

# Variability-aware Design Method for a Constant Inversion Level Bias Current Generator

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**Abstract**—A model for estimating the dispersion in the output of a MOS-only, constant inversion level, current reference is presented. Based on such model, a design method is introduced that allows to minimize dispersion for a given layout complexity. The method was applied to five different designs producing currents from 2.4 nA to 4.2 nA (@ 300 K) and simulated dispersions ranging from 1.3% to 1.5% ( $1 \sigma$ ) while the model predicted dispersions from 1.2% to 1.4%.

**Keywords**—Current References, MOS transistors, mismatch, ACM, temperature effects.

## I. INTRODUCTION

Current references are used in every kind of analog circuit for biasing purposes. Although constant or PTAT current references are mostly used, some circuits benefit from current references working at a constant inversion level as shown in several recent publications [1]–[8]. A MOS-only simple circuit [9]–[11] generates a constant inversion level current. Although insights on the design of these current reference circuits have been previously published [11]–[13], the dispersion of the bias current due to mismatch among the involved transistors has received so far little consideration. Moreover, in the past, the authors have come across designs with both excellent and very poor dispersion. Thus, there is a need to introduce a dispersion model in the design methodology for these circuits, which is the main purpose of this paper.

Section II reviews known results about the constant inversion level current generator. Section III introduces a mismatch based variability model for the current while Section IV presents a design method based on that model. Section V validates the design method by comparing some designs with simulation results. Finally, we draw some conclusions in section VI.

## II. CIRCUIT ANALYSIS

This section follows [11] in describing the constant inversion level current bias generator while defining notation. The circuit analysis is based on the ACM model [14], [15] for the long channel MOSFET, accurate through all inversion levels. In ACM, the drain current of a long channel MOSFET is expressed as:

$$I_D = S I_{SQ} (i_f - i_r), \quad (1)$$

where  $i_f$  and  $i_r$  are the forward and reverse inversion coefficients and  $S = W/L$ ,  $W$  and  $L$  being respectively the effective width and length of the MOSFET. The sheet

normalization current  $I_{SQ}$  is a technology parameter proportional to  $\mu C'_{ox}$ . The inversion coefficients are related to the normalized surface charge densities at source and drain  $q_{S(D)} = -Q'_{S(D)}/nC'_{ox}U_T$ , by [14]:

$$i_{f(r)} = q_{S(D)}(q_{S(D)} + 2) \iff q_{S(D)} = \sqrt{1 + i_{f(r)}} - 1. \quad (2)$$

The circuit in Fig. 1 is a constant inversion level bias current generator. It was first proposed in [9], [10], and later extended in [12]. It was fully modelled in [11] where it is shown that it produces a particular temperature dependence on the bias current which yields a constant inversion coefficient ( $i_f$ ) independent of temperature. Moreover, it depends only on ratios of the aspect ratios  $S_1..S_4$  of MOSFETs  $M_1..M_4$  (Fig. 1), as well as on the copy ratios of the mirrors. For simplicity and lowest consumption, these are considered to be 1:1 mirrors. It must be noted that  $M_1..M_3$  are saturated, while  $M_4$  is not. From Eq. 1, the generated bias current depends on  $I_{SQ}$ ,  $S$  and  $i_f$  of any of the saturated MOSFETs, e.g:  $I = I_{sq}i_{f2}S_2$ . Among the many possible sets of ratios involving  $S_1..S_4$ , the following have been chosen for circuit analysis:

$$B = \frac{S_2}{S_1}, \quad M = \frac{S_2}{S_3}, \quad D = \frac{S_4}{S_4 - S_3}. \quad (3)$$

As already noted [11], choosing  $B$ ,  $D$  and  $M$  determines the inversion levels of  $M_1..M_4$ :  $i_{f1}$ ,  $i_{f2}$ ,  $i_{f3} = i_{f4}$ ,  $i_{r4}$ . Their value must be computed numerically as shown in [11]. Usually, for ultra low power applications,  $S_1 > 1$ ,  $S_2 > 1$ ,  $S_3 < 1$  and  $S_4 < 1$ . Also, in order to minimize mismatch effects the transistors are designed as parallel ( $M_1, M_2$ ) or series ( $M_3, M_4$ ) associations of a unit MOSFET  $M_u$  with aspect ratio  $S_u = W_u/L_u$ . Thus, we define  $N_1, N_2, N_3, N_4$  such that  $S_1 = N_1S_u$ ,  $S_2 = N_2S_u$ ,  $S_3 = S_u/N_3$ ,  $S_4 = S_u/N_4$ . Therefore,  $N_2, N_3, N_4$  are obtained from  $N_1, B, D, M$  as:

$$N_2 = N_1B, \quad N_3 = \frac{M}{N_2}, \quad N_4 = N_3 \frac{D-1}{D}. \quad (4)$$

