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NEW HISTORICAL ESTIMATES OF THE HUMAN DEVELOPMENT INDEX

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"New Historical Estimates of the Human Development Index"

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Resumen

Este documento discute diferentes alternativas para construir el convencional Índice Histórico de Desarrollo Humano con considera tres dimensiones: ingreso, salud y educación. Se discuten los resultados de diferentes modelos en términos de mejoras totales absolutas y relativas en desarrollo humano, los rankings que arrojan, la contribución de los diferentes componentes al crecimiento y las compensaciones entre las tres dimensiones.

El objetivo del artículo es proponer el uso del índice que se considera mejor refleja el desarrollo histórico y que ofrece las menores brechas posibles entre las compensaciones de los diferentes componentes.

Se trabaja con una muestra de 18 países de siete regiones entre 1900 y 2010.

Abstract

This paper discusses different alternatives to construct the conventional Historical Human Development Index that considers three dimensions: income, health, and education. We discuss the outcome of different models in terms of aggregated improvements in human development, the rankings of performance, relative growth, the contributions to performance of the different dimensions, and the tradeoffs between the three dimensions.

The purpose of the paper is to propose an index that we consider better fits historical development and that provides the less possible gaps in the tradeoffs between the different components of the index. Such an index can be considered the best proxy on which to base policy recommendations.

The paper works with a sample of 18 countries of seven regions for 1900-2010.

Key words: human development, tradeoffs, convergence, income, education, health

JEL: E01, I15, I25, I31, N10, 05

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Introduction

The Human Development Index (HDI) constructed and used by the United Nations Development Program (UNDP) is already 30 years old. Along its history, it has been the subject of numerous discussions, criticisms, and improvements in many different directions. Some discussions have been centered on the components of the index, the way in which components are aggregated and their function is shaped, and the data and proxies used for each component. This last aspect is crucial for the present paper. While the UNDP usually works with data generated since the index was created and with more recent data covering a very wide range of variables and countries, our interest is in the use of the HDI to catch long-run development, since the beginning of the 20th century. This is the frame that constrains our discussion in this paper. We will work with the three conventional dimensions: income, health, and education.

The different assumptions made in the construction of the index, and the selection of variables and proxies, have clear and relevant impacts on the results and their consistency, as well as on the implications in terms of development policy. Our aim is to clearly present these outcomes.

In this paper we will first present the original UNDP HDI and its foundations. We will also review the discussion that followed on how to construct the index, the results obtained and its implications, focusing on long-run development. We will mainly concentrate on the contributions by the UNDP, Prados de la Escosura (2007 and 2021), and Bértola et al. (2010 and 2013). We will also consider the works by Ravallion (2012a and 2012b), Chakravarty (2003 and 2011), and Zambrano (2014 and 2017). Second, based on the previous discussion, we will select a set of indices to test the outcomes and contrast their implications. The selection will represent the different approaches discussed, as well as new choices of our own. Third, we will present the results of applying the different indices using the database previously constructed by Bértola et al. (2013), with some adjustments. This database covers 18 countries of Africa, the Americas, Asia, Australasia, and Europe. The use of this database will allow us to, in later works, include the estimation of inequality of the different components, as in Bértola et al. (2013). The discussion of the results will be organized as follows. Section 1 discusses the background. Section 2 presents the different models and their results: performance of the index; relative performance between countries and regions; contributions of the different components to performance and relative performance; tradeoffs between the components; and differences in the rankings according to the different models. Section 3 presents a short story told using the model we consider to be the best. Finally, a summary and the conclusions are included.

1. Background

a. The Original UNDP Estimates and Its Evolution

As it is well known, the HDI is linked to the concept of human development, mainly as it was developed by Sen. The idea behind the HDI is to measure different components that represent different capabilities to live a meaningful life. The HDI has been mainly composed by three equally weighted, normalized variables: per capita income (y); life expectancy at birth (h); and education (e).

The indices for each component Ih and Ie were constructed using linear functions and maximum (M) and minimum (m) possible values. The indices were constructed in the following way:

$$Ix=(x-x\text{m})/(x\text{M}-x\text{m}) \qquad (1),$$

where x is health (h) or education (e), M is the maximum possible value, and m is the minimum possible value, according to historical evidence. In other words, the index is 1 when actual value = M and 0 when actual value = m. h is obtained from life tables. In the case of education, the variables used went through important changes. Originally, 2/3 of the index was based on literacy rates, and 1/3 of the index was constructed considering enrollment rates in primary and secondary education. As data availability improved, average years of education of adult population and expected years of education of younger generations were introduced instead.

Contrary to both variables considered above, per capita income (y) was not measured linearly, but it was transformed into logarithms. This was based on three main explicit or implicit arguments. First, on the basis of a marginalist approach, increasing income was considered to yield decreasing improvements in welfare. Besides, at higher levels of income, an increasing share is invested rather than consumed. Second, the HDI was mainly aimed at measuring basic capabilities; therefore, income clearly above what is necessary to cover basic needs was not considered relevant. This was also in line with giving literacy rates such an important role in measuring educational achievement. Third, through the transformation of per capita income, the weight of the other two components was increased as income level grew, thus legitimating the use of the HDI. The normalized income function was the following:

$$Iy=(\log y-\log y\text{m})/(\log y\text{M}-\log y) \qquad (2)$$

b. Historical Human Development Indices: Some Background

There have been many attempts to construct long-run series of the HDI. Crafts (1996) was a pioneer in this matter. Some attempts were also made for Latin America (Astorga and Fitz-Gerald, 1998, see also, Thorp, 1998; Camou and Maubrigades, 2005; Bértola, Camou, Maubrigades, and Melgar, 2010).

Prados de la Escosura pioneered the construction of worldwide historical databases. His 2007 paper introduced several innovations in the construction of the index. The two main innovations were: (i) a convex achievement function for Ie and Ih, arguing that the achievement implied by increasing one year of education or life is higher, the higher the value of e and h; and (ii) the geometric average, instead of the arithmetic average, of the indices, to punish the index if one component rises but the others do not. Otherwise, Prados de la Escosura kept the logarithmic transformation of the income.

Bértola, Hernández, and Siniscalchi (BHS 2010) constructed a Historical HDI (HHDI) for the different Latin American countries, and for other countries of Europe, the Americas and Australasia (see also Bértola and Ocampo, 2013, Appendix Table 4). The authors introduced some innovations: they added a new variable, representing institutional achievements (democracy, stability, pluralism); following Prados de la Escosura (2010), they used the geometric average of the components; they tested different models, but they argued against the use of the logarithmic transformation of per capita income. Their main argument was that, if used for long-run comparisons of performance between countries, the logarithmic transformation implied a reduction of the income gap between rich and poor countries, the higher were the levels of per capita income on average. In other words, it implied an underestimation of the role of per capita income to improve human development and to measure inequality in human development between countries at higher levels of income.

Bértola, Hernández, Siniscalchi, and Rodríguez Weber (BHSR 2013), following Hicks (1999), and based on BHS (2010), estimated Gini coefficients of the three components of the HDI to construct an Inequality-Adjusted Historical Human Development Index (IAHHDI). The HHDI was multiplied by the geometric average of the Gini coefficients of the different dimensions. An interesting result was that inequality was steadily reduced in all regions during the whole 20th century, but that, as expected, Latin American inequality was higher, and Nordic inequality was lower than elsewhere, with the consequent impact on relative development of the IAHHDI.

The UNDP HDI for 2010 introduced the geometric average of the components. More interestingly, the 2014 HDI presented an inequality-adjusted HDI for the 1990-2010 period for a group of countries. When doing that, the per capita income index was not transformed into logarithms, as otherwise. It can be said that this is in line with all inequality estimates, for which income is never transformed into logarithms.

