A bargaining model with uncertainty and varying outside opportunities

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Abstract

The main purpose of this paper is to propose an alternative way of explaining – within the bargaining theory framework – the stylised fact of flat wages and employment bearing all the adjustment to shocks. Standard models predict this behaviour under the assumptions of a constant elasticity production function and a reservation wage independent of shocks. Once the latter is removed, however, the result holds no more. The proposed two-stage model, in which the second stage involves negotiations over employment after the state of nature is revealed, would allow to recover it as a consequence of the uncertainty agents face when bargaining over wages in the first stage. The model nests other formulations and allows for Pareto efficient and inefficient outcomes, depending on union power, agents' beliefs and the observed state of nature.

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Introduction

When doing empirical research, the choice of the theoretical model is a quite important matter, as the use of a non-adequate framework may distort the results obtained or at least confound the conclusions at which one arrives. Multi-stage models appear in this context as a good strategy in order not to impose some unnecessary restrictions on the way negotiations are carried out. Another interesting issue relates to the fact that uncertainty is commonly observed in all situations in which a conflict between negotiating parties may arise. As a consequence, it seems natural to include it in a model of bargaining and ask which would be the theoretically expected outcomes in a two-stage bargaining model. One of the predictions obtained in models under certainty is that wages would be sticky while employment would fluctuate when there are fluctuations in demand, which is also an observed stylised fact. However, not all labour markets have worked in this way, so that proposing a model in which the result would be observed under certain circumstances shows up as appealing.

The aim of this paper is to propose an alternative explanation of wages and employment fluctuations within the framework of the bargaining process between unions and management². In many economies, it is often observed that employment bears most of the adjustment to shocks while wages are relatively rigid. The result is justified in terms of the insider-outsider approach, as due to the bargaining power unionised workers have. However, its validity relies on both the elasticity of demand and the reservation wage being constant. The issue addressed in the paper is how to recover the observed stylised fact once the latter assumption is removed, although the former is kept. The proposed answer is linked to agents facing uncertainty on the future evolution of the economy when bargaining over the wage. Hence, expectations would be in the root of different possible patterns characterising the evolution of wages and employment.

¹ I gratefully acknowledge comments from John Driffill, Alvaro Forteza, Robin Naylor, Andrew Oswald and Marcel Vaillant. The usual disclaimer holds.

² The other two standard approaches are the efficiency wages and the implicit contracts theories.

The bargaining models proposed in the literature postulate that unions maximise the utility of their members. The individual utility function depends on the bargained wage for those that remain employed and on the alternative income that the worker would receive if fired for those that lose their job (what the individual can earn working at another firm and/or the unemployment benefit). This alternative wage is assumed fixed or given. The assumption, although simplifying, is not very reliable if the economy is subject to external shocks. If shocks have an impact on the whole economy, no matter how centralised/decentralised bargaining is, the hypothesis is not sustainable. The probability of finding a job will change with the observed state of nature and so will the expected pay. If negotiations are at the industry level, it is still an implausible assumption, as even sectoral shocks would have externalised level, say the firm, and the shock is specific to one or few sectors, might the hypothesis of fixed reservation wage be reliable.

The consequences of the assumption are not negligible when analysing the reaction of bargained wages and employment to shocks. Once it is removed, the widely studied pattern of constant wage and employment that bears all the adjustment to shocks (Blanchard and Fischer, 1993) no longer holds. Instead, wages would vary with shocks, as they are a mark-up over the now variable reservation wage while employment would be more or less flat.

The role of uncertainty has been discussed mainly in the framework of the implicit contracts theory. Stable wages along the economic cycle are the result of workers being risk averse, so that they would prefer lower wages in good states of nature but higher levels of pay in downturns. However, even if the need of insurance were absent from the union's objective function, there could be other explanations for the inclusion of uncertainty in the bargaining process. Assuming negotiations over wages have a fixed deadline and that unions care about employment, the possibility of a changing economic environment during the period in which the wage is fixed would induce uncertainty. Unless there is an explicit agreement on how will employment vary, the firm may unilaterally change its level when a shock takes place. Hence, agents would want to make use of any available information on the future state of nature when setting the wage. In order to do so, they must assign a probability to the occurrence of 'good' and 'bad' states, thus allowing their beliefs to alter the outcome of bargaining. Negotiations would not be contingent on future economic performance or employment but the probability distribution of shocks would play a role in wage bargaining.

The above mechanism could be thought of in the framework of multi-level bargaining. Wage negotiations would be done first, taking into account possible effects on employment. In a second stage, after the shock is observed, bargaining could take place over employment, possibly at a different level (such as the firm).

How would expectations influence the outcome of bargaining under the above hypotheses? When agents are optimistic, the expected rents to be shared are large and the probability of losing a job is small. Further, outside opportunities should be large and/or the income one could get should be high. Then it is quite possible that the union pushes up the wage more than if it had to worry about its members keeping their jobs. On the contrary, when a 'bad' shock is likely, moderate wage demands should be expected. Hence, the magnitude of wage variations will depend on the distribution of shocks, although not on their realisation.

Little work has been done along this line but there are some notable exceptions. Oswald (1982) proved that including uncertainty in a monopoly union model does not change its qualitative predictions, no matter which is the variable in the objective function agents are uncertain about. However, changes in the degree of uncertainty do modify the optimum wage. Another paper is Naish's (1988) in which the union is uncertain as to what price level will prevail when setting the wage in a monopoly union model. It is shown that the choice of the wage is linked to the shape of the utility function of the union and to the distribution of the price level. Hence, the degree of confidence on the price forecast would play a role in determining wages.

Before developing the model itself, it is worth to summarise two particular points that have been extensively discussed in the literature and that will be used in this paper. Firstly, it is quite generally agreed that the specification of the utility function maximised by the union with a kink at full employment of members, as proposed by Carruth and Oswald (1987) is adequate³. The assumption of flat indifference curves when all members are employed would explain why some unions are not concerned about employment at some point. Further, it emphasises the importance of the determination of membership for modelling purposes while it is a suitable benchmark to understand why sometimes, but not always, the outcome of bargaining is on the labour demand curve.