## III. MODELLING THE VARIABILITY OF THE BIAS GENERATOR

Mismatch in both the multiple output current mirror and among the active transistors  $M_1..M_4$  affect the temperature independent inversion coefficients ( $i_{fj}$ ). The absolute dispersion in the threshold voltage  $V_T$  and the relative dispersion in  $I_{SQ}$ , which is that of  $\mu C'_{ox}$ , were modelled following [16].

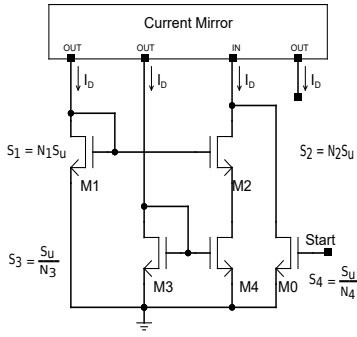


Fig. 1: MOS-only constant inversion level bias generator

We also consider the effect of channel width variation through  $A_{CWV}$  [17]. For each  $M_j$ ,  $j = 1, 2, 3, 4$ :

$$\delta V_{Tj}^2 = \frac{A_{VT}^2}{2W_u L_u} \frac{1}{N_j}; \quad \left( \frac{\delta \beta_j}{\beta_j} \right)^2 = \frac{A_\beta^2}{2W_u L_u} \frac{1}{N_j} + \frac{A_{CWV}^2}{2W_u^2 L_u} \frac{1}{N_j}, \quad (5)$$

where  $A_{VT}$  [ $V\mu m$ ],  $A_\beta$  [ $\mu m$ ] and  $A_{CWV}$  [ $\mu m^{3/2}$ ] are technology parameters and the dispersions are expressed relative to a mean transistor [18].

Fig. 2 shows how each transistor contributes to the dispersion in  $I_2$ , the branch current through  $M_2$  and  $M_4$ , by considering dispersion as a small DC signal [19]. Due to the self biased topology, the signal diagram in Fig. 2 has the shape of a closed loop.

The rest of this paper focuses on the dispersion introduced by  $M_1..M_4$ . The multiple output current mirror can be designed separately with proven techniques.

The dispersion in  $I_2$  will be computed with the help of sensitivity coefficients,  $S_{VTj}$  and  $S_{\beta_j}$  defined as:

$$S_{VTj} = \frac{\partial (\delta I_2 / I_2)}{\partial V_{Tj}}, \quad S_{\beta_j} = \frac{\partial (\delta I_2 / I_2)}{\partial (\delta \beta_j / \beta_j)}. \quad (6)$$

Expressions for  $S_{VTj}$  and  $S_{\beta_j}$  are shown in Appendix A. The sensitivities are determined from the transfer function corresponding to the diagram in Fig. 2. They can be expressed in terms of  $g_m / I_D$  and  $g_{md} / I_D$  ratios. As these ratios are functions of  $i_f$  and  $i_r$  which are in turn determined by B, D and M [11], then the sensitivities depend only on these geometrical ratios, the subthreshold slope ( $n$ ) and  $U_T$ .