A recent paper by Prados de la Escosura (2021) further develops the author's 2007 paper. His main innovation is, following BHS (2010), the introduction of the institutional variable, even if based on different institutional proxies. Otherwise, he kept the logarithmic transformation of the per capita income index and the convex achievement function for *e* and *h*. However, his arguments, at least with reference to *h*, are different: he now states that the reason for doing that is that quality of life improves as average age rises, marking a difference with the previous argument concerning achievement in terms of efforts (which seemed contradictory to the argument used in favor of the logarithmic transformation of income).

c. Tradeoffs and Aggregation Problems: A Transversal Discussion of Paradoxes

The HDI gives us the opportunity to see the implicit cost-benefit relations in the index, as one of the variables is expressed in monetary units. We can see which is the cost in terms of income of increasing one year of education or one year of life. Both the 2010 and 2014 HDI use the geometric average of the three dimensions. The geometric average is adopted because the score is lower than that of the arithmetic average when the different components show disparate performances. In this way, the index favors a balanced growth. Chakravarty (2011), Ravallion (2012b), and Zambrano (2016), point to the fact that the cost-benefit relations, implicit in the 2010 and 2014 HDI, present serious problems, as they have a strong impact on the tradeoffs between the different dimensions measured. For instance, the marginal rate of substitution between *h* and *y* is 240 times larger in the ten richest countries than in the ten poorest countries. Something similar happens in the relation between education and income: it is 170 times larger. The implications are serious: "These extraordinary differences in the tradeoffs across rich and poor countries become a problem if they are used as a guide for resource allocation, as they imply that the poorest countries should only be willing to spend about one half of one percent of what the richest countries would spend for the same gain in health or education capabilities" (Zambrano, 2016, p. 2).

Confronted with that situation, Ravallion proposed to use the Chakravarty (2003) index with a similar function for each of the components and the arithmetic average. The Chakravarty Index is as follows:

$$Ix = (x^r - x^r m)/(x^r M - x^r), \text{ where } r \in (0,1) \quad (3)$$

When r=1, the function is linear, as in the current UNDP functions for *h* and *e*. If r is below 1, the effect is like assuming diminishing marginal scores for a one-unit improvement. According to Ravallion, for a value of r=0.5, the cost-benefit differences according to the 2010 UNDP scores between the ten richest countries and the ten poorest countries, are significantly reduced to 9 to 1 in the case of *h* and to 8 to 1 in the case of *e* (vs. 240 and 170, respectively).

Zambrano, however, finds a paradox: while the UNDP functions strongly affect the tradeoffs, Ravallion´s proposal produces important shifts in the rankings between countries. For instance, Germany and Sweden disappear from the top ten list, while their place is taken by Qatar and Singapore, due to their higher per capita income and lower life expectancy at birth and education. Thus, Ravallion gets better tradeoffs and the UNDP gets better rankings.

When using the geometric average, one of the problems is that when one of the components approaches the lower bound value (the subsistence income level, for instance), the value of the aggregated index tends to zero, regardless of the performance of the others. If, instead, the arithmetic mean is used, as in Chakravarty (2011), it becomes possible for a country with high income to rank well, despite poor records in *h* and *e*.

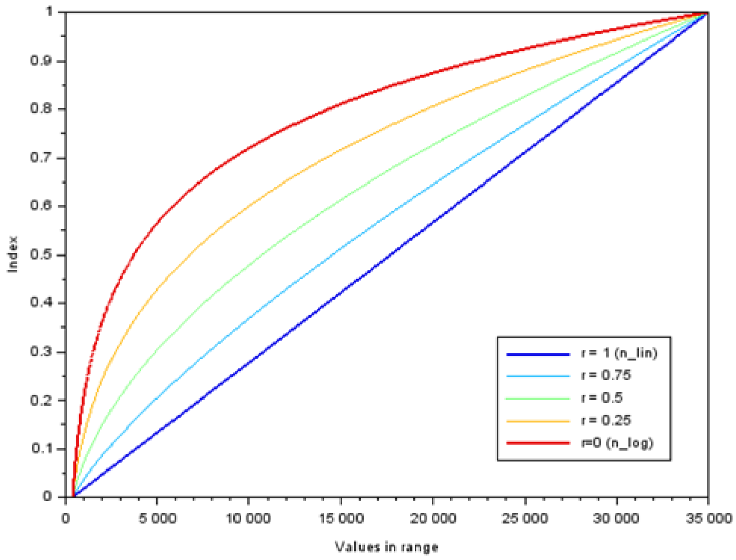
To choose the lower possible bias, Zambrano (2016) compares the 1990 and 2010 UNDP functions estimating the cost-benefit changes and the rankings between both. Interestingly, he finds that the main distortions produced in the cost-benefit relations and in the relative rankings, were most importantly due to the use of the logarithmic transformation of *y*, rather than to the geometric or arithmetic mean. The impact of the logarithmic transformation is five times higher than the impact of the geometric mean.

Zambrano, thus proposes the adoption of a hybrid index, following Zambrano (2015), consisting of the geometric mean of the indices. He assumes that improvements in health produce proportional improvements in life expectancy, and something similar occurs in the case of educational capabilities. Thus, he chooses an affine function. On the other hand, accepting the fact that income growth adds capabilities only on a fraction, the crucial question is how to choose the value of r. He strongly disregards the extreme solution of assigning the value r=0, i.e., the logarithmic transformation of income.

The family of functions chosen is $C_r: [y_m, y_M] \rightarrow [0,1]$ with

$$C_r(y) = \begin{cases} \frac{y^r - y_m^r}{y_M^r - y_m^r} & with\ \ r \in (0,1] \\ \frac{\log y - \log y_m}{\log y_M - \log y_m} & with\ \ r = 0 \end{cases} \quad (4)$$

Figure 1. Zambrano´s functions for different r-values (0,1) for income ranging from 0 to 35,000



By choosing r=0.5, Zambrano reduces the range of variation of the tradeoffs between health and income to a fourth or fifth of the variation arising from the UNDP method. Similar or even greater reductions are obtained in the case of the tradeoffs between education and income. The results are also much better in relation to the rankings, as compared to Ravallion´s results.

Prados de la Escosura (2021) rejects the use of the Chakravarty functions proposed by Zambrano, because he considers the choice of the r-value to be an arbitrary decision. However, he does not seem to notice that his own decision to use the logarithmic transformation of income is a similarly arbitrary (and extreme) choice: r=0.

While Bértola et al. (2013) constructed a hybrid index, intuitively similar to Zambrano's, they failed to recognize that the contribution of income growth to well-being is marginally decreasing. They focused on disregarding the logarithmic transformation, as Zambrano did, but did not offer an acceptable alternative and used the value of r=1, thus overestimating the role of income growth at higher levels of income. However, as they accepted in one of their models, the convex achievement function of *h*, both effects counteracted each other to some extent, as we will see below.

d. The Models to Be Compared

Given the background, we will choose to work with four alternatives to normalize a variable *p* within the range [*p*m,*p*M], where n and M are minimum and maximum values, respectively:

- Linear function: $n_{lin(p)} = \frac{p-pm}{pM-pm}$

- Lognormalized function: $n\log(\mathrm{p}) = \frac{logp-logpm}{logpM-logpm}$

- Potential-r function: $n_{pot(p)} = \frac{p^r-pm^r}{pM^r-pm^r}$

- Convex function: $n_{conv(p)} = \frac{\log(pM-pm)-\log(pM-p)}{\log(pM-pm)} = 1 - \frac{\log\,(pM-p)}{\log\,(pM-pm)}$

Combining the previous normalizing alternatives, now we will test the results of five models, as in Table 1. Models 1-3 represent the models used so far to construct long-run estimates of the HDI. Models 4 and 5 follow Zambrano, with r-values of 0.5 and 0.75, respectively. The idea is to cover an important variation of models to register the differences in outcomes.

Table 1. Models to Construct the Historical Human Development Index

Models	Iy	Ih	Ie	Authors
1	(LOGy-LOGym)/(LOGyM-LOGym)	(h-hm)/(hM-hm)	(e-em)/(eM-em)	UNDP
2	(LOGy-LOGym)/(LOGyM-LOGym)	LOG(85-20)-LOG(85-h))/LOG(85-20)	(LOG(16)-LOG(16-e))/LOG(16)	Prados de la Escosura
3	(y-ym)/yM-ym	LOG(85-20)-LOG(85-h))/LOG(85-20)	(LOG(16)-LOG(16-e))/LOG(16)	Bértola et. al., 2013
4	(y^0,5-ym^0,5)/(yM^0,5-ym^0,5)	(h-hm)/hM-hm)	(e-em)/(eM-em)	Zambrano 1
5	(y^0,75-ym^0,75)/(yM^0,75-ym^0,75)	(h-hm)/hM-hm)	(e-em)/(eM-em)	Zambrano 2

In all cases, we will use the geometric average of the components, i.e., the third square of the product of the three: I= (I*y**I*h**I*e*)^(1/3).

Another important decision to make has to do with maximum (M) and minimum (m) values. In the case of income, previous works used the values 40,000 and 100 for M and m, respectively. We consider these values to be extreme and unnecessarily distant from real values. In our database, the highest income is 31,654 (USA 2010) and the lowest is 598 (Brazil 1900) (both figures in 1990 US dollars). As it is not a good choice to take the extreme values too close to the historical records, we will choose the M and m values to be 35,000 and 400, respectively. The other two cases are less controversial.