Secondly, an extensively discussed issue is that of the efficiency of the wage-employment optimum pair⁴. While early models assuming the firm has the right-to-manage imply that the outcome is not Pareto-optimal, efficient contracts models impose efficiency but on the assumption that employment and wages are negotiated simultaneously. However, the empirical evidence renders the inclusion of employment in the bargaining agenda quite implausible, at least when bargaining is not fully decentralised. Further, if it were included it is rarely considered as an issue to be negotiated over at the same time or with the same weight than over wages. Some authors have proposed different ways of avoiding the theoretical dilemma of inefficiency. The specification of bargaining as a repeated game (Espinoza and Rhee, 1989) is one alternative. Efficiency, according to this formulation, would depend on the discount rate agents use to calculate the present value of their expected utility. Another possibility is to postulate that although there is no explicit bargaining over employment, unions do negotiate indirectly by establishing manning practices (Johnson, 1990). Other authors have proposed a similar hypothesis, based on the idea that being bargaining a repeated interaction, it could be implicitly agreed that the outcome has to be efficient and that to prevent agents to cheat it is just needed that the punishment for future negotiations is hard enough (Schultz, 1994). A final option is that of multistage models (Manning, 1987), in which it is assumed that wages are set in a first stage with a given bargaining power while in a second stage employment is determined. This could be thought of as a relatively simple proposal, nesting other formulations without restricting their outcome to be efficient or inefficient. The issue is thus left as a hypothesis to be verified empirically. However, sometimes their analytical complexity renders them

³ See textbooks as Booth (1995) or Pencavel (1991).

⁴ The topic has been included in textbooks as the ones cited in the previous footnote. See also Layard, Nickell and Jackman (1991).

intractable at a general level, so that very simplifying assumptions on the technology and/or the union's utility function have to be made to draw conclusions.

In what follows a model incorporating the elements discussed above will be proposed. Its main implication is that the stylised fact of flat wages and fluctuating employment can be recovered, once the assumption of constant reservation wage is removed, by incorporating uncertainty in bargaining. It will be shown that the pattern is not the only possible one to observe and that it is not the consequence of union power itself, as in Manning (1987), but of the impossibility of fully anticipate the state of nature that will prevail.

The model is developed incorporating uncertainty at the outset. However, the analogous result in the case of fully anticipated shocks (no uncertainty) is also derived. After describing the assumptions involved and the outcomes of bargaining obtained, the implications on wage rigidity are discussed. Finally, the efficiency of the outcomes and how the model nests other formulations are presented.

A model with uncertainty and varying outside opportunities

Bargaining is assumed to take place between one union and one employer or association of employers. The union represents a given percentage of the total workforce. Negotiations are carried out in two stages. In the first stage wages are determined, while in the second the employment level is set. The structure of bargaining is such that at each stage a sequence of offers and counteroffers occurs until an agreement is reached depending on the relative bargaining powers of the parties, so that the generalised Nash bargaining solution applies (Binmore, Rubinstein and Wolinsky, 1986). Union and management maximise utility functions defined over wages and employment. It is assumed that the objective function of the union has a 'kink' point at employment equal to membership as proposed by Carruth and Oswald (1987). Management maximises profits that do not include adjustment costs of employment and the production function is concave. A demand shock to the economy takes place before negotiations over employment. It is assumed that the shock can be of one of two types, 'good' (θ_g) or 'bad' (θ_b). The shock alters the revenue product and the reservation wage. Prices are normalised to unity.

The optimisation problem in the first stage is solved conditionally on its effects over the second stage of bargaining. The problem can be expressed in the following way:

Stage 1: Max
$$\Phi_1 = E_{\theta} [(\Gamma - \Gamma_0)]^{\alpha} E_{\theta} [(\Pi - \Pi_0)]^{1-\alpha}$$

w
Stage 2: Max $\Phi_2 = (\Gamma - \Gamma_0)^{\beta} (\Pi - \Pi_0)^{1-\beta}$
L

Where E_{θ} is the expected value operator; θ is the shock; $\Gamma(w,L,M,r)$ is the union's utility function; *w* is the wage; *L* is the employment level; $r = r(\theta)$ is the reservation wage; *M* is membership; $\Gamma_0 = Mu(r)$ is the fall-back position of the union; $u(\cdot)$ is the utility function of the individual member; $\Pi(w,L,\theta)$ is the value of profits for the firm and Π_0 its fall-back position that will be assumed to be zero (no production and no operating costs); while α and β are the bargaining powers of the union in the first and second stage of negotiations, respectively. Two cases can be distinguished. First, when the shock is fully anticipated by the parties, so that the expected utilities are equal to their actual values⁵. Second, when it is common knowledge to the parties that the shock will occur but there is no full anticipation of its value. The assumption to be used is that both parties assign the same probability to the two possible realisations - 'good' or 'bad'. The probability of observing a 'good' shock is *p* and, as there are only two states of nature, agents assign a probability of (1-p) to the event of a 'bad' shock. In this case the optimisation problem in the first stage takes place in an *a priori* unknown state of nature. Note that the case in which the shock is fully anticipated by the parties is the result of *p* being equal to 1 or to 0.

The specification of the utility functions is such that:

$$\begin{split} &\Gamma_{i}(w, L_{i}, M, r_{i}) = L_{i} \left[u(w) - u(r_{i}) \right] + M u(r_{i}) & \text{if } L_{i} < M \\ &\Gamma_{i}(w, L_{i}, M, r_{i}) = M u(w) & \text{if } L_{i} \ge M \\ &\Pi_{i}(w, L_{i}, \theta_{i}) = \theta_{i} f(L_{i}) - w L_{i} & ''L_{i} \end{split}$$

With: $\partial u / \partial w = u'_w \ge 0$ $r_i = r(\theta_i)$ $\partial r / \partial \theta_i = r_i' \ge 0$ and i = g, b

⁵ This is equivalent to postulating that the shock takes place before bargaining over the wage level.

The utility function of an employed union member is u(w) while $u(r_i)$ is that of an unemployed union member. The reservation wage - r - is assumed to be a linear non-decreasing function of the shock. If the shock is to the economy, what is being postulated is that good states of nature would increase -or keep constant- the expected income to be obtained in other activities. Sectoral shocks might have the opposite effect, though.

The expected utility functions are:

$$E_{\theta}(\Gamma - \Gamma_{0}) = pL_{g}[u(w) - u(r_{g})] + (1-p)L_{b}[u(w) - u(r_{b})] \quad \text{if} \quad L_{i} < M$$

$$E_{\theta}(\Gamma - \Gamma_{0}) = pM[u(w) - u(r_{g})] + (1-p)M[u(w) - u(r_{b})] \quad \text{if} \quad L_{i} \ge M$$

$$E_{\theta}(\Pi - \Pi_{0}) = p[\theta_{g}f(L_{g}) - wL_{g}] + (1-p)[\theta_{b}f(L_{b}) - wL_{b}] \quad ''L_{i}$$

$$r_{g} = r(\theta_{g}) \quad r_{b} = r(\theta_{b})$$

The outcome of the two-stage bargaining is obtained by backwards induction: first the level of employment - $L_i^*(w^*, \beta, \theta_i)$ for i = g, b - is obtained in the second stage. The resulting expression is substituted into the utility functions in order to solve for the optimum wage level - $w^*(\alpha, p, \theta_i, r_i)$ - in the first stage.