Once the sensitivity coefficients are known, the relative dispersion in  $I_2$  can be expressed as:

$$\sigma_{\frac{\delta I}{I}}^2 = \sum_{j=1}^4 \left( \delta V_{Tj}^2 \cdot S_{VTj}^2 + \left( \frac{\delta \beta_j}{\beta_j} \right)^2 \cdot S_{\beta_j}^2 \right) = \sum_{j=1}^4 \frac{f_{Mj}^2}{W_u L_u} \frac{1}{N_j}, \quad (7)$$

where, considering Eq. 5,  $f_{Mj}$  can be expressed as:

$$f_{Mj}^2 = \frac{A_{VT}^2 S_{VTj}^2 + A_\beta^2 S_{\beta_j}^2}{2} + \frac{A_{CWV}^2}{2W_u} S_{\beta_j}^2, \quad j = 1, 2, 3, 4. \quad (8)$$

From Eq. 7, and expressing the total area of each transistor in terms of B, D, M and  $N_1$  (introduced in Section II), we define  $\mathcal{G} \equiv \sigma_{\frac{\delta I}{I}}^2 W_u L_u$ . Thus,

$$\mathcal{G} = \frac{f_{M1}^2}{N_1} + \frac{f_{M2}^2}{N_1 B} + \frac{f_{M3}^2}{M} N_1 B + \frac{f_{M4}^2}{M} N_1 B \frac{D}{D-1}, \quad (9)$$

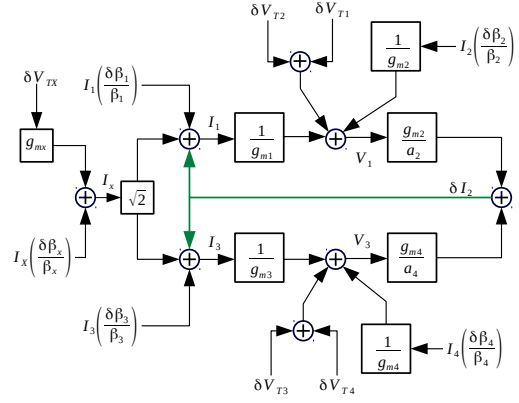


Fig. 2: Closed loop diagram for the constant inversion level bias current generator, showing dispersion in  $I_2$  as a function of small signal and mismatch parameters.

which is the squared relative dispersion multiplied by the gate area of a unit transistor ( $A_u = L_u W_u$ ).  $\mathcal{G}$  represents dispersion normalized to the unit transistor gate area and it does not depend on  $W_u$  or  $L_u$ . This is a very strong result and the basis for the design method described in Sec. IV.

We also define the normalized area:

$$\mathcal{A} = N_1(1 + B) + \frac{M}{BN_1} \left( 1 + \frac{D-1}{D} \right), \quad (10)$$

which is the total number of unit transistors in  $M_1..M_4$ . When  $\mathcal{A}$  is multiplied by  $A_u$  it becomes the total gate area ( $A_T$ ) for  $M_1..M_4$ .

#### IV. CURRENT SOURCE DESIGN

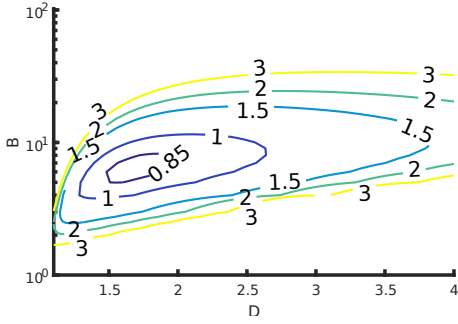
Combining Eq. 9 and Eq. 10, it is possible to express the square of relative dispersion times the total area as:

$$\sigma_{\frac{\delta I}{I}}^2 A_T = \mathcal{G} \mathcal{A}(B, D, M, N_1). \quad (11)$$

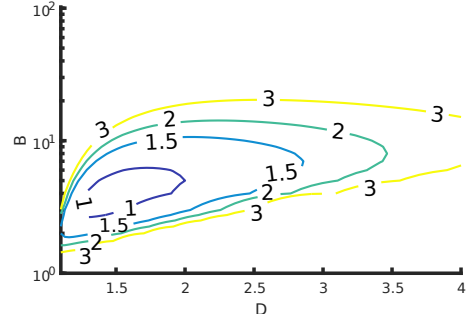
This expression is the dispersion introduced by  $M_1..M_4$  normalized to the total gate area, which is still independent of  $W_u$  and  $L_u$ . A lower value of  $\mathcal{G} \mathcal{A}$  corresponds to a lower dispersion for a given area.

