...,T \tag{4}$$

$$M\delta_t^r \geq r_t \qquad \forall t = 1,2,...,T \tag{5}$$

$$M\delta_t^d \geq d_t \qquad \forall t = 1,2,...,T \tag{6}$$

$$y_0^s = y_0^u = 0 \tag{7}$$

$$r_t = 0 \qquad \forall t \notin F \subseteq \{1,2,...,T\} \tag{8}$$

$$\delta_t^p, \delta_t^r, \delta_t^d \in \{0,1\} \;\; p_t, r_t, d_t, y_t^s, y_t^u \geq 0 \qquad \forall t = 1,2,...,T \tag{9}$$

Where *M* is a big number at least as large as $\max\{D_{1T}, R_{1T}\}$. We note that the ELSR-F can be considered as an extension of the traditional ELSR, if we consider the case in which $|F| = T$, and then it is NP-hard in general (van den Heuvel, 2004). As we will discuss in the following sections, the problem of determining the optimal remanufacturing quantity for the case that $|F| = 1$ can be solved in polynomial time and the NP-hard result remains valid for those cases for which $1 < |F| < T$.

3. The single-period case

In this section we address the problem of determining the remanufacturing quantity of the ESLR with only one period *i* fixed as positive remanufacturing period, with $1 \leq i \leq T$. We refer to this problem as ELSR-$\{i\}$. According to our best knowledge, the first to tackle this problem were Piñeyro and Viera (2012) for the case that it is profitable to maximize the remanufacturing quantity in the period fixed and assuming that at least one unit of used items is remanufactured. Here we relax these assumptions. The remanufacturing quantity in the period fixed can be either zero or a positive quantity. In addition, we only consider that remanufacturing is profitable according to the conditions given in the following assumption, used also in van den Heuvel (2004) and Piñeyro and Viera (2012).

Definition 1. We say that the costs of the returns are at most equal to the costs of the new items when the expressions below are fulfilled by the cost components:

$$K_i^r \leq K_j^p, \tag{10.1}$$

$$c_i^r \leq c_j^p, \tag{10.2}$$

$$h_i^u \leq h_j^s, \tag{10.3}$$

for any couple of periods *i* and *j* in $1,...,T$.

Consider first the case that the number of available returns in period *i* is at most equal to the demand of the period, i.e., $0 \leq R_i + y_{i-1}^u \leq D_i$. Then, assuming that the conditions of Definition 1 are fulfilled, the optimal remanufacturing quantity is either $r_i = 0$ or $r_i = R_i + y_{i-1}^u$. On the other hand, if the number of available returns is greater than the demand requirement of the period, we must determine the last period *j* within the planning horizon for which it is more profitable to satisfy its demand by remanufacturing in period *i* rather than by producing in period *j* or in a previous period *t*, with $1 \leq i \leq j \leq T$ and $1 \leq t \leq j \leq T$.The following result is about the profitability of remanufacturing in an optimal solution of the ELSR-$\{i\}$.

Proposition 1. Consider an ELSR-$\{i\}$ instance. If it is profitable to meet one unit of the demand of certain period *j* by remanufacturing in *i*, then it is profitable to satisfy as much as possible the demand of period *j* by remanufacturing in period *i*, with $1 \leq i \leq j \leq T$.

Proof. Without loss of generality, consider an ELSR-$\{i\}$ instance with $D_j \geq 2$ and a solution for which $r_i = D_{i(j-1)} + 1 < R_i + y_{i-1}^u$, with $1 \leq i \leq j \leq T$. We note that the remaining demand of period *j* is satisfied by producing only in certain period *t*, with $1 \leq t \leq j \leq T$. In addition, if it is profitable to meet one unit of the demand of period *j* by remanufacturing in period *i* rather than by producing in period *t*, then the expression $c_i^r + \sum_{t=i}^{(j-1)} h_\tau^s \leq c_t^p + \sum_{\tau=t}^{(j-1)} h_\tau^s$ must be fulfilled. Therefore, for each unit that we increase the remanufacturing quantity at period *i* for satisfying the demand of period *j*, we are reducing the cost of the current solution in at least $c_t^p + \sum_{t=i}^{(j-1)} h_\tau^s - c_i^r - \sum_{\tau=t}^{(j-1)} h_\tau^s \geq 0$ and then we obtain a new solution which fulfills that as much as possible of the demand of period *j* is satisfied by remanufacturing in period *i*. We note that the analysis above is also valid for the case that there is a period *k* of positive final disposing after period *i*, with $1 \leq i \leq k \leq T$.

■

Proposition 1 means that in order to determine the optimal remanufacturing quantity for the only period *i* of the ELSR-$\{i\}$ we must consider the periods one by one from period *i* onwards until we find a period *j* for which either it is not profitable to meet at least one unit of its demand requirement by remanufacturing at period *i*, or the returns in period i has been exhausted. Having this result in mind we provide in Figure 1 a pseudocode of a procedure for solving the ELSR-$\{i\}$.

```
01. r_t = 0,   ∀t = 1...T
02. (p,d) = ELSP_solver(r)
03. s = (p,r,d)
04. α = R_i + y^u_{i-1}
05. t = i
06. stop = 0
07. while α ≥ 0 and t ≤ T and stop = 0 do
08.     r_i = min(α, D_it)
09.     (p,d) = ELSP_solver(r)
10.     if cost(p,r,d) < cost(s)
11.         s = (p,r,d)
12.         α = α − r_i
13.         t = t + 1
14.     else
15.         stop = 1
16.     endif
17. enddo
18. return s
```

Figure 1. Pseudocode of an algorithm for solving the ELSR-{*i*}.

In the pseudocode above $s=(p,r,d)$ makes reference to the ELSR solution s with a production plan p, remanufacturing plan r, and final disposing plan d, respectively. We note that the case $\alpha=0$ is for taking into account the case where no remanufacturing is the optimal decision. Procedure ELSP_solver of lines 02 and 10 can be implemented by any of the well known algorithms for solving the ELSP like the $O(T^2)$ time algorithm of Wagner and Whitin (1958) or faster algorithms of $O(T\log T)$ time of Federgruen and Tzur (1991), Wagelmans et al. (1992) or Aggarwal and Park (1993).

We analyze now the computational complexity of the procedure of Figure 1. First we note that we must consider at most $(T-i+1)$ periods if the period i is fixed as the single period with positive remanufacturing of the ELSR. Then, the worst case is $i=1$ since we must consider T different periods. For each period under consideration we need to compute the optimal production and final disposing plans in order to obtain the ESLR solution, solving two independent ELSP instances. We also assume that the time for computing the cost of an ELSR solution can be neglected. Therefore, the ELSR-$\{i\}$ can be solved in $O(T^3)$ time if the algorithm of Wagner and Whitin (1958) is used or in $O(T^2\log T)$ time if faster algorithms are used for the ELSP like the algorithms of $O(T\log T)$ time mentioned above. We also note that the optimal single period for remanufacturing of an ELSR instance can be computed in $O(T^4)$, or

$O(T^3 \log T)$ time if faster algorithms are used for solving the ELSP subproblems, since we must consider each one of the *T* different periods.

4. The multi-period case

In this section we consider the ELSR-F with more than one period fixed as positive-remanufacturing period, i.e. $1 < |F| \leq T$. We note that the remanufacturing quantity in the periods fixed can be either zero or positive. We begin analyzing the computational complexity of the problem. As we noted in Section 2, the case $|F| = T$ (i.e. all the periods are fixed as positive-remanufacturing periods) is equivalent to the traditional ELSR, and then it is NP-hard for the same cases. We show below that the problem remains NP-hard for the case $1 < |F| < T$, i.e. the case with at least one period that is not fixed as positive-remanufacturing period. Secondly, we suggest a recursive algorithm for solving the ELSR-F in the general case of the number of periods fixed as positive-remanufacturing periods.

Proposition 2. The ELSR-F with $1 < |F| < T$, is NP-hard.

Proof. We prove the proposition by showing that a particular instance of the ELSR-F is at least as hard to solve as an ELSR instance which is known an NP-hard problem. Consider a particular ELSR-F instance of *T* periods, with $1 < |F| < T$. Let *i* be the first period in *F*, i.e., $i = \min\{t : t \in F\}$. Then define the components of this particular instance as follows:

- $D_t = R_t = 0$ if $t \notin F$.
- $h_t^s = +\infty$ if $1 \leq t < i$.
- $h_t^s = 0$ if $i < t \leq T$ and $t \notin F$.
- $h_t^u = 0$ if $t \notin F$.
- $K_t^p = c_t^p = +\infty$ if $i < t \leq T$ and $t \notin F$.
- $K_t^d = c_t^d = +\infty$ for all *t* in $1 \leq t \leq T$.