The first order condition (f.o.c.) for the second stage problem - generally known as the rent division curve (RDC) - is:

$$\beta \Pi / (\Gamma - \Gamma_0) = - (1 - \beta) \Pi_L / (\Gamma - \Gamma_0)_L$$

The conditional solution for the first stage maximisation problem is given by:

$$\beta(1-\alpha)E_{\theta}[\Pi_w]/E_{\theta}[(\Gamma-\Gamma_0)_w] = \alpha(1-\beta)E_{\theta}[(\Gamma-\Gamma_0)\Pi_L/(\Gamma-\Gamma_0)_L]/E_{\theta}(\Gamma-\Gamma_0)$$

The second order condition for a maximum in the second stage optimisation problem holds. That for the first stage problem holds for risk-neutral and risk-averse players. If workers are risk loving, on the other hand, some additional restrictions ought to be satisfied. Given the definition of the utility function of the union, two cases have to be distinguished. Firstly, that in which shocks and bargaining are such that the resulting changes in employment maintain its level below membership $(L_i^* < M)$ no matter the shock is 'good' or 'bad'. Secondly, the case in which the optimum level of employment bargained is equal to or greater than the number of members $(L_i^* \ge M)$.

Case I: $L^* < M \quad \forall \theta$

The f.o.c. for the second and first stage optimisation problems can be respectively rewritten given the assumed utility functions as:

$$w^* = \beta \theta_i f(L_i)/L_i + (1-\beta)\theta_i f_L(L_i) \qquad i = g,b \qquad (1)$$

$$w^{*} = \frac{\alpha(1-\beta)E_{\theta}(\theta f_{\underline{L}}L)\{E_{\theta}[Lu_{w}] + E_{\theta}[(u(w) - u(r))L_{w}]\}}{\alpha(1-\beta)E_{\theta}(L)\{E_{\theta}[Lu_{w}] + E_{\theta}[(u(w) - u(r))L_{w}]\} - \beta(1-\alpha)E_{\theta}\{[u(w) - u(r)]L\}E_{\theta}(L_{w})}$$

$$-\frac{\beta(1-\alpha)E_{\theta}\{[\mathbf{u}(w)-\mathbf{u}(r)]L\}E_{\theta}[\theta L_{w}\mathbf{f}_{L}-L]}{\alpha(1-\beta)E_{\theta}(L)\{E_{\theta}[L\mathbf{u}_{w}]+E_{\theta}[(\mathbf{u}(w)-\mathbf{u}(r)]L_{w}]\}-\beta(1-\alpha)E_{\theta}\{[\mathbf{u}(w)-\mathbf{u}(r)]L\}E_{\theta}(L_{w})}$$
(2)

Case II: $L^* \xrightarrow{s} M \quad \forall \theta$

If
$$\Gamma = M\mathbf{u}(w) \implies \Gamma - \Gamma_0 = M[\mathbf{u}(w) - \mathbf{u}(r)]$$
 and $\partial(\Gamma - \Gamma_0)/\partial L = 0$

Hence the f.o.c. for the second stage problem is just $\Pi_L = 0$ and β has no influence on the employment level. Being all union members employed, unions should not care about the employment level. The results in this case, following the same steps as before, are:

$$w^* = \theta_i f_L(L_i) \qquad i = g, b \qquad (3)$$
$$w^* = E_{\theta}[\theta f(L)] / E_{\theta}(L) - [(1-\alpha)/\alpha] E_{\theta}[u(w) - u(r)] / u_w \qquad (4)$$

Comparing equations (1) and (3) it is seen that for a given wage level, rule I determines a higher level of employment than that stemming from the use of rule II. The result is the

expected one if there are unemployed members and unions care about and bargain over employment.

It could be the case that the above two rules do not cover all possible situations. One could imagine that the shock is such that $L^* > M$ if rule I is used and $L^* < M$ if rule II is used. If this were the case, it would be sensible to assume that the union would bargain conditional on full employment of its members ($L^* = M$), so that rule II should be used. This yields the following optima:

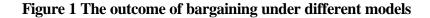
Case III:
$$L^* = M \quad \forall \theta$$

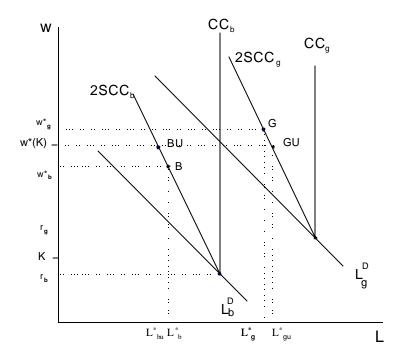
 $L_i^* = M$ (5)
 $w^* = E_{\theta}(\theta) f(M) / M - (1 - \alpha) / \alpha E_{\theta}(r)$ (6)

Equations (1) to (6) show that bargained wages depend on the distribution of shocks, while the negotiated employment level depends also on the realisation of the shock, except for *Case III*, in which employment is fixed at membership. Thus, wages would be rigid and employment would bear most of the adjustment to shocks. But this is so due to the shock not being fully anticipated and not because of a fixed reservation wage. As agents are uncertain as to what state of nature they will face, wage claims will be somewhere in between the levels that would be accepted if the 'bad'/'good' character of the shock were known. The point is illustrated in Figure 1 comparing the results with those stemming from both an efficient contract and a two-stage bargaining model with no uncertainty when employment is below membership. The additional assumptions used are that union members are risk-neutral; the utility function of the union is utilitarian; and the production function of the firm is quasi Cobb-Douglas. The expressions for i = g, b are thus:

$$u(w_{i}) = w_{i} \quad ; \quad \Gamma_{i}(w_{i}, L_{i}, M, r_{i}) - \Gamma_{0} = L_{i}(w_{i} - r_{i}) \quad ; \quad f(L_{i}) = L_{i}^{\gamma} \quad ; \quad \Pi_{i}(w_{i}, L_{i}, \theta_{i}) = \theta_{i}L_{i}^{\gamma} - w_{i}L_{i}$$
$$L_{i}^{*} = \left\{ \left[\left(\alpha + \gamma(1 - \alpha) \right) / \gamma \left(\beta + \gamma(1 - \beta) \right) \right] \left[E_{\theta}(Lr) / \theta_{i}E_{\theta}(L) \right] \right\}^{1/(\gamma - 1)}$$
$$w^{*} = \left\{ \left[\alpha + \gamma(1 - \alpha) \right] / \gamma \right\} \left[E_{\theta}(Lr) / E_{\theta}(L) \right]$$

With $E_{\theta}(r) = pr_{g} + (1-p)r_{b}$ and $E_{\theta}(Lr) = pL_{g}r_{g} + (1-p)L_{b}r_{b}$





K is the ratio $E_{\theta}(Lr)/E_{\theta}(L)$, so that $w^*(K)$ is the optimum wage under uncertainty in *Case I*. L_b^{D} , L_g^{D} are the labour demand curves after a 'bad' and a 'good' shock, respectively. CC_b, CC_g are the corresponding contract curves, while 2SCC_b, 2SCC_g are possible two-stage contract curves in the event of 'bad' and 'good' shocks, respectively. Points B and G are the employment-wage optimum pairs that would result after a 'bad'/good' shock with no uncertainty (p=0/p=1), while BU and GU would be the outcomes of a model with uncertainty (0). Results are unchanged if*Case II*is used instead. The 2SCC collapses

to the labour demand function and points BU and GU will be to the left of those here $drawn^{6}$.