The proposed design method attempts to minimize  $\mathcal{G} \mathcal{A}$  while keeping a reasonable number of unit transistors ( $\mathcal{A}$ ) for a feasible layout. The four variable minimization problem is split in two steps for dimension reduction. For each point in the plane  $(M, N_1)$ , the minimum of  $\sqrt{\mathcal{G} \mathcal{A}}(B, D)|_{M, N_1}$  is found. These minima define a surface in the  $(M, N_1)$  plane. Fig. 3 shows  $\sqrt{\mathcal{G} \mathcal{A}}$  contours in  $(B, D)$  planes for different sets of  $M$  and  $N_1$ . Fig. 4 shows contours in the  $(M, N_1)$  plane together with plots of the constant normalized area  $\mathcal{A}$ . It is clear that increasing the normalized area (layout complexity) decreases the dispersion. This plot helps the designer to manage the trade-off between dispersion and layout complexity.

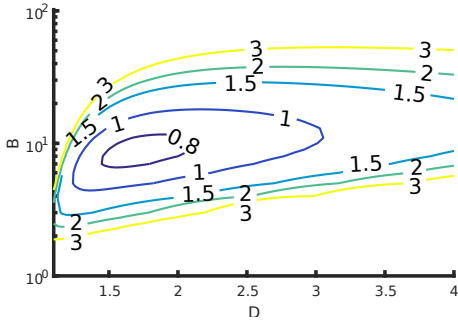
The plots depend only on technology mismatch parameters. So, the plot holds for any current source to be designed in a given technology thanks to the normalized approach, which is an outstanding result. The designer can start defining all



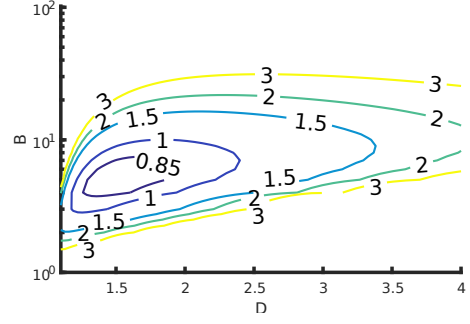
(a)  $M = 500, N_1 = 4$ .



(b)  $M = 500, N_1 = 8$ .



(c)  $M = 1000, N_1 = 4$ .



(d)  $M = 1000, N_1 = 8$ .

Fig. 3: Dispersion multiplied by the root of total area ( $\sqrt{\mathcal{G}\mathcal{A}}$ ) for different sets of  $M$  and  $N_1$ .

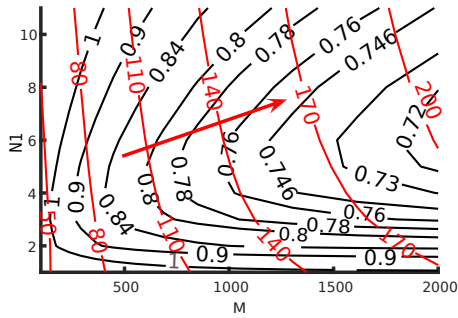


Fig. 4: Contours for  $\mathcal{A}$  in red and minimal  $\sqrt{\mathcal{G}\mathcal{A}}$  in black.

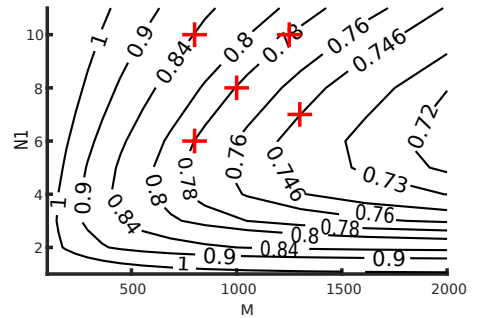


Fig. 5: Selected design points overlaid on the plot in Fig. 4.

the geometries without the need of choosing  $W_u$  and  $L_u$  until later on. The design method determines the relative size of transistors for optimum dispersion given a layout complexity. The size of the unit transistor controls the current value through  $\frac{W_u}{L_u}$  and the dispersion through  $W_u L_u$ .

## V. MODEL RESULTS VS SIMULATION

The design method was validated through five different designs on a 130 nm CMOS technology. Fig. 5 shows them as points overlaid on the contours in Fig. 4. In order to compare them to simulation results it was necessary to select  $W_u$  and  $L_u$ , arbitrarily chosen as  $W_u = 1 \mu\text{m}$  and  $L_u = 50 \mu\text{m}$ . This also determines the actual bias current. Table I presents the

results which show approximately 12 % of relative difference between the model used throughout the design and the simulation results. This is close enough and validates the design method.