The rest of the demand, return and cost values, i.e. the values for the periods in *F*, are arbitrary except that we assume that the cost components satisfy the conditions of Definition 1. Thus, for any optimal solution of this particular ELSR-F instance, the demand requirements of the periods in *F* are satisfied only by the production and remanufacturing of the periods in *F*. Therefore, this particular instance of the ELSR-F is equivalent to a traditional ELSR instance with a planning horizon length $|F|$ for which the demand, return and cost values are equal to the demand, return and costs values of the periods in *F*, taken in ascending order in the number of periods. Since solving the ELSR under the conditions of Definition 1 is NP-hard (van den Heuvel, 2004), we can conclude that the ELSR-F is NP-hard.

■

4.1. An exact algorithm for the ELSR-F

In this section we suggest a recursive algorithm for solving the ELSR-F in the general case based on the fact that the optimal plans of production and final disposing from a given remanufacturing plan can be obtained in polynomial time, since they can be formulated as traditional ELSP problems (Piñeyro and Viera, 2009). The algorithm receives five parameters: 1) the problem data denoted by *P* (planning horizon length, demand, return and costs values); 2) the set *F* of periods for remanufacturing; 3) the period *i* to consider; 4) the remanufacturing quantities determined to the moment and 5) the best ELSR-F solution to the moment denoted by *s*. Thus, for the period under consideration, we determine the range of feasible remanufacturing quantities taking into account: 1) the remanufacturing quantities to the moment, 2) the number of available returns and 3) the accumulative demand from the current period onward. If the period under consideration is not the last one in *F*, we update the remanufacturing plan to the moment with the remanufacturing quantity of period *i* and call the algorithm for the next period of *F* in the planning horizon. Otherwise, if the period under consideration is the last one, we determine the corresponding optimal production and final disposing plans in order to obtain a new solution of the problem. If the cost of the new solution is less than the cost of the best solution to the moment, we replace the best solution by the new one. The algorithm begins with the first period (*t* = 1) and a zero remanufacturing plan ($r_1 = ... = r_T = 0$). Since the algorithm considers all the feasible remanufacturing plans, the solution obtained is optimal. A pseudocode of the algorithm for solving the ELSR-F is given in Figure 2.

```
solve_ELSR(P,F,i,r,s)

01. u_i = min(R_1i − r_1(i−1), D_iT)
02. for r_i = 0 to u_i
03.    r ← r_i
04.    if i < last_period(F)
05.       j = next_period(F,i)
06.       solve_ELSR(P,F,j,r,s)
07.    else
08.       (p,d) = ELSP_solver(r)
09.       if cost(p,r,d) < cost(s)
10.          s = (p,r,d)
11.       endif
12.    endif
13. endfor
```

Figure 2. Pseudocode of an algorithm for solving the ELSR-F.

The function last_period(*F*) of line 04 returns the greater period of *F* and the function next_period(*F*,*i*) of line 05 returns the minor period of *F* that is greater than *i* or zero otherwise. The above algorithm solves the ELSR-F in at most $O(\min\{R_{1T}, D_{1T}\}^{|F|} \cdot T^2)$ time since there are most $O(\min\{R_{1T}, D_{1T}\}^{|F|})$ different remanufacturing plans, and for each one of them we must determine the optimal production plan and the optimal final disposing plan solving two independent ELSP problems by means of the ELSP_solver procedure of Figure 1, which takes at most $O(T^2)$ time (Wagner and Whitin, 1958).

The algorithm of Figure 2 can be applied for solving the traditional ELSR problem if we consider the ELSR-F with $F = \{1,..,T\}$. For the ELSR, Yang et al. (2005) provide a dynamic programming algorithm of $O(T \cdot \min\{R_{1T}, D_{1T}\} \cdot (R_{1T})^2 \cdot (D_{1T})^2)$ time. Although the order of complexity of our algorithm is worse than that of Yang et al. (2005), we note that it can be better for those cases in which $|F|$ is significantly less than T and/or the number of returns is significantly less than the number of demand requirements.

5. Conclusions and future research

In this paper we have analyzed the ELSR for which the periods where remanufacturing is allowed are known in advance. We refer to this problem as ELSR with fixed periods for remanufacturing (ELSR-F). For the case of only one period fixed as remanufacturing period, we derived a polynomial time procedure for obtaining the optimal remanufacturing quantity, assuming as it is common in the literature that the costs related to the used items are at most equal to the costs of new items. The procedure is based on the property that either zero or as much as possible of the demand of certain period must be satisfied by remanufacturing in the fixed period. We also note that the minor cost period for remanufacturing can be determined in polynomial time. For the general case of more than one period fixed as remanufacturing period, we showed that the problem is NP-hard even under particular structure on the costs. We provide a recursive algorithm of pseudopolynomial time that can be time-effective in practice if the number of total returns or the number of periods where remanufacturing is allowed is small.

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8 The economic lot-sizing problem with remanufacturing and one-way substitution

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Abstract. We investigate a lot-sizing problem with different demand streams for new and remanufactured items, in which the demand for remanufactured items can be also satisfied by new products, but not vice versa. We provide a mathematical model for the problem and demonstrate it is NP-hard, even under particular cost structures. With the aim of finding a near optimal solution of the problem, we suggest and evaluate a Tabu Search-based procedure. The numerical experiment carried out confirms the successful of the procedure for different cases.

Keywords: Economic Lot-Sizing Problem, Remanufacturing, One-way Substitution, Tabu Search.

1. Introduction

The economic lot-sizing problem with remanufacturing and final disposal options (ELSR) refers to the problem of finding the quantities to produce, remanufacture, and dispose in each period over a finite planning horizon such that all demand requirements of a single item are satisfied on time, minimizing the sum of all the involved costs. The main difference with the traditional economic lot-sizing problem (ELSP) is that the demand can be also satisfied by recovering used items returned to the origin. Governmental and social pressures as well as economic opportunities have motivated many firms to become involved with the return of used products for recovery (Gungor and Gupta, 1999; Guide, 2000; Fleischmann, 2001; Brito and Dekker, 2002). Remanufacturing can be defined as the recovery of returned products, after which the products are as good as new (Gungor and Gupta, 1999; Hormozi, 2003). Remanufacturing tasks often involve disassembly, cleaning, testing, part replacement or repairing, and reassembling operations. Products that are remanufactured include automotive parts, engines, tires, aviation equipment, cameras, medical instruments, furniture, toner cartridges, copiers, computers, and telecommunications equipment.

However, possible downgrading in the remanufactured products may cause that they are offered at an inferior market price than the new ones, i.e. they are not identical from the consumer's viewpoint. Then, it makes sense to assume different demand segments for remanufactured and new items. Industrial applications where segmented market for new and remanufactured occurs include photocopiers, tires and personal computers (Ayres et al., 1997; Ferrer, 1997b; Maslennikova and Foley, 2000; Inderfurth, 2004). Since the demand requirements must be fulfilled on time, the case where the available returned items in a certain period are not sufficient to meet the demand requirements for remanufactured products must be considered. To address this problem, a manufacturer's market strategy is to allow substitution of remanufactured products by new ones, possibly maintaining the selling price of the remanufactured products in order to avoid losing potential customers (Bayindir et al., 2007; Inderfurth, 2004). Thus, we can consider the substitution necessary rather than desirable. On the other hand, as we will see further in a numeric example of Section 3.1, allowing substitution can result in cost savings, even when the returns are sufficient to fulfill the requirements of remanufactured products in any period and the remanufacturing costs are favorable. As it is noted by Inderfurth (2004), when manufacturing and remanufacturing processes are sharing common manufacturer resources and/or the different markets are interconnected by substitution, it is necessary to coordinate manufacturing and remanufacturing decisions.