Table 2 presents the relation between maximum (M) and minimum (m) values that arises from the different models. Our choice has the advantage that the spread of the relations between M and m of the different variables is much lower.

Table 2. Ratios between Maximum (M) and Minimum (m) Values: Ours and Others

Ours	M	m	ratio		Previous	M	m	ratio
linear (r=1)	35000	400	87,5		linear (r=1)	40000	100	400,0
r=0,75	2559	89	28,6		r=0,75	2828	32	89,4
r=0,5	187	20	9,4		r=0,5	200	10	20,0
log (r=0)	4,5	2,6	1,7		log (r=0)	4,6	2,0	2,3

The Data

Our data is taken from Bértola et al. (2013), with minimal updates using the same sources as those referred to in the quoted work. One exception is the African and Asian countries in 1900-1950, for which the data was estimated from Prados de la Escosura (2021). The variables used are as in Table 3. The data is presented on a decennial basis.

Table 3. Variables Used as Proxies for the Different Dimensions

notation	dimension	variable
y	per capita income	per capita GDP
h	health	life expectancy at brith
e	education	average years of education
M	Maximun value	all
n	Minimun value	all

The countries included in the database are as follows:

- Latin American Southern Cone (LASC) (4): Argentina, Brazil, Chile, and Uruguay.
- Core countries (4): France, Germany, UK, and USA.
- Settler economies (2): Australia and New Zealand.
- Latin Europe (2): Italy and Spain.
- Nordic countries (2): Finland and Sweden.
- Asia (2): Japan and South Korea.
- Africa (2): Egypt and South Africa.

2. Results

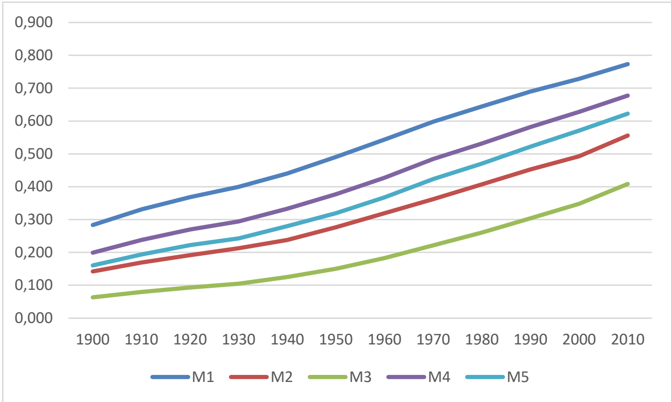
a. Performance of Aggregate Human Development

Figure 2 shows the aggregated weighted performance of world HDI according to the different models discussed.

For every combination of *y*, *h* and *e*, the following applies (except for the cases in which *h*>84 and *e*>15):

- $M1(y,h,e) > Mi(y,h,e)\ \forall i = 2,3,4,5,6$ (M1 always higher)
- $M3(y,h,e) < Mi(y,h,e)\ \forall i = 1,2,4,5,6$ (M3 always lower)
- $M1(y,h,e) > M4(y,h,e) > M5(y,h,e)$

Figure 2. World HDI Performance According to Different Models, 1900-2010



Note: Weighted average of the sample of 18 countries from 7 regions.

Table 4 presents the average total weighted increase in human development by decade. Some conclusions may be drawn:

- For the whole period, the range of variation in total performance between the models is 0.7-1. M1 shows the best and M3 the worst performance. M2 is between the two, while M4 and M5 are close to M1.
- The period 1900-1950 shows the highest variation between the results of the different models; a range of 0.42 to 1.0. On the contrary, in 1950-2010 the variation of performance is reduced to 0.85 to 1.0. The ranking is the same as in the whole period.
- All models show a better performance in 1950-2010 than in 1900-1950, but M1 is the one that shows the smaller increase between both periods, while M3 shows the highest.

Table 4. Total HDI Performance by Decade According to Different Models
(Weighted Averages, 18 countries, 1900-2010)

	M1	M2	M3	M4	M5
Average abolute increase by decade					
1900-1950	0,041	0,027	0,017	0,036	0,032
1950-2010	0,047	0,047	0,043	0,050	0,051
1900-2010	0,045	0,038	0,031	0,043	0,042
Average decennial increase in relation to best performer					
	M1	M2	M3	M4	M5
1900-1950	1,00	0,65	0,42	0,86	0,77
1950-2010	0,93	0,92	0,85	0,99	1,00
1900-2010	1,00	0,85	0,70	0,98	0,94

How can we explain these differences? We will develop this discussion in the coming sections, but we can advance two conclusions: (i) the lower the value of r, the higher the performance, due to the implicit reduction of the weight given to income, and the higher weight given to the other two variables (this can be easily seen comparing M3 with all the other models that transform income in some way); (ii) the use of the convex achievement functions also produces a reduction of performance, as both M2 and M3 show a lower performance than the other three models.

However, these conclusions are context-dependent and should be carefully considered, as these are weighted averages of quite different patterns of development. Let us now turn to uneven development.

b. Relative Performance between the Core and Lower HDI Regions

In this section we want to analyze how the different models reflect the relation between the core countries and the low HDI countries.

Figure 3 shows the HDI of LASC and Africa relative to that of the Core countries. Table 5 shows the changes in the relative positions in terms of index units, as well as the size of the gap in selected years.

The main conclusions to be drawn are as follows:

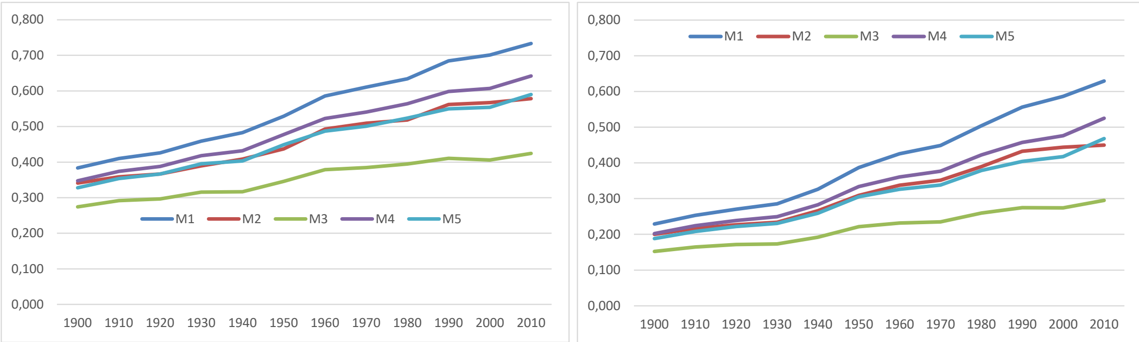
- One of the most important findings is that M1, the UNDP model, is the only one that shows rather constant levels of the gap (it rises until the 1950s and in 2010 diminishes to levels which are similar to the ones registered in 1900). All the other models show that the size of the gap is steadily increasing, both in Africa and Latin America.
- M1 (UNDP) shows the best relative position of countries with lower HDI, while M3 (Bértola et al.) shows the lowest values, both for Africa and LASC. Curiously enough, M5 (Zambrano 2, r=0.75) and M2 (Prados), show quite similar relative positions, both in Africa and LASC, while M4 is closer to M1. As we will see, the

similarities between M2 and M5 are hiding different contributions of the components of the HDI.

- All models show a process of convergence between LASC and Africa, on the one hand, and the core countries, on the other.

Figure 3. Latin American Southern Cone and Africa: HDI Relative to Core Countries, 1900-2010

a. Latin American Southern Cone b. Africa

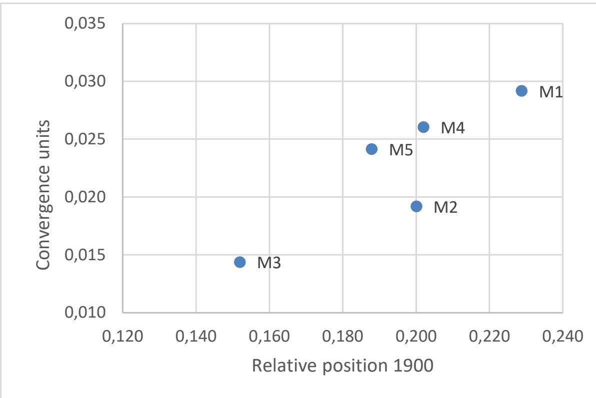


- The models that show the highest rate of convergence are those that show an overall higher relative position of the countries with lower HDI (see also Figure 4): M1 in first place (UNDP), followed by M4 and M5. M2 looks like an outlier, which depends on the use of convex achievement functions in education and health. This model, despite having relatively higher relative positions, shows a lower convergence. M3 is at the bottom, both because of the linear income function (r=1) and the use of convex achievement functions.