Being the wage set at a value in between those expected under certainty, employment will vary more than it would if the shock were fully anticipated. The result is thus analogous to that obtained if assuming a fixed reservation wage, the reasons behind it being however very different.

How much would employment adjust to shocks? It would depend -given the above- on the distribution of shocks, that is, on the ability agents have in foreseeing the future state of nature. When the distributional variance is small, agents are quite certain they will face a 'good'/'bad' state of nature $(p \rightarrow 1/p \rightarrow 0)$. If their beliefs turn out to be 'correct', the effect on the employment level could not be stated *a priori*, as the positive/negative effect of the shock could be offset by the increase/decrease in the wage level. However, if the shock turns out to be of the opposite sign to that expected by agents (θ_b/θ_g) , all the adjustment would be born by the employment level. On the other hand, when uncertainty is at its maximum (p=1/2), employment will bear most of the adjustment no matter the sign of the observed shock.

The above can be summarised in the following proposition:

Proposition

In a two-stage bargaining model where shocks take place after negotiating wages, the elasticity of demand is constant and the reservation wage is a non-decreasing function of the state of nature, the extent up to which employment adjusts to shocks will be determined by agents' expectations, the realised state of nature and its effect on the reservation wage.

How will changes in the distribution of shocks and in the reservation wage affect the outcome of bargaining? Although it is not possible to derive unconditionally the effects of

⁶ The optimum wage according to rule II is always higher than that resulting from rule I under the assumption of mean independence of *L* and *r*: $w_{I}^{*} = \{ [\alpha + \gamma(1-\alpha)]/\gamma \} E_{\theta}(r) \leq [\gamma(1-\alpha)/(\gamma-\alpha)] E_{\theta}(r) = w_{II}^{*}$.

changes in every parameter, some results can be stated. Table 1 shows the sign of the relevant derivatives, their explicit expressions being included in the appendix for the example that is being considered.

Derivative	Case I	Case II	Case III
$\partial w^* / \partial r_i$	+	+	+
$\partial w^* / \partial p$	+	+	+
$\partial w * / \partial \theta_{g}$	+	+	+
$\partial w * / \partial \theta_b$?	+	+
$\partial L_i * / \partial r_i$	-	-	0
$\partial L_i * / \partial r_j$	-	-	0
$\partial L_i * / \partial p$	-	-	0
$\partial L_i * / \partial \theta_i$	- iff C ₁	- iff C ₁	0
$\partial L_{\rm g}*/\partial \theta_{\rm b}$	- iff C ₂	_	0
$\partial L_{\rm b}*/\partial \theta_{\rm g}$	-	-	0

 Table 1 Changes in the optimum wage and employment levels

Condition C₁ is: $(\partial w^* / \partial \theta_i)(\theta_i / w^*) > 1$

Condition C_2 is: $\partial w^* / \partial \theta_b \ge 0$

Being a mark-up over the reservation wage, the optimum wage bargained rises and the employment level falls whenever there is an exogenous increase in the alternative income. Further, changes in the value of 'good' and 'bad' shocks would in turn influence the value of the reservation wage, thus reinforcing their direct effect.

If there are unemployed members and agents become more optimistic (increases in p) or if the possible states of nature improve (increases in θ_g and/or θ_b), the optimum wage will be set at a higher level, although in the latter case this will depend on the relative magnitude of the alternative wage under both states of nature. Thus, a distribution of shocks with a bigger mean (more to the right) will generate increases in the wage bargained, no matter what the change in the distributional variance is⁷. Unfortunately, the effects of changes in the variance, for a fixed distributional mean, on the outcome cannot be derived analytically. Some preliminary simulations were carried out but the results obtained were not conclusive. While increases in the variance due to a rise in p from 0 to 0.5 with fixed ε have a positive effect on the optimum wage, when p decreases from 1 to 0.5 the result will depend on the value of the parameters defining the utility function of agents. Thus, further work needs to be done in this area.

The level of employment bargained at the second stage will depend also on the realisation of the shock. Given a 'good' state of nature, employment will fall with increases in the probability assigned by agents to 'good' shocks. A rise in the value of θ_g , on the other hand, may result in a decrease or an increase in employment, depending on the change of the optimum wage relative to that of the shock (more or less than proportional). The same results are derived for realisations of 'bad' sates of nature. However, a rise in θ_g implies a decrease in employment if the observed shock turns out to be 'bad', while a rise in θ_b generates a decline in employment when the actual shock is 'good' only if it raises the wage.

Whether there are unemployed members or not will not change the direction of the above variations but their magnitude. As employment is not an argument in the objective function of the union when all members have a job, the wage level set and its rate of change will be always higher in this case than otherwise.

The model do invite for further developments, especially those related to its dynamic aspects. A first possible way of introducing dynamics would be to assume shocks are specific stochastic processes. If a sequence of periods is considered and bargaining with the timing proposed is assumed to take place in each period, states of nature that are not time-homogenous would result in wages being time dependent. If the stochastic process has

⁷The mean value and the variance of the distribution are: $\theta_{\rm m} = p\theta_{\rm g} + (1-p)\theta_{\rm b}$; $V(\theta) = p(1-p)\varepsilon^2$ with $\varepsilon = (\theta_{\rm g} - \theta_{\rm b})$