In this paper we investigate the economic lot-sizing problem with products returns under the circumstances described above, i.e., two independent demand streams for new and remanufactured items, and where the substitution for the remanufactured items is allowed but not vice versa. We refer to this problem as the Economic Lot-Sizing Problem with Remanufacturing and Final Disposal options and one-way Substitution (ELSR-S). We provide a mathematical model for the problem and show it is NP-hard, even under stationary cost parameters. We also show that in order to effectively solving the ELSR-S, we can apply a divide-and-conquer approach determining first the remanufacturing and substitution quantities at each period. Considering this last result, we suggest a Tabu-Search-based procedure for solving the

ELSR-S that can be considered as an extension of that presented in Piñeyro and Viera (2009) for the traditional ELSR, i.e. when substitution is not allowed. To the best of our knowledge, this is the first time that a metaheuristic, and in particular the Tabu Search, is used for solving this kind of problem.

The remainder of this paper is organized as follows. Section 2 is devoted to the literature review. In Section 3 we provide the problem definition and the respective mathematical model. We also present an analysis of the relevance that the remanufacturing plays in the ELSR-S resolution and certain effects of the substitution in its determination. In Section 4 we present the Tabu Search procedure suggested for solving the ELSR-S. The computational analysis is provided in Section 5. Finally, Section 6 is devoted to our conclusions and several directions for future research.

2. Literature review

To the best of our knowledge, the first to study a deterministic and dynamic inventory-system with product returns are Richter and Sombrutzki (2000). They provide an extension to the well-known algorithm of Wagner and Whitin (1958) for the particular case that the number of returns in the first period is sufficient to satisfy the total demand over the planning horizon. In Richter and Weber (2001), the previous work is extended for including variable costs. The same problem with less restrictive assumptions in the returns flow is analyzed in Golany et al. (2001). They introduce a Network Flow formulation for the problem and demonstrate that it is NP-hard for the case of general concave cost functions. They also provide an exact algorithm of $O(T^3)$ for the case of linear function costs. The NP-hard result is extended in Yang et al. (2005) for the case of stationary concave cost functions, and a heuristic procedure of $O(T^4)$ is proposed for the ELSR. Finally, van den Heuvel (2004) demonstrates that the problem is NP-hard in the particular case that the cost functions are composed of both setup and variable costs for the activities and variable cost for the holding inventory. This last result is valid even when the setup and variable costs are stationary. Teunter et al. (2006) suggest several heuristics for the economic lot-sizing problem with the remanufacturing option (final disposal is not considered). Two versions of the problem are analyzed: with joint and separate setup costs for production and remanufacturing, respectively. For the case of joint setup costs, an algorithm of $O(T^4)$ time based on a dynamic programming approach is provided. For the other case, the authors "conjecture that the problem with separate set-up costs is NP-hard". Finally, Piñeyro and Viera (2009) propose and evaluate a set of inventory policies specially designed for the ELSR, under the assumption that remanufacturing used items is more suitable than disposing of them and producing new items. In addition, a Tabu Search based-on procedure for the problem is developed. The policies as well as the Tabu Search procedure are based on the divide and conquer principle and exploiting the key role that remanufacturing plays in the ELSR resolution.

We also note that the continuous review version of the inventory problem with product returns has been received a lot of attention. Relevant and recent works examples are as follow. Minner and Kleber (2001) determine optimally conditions for a continuous time model along with an algorithm tested under different return scenarios. In Teunter (2004) new and simple formulas are derived for determining the optimal lot sizes of production and recovery. He also presents a detail analysis of different policies. Minner and Lindner (2004) analyze the continuous review inventory system with returns, showing for example that a policy with non-identical lot sizes may be better than those with identical lot sizes. In addition, more realistic and complex situations have been considered recently. Examples are the papers of Konstantaras and Papachristos (2006) and Pan et al. (2009). They analyze the inventory problem with returns allowing demand backlogging and considering capacity constraints, respectively. However, substitution is not allowed.

Since the ELSR is a relatively new problem, the works dealing with both remanufacturing and substitution are very scarce. Inderfurth (2004) suggests and analyzes a profit model for the single-period hybrid manufacturing/remanufacturing system with product substitution. Optimal policies are derived for the problem taking into account different initial inventory values, costs configurations and positive lead-times values. Bayindir et al. (2005) and Bayindir et al. (2007) propose profit models in order to investigate the effect of substitution on the optimal utilization of remanufacturing option under capacity constraint. Several observations and managerial insights are derived for the numerical experiment carried out by the authors. A multi-product version of the problem tackled in this paper is study by Li et al. (2006), without considering the final disposal of used items and without distinction among produced and remanufactured items. They provide a dynamic programming approach to obtain the optimal solution for the particular case of large quantities of returned products. Based on this approach, an approximate procedure is suggested for the general case of $O(TQ)$ time, where T is the number of periods and Q the number of products. A detailed analysis of the procedure is reported, considering also the effect of the substitution.

The main contribution of this paper is to analyze a deterministic and dynamic inventory-system with product returns and one-way substitution, considering 1) multi-period, 2) no special constraints about the returns flow, 3) different storages for produced and remanufactured items, and 4) the final-disposal option.

3. Problem definition

We investigate a single-item economic lot-sizing problem with remanufacturing and final disposal options and different demand streams for new and remanufactured products, where in addition the requirements for the remanufactured items can be also satisfied by new items, but not vice versa (i.e. one-way substitution). Figure 1 shows a sketch of the flow of items for this inventory system.

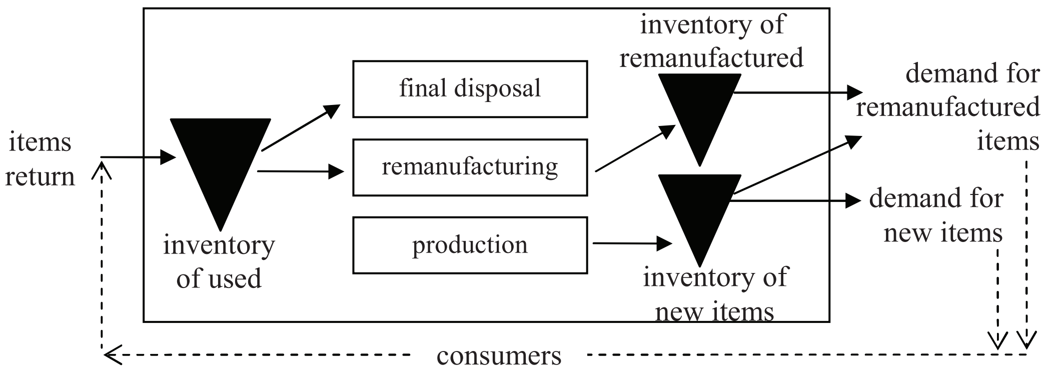

FIGURE 1. Flow of items in the system

We assume that the demand and return values are known in advance for each period over the finite planning horizon of long $T > 0$, and no relationships are assumed among them. Two different demand streams are considered. One of them is for new items, and the other for remanufactured items. This means that new and remanufactured items are not identical for the consumer's viewpoint. It is also assumed that the customer accepts substitution, i.e. the demand for remanufactured items can be satisfied by remanufacturing used items returned to origin and/or producing new ones. Backlogging demand is not allowed for both new and remanufactured items. Infinite capacity for producing, remanufacturing and disposing is assumed. Nevertheless, we note that the sum of the remanufacturing and final disposal quantities in a certain period is bounded by the amount of used items available in that period. The producing, remanufacturing, and final disposal lead-times are assumed to be zero. The inventory level in a certain period for used, remanufactured or new products is determined at zero-time and after all activities. When a positive amount is produced, remanufactured, or disposed in a certain period, set-up and unit costs are incurred for each activity. When substitution occurs in a certain period unit costs are incurred. In the meantime, holding costs are incurred for carrying ending positive inventory from one period to the next. All decisions occur at the beginning of the period and all values of the problem (i.e. demands, returns, and components of the cost functions) are assumed to be non-negative, dynamic, and independent. Finally, we assume that 1) all initial stocks are zero and 2) there is at least one type of demand with a positive requirement in the first period. The objective is to determine the amounts to produce, remanufacture, and dispose for each period in the planning horizon such that all demand requirements are satisfied on time, minimizing the sum of all the involved costs. We use the following notation for the problem throughout the remainder of the paper.