Table 5. Latin American Southern Cone and Africa vs. Core Countries: 1900-2010 (HDI Units, 1900-2010)

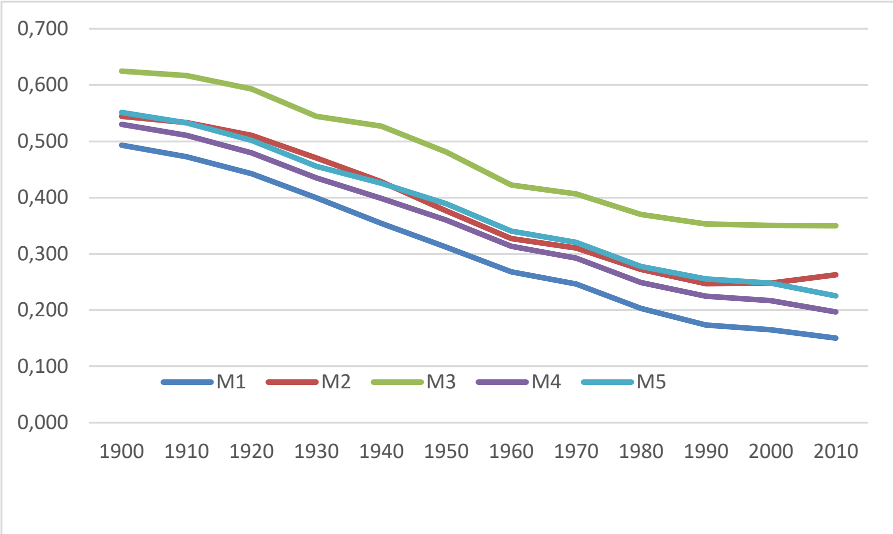
	M1	M2	M3	M4	M5
a) Changes in relative HDI by decade					
LA Southern Cone/Core					
1900-1950	0,029	0,019	0,014	0,026	0,024
1950-2010	0,034	0,024	0,013	0,027	0,024
1900-2010	0,032	0,022	0,014	0,027	0,024
Africa /Core					
1900-1950	0,032	0,022	0,014	0,026	0,023
1950-2010	0,040	0,023	0,012	0,032	0,027
1900-2010	0,036	0,023	0,013	0,029	0,025
b) Size of the gap					
LA Southern Cone/Core					
1900	0,266	0,15	0,094	0,222	0,196
1950	0,300	0,22	0,171	0,281	0,267
2010	0,241	0,31	0,396	0,315	0,355
Africa /Core					
1900	0,333	0,18	0,109	0,271	0,237
1950	0,391	0,27	0,204	0,359	0,336
2010	0,335	0,40	0,485	0,418	0,460

Figure 4. Convergence Units 1900-2010 and Relative Position 1900, Africa vs. Core Countries, All Models



Another way in which we can test convergence are changes in the coefficient of variation of the country values (Figure 5). Also as expected, M1 shows the lowest values and the more noticeable and continued decline, while M3 is the opposite: convergence ends by the 1990s. This latter change is also noticeable in M2.

Figure 5. Coefficient of Variation of HDI According to Different Models (18 Countries), 1900-2010



c. Contribution of the Components to Performance

Figure 6 presents the total increase in HDI units and the contribution of each component to this increase, according to the five different models. Table 6 presents the data for every region, as well as the averages of contributions across models (last column) and across regions (last rows).

Figure 6 shows that M1 and M4 estimate a similar total HDI increase. However, the contribution of the different components is quite different. Income contributes with 43% in M1 and with 64% in M4. In short, the models that transform income into logarithms show a clear reduction of the contribution of this variable to total growth, not unexpectedly. On the other hand, the use of convex achievement functions for health and education reduces total growth, as in M2 and M3. If combined with logged income growth, the contribution of health and education increases significantly. The use of the linear income function, more than compensates this effect in M3, showing the lowest total HDI increase and a high contribution of income.

If we look across the models, we see that in low HDI regions, like Africa and Latin America, income makes a low contribution to HDI growth. Africa is the extreme case: no matter the model, education undoubtedly makes the highest contribution to HDI growth; always clearly above 50%. In M1 and M2, health follows, while in M3 and M5, income follows. In M4, income and health make similar contributions. On average, the models assign 65% of growth to education. In the case of Latin America, income accounts for a relatively low contribution of 26% on average, the rest being equally distributed. In all the other regions, and on average, income makes the highest contribution to growth (above 40% on average), followed by health in Core countries and Latin Europe, and by education in Settlers, Asia, and Nordic countries.

Figure 6. HDI Growth (HDI Units) and the Contribution of Each Component (%), by Regions and Total According to M1-5, 1900-2010

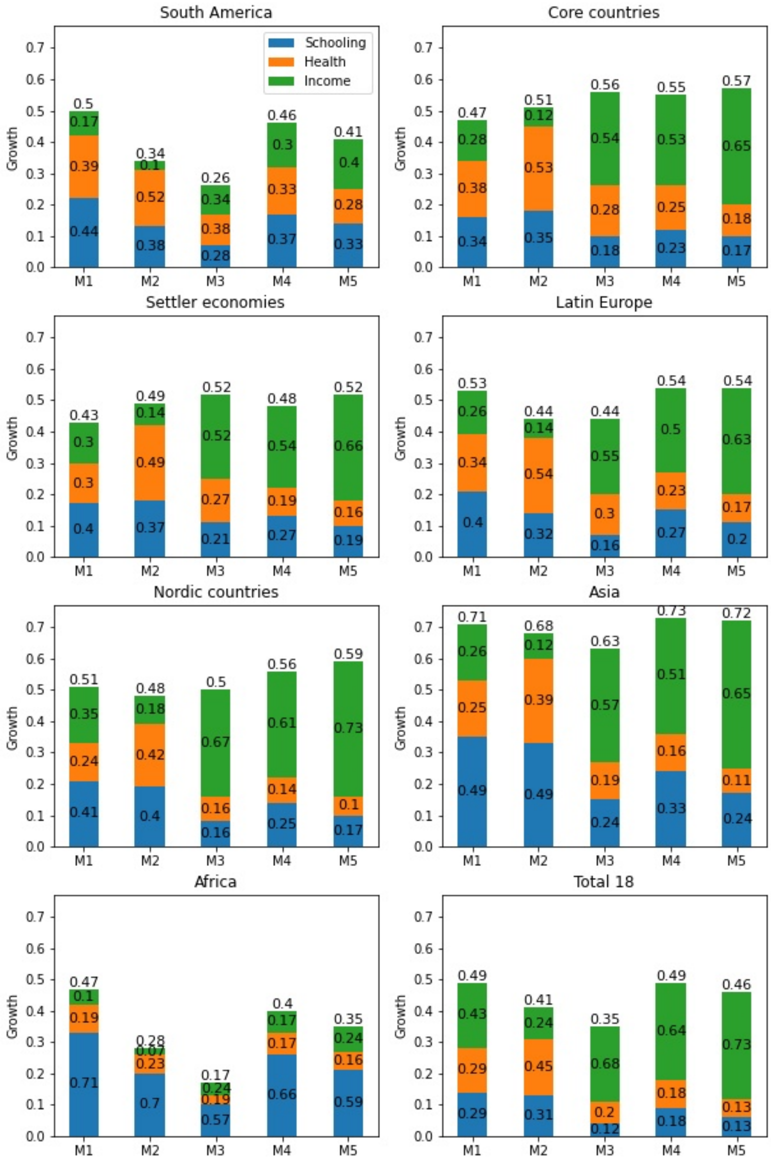


Table 6. Contributions of the Three Dimensions to HDI Growth According to Models 1-5 (%)