'memory' (shocks are not independent) there is also scope for persistent effects on employment. The model can be expressed, oversimplifying, as:

$$w_t^* = a_0 + a_1 E_{t-1}(r_t)$$
 $L_t^* = b_0 + b_1 \theta_t - b_2 w_t^*$

If $E_{t-1}(\theta_t) = \theta_{m,t}$ and $E_{t-1}(r_t) = r_{m,t}$, then w^* would not be constant anymore when agents are assumed to take into account all the past relevant information. Wages are not responsive to the realisation of the shock, but as states of nature have a different distributional mean, employment adjustment might be smoother than otherwise. Moreover, if states were autocorrelated, the past history would influence directly the outcome:

$$\begin{aligned} \theta_{t} &= \lambda \theta_{t-1} + \varepsilon_{t} \quad E_{t-1}(\theta_{t}) = \lambda \theta_{t-1} \quad \text{and} \quad E_{t-1}(r_{t}) = \eta r_{t-1} \\ \Rightarrow & w_{t}^{*} = a_{0} + a_{1} \eta r_{t-1} \qquad L_{t}^{*} = b_{0} + b_{1} \lambda \theta_{t-1} - b_{2} w_{t}^{*} + b_{1} \varepsilon_{t} \\ \Rightarrow & b_{1} \lambda \theta_{t-1} = \lambda L_{t-1}^{*} - \lambda b_{0} + b_{2} \lambda w_{t-1}^{*} \implies L_{t}^{*} = b_{0}(1-\lambda) - b_{2} \lambda w_{t}^{*} + b_{2} \lambda w_{t-1}^{*} + \lambda L_{t-1}^{*} + b_{1} \varepsilon_{t} \end{aligned}$$

Further extensions could be analysed under different assumptions. If the distribution of shocks is not known but should be forecasted instead, the behaviour of the variables used to predict the parameters and/or past realisations of shocks would influence the outcome. A Bayesian approach could also be considered in a multi-period framework, so that agents would update their subjective beliefs using all the available information and thus generate dynamics.

Fully anticipated shocks

If there were no uncertainty and outside opportunities depended on the realisation of the shock, the set of outcomes (2), (4) and (6) would be the following:

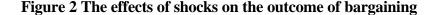
$$w_i^* = \theta_i f_L(L_i) + \underline{\beta(1-\alpha)L_i[u(w_i) - u(r_i)]} \qquad i = g,b$$

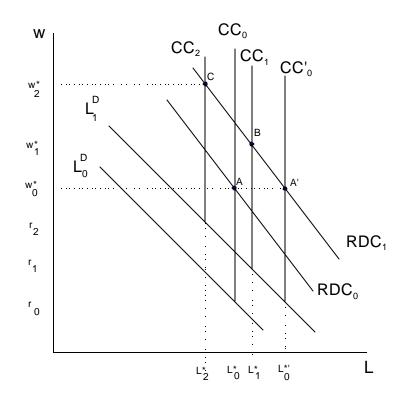
$$\alpha(1-\beta)u'(w_i)L_i + (\alpha-\beta)[u(w_i) - u(r_i)]L_w$$
(2)

$$w_i^* = \theta_i f(L_i) / (L_i) - [(1 - \alpha) / \alpha] [u(w_i) - u(r_i)] / u_w \quad i = g, b$$
(4)'

$$w_i^* = \alpha \theta_i f(M)/M + (1 - \alpha)r_i \qquad i = g, b \qquad (6)'$$

Equation (2)' is the two-stages contract curve (2SCC) as derived in Manning (1987). The bargaining outcomes in this case clearly show that wage stickiness will not be observed, being the magnitude of its adjustment dependant on how the alternative income is affected by shocks. Employment might then fluctuate less, as the positive/negative impact of shocks will be partially offset by the relative increase/decrease in the wage level. This could mean that, in the event of a 'good' shock for example, the employment level could remain unaltered or even decrease, depending on the ratio r_g/θ_g . The shock shifts the RDC curve and the 2SCC to the right. An increase in the reservation wage, on the other hand, does not move the RDC but causes the 2SCC to shift in. Hence, with shocks altering the reservation wage the final result on the 2SCC cannot be asserted *a priori*. The result is depicted in Figure 2 using the same simple example as before.





The figure is drawn assuming $\alpha = \beta$ (so that the two-stage model is equivalent to the efficient contracts formulation). Point A describes the initial wage-employment optimum pair, with reservation wage r_0 , labour demand curve L_0^{D} , rent division curve RDC₀ and contract curve CC₀. Point A' shows the optimum pair after the occurrence of a 'good' shock θ that shifts the labour demand curve to L_1^{D} and the rent division curve to RDC₁ but does not affect the reservation wage. Point B is the optimum pair that would result if the shock affects the reservation wage, so that its new level is r_1 . Point C is the optimum pair that would result if the shock on the reservation wage were such that its new level was r_2 .

The behaviour of wages and employment in the different cases analysed - $L^* \xrightarrow{\circ} M$, $L^* < M$ is almost analogous to the one resulting from a 2-stages model with constant reservation wage, as in Manning (1987), that is, higher mark-up of w over r and smaller employment level for each wage when bargaining takes place according to the rule prevailing for $L^* \xrightarrow{\circ} M$ than when $L^* < M$. With constant r, wages will always be higher when there is full employment of members than when there are members unemployed. Although the result is what one would expect, it is often observed in some economies that although unemployment is high, wage increases do not slow down. The model proposed here would give an explanation that is quite plausible around the point at which $L^* = M$. 'Good' shocks when there are unemployed members might generate a higher wage level compared to the one got in a 'bad' state of nature with full employment of members, provided the difference between the reservation wages in both states is big enough:

 $w_b^*(L^{\mathcal{B}}M) \, \mathbf{f} \, w_g^*(L < M) \quad \text{iff} \quad 1 - r_b \, / r_g^{\mathcal{B}} \, \alpha^2 (1 - \gamma) / \gamma^2 \, (1 - \alpha)$

Higher wages with a lower employment level in 'good' states relative to the values observed in a 'bad' state could be possible, given the above, as a consequence of the bargaining process. The odds of observing such result increase with the reaction of the reservation wage to shocks. The above situation links the character of the shock with the preferences of the union in a way such that it can explain why unions seem to react differently when employment is to be increased than when it is to be lowered. In 'good' states wages will rise more than in 'bad' states, thus allowing that employment could even remain unchanged in the former situation while in the latter the decrease in the number of jobs could be less significant. Economies and/or sectors in which most employees are unionised could be thought of as being well described by this case, since increases in the employment level would always mean hiring non-members while decreases in the number of workers would be linked to firing members. Hence, 'good' shocks would always generate relative wage inflation and employment stagnation while 'bad' shocks would be accompanied by moderate wage increases and a relatively smaller employment adjustment.