## VI. CONCLUSIONS

A constant inversion level bias current source was presented along with a thorough model of mismatch effects. The design problem was addressed and a design method was proposed that considers the trade-off between dispersion and layout complexity. The method was validated by comparing the predicted dispersion with simulation results which were just

Design Dimensions				I [nA]	$\frac{\sigma(I)}{I}$ (%)		
N <sub>1</sub>	N <sub>2</sub>	N <sub>3</sub>	N <sub>4</sub>		Model	Simulation	Relative Difference [%]
10	40	20	6	2.62	1.36	1.54	11.9
7	43	30	11	4.17	1.11	1.24	10.9
8	42	24	8	2.49	1.22	1.36	10.5
10	48	26	8	2.43	1.15	1.29	11.0
6	35	23	8	2.76	1.30	1.44	9.9

TABLE I: Model results versus simulation results for dispersion.

12 % off. The obtained results define a design methodology for this kind of current source.

#### APPENDIX A SENSITIVITY COEFFICIENTS

The sensitivity coefficients are found by solving the transfer functions determined by Fig. 2. The inputs are the  $\beta$  and  $V_T$  mismatch parameters in each transistor and the output is the dispersion in  $I_2$ .

Transistor	Sensitivities for $\beta$	Sensitivities for $V_T$
$M_1$	$S_{\beta_1} = \frac{\alpha_1}{1-A}$	$S_{V_{T1}} = \frac{1/\alpha_2}{1-A} \frac{g_{m2}}{I_2}$
$M_2$	$S_{\beta_2} = \frac{1/\alpha_2}{1-A}$	$S_{V_{T2}} = \frac{1/\alpha_2}{1-A} \frac{g_{m2}}{I_2}$
$M_3$	$S_{\beta_3} = \frac{\alpha_2}{1-A}$	$S_{V_{T3}} = \frac{g_{m4}/\alpha_4}{1-A} \frac{1}{I_2}$
$M_4$	$S_{\beta_4} = \frac{1/\alpha_4}{1-A}$	$S_{V_{T4}} = \frac{g_{m4}/\alpha_4}{1-A} \frac{1}{I_2}$

TABLE II: Sensitivity coefficients for each transistor.

Table II shows all sensitivities in terms of the expressions defined in Eq. 12 through Eq. 18. They depend on the geometrical relations B, D, M (Eq. 3) and the normalized charge densities  $q_{S1}$ ,  $q_{S2}$ ,  $q_{S4}$ ,  $q_{D4}$ . The latter are related to the inversion coefficients through Eq. 2 which are in turn defined by B, D, M [11].

$$A = \alpha_1 + \alpha_2 \quad (12)$$

$$\alpha_1 = \left( \frac{1}{g_{m1}} \cdot \frac{g_{m2}}{a_2} \right) = B \frac{q_{S2}}{q_{S1}} \frac{1}{2a_2} \quad (13)$$

$$\alpha_2 = \left( \frac{1}{g_{m3}} \cdot \frac{g_{m4}}{a_4} \right) = \frac{S_4}{S_3} \frac{1}{2a_4} \left( 1 - \frac{q_{D4}}{q_{S4}} \right), \quad \frac{S_4}{S_3} = \frac{D}{D-1} \quad (14)$$

$$a_4 \approx 1 + \frac{g_{m4}}{ng_{m2}} = 1 + \frac{S_4}{S_2} \frac{q_{D4}}{q_{S2}}, \quad \frac{S_4}{S_2} = \frac{D}{D-1} \frac{1}{M} \quad (15)$$

$$a_2 \approx 1 + \frac{ng_{m2}}{g_{m4}} = 1 + \frac{S_2}{S_4} \frac{q_{S2}}{q_{D4}} \quad (16)$$

$$\frac{g_{m2}}{I_2} = \frac{2}{nU_T(q_{S2} + 2)} \quad (17)$$

$$\frac{g_{m4}}{I_2} = \frac{S_4}{S_2} \frac{2}{nU_T} \frac{q_{S4}}{q_{S2}} \frac{1 - \frac{q_{D4}}{q_{S4}}}{q_{S2} + 2} \quad (18)$$

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