		M1	M2	M3	M4	M5	Average
LASC	ly	0,17	0,10	0,34	0,30	0,40	0,26
	le	0,44	0,38	0,28	0,37	0,33	0,36
	lh	0,39	0,52	0,38	0,33	0,28	0,38
	total	0,50	0,35	0,26	0,45	0,41	0,39
Core	ly	0,28	0,12	0,54	0,53	0,65	0,42
	le	0,34	0,35	0,18	0,23	0,17	0,25
	lh	0,38	0,53	0,28	0,25	0,18	0,32
	total	0,47	0,51	0,56	0,54	0,57	0,53
Settlers	ly	0,30	0,14	0,52	0,54	0,66	0,43
	le	0,40	0,37	0,21	0,27	0,19	0,29
	lh	0,30	0,49	0,27	0,19	0,16	0,28
	total	0,42	0,48	0,52	0,48	0,51	0,48
Latin Europe	ly	0,26	0,14	0,55	0,50	0,63	0,42
	le	0,40	0,32	0,16	0,27	0,20	0,27
	lh	0,34	0,54	0,30	0,23	0,17	0,32
	total	0,52	0,45	0,43	0,54	0,54	0,50
Nordic	ly	0,35	0,18	0,67	0,61	0,73	0,51
	le	0,41	0,40	0,16	0,25	0,17	0,28
	lh	0,24	0,42	0,16	0,14	0,10	0,21
	total	0,51	0,48	0,51	0,56	0,59	0,53
Asia	ly	0,26	0,12	0,57	0,51	0,65	0,42
	le	0,49	0,49	0,24	0,33	0,24	0,36
	lh	0,25	0,39	0,19	0,16	0,11	0,22
	total	0,71	0,68	0,64	0,72	0,72	0,69
Africa	ly	0,10	0,07	0,24	0,17	0,24	0,16
	le	0,71	0,70	0,57	0,66	0,59	0,65
	lh	0,19	0,23	0,19	0,17	0,16	0,19
	total	0,47	0,28	0,18	0,39	0,35	0,33
Total	ly	0,43	0,24	0,68	0,64	0,73	0,54
	le	0,29	0,31	0,12	0,18	0,13	0,21
	lh	0,29	0,45	0,20	0,18	0,13	0,25
	total	0,49	0,41	0,35	0,48	0,46	0,44
Non-weighted average of contributions							
	ly	0,25	0,12	0,49	0,45	0,57	
	le	0,46	0,43	0,26	0,34	0,27	
	lh	0,30	0,45	0,25	0,21	0,17	
	Total	1,00	1,00	1,00	1,00	1,00	

d. Tradeoffs between the Three Dimensions

We have already discussed the different outcomes of the different models in terms of absolute and relative growth, and in terms of the contributions of the different dimensions to HDI improvements. In this section we will consider the different tradeoffs between the dimensions of the HDI, arising from the different models.

In the HDI, the marginal rate of substitution between life expectancy and income (MRS_h), or expected years of educational attainment and income (MRS_e), was always variable for different combinations of dimensional achievement. Since 2010, when the UNDP introduced the geometric mean as aggregation function, this effect was amplified. As shown in Ravallion (2012b), the MRS is higher in countries with higher income and life expectancy. In other words, the rates at which each country can substitute life expectancy for income, keeping the HDI unchanged, are higher at higher levels of income and life expectancy at birth.

Although the geometric average produces an increase in MRS, we have two reasons to use this alternative: (i) we want to punish the index if its increase relies on one dimension alone (high income at the expense of education and life expectancy, for instance); and (ii) because Zambrano (2017) showed that changes in MRS were 5 times bigger due to the use of the log transformation of income than due to the choice of the geometric mean.

Table 7 shows our results. Regarding the value of the tradeoffs, we can say the following:

- Either we look at countries or regions, M2 shows the highest MRSh and MRSe values. On the contrary, M5 always shows the lower tradeoffs, closely followed by M4.
- Either we look at countries or regions, M2 shows the highest rate between maximum and minimum values. On the contrary, M5 always shows the lowest rate, closely followed by M4. M2's rate is about 10 times higher than M5's.
- On average, the MRSe are higher than the MRSh, either we look at countries or regions. This means that one more year of education is more expensive than one more year of life. However, the spread between maximum and minimum values is higher in *h* than in *e*, i.e., the cost of achieving one more year of life varies more than the cost of achieving one more year of education.
- Either we look at the regional or the country data, the coefficient of variation of the minimum values of the different models is lower than that of the maximum values. Thus, differences in the spread of outcomes are in direct relation to increases in general performance.
- Obviously, if country data are considered instead of regional aggregation data, the above observations are magnified.

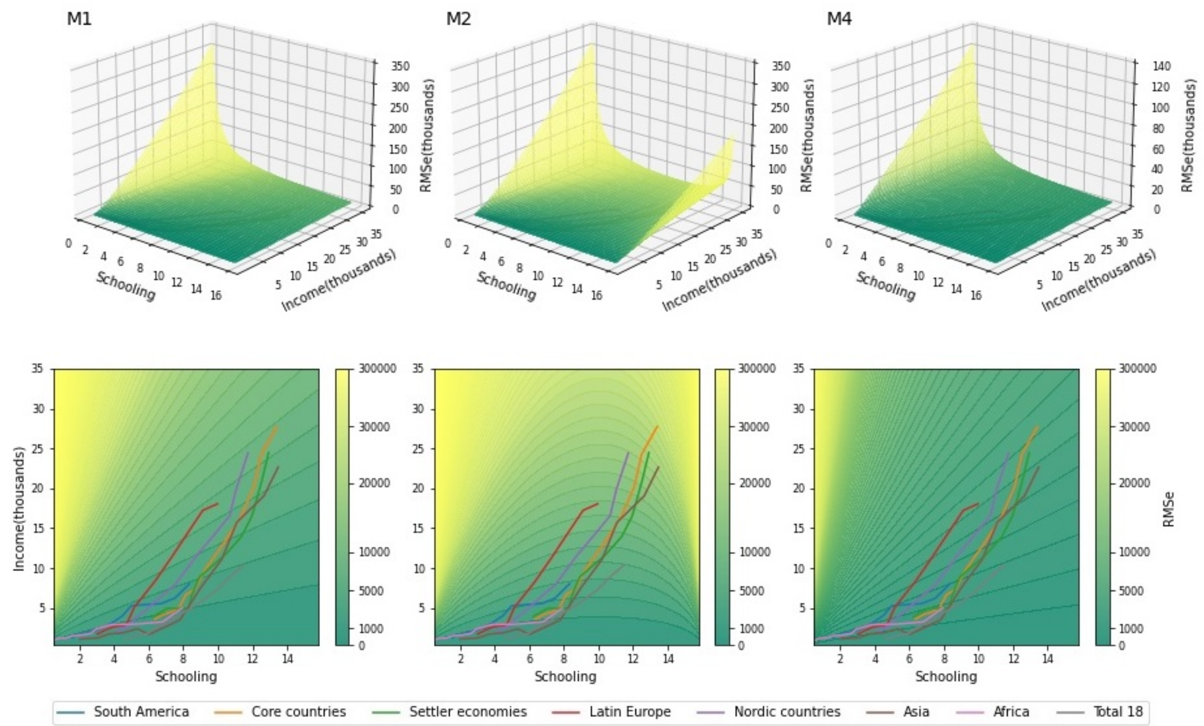
It is worth mentioning that the tradeoffs of M2 and M3 have an exponential growth, ratcheting as they approach the upper bounds. Figure 7 shows the case of the education-income tradeoff for M1, M2, and M4 (a similar result is obtained for the health-education tradeoff). In all cases, when education tends to zero and income grows, the tradeoffs tend to infinite. This is not a problem, though, as this is not a real case. However, when both variables increase, the models show different results. According to M1 and M4 (see second-row figures), the different regions show a close to linear trajectory; the tradeoffs grow, but in a moderate way. On the contrary, M2 presents a serious problem, as the tradeoffs increase significantly as maximum values are approached. A similar problem is found in M3, due to the convex achievement function used.

Table7. Marginal Rates of Substitution (MRS) between Income and One More Year of Life or Education, According to Different Models (1990 Geary-Khamis dollars)

Health (MRSh)						
Regions				Countries		
	Max	Min	Rate	Max	Min	Rate
M1	1951	71	27	2306	23	100
M2	10530	83	127	13088	29	454
M3	2518	52	49	3187	22	147
M4	810	56	14	937	19	49
M5	588	50	12	677	18	38
Mean	3280	62	46	4039	22	158

Education (MRSe)						
Regions				Countries		
	Max	Min	Rate	Max	Min	Rate
M1	8762	572	15	10479	162	65
M2	24960	613	41	38656	170	228
M3	5803	383	15	9388	140	67
M4	3638	447	8	4256	146	29
M5	2646	399	7	3076	140	22
	9162	483	17	13171	152	82

Figure 7. Income-Education Tradeoffs in the Range between Maximum and Minimum Values, According to M1, M2, and M4



Note. First row: Tradeoffs between education and income in M1, M2, and M4, for all combinations of values, between M and m. Second row: Top view of first row and trajectories of the seven regions.