- $T > 0$: Long planning horizon, with $T < +\infty$.
- $DP_t \geq 0$: Number of new items demanded in period t, with $1 \leq t \leq T$.
- $DP_{ij} \geq 0$: Accumulated demand of new items between periods i and j, with $1 \leq i \leq j \leq T$.
- $DR_t \geq 0$: Number of remanufactured items demanded in period t, with $1 \leq t \leq T$.
- $DR_{ij} \geq 0$: Accumulated demand of remanufactured items between periods i and j, with $1 \leq i \leq j \leq T$.
- $R_t \geq 0$: Number of used items returned in period t, with $1 \leq t \leq T$.
- $R_{ij} \geq 0$: Accumulated returns between periods i, j with $1 \leq i \leq j \leq T$.
- $p_t \geq 0$: Number of items produced in period t, with $1 \leq t \leq T$.
- $r_t \geq 0$: Number of items remanufactured in period t, with $1 \leq t \leq T$.
- $r_{ij} \geq 0$: Accumulated remanufacturing quantity between periods i and j, with $1 \leq i \leq j \leq T$.
- $d_t \geq 0$: Number of items disposed in period t, with $1 \leq t \leq T$.
- $y_t^p \geq 0$: Inventory level of new items during period t, with $1 \leq t \leq T$.
- $y_t^r \geq 0$: Inventory level of remanufactured items during period t, with $1 \leq t \leq T$.
- $y_t^u \geq 0$: Inventory level of used items during period t, with $1 \leq t \leq T$.

- $s_t \geq 0$: Number of remanufactured items satisfied by substitution in period *t*, with $1 \leq t \leq T$.
- $K_t^p > 0$: Fixed cost of production in period *t*, with $1 \leq t \leq T$.
- $K_t^r > 0$: Fixed cost of remanufacturing in period *t*, with $1 \leq t \leq T$.
- $K_t^d > 0$: Fixed cost of final disposing in period *t*, with $1 \leq t \leq T$.
- $c_t^p \geq 0$: Unit cost of production in period *t*, with $1 \leq t \leq T$.
- $c_t^r \geq 0$: Unit cost of remanufacturing in period *t*, with $1 \leq t \leq T$.
- $c_t^d \geq 0$: Unit cost of final disposing in period *t*, with $1 \leq t \leq T$.
- $c_t^s \geq 0$: Unit cost of substitution in period *t*, with $1 \leq t \leq T$.
- $h_t^p \geq 0$: Unit cost for holding inventory of new items during period *t*, with $1 \leq t \leq T$.
- $h_t^r \geq 0$: Unit cost for holding inventory of remanufactured items during period *t*, with $1 \leq t \leq T$.
- $h_t^u \geq 0$: Unit cost for holding inventory of used items during period *t*, with $1 \leq t \leq T$.

The lot-sizing problem described above can be modeled as the following Mixed Integer Linear Programming (MILP) problem:

$$\min \sum_{t=1}^{T} \left\{ K_t^p \delta_t^p + c_t^p p_t + c_t^s s_t + K_t^r \delta_t^r + c_t^r r_t + K_t^d \delta_t^d + c_t^d d_t + h_t^p y_t^p + h_t^r y_t^r + h_t^u y_t^u \right\} \quad \text{(P)}$$

subject to :

$$y_t^p = y_{t-1}^p + p_t - s_t - DP_t \qquad \forall t = 1,2,...,T \quad (1)$$

$$y_t^r = y_{t-1}^r + r_t + s_t - DR_t \qquad \forall t = 1,2,...,T \quad (2)$$

$$y_t^u = y_{t-1}^u + R_t - r_t - d_t \qquad \forall t = 1,2,...,T \quad (3)$$

$$s_t \leq DR_t \qquad \forall t = 1,2,...,T \quad (4)$$

$$M\delta_t^p \geq p_t \qquad \forall t = 1,2,...,T \quad (5)$$

$$M\delta_t^r \geq r_t \qquad \forall t = 1,2,...,T \quad (6)$$

$$M\delta_t^d \geq d_t \qquad \forall t = 1,2,...,T \quad (7)$$

$$y_0^s = y_0^r = y_0^u = 0 \quad (8)$$

$$\delta_t^p, \delta_t^r, \delta_t^d \in \{0,1\} \;\; p_t, s_t, r_t, d_t, y_t^p, y_t^r, y_t^u \geq 0 \qquad \forall t = 1,2,...,T \quad (9)$$

Constraints (1) – (3) are the inventory balance equations for new, remanufactured and used items, respectively. Constraint (4) states that the substitution quantity in a certain period cannot be greater than the demand for remanufactured items of the current period. Otherwise, the inventory of remanufactured items would increase with the production of new items. Constraints (5) – (7) indicate that a set-up is made whenever an activity is carried out in a period for a positive quantity, where *M* is a large natural number with $M \geq \max\{D_{1T}, R_{1T}\}$. Constraint (8) states that the initial inventory-level of each type of item must be zero. Finally, the set of possible values for each decision variable is specified by constraint (9).

The ELSR-S modeled above can be considered as an extension of the traditional ELSR. To see this, consider a particular instance of the ELSR-S without positive demand for new items, zero substitution costs, and equal inventory holding costs for both new and remanufactured items. Thereby, this last ELSR-S instance is equivalent to an ELSR instance with the same costs for the activities and storage. The ELSR is an NP-hard problem for different cost function structures, even for those used in this paper and considering stationary parameters (Golany et al., 2001; van den Heuvel, 2004; Yang et al., 2005). Therefore, the ELSR-S is also NP-hard for the same cost function structures and it is unlikely that we can develop a time-effective algorithm to solve it optimally. Instead, we propose a Tabu-Search-based heuristic for solving the ELSR-S, extending that of Piñeyro and Viera (2009) for the ELSR. The procedure is based on the divide-and-conquer approach and considering the key role that remanufacturing plays in the structure of an optimal solution of the traditional ELSR. In the following section we analyze this last fact along with certain effects of the substitution in the optimal remanufacturing-plan determination.

3.1. Problem analysis

In this section we investigate the connection among the different problem activities. Let us assume that the remanufacturing and substitution quantities at each period are known in advance. Then, the production and final-disposal subproblems can be determined separately and in a time-effective way, as in the traditional ELSR (Piñeyro and Viera, 2009). This last observation is based on the fact that we can formulate the production and the final-disposal subproblems as separate single-source uncapacitated minimum concave-cost network flow problems (Zangwill, 1968; Guisewite and Pardalos, 1991). Figure 2 gives the network flows formulation for both subproblems.

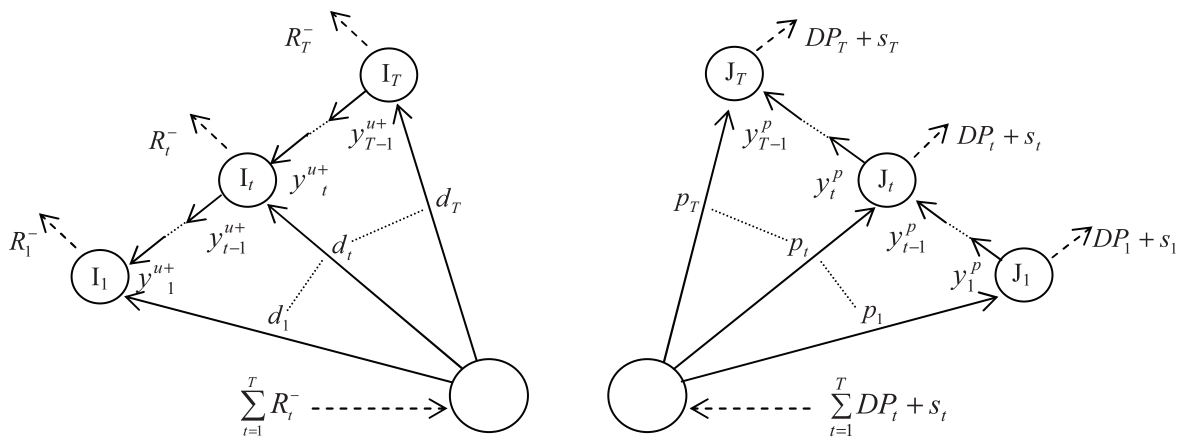

FIGURE 2. Network flow formulation for the production and final-disposal subproblems of the ELSR-S

Values $DP_t + s_t$, with $1 \leq t \leq T$, are the total demand requirements that we must satisfy by producing new items at each period, i.e., the sum of the demand for new products and the substitution quantity for remanufactured products. On the other hand, the return values $R_1^-, R_2^- ... R_T^-$ are the portion of the returns not remanufactured that we consider for disposal in each period. In addition, variables y_t^{u+}, with $1 \leq t \leq T$, represent the numbers of used items that we must add to the inventory level determined from remanufacturing in order to obtain the inventory level for the entire problem.