Finally, it is interesting to note that the three cases that have been defined along the paper - depending on employment being less than, equal to, or greater than membership- can be restated for the example used in terms of the relative magnitude of θ and $r(\theta)^8$:

 $\begin{array}{ll} Case \ I: & \theta_i/r_i \leq \ [\alpha + \gamma(1-\alpha)]M^{(1-\gamma)}/\gamma[\beta + \gamma(1-\beta)] \\ Case \ II: & \theta_i/r_i > (1-\alpha)/(\gamma-\alpha)]M^{(1-\gamma)} \\ Case \ III: & [\alpha + \gamma(1-\alpha)]M^{(1-\gamma)}/\gamma[\beta + \gamma(1-\beta)] \leq \ \theta_i/r_i < (1-\alpha)/(\gamma-\alpha)]M^{(1-\gamma)} \end{array}$

Rephrasing the problem in this way what is being defined are two thresholds for θ that would determine employment being smaller or greater than membership. This allows one to think of an asymmetric behaviour of unions depending on the magnitude of exogenous shocks. There is a zone in between both values, however, that is not determined. There is thus an economic environment in which unions would bargain subject to all members having a job (*Case III*).

Efficiency of the outcomes

The proposed formulation allows for different results depending not only on the bargaining power of the parties at the two stages of negotiations but also on the evolution of the reservation wage after the shock.

⁸ Alvaro Forteza kindly suggested this point to me.

The outcome of bargaining would be efficient whenever the isoprofit and the indifference curves are tangent, that is, when the following equality holds:

$$\theta_{i}f_{L}(L_{i}^{*}) - w^{*} = -[u(w^{*})-u(r)]/u'(w^{*})$$

With the assumed utility function of the individual member the above equation becomes: $\theta_i f_L(L_i^*) = r_i$. Assuming a quasi-Cobb-Douglas production function and a utilitarian objective function for the union, the conditions under which an efficient outcome is obtained can be stated for *Cases I* and *II*. In *Case III* the outcome will not be efficient by construction.

Case I:
$$L^* < M \quad \forall \theta$$

 (w^*, L^*) is efficient iff $\{[\alpha + \gamma(1-\alpha)]/[\beta + \gamma(1-\beta)]\}[E_{\theta}(Lr)/E_{\theta}(L)] = r_i \quad i = g, b$

If the reservation wage is an increasing function of the shock -an assumption that would not necessarily hold if decentralised bargaining is considered or if shocks are sector specific-then:

$$r_{\rm b} \leq E_{\theta}(Lr)/E_{\theta}(L) \leq r_{\rm g}$$

This implies a different necessary condition depending on the nature of the actual shock:

If
$$\theta = \theta_{g}$$
: { $[\alpha + \gamma(1-\alpha)]/\gamma[\beta + \gamma(1-\beta)]$ } $[E_{\theta}(Lr)/E_{\theta}(L)] = r_{g}$
 $\Rightarrow [\alpha + \gamma(1-\alpha)]/\gamma[\beta + \gamma(1-\beta)] \ge 1 \qquad \Leftrightarrow \alpha \ge \beta$

If
$$\theta = \theta_{\rm b}$$
: { $[\alpha + \gamma(1-\alpha)]/\gamma[\beta + \gamma(1-\beta)]$ } $[E_{\theta}(Lr)/E_{\theta}(L)] = r_{\rm b}$
 $\Rightarrow [\alpha + \gamma(1-\alpha)]/\gamma[\beta + \gamma(1-\beta)] \le 1 \qquad \Leftrightarrow \alpha \le \beta$

Hence, if there are unemployed members but still the bargaining power of the union over wages is greater than over employment, efficient outcomes can only be observed in 'good'

states. If the opposite holds, efficiency can be attained only in 'bad' states, while if bargaining powers are equal in the two rounds of negotiations the outcome can be efficient when shocks are both 'good' and 'bad'. Since the expected reservation wage is always greater than that observed in a fully anticipated 'bad' state of nature, and the mark-up over it is greater than 1 if bargaining power over wages is higher than over employment and there are members unemployed, the union/firm could have always been better off, without negatively affecting the other party, bargaining a smaller wage and getting a higher employment level in the second stage if a 'bad' shocks takes place. However, if the shock turns out to be 'good', there is still the chance that the output is efficient, given the combined effect of shocks and wage level on the level of employment and hence on the level of benefits. The analogous reasoning applies to the case in which bargaining power over employment is greater than over wages.

The result is different from that obtained in Manning's (1987) two-stage model. While there the sufficient condition for efficiency is $\alpha = \beta$, in the present formulation it is not the only one, because of agents bargaining over wages subject to the expected shock and its effects on the reservation wage. Further, the same argument allows for Pareto optimality also when $\alpha \neq \beta$, so that inefficiencies would arise not only because of different bargaining powers as in the cited paper but also depending on the state of nature in which employment negotiations take place and on how 'correctly' agents are able to predict it.

Finally, underemployment is observed when the marginal labour product exceeds the competitive wage and if this difference is negative there is overemployment. Assuming that the alternative income is a good approximation to the above competitive wage, one can draw some conclusions by analysing the model's optima. When union power in negotiating wages is greater than or equal to that when bargaining over employment ($\alpha > \beta$) and a 'bad' shock takes place ($\theta = \theta_b$) overemployment is not possible, while if $\alpha \le \beta$ and $\theta = \theta_g$ underemployment cannot be observed. However, for the combination of $\alpha > \beta$ and $\theta = \theta_g$ and for $\alpha \le \beta$ and $\theta = \theta_b$ both results are possible. The latter conclusion differs from the one arising from a two-stage model with no uncertainty in which underemployment will necessarily occur when $\alpha > \beta$ and overemployment only in the opposite situation.

Case II: $L^* \overset{\mathfrak{s}}{\to} M \quad \forall \theta$

When employment exceeds membership, the efficiency condition becomes:

$$(w^*, L^*)$$
 is efficient iff $[\gamma(1-\alpha)/(\gamma-\alpha)]E_{\theta}(r) = r_i$ $i = b,g$

Given that $[\gamma(1-\alpha)/(\gamma-\alpha)] \stackrel{\circ}{\rightarrow} 1 \quad \forall \alpha$ - the mark-up over the reservation wage is always greater than 1 - efficiency could only be attained in 'good' states if $r_b \, \mathbf{\pounds} \, r_g$ holds. Moreover, in 'bad' states there will always be underemployment. This result is the consequence of the union not caring about employment when all members have a job. However, it should be assumed that firms faced with a 'bad' shock are laying off non-union members first. On the contrary, in 'good' states it is possible to observe both over and underemployment, depending on the probabilities assigned to each state of nature, the bargaining power over wages and the elasticity of output with respect to employment.

Summarising, the main results on efficiency relate to the possibility of obtaining Pareto optimality of the outcome without imposing that the union's bargaining power over wages and employment should be equal. The conditions under which efficiency is possible depend, however, on the nature of the shock, the existence or not of unemployed union members and on the accuracy of agents' predictions.