Let us now see two examples for the year 2010.

- According to M2, the MRSh for Africa (531,2 1990 PPPUS$) is 20 times lower than that of the Settlers (10,530.4 1990 PPPUS$). Africa must resign 10,5% of its GDP (5,014 1990-PPPUS$) to gain one year of life (from 62.5 to 63.5) keeping the HDI unchanged. On the other hand, the settler societies must resign 44% of their income (24,473 1990 PPPUS$) to increase life expectancy by one year (from 81.8 to 82.8).

- The Korean MRSe is 38,656 dollars in 2010, while its per capita income is 22,590 dollars.

Based on these conclusions, we think it is better to avoid the use of the convex achievement functions.

e. Rankings

As it was mentioned before, the main changes in the rankings, as noticed by Zambrano, were produced when the Chakravarty index was applied to the three dimensions of the index. This is not the case of any of the models we are working with.

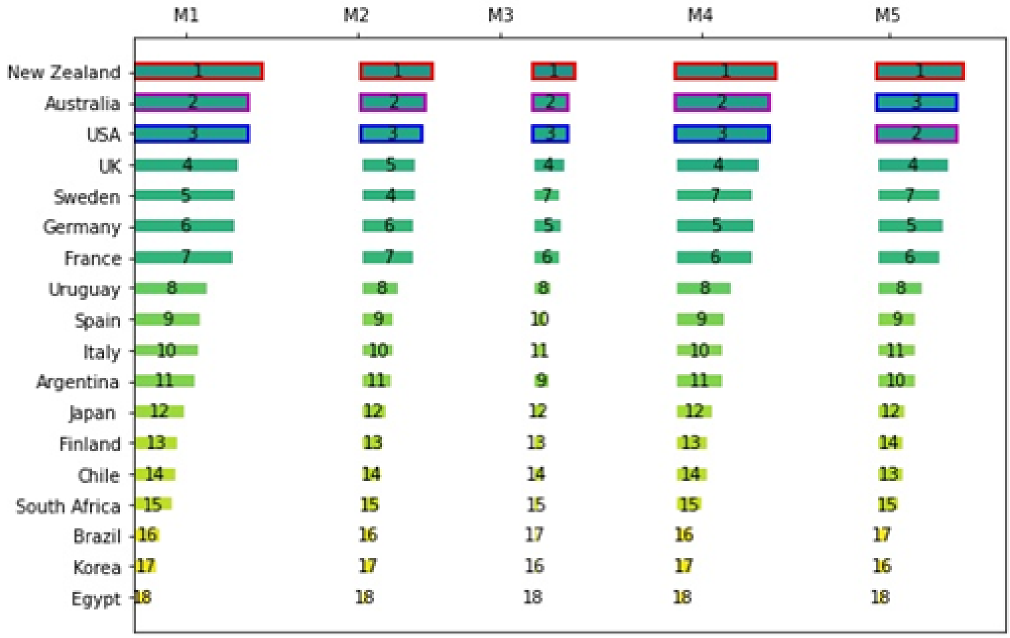
Figure 8 shows the rankings for the years 1900 and 2010. We can only find some important differences at higher levels of development. Indeed, differences in the year 1900 are small. If we look at medium development countries, M3 is more similar as M4 and M5 than to M1 and M2, because at low levels of income the impact of the logarithmic transformation of income is lower.

However, some differences can be noticed among the top ranked countries in 2010: the same six countries are at the top according to all the models, but in different order. For example: Korea ranks 1st according to M1, but more clearly in M2 and M3 (due to higher educational achievements, despite its 5th position in income, and its 8th position in health). M4 and M5 place USA first (4th in education, and similar as Korea in health).

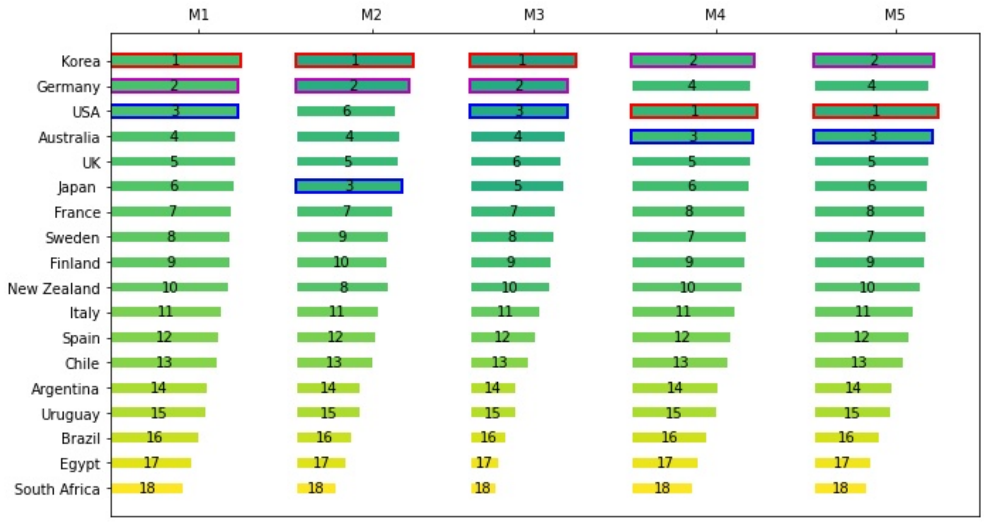
In short, M2 is the one that shows more differences with respect to the others, but the rankings are not a crucial aspect to make the final choice of a model.

Figure 8. Ranking of Human Development According to Different Models, 1900 and 2010

a. 1900



b. 2010



f. In Short

All the indices discussed in this paper show problems. It is difficult to find a way to smoothly approach the complex interaction between different dimensions in the process of development.

The different indices highlight different aspects. Nevertheless, we do not want to avoid the choice of what we think is the best (even if imperfect) alternative to approach development.

Table 8 summarizes our results.

Our first conclusion is that the traditional way to construct the HDI shows many problems, mainly due to the use of the logarithmic transformation of income. Comparing the outcomes of the different models, M1 (UNDP's traditional way), shows extremely high absolute levels, quite high performance, and the highest level of convergence among regions and countries. Low HDI countries converge significantly driven by education and health improvements, and thanks to the compressed weight given to income. Moreover, M1 is the only model that shows that the gap between core countries and less developed regions remains constant, while all the other models show an increasing gap. In our opinion, the logarithmic transformation of income is an extreme choice, which, in terms of Zambrano's approach, implies to choose r=0. We are aware that income is not everything and that there has been a steady attempt to emphasize other components of development. This is done from the very starting point, by weighting the three components in a similar way. However, the extreme transformation of income produces what we think is an extreme reduction of the role of income in explaining differences among countries and regions, giving the dubious expectation that significant catching up in HDI can be achieved despite weak economic growth. Besides, the UNDP model produces relatively high tradeoffs between the three components.

It is important to further develop this discussion. Income is the demand side of production. If we reduce the importance of income, we are reducing the importance of production. The black box of production has been neglected by many strands of economic thought. The Human Development approach has very recently tackled the problem of production for the first time, mainly because of environmental concerns, as shown by the new versions of the Human Development Index (UNDP, 2020). However, the productive capacities of societies are crucial to understand not only income growth, but also the efforts that can be made to improve the number of years of education and the extension of life. Even if the HDI does not aim to study the interaction between these variables, the extreme choice of r=0 seems to help to neglect one crucial aspect of development and lead to inaccurate policy recommendations.

Prados de la Escosura's attempt to improve the HDI by means of the convex achievement function for education and health (M2) produces paradoxical results. First, it reduces even more the role of income to explain HDI growth, with all the problems we have already mentioned. Second, it creates extremely high tradeoffs between the components of the HDI, thus giving controversial signals to policy-making. The underlying assumptions for doing that are dubious and the implications in terms of tradeoffs are extremely problematic, particularly when the real values approach the M-values. The paradoxical outcome is that it finally recommends improving development through progress in education and health, but the gap in these aspects is increased through the convex functions. Therefore, growth and convergence is lower than those arising from M1, despite giving even more weight to education and health.

Bértola et al.'s attempt to correct the UNDP model (M3) using the linear income function, and adopting the convex achievement function, generates extremely low results in terms of absolute and relative achievements of developing regions. The model emphasizes the role of income, but it fails to acknowledge that the marginal contribution of income to well-being is, to some extent, decreasing. Besides, by using the convex achievement functions it generates rather high tradeoffs.