Note that each of these two networks is equivalent to that for the economic lot-sizing problem (Zangwill, 1968). Therefore, if the remanufacturing and substitution plans are given, the production and final disposal problems can be optimally solved separately in $O(T^2)$ time by means of a Wagner-Whitin (W-W) algorithm type. For the networks formulation we have assumed a final-inventory-level equal to zero for both the used and new items. This assumption can be eliminated without loss of generality by the same arguments discussed in Piñeyro and Viera (2009).

Therefore, we can solve the ELSR-S by means of finding first the remanufacturing and substitution plans of perfect cost, i.e. the remanufacturing and substation quantities at each period that allows us to determine in a time-effective way the production and final disposal plans of an optimal solution for the ELSR-S. We note that is last result is also valid for the case of general concave-cost functions. Since the ELSR-S is NP-hard, it is unlikely that we can develop any efficient time procedure for determining remanufacturing and substitution plans of perfect cost. In addition, we note that the approach of Piñeyro and Viera (2009) for the traditional ELSR of maximizing the total remanufacturing quantity cannot be a nice decision for the ELSR-S. Consider a particular ELSR-S instance with $T = 5$, where demand and return values are equal to 10 for each period. The costs are assumed stationary with the following values: $K_t^p = K^p = 200$, $c_t^p = c^p = 40$, $c_t^s = c^s = 10$, $K_t^r = K^r = 150$, $c_t^r = c^r = 20$, $K_t^d = K^d = 150$, $c_t^d = c^d = 20$, $h_t^p = h^p = 10$, $h_t^r = h^r = 3$, $h_t^u = h^u = 1$, $\forall t = 1,\ldots,5$. The optimal solution for this particular instance is $\{p_1 = 30,\ p_3 = 20,\ p_5 = 10,\ r_2 = 20,\ r_4 = 20\}$ with the remainder values for the activities equal to zero, and an optimal value of 4490. Note that the total remanufacturing quantity of the optimal solution is less than the total returns quantity. On the other hand, if substitution is not allowed, the optimal solution for the same instance is $\{p_1 = 30,\ p_4 = 20,\ r_t = 10,\ \forall t = 1,\ldots,5\}$, with an optimal value of 4550. Note that this last problem is feasible because the returns are useful (Piñeyro and Viera, 2009). Therefore, when one-way substitution is allowed, maximizing the total remanufacturing amount may not be the best option, even when the remanufacturing costs are favorable, as in the example above. In addition, we note that allowing substitution can lead in costs savings.

4. A Tabu Search procedure for the ELSR-S

In this section, we suggest a Tabu Search-based heuristic for the ELSR-S that can be considered an extension of that proposed in Piñeyro and Viera (2009) for the ELSR. The metaheuristic of Tabu Search is an iterative exploration process based on information stored in memory. This allows the search to escape from the trap of local optima. The procedure repeatedly moves from a current solution to the best among neighboring solutions until an aspiration criterion is fulfilled. In order to prevent cycling, the procedure stores recently visited solutions (or related information about them) in a continuously updated "tabu list". Thus, previously visited solutions are discarded for the next steps of the procedure as long as they are in the tabu list. Additionally, the notions of long and short-term memory, along with intensification and diversification strategies, are commonly used in order to make the Tabu Search an effective and robust procedure. This metaheuristic has been applied successfully in a wide range of optimization problems (Glover, 1990).

As it was further discussed in Section 3.1, obtaining an optimal solution of the ELSR-S can be reduced to the problem of finding the remanufacturing and substitution plans of perfect-cost. Although it is not a simple task, since the ELSR-S is a NP-hard problem, this result allows us to focus only in the remanufacturing and substitution activities. The Tabu Search procedure defined in Piñeyro and Viera (2009) receives the periods where it is desirable to remanufacture, and determines the quantities according to a simple rule. After the remanufacturing plan is obtained, the optimal production and disposal plans are obtained by means of a W-W algorithm type. The rule employed in Piñeyro and Viera (2009) to determine the remanufacturing quantity can be easy adapted for the case of one-way substitution, as follows.

$$r_t = \min(DR_{t(j-1)}, y^u_{t-1} + R_t) \quad 1 \le t < j \le T \tag{10}$$

Where periods t and j have been fixed as positive-remanufacturing periods, and $j = T$ if t is the last positive-remanufacturing period. Expression (10) means that the remanufacturing quantity in a certain period t is the minimum quantity between 1) all the available returns and 2) the accumulated demand from this period to the next one fixed as positive-remanufacturing period or to the end of the planning horizon. In other words, the remanufacturing plan obtained from (10) is the maximum among all the useful remanufacturing plans with the same positive-remanufacturing periods. A plan is useful when the accumulated remanufacturing quantity from any period $t \ge 1$ to the end-period T is at most equal to the accumulated demand (Piñeyro and Viera, 2009). This rule seems to be the fair option if we assume that remanufacturing used items is more suitable than disposing of them and producing new items. This last assumption is supported by both ecological as well as economic reasoning (Gungor and Gupta, 1999; Guide, 2000; Hormozi, 2003). Note that the remanufacturing plan of the optimal solution of the numeric example in Section 3.1 can be obtained applying (10) on periods 2 and 4. In order to apply the W-W algorithm for obtaining the production plan, we need to determine the substitution quantity for each period. We suggest the following rule for obtaining the substitution quantity at each period.

$$\begin{cases} s_t = \max\{(DR_{1t} - r_{1t} - s_{1(t-1)}), 0\}, & 1 \le t \le T \\ s_{1,0} = 0 \end{cases} \tag{11}$$

With s_{ij} the accumulated substitution between periods i and j, with $1 \le i \le j \le T$. Expression (11) means that the substitution quantity at each period is calculated as the portion of the remanufactured demand of the period that cannot be fulfilled by remanufactured items, i.e., the minimum among all the feasible substitution quantities. Thus, the demand for each period $t = 1,...,T$ that we must consider for the production subproblem is $DP_t + s_t$. In a similar way, for the final disposal problem, we need to determine first the portion of the used items, Δ_t, returned but not remanufactured. These amounts can be obtained as follows.

$$\begin{cases} \Delta_t = \max\{(R_{tT} - r_{tT} - \Delta_{(t+1)T}), 0\}, & 1 \le t \le T \\ \Delta_{(T+1)} = 0 \end{cases} \tag{12}$$

With Δ_{ij} the accumulated portion of returns that is not remanufactured between periods i and j, with $1 \le i \le j \le T$. We must enlarge the planning horizon by one period (T + 1) in order to represent the situation where not-disposal is the most suitable option (Piñeyro and Viera, 2009). By completeness reasons we present it in the following sections the different components of the Tabu Search procedure developed in Piñeyro and Viera (2009).

4.1. Solution representation

A feasible solution of the ELSR-S is represented by means of a (0,1) T-tuple. A value of 1 in position t means that remanufacturing is positive in period t, with $1 \leq t \leq T$. On the other hand, a value of 0 in position t means remanufacturing is zero. Thus, the tabu list stores a set of (0,1) T-tuples, each of which is associated with a feasible ELSR solution. For the remainder of the paper, optionally we refer to an ELSR-S solution only by the corresponding (0,1) T-tuple of its remanufacturing activity, since the optimal production and final disposal plans, and the substitution quantities can be determined as we explained in Section 3.1 and Section 4, respectively.

4.2. Neighborhood and moves

The neighborhood notion is based on the Hamming distance between two ELSR-S solutions represented by the (0,1) T-tuple. Thus, a solution u_d is d-neighboring to solution u_0 when the Hamming distance between them is exactly d, with $d > 0$. In order to explore the neighborhood of a current solution, we use a swap move. The move consists of swapping 0 with 1 (or vice versa) in a selected group of positions of the (0,1) T-tuple. For a given value $d > 0$, we must swapping d times in d different positions in order to obtain one of the neighboring solutions. An odd number of d must be employed to ensure that any of the (0,1) T-tuples can be generated regardless of the initial solution.

4.3. Aspiration criterion

The stop conditions are given by either the total number of iterations or a maximum number of iterations without improvement. The procedure returns the best among all of the evaluated solutions (including the initial solution).