Nesting existing models

For different combinations of the values of the parameters of the proposed model, various standard formulations are derived. Firstly, if the probability of occurrence of a 'good' shock is set equal to 1 or 0, the formulation becomes the two-stage model with varying outside opportunities sketched previously. That is, the case in which the shock is fully anticipated.

If it is further assumed that the reservation wage is independent of shocks, the standard two-stage model results. Note, however, that this same model can be obtained if keeping uncertainty but with a constant reservation wage and a production function with constant elasticity.

Adding the assumption that bargaining powers are equal in both stages determines that the model collapses to the efficient contracts formulation while if β is set equal to 0 the right-to-manage model is obtained. Finally, imposing the restrictions that $\beta=0$ and $\alpha=1$, the model becomes the monopoly union.

As stated in Manning (1987), the advantages of having a general formulation are obvious. In encompassing different possible bargaining structures, it allows for testing, instead of imposing, the restrictions that would yield a simpler model. However, it must be noted that there might exist identification problems that would severely reduce the practical viability of the testing procedures.

Concluding remarks

The bargaining model proposed in this paper is intended to analyse how standard results would be affected by the inclusion of agents' beliefs and varying outside opportunities. This is thought to be relevant not only because of being a better approximation of real world but also because of its consequences on the expected behaviour of wages and employment.

The results obtained show that once the alternative income is allowed to vary with shocks, wages are not sticky anymore and the employment level fluctuates less than according to standard models. If uncertainty is included, however, wage rigidity is recovered while the extent of employment fluctuations will depend on the distribution of shocks, their realisation and the evolution of the reservation wage. If uncertainty is high and/or if the state of nature turns out to be of the opposite sign of the most expected one by agents, the employment adjustment is maximum. Observed shocks of the same sign than those expected by agents and/or scarce uncertainty generate small employment adjustment to shocks. Moreover, the relative responsiveness of wages and employment will depend on the existence or not of unemployed members.

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According to the proposed model, there is not a unique prediction regarding the Pareto optimality of the outcome of bargaining. In contrast to other formulations, it allows for both efficient and inefficient wage-employment pairs, depending not only on union strength but also on agents' beliefs and the observed state of nature. Even if unions were not concerned about employment, as many authors claim to be the case, it would be possible to attain efficient outcomes when faced to 'good' states of nature in this framework. Thus, the model provides a way of overcoming one of the points confronting right-to-manage and efficient contracts models.

Further theoretical work must be done, however, analysing how sensitive the results are to hypotheses such as the risk neutrality of agents. Given that it is not possible to derive analytically the outcome of bargaining when individuals are risk averse, simulations should be carried out in order to shed some light on this issue.

Moreover, it would be interesting to compare the results derived with those that would be obtained in a setting with decentralised bargaining, especially because the relevant reservation wage could evolve differently depending on the shock being sectoral or economy-wide.

Finally, given that the predictions of the model regarding employment adjustment depend on the responsiveness of the reservation wage to shocks, an interesting extension would be to endogenise the alternative income in a general equilibrium model of the labour market. Considerations relative to the behaviour of the unemployed individuals - such as effectiveness of their job search; duration of unemployment; or availability of information on vacancies- would be thus included.

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Appendix: Derivatives Table 1

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$$\begin{aligned} Case I: L^* < M^{"} \mathbf{q} \\ \partial w^* / \partial r_g &= \left\{ [\alpha + \gamma(1 - \alpha)] / \gamma A^2 \right\} p \theta_g^{1/(\gamma - 1)} \ge 0 \quad \partial w^* / \partial r_b &= \left\{ [\alpha + \gamma(1 - \alpha)] / \gamma A^2 \right\} (I - p) \theta_b^{1/(\gamma - 1)} \ge 0 \\ &\text{with } A = p \theta_g^{1/(\gamma - 1)} + (I - p) \theta_b^{1/(\gamma - 1)} \ge 0 \\ \partial w^* / \partial p &= \left\{ [\alpha + \gamma(1 - \alpha)] / \gamma A^2 \right\} (r_g - r_b) (\theta_g \theta_b)^{1/(\gamma - 1)} \ge 0 \\ \partial w^* / \partial \theta_g &= \left\{ [\alpha + \gamma(1 - \alpha)] / (1 - \gamma) \gamma A^2 \right\} p \theta_g^{\gamma/(\gamma - 1)} [(1 - \gamma) A \theta_g \partial r_g / \partial \theta_g + (I - p) (r_g - r_b) \theta_b^{1/(\gamma - 1)} \right] \ge 0 \\ \partial w^* / \partial \theta_b &= \left\{ [\alpha + \gamma(1 - \alpha)] / (1 - \gamma) \gamma A^2 \right\} (I - p) \theta_b^{\gamma/(\gamma - 1)} [(1 - \gamma) A \theta_b \partial r_b / \partial \theta_b + p (r_b - r_g) \theta_g^{1/(\gamma - 1)} \right] \ge 0 \\ \partial L_i^* / \partial r_i &= [\partial w^* / \partial r_i] (\theta_i \mu)^{1/(\gamma - 1)} w^* (2 - \gamma) / (\gamma - 1) / (\gamma - 1) \le 0 \quad \text{for } i = g, b \\ \partial L_i^* / \partial p &= \left[\partial w^* / \partial p \right] (\theta_i \mu)^{1/(\gamma - 1)} w^* (2 - \gamma) / (\gamma - 1) / (\gamma - 1) \le 0 \quad \text{for } i, j = g, b \\ \partial L_i^* / \partial \theta_i &= \left[(\partial w^* / \partial \theta_i) \theta_i - w^* \right] (\theta_i \mu)^{1/(\gamma - 1)} w^* (2 - \gamma) / (\gamma - 1) / (\gamma - 1) \theta_i \ge 0 \quad \text{for } i = g, b \\ \partial L_g^* / \partial \theta_b &= \left[\partial w^* / \partial \theta_b \right] (\theta_g \mu)^{1/(\gamma - 1)} w^* (2 - \gamma) / (\gamma - 1) / (\gamma - 1) \right] \ge 0 \end{aligned}$$