In our opinion, M4 and M5 are the best approaches: they show the lowest tradeoffs, without ignoring the fact that these tradeoffs do change during the process of development. In terms of absolute levels, absolute and relative performance, the results are not extreme. They do show income as the main driver of HDI growth, which we believe is in line with historical evidence. They also show that the contributions to growth in low HDI regions are more evenly distributed, hence income growth is an important explanation of the remaining gap. Thus, economic growth is still a crucial policy target in developing regions.

Doomed to choose between M4 and M5, we find M4 to be less controversial, as it shows a more moderate role of income.

Table 8. Outcomes of the Different HDI Models, 1900-2010

	Absolute levels	**Absolute performance (as compared to other models)**	**Relative perfomance (bigger or smaller convergence)**	**Size of the gap**	**Drivers**	**Tradeoffs (the lower the better)**
M1	Extremely high	Good or very good performer	Highest convergence and smalest gap	Stable	Evenly distributed; *y* leads. More even contributions in rich countries	Medium
M2	Medium	Relatively low performer	Medium. *h* and *e* lead convergence	Increasing	*h* and *e* lead in less developed regions; more evenly distributed in highly developed regions	Extremely high
M3	Extremely low	Worst or almost worst performer	Extremely low. Income leads divergence. Biggest gap	Increasing	*y* the great performer (linear). More even in poor countries	Medium
M4	Medium	Close to best performer	Medium to high convergence, relatively low gap. Income reduces convergence	Increasing	*y* leads. More even in poor countries, education in Africa	Low
M5	Medium	Close to best performer	Medium convergence, medium gap. Income reduces convergence	Increasing	*y* leads. More even contributions in poor countries, education in Africa	Lowest

3. A Short Story Told by M4

The story we can tell based on Model 4 is as follows.

World HDI was at a level of 0.2 by 1900, and more than tripled during the past 110 years, reaching a level of 0.67. This means that, according to the current goalposts, there is still much room for improvement.

A process of convergence in relative terms can be noticed in all regions in relation to the core countries; however, the Settlers converged from above (Table 9). With respect to the less developed regions, convergence took place mainly in health and education. Income convergence took place in Latin America at the beginning of the 20th century but remained fluctuating around a stable level afterwards. African income remained stable in relative terms during the whole period. Income shows a much worse relative position than the two other variables in the two less developed regions. This confirms the idea that bridging the gap implies an important effort in terms of income growth.

Nevertheless, if instead of looking at relative positions, we look at the size of the gap, a much less favorable view arises. The size of the gap in relation to the developed world has increased both in Africa and Latin America. Even the case of Latin Europe looks less favorable, as the gap in relation to the core remained constant.

Table 9. Relative HDI by Regions, 1900-2010 (core=1)

Relative dev	LASC	Core	Settlers	Lat. Eu	Nordic	Asia	Africa	Total 18
1900	0,35	1,00	1,10	0,59	0,77	0,40	0,20	0,59
1910	0,37	1,00	1,13	0,60	0,77	0,44	0,22	0,62
1920	0,39	1,00	1,11	0,65	0,74	0,50	0,24	0,65
1930	0,42	1,00	1,07	0,67	0,80	0,55	0,25	0,66
1940	0,43	1,00	1,07	0,62	0,82	0,59	0,28	0,68
1950	0,48	1,00	1,01	0,66	0,85	0,56	0,33	0,70
1960	0,52	1,00	0,98	0,71	0,90	0,73	0,36	0,72
1970	0,54	1,00	0,98	0,78	0,93	0,84	0,38	0,73
1980	0,56	1,00	0,97	0,81	0,93	0,90	0,42	0,73
1990	0,60	1,00	0,97	0,83	0,93	0,93	0,46	0,75
2000	0,61	1,00	0,98	0,85	0,93	0,95	0,48	0,76
2010	0,64	1,00	0,97	0,84	0,94	0,97	0,53	0,77

Siez of the gap	LASC	Core	Settlers	Lat. Eu	Nordic	Asia	Africa	Total 18
1900	0,22	0,00	-0,04	0,14	0,08	0,20	0,27	0,14
1910	0,24	0,00	-0,05	0,15	0,09	0,21	0,30	0,14
1920	0,25	0,00	-0,04	0,15	0,11	0,21	0,31	0,14
1930	0,26	0,00	-0,03	0,15	0,09	0,20	0,34	0,15
1940	0,28	0,00	-0,03	0,19	0,09	0,20	0,35	0,16
1950	0,28	0,00	0,00	0,19	0,08	0,24	0,36	0,16
1960	0,28	0,00	0,01	0,17	0,06	0,16	0,38	0,17
1970	0,30	0,00	0,01	0,15	0,05	0,11	0,41	0,18
1980	0,32	0,00	0,02	0,14	0,05	0,07	0,42	0,19
1990	0,31	0,00	0,03	0,13	0,06	0,05	0,42	0,20
2000	0,32	0,00	0,01	0,12	0,06	0,04	0,43	0,20
2010	0,31	0,00	0,02	0,14	0,06	0,02	0,42	0,20

The drivers of development are shown in Figure 9. Asia stands out as best performer with income as its main driver; this applies to all highly developed countries. As it was already mentioned, the drivers of HDI growth in least developed countries (LDC) are education and health, while income makes a small contribution.

Figure 9. HDI Growth (HDI Units) and Contributions of the Different Components (%), 1900-2010, According to Model 4

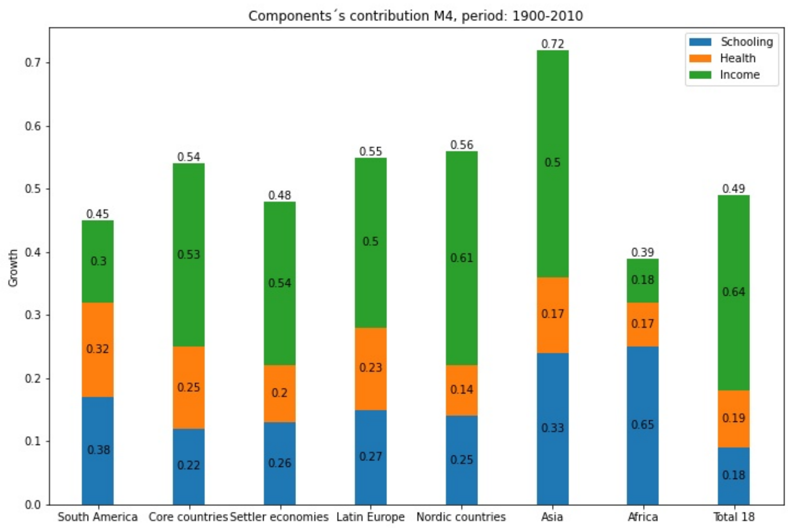


Table 10. Relative HDI by Regions and Size of the Gap (in HDI Units), 1900-2010 (core=1)

	y		h		e		Total	
Relative HDI	LASC	Africa	LASC	Africa	LASC	Africa	LASC	Africa
1900	0,39	0,27	0,43	0,35	0,25	0,09	0,35	0,20
1910	0,45	0,31	0,45	0,34	0,26	0,11	0,37	0,22
1920	0,45	0,32	0,46	0,34	0,29	0,13	0,39	0,24
1930	0,47	0,30	0,49	0,33	0,32	0,16	0,42	0,25
1940	0,43	0,31	0,53	0,37	0,35	0,20	0,43	0,28
1950	0,48	0,31	0,59	0,49	0,39	0,24	0,48	0,33
1960	0,46	0,29	0,74	0,56	0,42	0,29	0,52	0,36
1970	0,46	0,30	0,80	0,63	0,43	0,28	0,54	0,38
1980	0,49	0,32	0,83	0,71	0,44	0,34	0,56	0,42
1990	0,45	0,28	0,86	0,77	0,56	0,43	0,60	0,46
2000	0,43	0,27	0,86	0,72	0,60	0,54	0,61	0,48
2010	0,48	0,35	0,89	0,71	0,62	0,59	0,64	0,53

Size of the gap	y		h		e		Total	
1900	0,15	0,17	0,24	0,27	0,30	0,36	0,22	0,27
1910	0,15	0,19	0,27	0,32	0,32	0,38	0,24	0,30
1920	0,16	0,19	0,30	0,36	0,33	0,40	0,25	0,31
1930	0,16	0,21	0,31	0,41	0,33	0,41	0,26	0,34
1940	0,21	0,26	0,30	0,40	0,33	0,41	0,28	0,35
1950	0,21	0,27	0,30	0,38	0,33	0,41	0,28	0,36
1960	0,25	0,33	0,20	0,34	0,34	0,41	0,28	0,38
1970	0,31	0,40	0,16	0,29	0,37	0,47	0,30	0,41
1980	0,33	0,44	0,14	0,24	0,39	0,47	0,32	0,42
1990	0,40	0,52	0,12	0,20	0,33	0,43	0,31	0,42
2000	0,46	0,59	0,13	0,25	0,31	0,36	0,32	0,43
2010	0,46	0,57	0,10	0,27	0,32	0,34	0,31	0,42

Table 10 shows the relative performance of each component, as well as the size of the gap in each of them. What we see is that the relative position of less developed regions improves significantly in education and health but remains rather constant in income. However, when we look at the size of the gap, we see that the income gap widens all the time and triples during the whole period in both Africa and Latin America. Only in terms of life expectancy at birth in the Latin American Southern Cone can we see an important reduction of the gap. This variable remains rather constant in Africa. Regarding education, the gap remains constant in both regions.