4.4. Tabu list management

The tabu list has a fixed size $K > 0$, and is managed in a cyclical way using the FIFO strategy. Thus, a solution in the position k of the tabu list moves to position $(k + 1)$ when a new solution is entered into the list. When the position $(k + 1)$ is greater than the value K, the solution in $(k + 1)$ is eliminated.

4.5. Procedure description

The procedure receives four parameters: the total number of iterations, the maximum number of iterations without improvement, the size of the tabu list, and the initial (0,1) T-tuple corresponding to the positive remanufacturing periods. First, we determine the initial ELSR-S solution, applying the expression (10) in order to obtain the quantities to remanufacture for each period with value 1 in the initial (0,1) T-tuple. Then, the corresponding optimal plans for both production and final disposal activities are determined by means of the W-W algorithm, considering the expression (11) and (12) respectively. The cost of the initial solution is calculated. The search process is as follows. We first construct the set of neighboring solutions for the current solution by means of the swap move with a Hamming distance of value equal to 1. Observe that with this value, we can compute the whole neighborhood of the current solution in $O(T)$ time. For each neighboring solution, we control whether that solution belongs to the tabu list. If it does not, we add the neighboring solution to the tabu list; otherwise, we discard it. For this new neighboring solution, we determine the corresponding ELSR-S solution by means of the expressions (10) – (12) and the W-W algorithm. We compare the cost of this new solution with the best cost at the moment. If the cost of the new solution is smaller, we mark it as the new best neighboring solution. We proceed in the same way for each neighboring solution. After the best neighboring solution is determined, we compare its cost with the global best cost at that moment. If the cost of the selected neighboring solution is again smaller, we mark it as the new best global solution and reset the number of iterations without improvement. The iterative section continues until either the number of iterations is greater than the total number of iterations allowed or the number of iterations without improvement is greater than the maximum allowed. The procedure returns the best global solution evaluated.

5. Computational analysis

In this section, we report the numerical experiment carried out for the Tabu Search procedure suggested for solving the ELSR-S in the previous section. Three different demand-return relationships are tested in order to reflect situations with low, medium, and high return rates. Additionally, three different relationships between the new and remanufactured demand-streams are considered. Demands and return values are generated from the Poisson random distribution with means λ_{DP}, λ_{DR} and λ_R, respectively. We use the following values: $\lambda_{DP} = 10$, $\lambda_{DR} = (5.0, 7.5, 10)$ and $\lambda_R = (2.5, 5.0, 7.5)$. For the costs values also three different situations are considered for each demand-return relationship. The different costs components are obtained from a continuous uniform distribution. The values for the production costs are as fallows. The production setup cost K_t^p is uniformly distributed in the interval $[300, 500]$; the unit production cost c_t^p is uniformly distributed in the interval $[30, 50]$; the unit cost of holding produced items h_t^p is uniformly distributed in the interval $[10, 20]$. As it is common in practice, we assume that the return costs are at most equal to the serviceable costs (Guide, 2000; Richter and Weber, 2001; Brito and Dekker, 2002). For substitution costs, we assume the opposite behavior compared with the costs of returns, i.e. they are low when the costs of returns are high, and vice versa. This last decision is supported by the assumption that when the tasks related to the returns are expensive, the gap with new products should be lower. The different intervals of costs for both the activities concerned with returns and substitution are listed below in Table 1.

Costs	Low case	Medium case	High case
K_t^r	$[30, 60]$	$[60, 100]$	$[100, 150]$
c_t^r	$[10, 20]$	$[20, 30]$	$[30, 40]$
K_t^d	$[10, 20]$	$[30, 40]$	$[60, 80]$
c_t^d	$[2, 5]$	$[5, 10]$	$[10, 15]$
h_t^r	$[5, 8]$	$[8, 12]$	$[12, 15]$
h_t^u	$[1, 3]$	$[3, 5]$	$[5, 8]$
s_t	$[10, 15]$	$[5, 10]$	$[1, 5]$

TABLE 1. Intervals for the return and substitution costs

Combining all the previous cases for the demands, returns and costs, we have a total of 27 benchmarks. For each one of the benchmarks, we execute ten different cases using planning horizon values of $T = 5$ and $T = 15$, respectively. Thus, we test a total of 540 different cases. For the Tabu Search procedure, we use the following configuration. The size of the tabu list is fixed at 5000. The total number of iterations is 500, and the maximum number of iterations without improvement is 250. The initial solution is always the (0) T-tuple (i.e. without remanufacturing at any period). These parameter values are arbitrary, and we aim to maintain a large number of visited solutions in the tabu list. Our results are presented in Tables 2, 3 and 4. All of the entries are the sample means from ten cases. For each table, we indicate the percentage difference with respect to the cost of the optimal solution obtained with GAMS using the CPLEX solver. We mark in bold the minimum and maximum gap; the final column contains the average gap.

$\lambda_R = 2.5$	Low case (%)		Medium case (%)		High case (%)		Average (%)
	$T = 5$	$T = 15$	$T = 5$	$T = 15$	$T = 5$	$T = 15$	
$\lambda_{DR} = 5$	0.19	**0.62**	0.26	0.35	**0.09**	0.37	**0.31**
$\lambda_{DR} = 7.5$	**0.74**	0.42	0.62	0.34	**0.06**	0.34	**0.42**
$\lambda_{DR} = 10$	0.15	**0.34**	0.10	0.25	**0.07**	0.26	**0.20**

TABLE 2. Results for the case of low return rate

$\lambda_R = 5$	Low case (%)		Medium case (%)		High case (%)		Average (%)
	$T = 5$	$T = 15$	$T = 5$	$T = 15$	$T = 5$	$T = 15$	
$\lambda_{DR} = 5$	0.36	0.79	**0.13**	0.24	0.35	**1.21**	**0.51**
$\lambda_{DR} = 7.5$	0.88	**1.16**	0.39	0.89	**0.19**	0.45	**0.66**
$\lambda_{DR} = 10$	0.64	**0.85**	0.42	0.56	**0.32**	0.45	**0.54**

TABLE 3. Results for the case of medium return rate

$\lambda_R = 7.5$	Low case (%)		Medium case (%)		High case (%)		Average (%)
	$T = 5$	$T = 15$	$T = 5$	$T = 15$	$T = 5$	$T = 15$	
$\lambda_{DR} = 5$	0.44	0.20	0.18	0.19	**0.17**	**0.61**	**0.30**
$\lambda_{DR} = 7.5$	0.34	**0.71**	0.17	0.32	**0.16**	0.46	**0.36**
$\lambda_{DR} = 10$	1.09	**1.17**	0.58	0.48	0.46	**0.44**	**0.70**

TABLE 4. Results for the case of high return rate

From tables 2–4 we are able to conclude that the Tabu Search procedure shows an excellent behavior regardless of the case under consideration. For all cases the average gap is less than one percent and the optimal value was attained for 153 cases; this is 28.33% of the total tested cases. In 58.89% cases the gap among the cost of the solution obtained and the optimal solution was positive and less than 1%. Only in 13% of the total tested cases the gap was superior to 1% and always less than 5%. Referring to the procedure efficiency we note that the solution was found in the first

20 iterations for all the tested instances, and the running time was less than 250 milliseconds.

The successful of the procedure can be justified on the reasons explained below. First, the production and final disposal plans are the optimal for the remanufacturing and substitution plans determined. As the major costs are related to the production activity, it makes sense to produce in an intelligent way. Second, the rule applied for obtaining the quantities for the fixed remanufacturing periods. By maximizing the remanufacturing in those periods fixed as positive-remanufacturing periods we are reducing the number of items substituted, and thus the production-related costs. In addition, we are exploiting in practice the common situation that remanufacturing used items is more suitable than disposing of them and producing new items. Therefore, we are able to conclude that for cases where the remanufacturing costs are favorable, the divide and conquer is an effective technique for obtaining the different plans. We note that further analysis about how to identify the periods of positive remanufacturing, and conditions under which the above rule is optimal must be done in order to obtain high quality solutions by means of the divide and conquer technique. This analysis should take into account the impact of the substitution.