$$\frac{\partial w^*}{\partial r_g} = p\gamma(1-\alpha)/(\gamma-\alpha) \ge 0 \qquad \frac{\partial w^*}{\partial r_b} = (1-p)\gamma(1-\alpha)/(\gamma-\alpha) \ge 0 \\ \frac{\partial w^*}{\partial \theta_g} = (r_g - r_b)\gamma(1-\alpha)/(\gamma-\alpha) \ge 0 \qquad \frac{\partial w^*}{\partial \theta_b} = (\frac{\partial r_b}{\partial \theta_b})(1-p)\gamma(1-\alpha)/(\gamma-\alpha) \ge 0 \\ \frac{\partial L_i^*}{\partial r_i} = [\frac{\partial w^*}{\partial r_i}](\theta_i\gamma)^{1/(\gamma-1)} w^{*(2-\gamma)/(\gamma-1)}/(\gamma-1) \le 0 \quad \text{for } i = g,b \\ \frac{\partial L_i^*}{\partial r_j} = [\frac{\partial w^*}{\partial r_j}](\theta_i\gamma)^{1/(\gamma-1)} w^{*(2-\gamma)/(\gamma-1)}/(\gamma-1) \le 0 \quad \text{for } i,j = g,b \quad i \downarrow j \\ \frac{\partial L_i^*}{\partial \theta_i} = [(\frac{\partial w^*}{\partial \theta_i})\theta_i - w^*](\theta_i\gamma)^{1/(\gamma-1)} w^{*(2-\gamma)/(\gamma-1)}/(\gamma-1)\theta_i ? 0 \quad \text{for } i = g,b \\ \frac{\partial L_i^*}{\partial \theta_j} = (\frac{\partial w^*}{\partial \theta_j})(\theta_j\gamma)^{1/(\gamma-1)} w^{*(2-\gamma)/(\gamma-1)}/(\gamma-1)\theta_i \le 0 \quad \text{for } i = g,b \quad i \downarrow j \\ \frac{\partial L_i^*}{\partial \theta_j} = (\frac{\partial w^*}{\partial \theta_j})(\theta_j\gamma)^{1/(\gamma-1)} w^{*(2-\gamma)/(\gamma-1)}/(\gamma-1)\theta_i \le 0 \quad \text{for } i = g,b \quad i \downarrow j \\ \frac{\partial L_i^*}{\partial \theta_j} = (\frac{\partial w^*}{\partial \theta_j})(\theta_j\gamma)^{1/(\gamma-1)} w^{*(2-\gamma)/(\gamma-1)}/(\gamma-1)\theta_i \le 0 \quad \text{for } i = g,b \quad i \downarrow j \\ \frac{\partial L_i^*}{\partial \theta_j} = (\frac{\partial w^*}{\partial \theta_j})(\theta_j\gamma)^{1/(\gamma-1)} w^{*(2-\gamma)/(\gamma-1)}/(\gamma-1)\theta_i \le 0 \quad \text{for } i = g,b \quad i \downarrow j \\ \frac{\partial L_i^*}{\partial \theta_j} = (\frac{\partial w^*}{\partial \theta_j})(\theta_j\gamma)^{1/(\gamma-1)} w^{*(2-\gamma)/(\gamma-1)}/(\gamma-1)\theta_i \le 0 \quad \text{for } i = g,b \quad i \downarrow j \\ \frac{\partial L_i^*}{\partial \theta_j} = (\frac{\partial w^*}{\partial \theta_j})(\theta_j\gamma)^{1/(\gamma-1)} w^{*(2-\gamma)/(\gamma-1)}/(\gamma-1)\theta_i \le 0 \quad \text{for } i = g,b \quad i \downarrow j \\ \frac{\partial L_i^*}{\partial \theta_j} = (\frac{\partial w^*}{\partial \theta_j})(\theta_j\gamma)^{1/(\gamma-1)} w^{*(2-\gamma)/(\gamma-1)}/(\gamma-1)\theta_i \le 0 \quad \text{for } i = g,b \quad i \downarrow j \\ \frac{\partial L_i^*}{\partial \theta_j} = (\frac{\partial w^*}{\partial \theta_j})(\theta_j\gamma)^{1/(\gamma-1)} w^{*(2-\gamma)/(\gamma-1)}/(\gamma-1)\theta_i \le 0 \quad \text{for } i = g,b \quad i \downarrow j \\ \frac{\partial L_i^*}{\partial \theta_j} = (\frac{\partial w^*}{\partial \theta_j})(\theta_j\gamma)^{1/(\gamma-1)} w^{*(2-\gamma)/(\gamma-1)}/(\gamma-1)\theta_i \le 0 \quad \text{for } i = g,b \quad i \downarrow j \\ \frac{\partial L_i^*}{\partial \theta_j} = (\frac{\partial w^*}{\partial \theta_j})(\theta_j\gamma)^{1/(\gamma-1)} w^{*(2-\gamma)/(\gamma-1)}/(\gamma-1)\theta_i \le 0 \quad \text{for } i = g,b \quad i \downarrow j \\ \frac{\partial L_i^*}{\partial \theta_j} = (\frac{\partial w^*}{\partial \theta_j})(\theta_j\gamma)^{1/(\gamma-1)} w^{*(2-\gamma)/(\gamma-1)}/(\gamma-1)\theta_j \le 0 \quad \text{for } i = g,b \quad i \downarrow j \\ \frac{\partial L_i^*}{\partial \theta_j} = (\frac{\partial W_i^*}{\partial \theta_j})(\theta_j\gamma)^{1/(\gamma-1)} w^{*(2-\gamma)/(\gamma-1)}/(\gamma-1)\theta_j \le 0 \quad \text{for } i = g,b \quad i \downarrow j \\ \frac{\partial L_i^*}{\partial \theta_j} = (\frac{\partial W_i^*}{\partial \theta_j})(\theta_j\gamma)^{1/(\gamma-1)} w^{*(2-\gamma)/(\gamma-1)}/(\gamma-1)\theta_j \le 0 \quad \text{for } i = g,b \quad i \downarrow j \\ \frac{\partial L_i^*}{\partial \theta_j} = (\frac{\partial W_i^*}{\partial \theta_j})(\theta_j)(\theta_j)^{1/(\gamma-1)} w^{*(2-\gamma)$$

Case III:
$$L^* = M$$
 "q

$$\frac{\partial w^{*}}{\partial r_{g}} = p (1-\alpha) \geq 0 \qquad \qquad \partial w^{*} / \partial r_{b} = (1-p)(1-\alpha) \geq 0 \\ \frac{\partial w^{*}}{\partial p} = (r_{g}-r_{b})(1-\alpha) + (\theta_{g}-\theta_{b})\alpha p M^{(\gamma-1)} \geq 0 \\ \frac{\partial w^{*}}{\partial \theta_{g}} = (\partial r_{g} / \partial \theta_{g})p(1-\alpha) + p M^{(\gamma-1)} \geq 0 \qquad \qquad \partial w^{*} / \partial \theta_{b} = (\partial r_{b} / \partial \theta_{b})(1-p)(1-\alpha) + (1-p)M^{(\gamma-1)} \geq 0 \\ \frac{\partial L_{i}^{*}}{\partial x} = 0 \quad \text{for } x = r_{g}, r_{b}, p, \theta_{g}, \theta_{b}$$