4. Summary and Conclusions

The HDI is now more than 30 years old. It has been a great contribution to the concept and measurement of development, becoming some kind of super-ideology.

Along its history, the HDI went through important changes and improvements. It also generated an intense debate about how to construct it.

This paper is centered on the application of the HDI to measure long-run development. The discussion has therefore been constrained by data availability. Our main goal was to compare the different models that have been proposed, and to choose the one we think better helps to describe human development.

We have worked with the three conventional variables: income, proxied by per capita GDP; health, proxied by life expectancy at birth; and education, proxied by average years of education.

To simplify our analysis, we constructed a sample of 18 countries from seven different regions: Core countries (France, Germany, UK and US), Settler economies (Australia and New Zealand); Nordic countries (Sweden and Finland); Latin Europe (Italy and Spain); Asia (Japan and South Korea); Africa (Egypt and South Africa); and the Latin American Southern Cone (Argentina, Brazil, Chile and Uruguay).

Following UNDP and Zambrano's conclusion (2015), we decided to maintain the geometric average of the components. This decision strengthens the importance of development of the three components, as they are not only weighted equally, but the index is punished if one component grows at the expense of the others.

Based on this general agreement, the models differ in the way in which the different functions are constructed. While the UNDP and Prados de la Escosura use the logarithmic transformation of income, Bértola et al. use the linear function. Zambrano, while accepting that income growth has marginal decreasing returns in terms of well-being, considers the log-transformation an extreme fall, and proposes intermediate alternatives. In the case of education and health, Prados de la Escosura, followed by Bértola et al., chooses a convex achievement function, implying that additional years of life and education have increasing marginal benefits in terms of welfare. This alternative is not used by the UNDP and is strongly rejected by Zambrano.

We have compared and analyzed five different models: UNDP (M1: log of income, linear functions for education and health); Prados de la Escosura (M2: log of income, convex achievement functions for education and health); Bértola et al. (M3: linear income; convex achievement function for education and health); Zambrano 1 (M4: r=0.5 for income, linear functions for education and health); and Zambrano 2 (M5: r=0.75 for income, linear functions for education and health).

We conclude that the extreme choice of the log-transformation of income (r=0), as in M1 and M2, produces a reduction of the differences in development between core regions and less developed ones. These results are due to a compression of the role of income in development and the higher weight tacitly given to the other two components. M1 also yields relatively high tradeoffs between the different components of the index. In M2, the effect on the gap is counteracted by the effect of the convex achievement functions for education and health. However, this model yields two problematic results: an extremely high contribution of education and health to HDI growth, and extremely high tradeoffs between the variables. The policy implications of these results, as well as those of M1, may be problematic, as they seem to conclude that more efforts must be carried out in terms of education and health, rather than on income growth. Even if this sounds right from the point of departure of the human development approach, it may be misleading, as it tends to neglect the huge difficulties that developing countries are facing in their economic performance, which, by the way, is closely correlated with the potential achievements in terms of health and education.

M5's attempt to correct the effects of the log-transformation of income is also an extreme solution, which yields poor results in terms of growth and relative development. The combination of this model with the convex achievement function for education and health adds more problems, in terms of high tradeoffs.

We think the two versions of Zambrano's model are the ones that better describe the human development. We chose M4 because it is less controversial in terms of the contribution of income to growth. The story told by this model is one of an important growth in human development during the 110 years under study, and shows that income growth made, on average, the highest contribution to growth. The model shows that less developed regions improved their levels of development in relation to the core countries, due to education and health improvements, not income. However, when we look at the size of the gap between more and less developed regions, we find that the gap steadily increases, particularly in terms of income. Less developed regions show limited success in terms of relative income growth.

The final idea we want to stress is that theories of economic growth and development have often neglected the black box of production. Until the emergence of endogenous growth theories, this has been a common feature of the neoclassical research tradition. The human development approach puts lots of energy on the development of capabilities, mainly from the point of view of individuals, but it also maintains the lack of interest in the production process inherited from neoclassical thinking. Human development has been an important contribution, as it helps to change the idea that economic growth alone is enough to secure the well-being of the population. However, less developed regions have faced huge barriers in their growth process, and only a few countries have succeeded to catch up with the leaders. Thanks to the increasing attention paid to sustainable development, especially in relation to the environment, the black box of production is being the subject of an increasing interest, joining the efforts traditionally made by Post-Keynesian, Evolutionary, Structuralist, and Neo-Schumpeterian approaches. The famous contributions by Thomas Piketty have similar shortcomings inherited from neoclassical thinking. The distributional outcomes of economic growth are correctly highlighted. However, too little is said about their impact on production and the growth process.

References

Astorga P. and V. Fitz-Gerald (1998), "The Standard of Living in Latin America during the Twentieth Century", in Thorp, R. (1998) *Progress, poverty and exclusion: An Economic History of Latin America in the 20th century*, statistical appendix, Interamerican Development Bank, Washington.

Bértola, L., M. Camou, S. Maubrigades, and N. Melgar (BCMM) (2010), "Human Development and Inequality in the Twentieth Century: The Mercosur Countries in a Comparative Perspective" in: Salvatore, R., Coatsworth, J. H. y Challú, A. E. (Org.), *Living Standards in Latin American History: Height, Welfare and Development, 1750-2000*, Cambridge, David Rockefeller Center for Latin American Studies, Harvard University.

Bértola, L., M. Hernández, J. Rodríguez Weber, and S. Siniscalchi (2013), "A century of Human Development and Inequality".

Bértola, L. and J. A. Ocampo (2012), *The Economic Development of Latin America since Independence*, Oxford University Press.

Bértola, L., M. Hernández, and S. Siniscalchi (2010), "Un Índice Histórico de Desarrollo Humano de América Latina y algunos países de otras regiones: metodología, fuentes y bases de datos", Documento de Trabajo, Programa de Historia Económica y Social, Facultad de Ciencias Sociales, Montevideo.

Camou, M.& S. Maubrigades (2005), "La calidad de vida bajo la lupa: 100 años de evolución de los principales indicadores" in *Boletín de Historia Económica*, No.4.

Chakravarty, S.R. (2003), "A Generalized Human Development Index", *Review of Development Economics* 7: 99–114.

Chakravarty, S.R. (2011), "A Reconsideration of the Tradeoffs in the New Human Development Index", *Journal of Economic Inequality* 9: 471-474.

Crafts N.F.R. (1996), "The Human Development Index: Some Historical Comparisons", Working Paper No. 33/96 Department of Economic History, London School of Economics, July 1996.

Prados de la Escosura, L. (2010), "Improving Human Development: a long-run view", CEPR Discussion Paper No. 7982.

Prados de la Escosura, L. (2021), "Augmented human development in the age of globalization", *Economic History Review*, 0, 0 (2021), pp. 1–30.

Ravallion, M. (2012a), "Mashup Indices of Development", *World Bank Research.*

Ravallion, M. (2012b), "Troubling Tradeoffs in the Human Development Index", *Journal of Development Economics* 99: 201–209.

Sen, A. (1985), *Commodities and Capabilities*, North-Holland, Amsterdam, 1985.

Sen, A. (2000), *Development as Freedom*, Anchor Books, New York.

Sen, A. (1988), “The Concept of development”, *Handbook of Development Economics*, Volume I, Edited by H. Chenery and T.N. Srinivasan, Elsevier Science Publishers B.V.

United Nations Development Program (1990-2020), *Human Development Report.* New York

Zambrano, E. (2014), “An Axiomatization of the Human Development Index”, *Social Observer* 27 (1): 1-32.

Zambrano, E. (2017); “The ‘Troubling Tradeoffs’ Paradox and a Resolution”, *Review of Income and Wealth* 63 (3): 520-541.