6. Conclusions and future research

In this paper we investigate the economic lot-sizing problem with product returns and one-way substitution (ELSR-S). We provide a mathematical model for the problem and show that it can be considered an extension of the ELSR, and then it is also NP-hard for the same cases. By means of a numerical example, we show that unlike the ELSR, when substitution is allowed, maximizing the total remanufacturing quantity cannot be the most suitable option. However, we show that we can apply a divide-and-conquer approach for solving the ELSR-S, similar to that applied in Piñeyro and Viera (2009) for the ELSR. In the case of the ELSR-S, if the remanufacturing and substitution quantities are known in advance, the optimal production plan and the optimal final disposing plan can be determined separately and in time-effective way. Thus, we suggest a Tabu-Search-based procedure for solving the ELSR-S, extending that of Piñeyro and Viera (2009) for the ELSR. The procedure explores different remanufacturing plans, guided by the rule of maximizing the useful remanufacturing quantity for each period fixed as positive remanufacturing-period. The substitution at each period is determined as the portion of the remanufactured demand that cannot be fulfilled by remanufactured items. The corresponding optimal production and final disposal plans are obtained by means of the Wagner-Whitin algorithm. The experiment conducted shows that the suggested procedure will be cost-effective for a wide-range of problem instances. We note that the optimal solution was found for nearly one third of tested cases and for most cases the gap with the optimal solution was less than 1%. We conclude that to maximize the useful remanufacturing quantity rather than to maximize the total remanufacturing quantity seems to be the best option when substitution is allowed. However, how to identify the periods with positive remanufacturing is not even clear.

Despite the low running-time obtained for the procedure, we note that for larger problems, it could be necessary to replace the Wagner and Whitin algorithm for any of the new faster algorithm of $O(TlogT)$ time developed by Federgruen and Tzur (1991), Wagelmans et al. (1992) or Aggarwal and Park (1993). From a theoretical point of view, a detailed analysis about the structural properties of the ELSR-S optimal solutions must be done. In this sense, further analysis on the problem of obtaining the perfect-cost remanufacturing plan seems an important goal. Other possible direction is to extend the analysis of Yang et al. (2005) on the extreme-point optimal solutions of the minimum concave-cost network flow formulation for the ELSR in order to include substitution. Also the analysis of stability regions as in Konstantaras and Papachristos (2007) for the case of constant parameters and large returns available in the first period should be extended in order to cover more general situations as discussed in this paper.

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9 An $O(T^3)$ algorithm for the capacitated economic lot-sizing problem with stationary capacities and concave cost functions with non-speculative motives

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Abstract. We consider the capacitated economic lot-sizing problem (CLSP) with stationary capacities and concave cost functions with non-speculative motives. Under these assumptions we show that there is an optimal solution of the problem that is composed only by subplans that can be computed in linear time, which means that the problem can be solved in $O(T^3)$ computation time.

Keywords: Capacitated Economic Lot-Sizing Problem; Inventory Control; Optimization.

1. Introduction

The capacitated economic lot-sizing problem (CLSP) refers to the problem of determining the quantities to produce at each period in order to meet the demand requirements of a single product on time, minimizing the sum of the costs involved. The number of units that can be produced at each period are limited by a maximum value. The CLSP is an NP-hard problem in general, and even for special cases on the cost functions and/or the capacity pattern (Bitran and Yanasse, 1982; Florian et al. 1980). For the case of concave cost functions and stationary capacities (i.e., equal capacity upper-bounds for each period) Florian and Klein (1971) propose an effective algorithm of $O(T^4)$ time. More recently, faster algorithms of $O(T^3)$ and $O(T^2 \log T)$ times have been suggested by van Hoesel and Wagelmans (1996) for the case of linear inventory holding costs and by Van Vyve (2003) for the case of linear costs with non-speculative motives, respectively. Bitran and Yanasse (1982) propose polynomial time algorithms for the cases NI/G/NI/ND, NI/G/NI/C, C/Z/ND/NI, and ND/Z/ND/NI of the CLSP, where the notation $\alpha/\beta/\gamma/\delta$ represents the set-up costs, the holding costs, the unit production costs, and the capacity pattern, respectively. Letters G, C, ND, NI, Z are used to indicate arbitrary pattern, constant, non-decreasing, non-increasing and zero, respectively. For the case NI/G/NI/ND of the CLSP, Chun and Lin (1988) provide an algorithm of $O(T^2)$ time. van den Heuvel and Wagelmans (2006) also consider the NI/G/NI/ND case, providing other $O(T^2)$ time algorithm which may run faster in practice. Chen et al. (2008) provide a pseudo-polynomial time algorithm for the same CLSP case but with more general cost functions. For surveys on the CLSP, we refer the readers to Brahimi et al. (2006) and Karimi et al. (2010).

The main contribution of this paper is to show that the subplans composing an optimal solution of the CLSP with stationary capacities and concave cost functions with non-speculative motives (i.e. when it is profitable to produce as late as possible) have a particular structure and can be obtained by means of a linear time procedure. This result implies that the running time of the well-known algorithm of Florian and Klein (1971) for the CLSP can be improved from $O(T^4)$ time to $O(T^3)$ time for the case of non-speculative motives on the costs. According to our best knowledge, our approach can be applied over situations that are not covered by previous related works in the literature. In addition, we want to note that our approach is simpler than the approach of Van Vyve (2003).

The remainder of the paper is organized as follows. Section 2 provides the notation used through the paper and the mathematical formulation for the CLSP. In Section 3 we describe the kind of production subplans that compose an optimal solution of the CLSP under the assumptions mentioned above. In Section 4 we describe the linear time procedure for obtaining this particular kind of production sequences. Finally, Section 5 concludes the paper.

2. Notation and mathematical formulation

We consider the CLSP with a finite planning horizon of length $T > 0$. For each period $t = 1,...,T$, there is a known customer demand $D_t \geq 0$ which must be satisfied on time by producing on the same period or in a previous period the quantity $x_t \geq 0$. Backlogging demand is not allowed and the production quantity at each period is limited by C_t

with $0 < C_t < +\infty$ and $t = 1,...,T$. There are costs for carrying out the production and for storing a positive quantity $y_t \geq 0$ at each period $t = 1,...,T$. Henceforth, we assume that the production cost function $f_t(\cdot)$ and the holding inventory cost function $h_t(\cdot)$ are non-decreasing concave functions on the interval $[0,+\infty)$, and equal to zero when its argument is zero or negative, with $t = 0,...,T$. It is also assumed that the initial inventory and the lead-time are equal to zero. The objective is to determine the quantities x_t to produce at each period in order to meet the demand requirements on time fulfilling the capacity constraints C_t and minimizing all the involved costs. The problem described above can be formulated as follows:

$$\min \sum_{t=1}^{T} \{f_t(x_t) + h_t(y_t)\} \qquad \text{(P)}$$

subject to:

$$y_t = y_{t-1} + x_t - D_t, \quad \forall t = 1,...,T \qquad (1)$$

$$y_0 = 0 \qquad (2)$$

$$x_t \leq C_t, \quad \forall t = 1,...,T \qquad (3)$$

$$x_t, y_t \geq 0, \quad \forall t = 1,...,T \qquad (4)$$

Constraints (1) state the well-known inventory equilibrium equation. Constraint (2) establishes that the initial inventory quantity must be zero. The capacity constraints are given in (3), and constraints (4) state the set of possible values for the decision variables. We note that the the decision variables y_t of the MILP above can be replaced by $(x_{1t} - D_{1t})$, where x_{ij} denotes the accumulated production and D_{ij} the accumulated demand between periods *i* and *j* respectively, with $1 \leq i \leq j \leq T$. Therefore, the problem formulated above reduces to find the set of feasible plans $x = (x_1,...,x_T)$. The set of feasible plans is not empty if and only if the accumulated demand of the first *t* periods does not exceed the accumulated capacities over these periods, formally:

$$\sum_{i=1}^{t} C_i \geq \sum_{i=1}^{t} D_i, \quad \forall t = 1,...,T \qquad (5)$$

Therefore, from now on we assume that expression (5) is fulfilled. Since the objective function of (P) is a concave function and the constraints (1) – (4) define a closed bounded convex set, there is an optimal solution of the CLSP that is an extreme point of this set. In addition, without loss of generality we can assume that the different feasible plans are composed by subplans $S_{ij} = (x_i,...,x_j)$ called *sequences* such that $y_i = y_j = 0$ and $y_t > 0$, for all *t* in $0 \leq i < t < j \leq T$. Periods *i* and *j* are commonly referred as *regeneration points*. Florian and Klein (1971) showed that the extreme-point solutions are composed only by sequences for which the production quantities of the periods are zero or equal to the capacity, except in at most one period, which is called the *fractional period*. This kind of sequence is referred as *capacity constrained sequence*. Based on this property, they provide an $O(T^4)$ time algorithm for solving the CLSP with stationary capacity-pattern. In the following sections we will see that the algorithm can be improved from $O(T^4)$ to $O(T^3)$ time if it is also assumed a non-speculative type of cost structure.

3. The ascending capacity constrained sequences of the CLSP

In this section we show that under the assumption of non-speculative motives on the costs and stationary capacity-pattern, there is an optimal solution of the CLSP that is composed only by a particular kind of subplans that we refer as ascending capacity constrained sequences, since the production level in these subplans is increasing over time.

Definition 1. We say that the cost functions of the CLSP are *non-speculative* if the expressions below are fulfilled:

$$f_i(a)+\sum_{t=i}^{j-1} h_t(b_t) \geq \sum_{t=i}^{j-1} h_t(b_t-a)+f_j(a), \quad \text{with } b_t, a>0, 1\leq i\leq t<j\leq T \tag{6.1}$$

$$f_i(a)+\sum_{t=i}^{j-1} h_t(b_t)+f_j(c) \geq f_i(a-\varepsilon)+\sum_{t=i}^{j-1} h_t(b_t-\varepsilon)+f_j(c+\varepsilon), \quad \text{with } a, b_t, c, \varepsilon>0, 1\leq i\leq t<j\leq T \tag{6.2}$$

Expression (6.1) states that it is profitable to transfer forward all the production quantity from one active period to another period initially inactive, and (6.2) that it is profitable to transfer forward at least one unit between two active periods. Expressions (6.1) and (6.2) are fulfilled in different settings of practical interest, e.g., when all the costs involved are concave functions and either stationary or non-increasing. We want to note that there may be other cost structures that satisfy the conditions stated in Definition 1 which are not covered by previous related works. For example, consider a CLSP instance of *T* periods where $f_t(x)=K_t^P+c_t^P x$, with $K_t^P=t$, $c_t^P=1$, the set-up and unit costs for production respectively, $h_t(x)=2x+\sqrt{x}$, and stationary capacity *C*, i.e., $x_t<C$, for each period $t=1,...,T$ respectively. We can verify that expressions (6.1) and (6.2) are fulfilled for this particular cost structure, if we assume also integral demand values. First we note that increasing set-up costs are not covered by the algorithms proposed in Chung and Lin (1988), van den Heuvel and Wagelmans (2006) and Chen et al. (2008). Secondly, the analysis of van Hoesel and Wagelmans (1996) and Van Vyve (2003) do not consider non-linear cost functions.

Since the CLSP is polynomial-time solvable in the case of stationary capacity (Florian and Klein, 1971), from this point on, for the remainder of the paper we consider a stationary capacity-pattern, i.e., $C_t=C$ for all $t=1,...,T$. Florian and Klein (1971) showed that in order to solve the CLSP we can focus only on those solutions composed only by capacity constrained sequences, i.e., sequences with at most one period below capacity. We show below that when the costs are non-speculative according to Definition 1, we can apply the algorithm of Florian and Klein (1971) to a particular kind of capacity constrained sequences defined below and introduced in Chung and Lin (1988) for the case NI/G/NI/ND of the CLSP.

Definition 2. We say that a capacity constrained sequence is an *ascending capacity constrained sequence* (ACC sequence) whenever the period with a positive quantity below capacity, if it exists, is the first among all the positive periods in the sequence.

Proposition 1. Assume that the cost functions of the CLSP are non-speculative according to Definition 1 and stationary capacities. Then, the solutions of the CLSP with plans composed only by ACC sequences are dominant, i.e., given a feasible solution of the CLSP for which there is at least one plan that is not composed by ACC sequences, we can determine a new feasible solution with all plans composed only by ACC sequences with at most the same cost than the original.

Proof. Consider a feasible solution $x = (x_1,...,x_T)$ of the CLSP composed only by capacity constrained sequences. Without loss of generality, suppose that x has only one sequence $S_{\alpha\beta}$ that is not an ACC sequence (the fractional period is not the first positive period in the sequence). This means that there are two consecutive periods i and j such that $C = x_i > x_j > 0$ with $1 \le \alpha \le i < j \le \beta \le T$. Then, define $\varepsilon = \min\{y_i, y_{i+1},..., y_{j-1}, C - x_j\}$ and consider the following definition of a new solution z:

$$\begin{cases} z_i = x_i - \varepsilon \\ z_j = x_j + \varepsilon \\ z_t = x_t, \quad t \ne i, t \ne j, 1 \le t \le T \end{cases}$$

We must note that z is also a feasible solution for the same CLSP instance that x is feasible. In addition note that at least one of the two following cases is fulfilled: 1) $z_j = C$; or 2) $y_t = 0$, for some t in $i \le t < j$. If case 1) is fulfilled, then the production quantity of period i in the new solution z is below capacity since $0 < \varepsilon < C$. In the case that period i is not the first positive period in the sequence, we can determine a new ε for period i and the immediately previous period k of the sequence such that $C = x_k > x_i > 0$, with $\alpha \le k < i \le \beta$. We repeat this process until the first positive period in the sequence is reached. On the other hand, if case 2) is fulfilled, we note that the sequence $S_{\alpha\beta}$ has been decomposed into two new sequences $S_{\alpha t}$ and $S_{t\beta}$ for some t with $i \le t < j$. We note that sequence $S_{t\beta}$ is an ACC sequence, since all the positive periods are at capacity. In the case of the sequence $S_{\alpha t}$, the period i is below capacity. If it is not the first positive period we proceed as we explained for case 1) for period i and the immediately previous period k of the sequence for which $C = x_k > x_i > 0$. Since we are assuming that the costs are non-speculative with respect to the transfer, the cost of the new solution z is at most equal to the cost of the original solution x. Thereby, we have constructed another feasible solution with at most the same cost as the original one but composed only by ACC sequences.

■

4. Computing the values of an ACC sequence

In this section we describe a procedure for determining the values of an ACC sequence in linear time. First, by Florian and Klein (1971), we note that for any capacity constrained sequence $S_{ij} = (x_i, x_{i+1},..., x_j)$, there are K periods at capacity, at most one positive period below capacity and the remaining periods equal to zero, with

$x_i + ... + x_j = D_{ij} = K.C + \varepsilon$, with $K \in \{1,2,...\}$ and $\varepsilon \geq 0$. Then, in order to compute the values of an ACC sequence between any pair of periods *i* and *j*, we must determine a sequence $A_{ij} = (x_i,...,x_j)$ satisfying 1) $x_{ij} = D_{ij} = K.C + \varepsilon$, with $K \in \{1,2,...\}$, $\varepsilon \geq 0$; and 2) $y_t > 0$ with $i \leq t < j$. Without loss of generality assume that $D_i > 0$. If $\varepsilon > 0$, then $x_i = \varepsilon$, otherwise $x_i = C$. The next positive period *t* at capacity, i.e., $x_t = C$, will be the earliest period *t* such that $D_{it} > x_i$, with $i < t \leq j$. We apply the same reasoning until all the *K* positive periods at capacity have been reached. In the cases that either $x_i = \varepsilon < D_i$ or for some period *t*, $x_{it} = D_{it}$, then there is not a feasible ACC sequence between periods *i* and *j*. As we are producing as late as possible, by (6.1) the ACC sequence obtained is of minimum cost. We also note that there is at most only one ACC sequence between any pair of periods. Since all the feasible ACC sequences are at most $(T + 1)T/2$ (Florian et al., 1980), the optimal solution of the CLSP can be determined in $O(T^3)$ time by means of the algorithm of Florian and Klein (1971), replacing the procedure for obtaining the production values of the capacity constrained sequences by the procedure described above for the ACC sequences.

5. Conclusions and future research

In this paper we show that for the CLSP under the assumptions of stationary capacities and concave cost functions with non-speculative motives, i.e. it is profitable to produce as late as possible, the algorithm of Florian and Klein (1971) can be improved from $O(T^4)$ time to $O(T^3)$ time. This result is supported by the fact that there is an optimal solution for which the plans are composed exclusively by a kind of sequences for which the only fractional period, if it exists, is the first among all the positive periods of the sequence. The type of cost structure that we assumed includes many cases of interest. In particular those cases for which the fixed costs of production are non-decreasing and non-linear functions, which according to our best knowledge, are not covered by the algorithms proposed in previous works in the literature.

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