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Efficient Bailouts?

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Abstract

This paper develops a non-linear DSGE model to assess the interaction between ex-post interventions in credit markets and the build-up of risk ex ante. During a systemic crisis, bailouts relax balance sheet constraints and mitigate the severity of the recession. Ex ante, the anticipation of such bailouts leads to an increase in risk-taking, making the economy more vulnerable to a financial crisis. The optimal policy requires, in general, a mix of ex-post intervention and ex-ante prudential policy. We also analyze the effects of bailouts on financial stability and welfare in the absence of ex-ante prudential policy. Our results show that the moral hazard effects of bailouts are significantly mitigated by making bailouts contingent on the occurrence of a systemic financial crisis.

Keywords: Bailouts, moral hazard, credit crunch, financial shocks, macro-prudential policy
JEL classification: E32, E44, F40, G18

Resumen

Este artículo propone un modelo cuantitativo de equilibrio dinámico para estudiar la interacción entre intervenciones en el mercado de crédito y la generación de riesgos sistémicos. Durante las crisis financieras, salvatajes financieros alivian restricciones financieras y mitigan la severidad de la recesión. Previo a las crisis, la anticipación de salvatajes financieros elevan la toma de riesgo, incrementando la vulnerabilidad a las crisis financieras. La política óptima requiere, en general, el uso de intervenciones durante las crisis así como una política macroprudencial durante las bonanzas. También analizamos los efectos de salvatajes financieros en la estabilidad financiera y bienestar, en la ausencia de política macroprudencial. El análisis muestra que los efectos de riesgo moral son mitigados sustancialmente cuando los salvatajes financieros son condicionales en una crisis sistémica.
Palabras Clave: Salvatajes financieros, riesgo moral, crisis financieras, política macroprudencial
Clasificación: E32, E44, F40, G18

1 Introduction

A common feature of financial crises is massive government intervention in credit markets. For example, the initial Troubled Assets Relief Program (TARP) required 700 billion dollars to provide credit assistance to financial and non-financial institutions. Related measures in the ongoing European crisis continue to spark an intense debate on the desirability of government intervention in credit markets. Many argue that bailouts are often necessary to prevent a complete meltdown of the financial sector, which would bring an extraordinary contraction in output and employment. An alternative view argues that bailouts create incentives to take even more risk ex ante, sowing the seeds for future crises. According to this view, regulations should introduce a strict limit on the government's ability to bail out the financial sector.

How does the expectation of a bailout impact the stability of the financial sector? Is it desirable to prohibit the use of public funds to bail out the financial sector? How large should bailout packages be? How critical are policies to prevent excessive risk-taking due to the anticipation of future bailouts?

This paper addresses these questions using a non-linear DSGE model in which credit frictions generate scope for bailouts during a financial crisis, but where the anticipation of bailouts generate more risk-taking before the crisis actually hits. One strand of the literature, summarized by Gertler and Kiyotaki (2010), has analyzed how credit policy can mitigate a credit crunch and the resulting recession ex post. At the same time, a growing theoretical literature investigates the moral hazard effects of government bailouts (e.g. Farhi and Tirole, 2012). However, there has been little work assessing the quantitative implications of the moral hazard effects. This paper contributes to fill in this gap by developing a quantitative equilibrium model to assess the interaction between ex-post interventions in credit markets and the build-up of risk ex ante in a unified framework. We use this framework to investigate the optimal bailout policy and evaluate its macroeconomic and welfare effects.

The model features a representative corporate entity that faces two frictions in its capacity to finance investment. First, debt contracts are not enforceable, giving rise to a collateral constraint that limits the amount that firms can borrow. Second, there is an equity constraint that imposes a minimum dividend payment that firms must make each period. In the stochastic steady state of the model, firms are able to finance the desired level of investment during normal economic conditions. However, when leverage is sufficiently high and an adverse financial shock hits the economy, firms are forced to cut down on investment, leading to a protracted recession. Anticipating that such episodes are costly, firms behave in a precautionary manner during normal times, balancing the desire to increase borrowing and investment today with the risk of becoming financially constrained in the future.

In our model, credit crunches are socially inefficient because firms remain undercapitalized, hindering economic recovery. From an individual point of view, households do not have an incentive to unilaterally transfer funds to firms, since this only entails costs for them. From a social point of view, however, a collective transfer to firms allows all households to obtain higher dividends and higher labor income in the future. When the credit crunch is sufficiently severe, these benefits outweight the efficiency costs of the transfers and bailouts make everyone better off. Ex-post interventions also have consequences for the optimality of ex-ante risk taking decisions, as we explain below.

Our normative analysis begins by considering a benevolent social planner that makes financial decisions subject to the same financial constraints as the private economy. We then study the policies that can decentralize the constrained-efficient allocations. We show that the optimal policy mix requires, in general, a combination of ex-post intervention and ex-ante prudential policies. Ex-post interventions, which we refer as bailouts, are necessary in order to redistribute resources from households to firms during financial crises.

The scope for ex-ante intervention, i.e., prudential policy, depends on the perception of how bailouts are implemented. When bailouts are implemented using lump-sum transfers conditional on the economy's aggregate states, individual financial decisions are not distorted ex-ante and there is no need for curbing borrowing before a crisis materializes. That is, although firms take more risk anticipating that the government will relax balance sheet constraints once a crisis occurs, this is an efficient response to the insurance provided by the government. This form of non-targeted bailout is not very practical, though, as the government needs to provide the same transfer to all entities regardless of their financial position. In a more realistic case in which bailouts are partially targeted, the government needs to use policies that curb borrowing to implement the constrained-efficient allocations. Intuitively, targeted bailouts cause borrowing costs to decline below the social value through implicit subsidies. This gives rise to a complementarity between targeted bailouts and macro-prudential policy.

For our quantitative investigation, we calibrate the model using the US economy as a reference. We match targets for leverage, volatility of investment, and the frequency and duration of financial crises. We find that the multiplier effects of bailouts: a bailout of one percentage point of GDP in a crisis comparable to the Great Recession can lead to cumulative output gains of about 8 percentage points. Moreover, the multiplier effects are time-varying as the effectiveness depends on the severity of the crisis. Bailouts also result in financial constraints becoming frequently more binding in the economy although the severity of these episodes becomes less severe. Hence, while the planner's intervention induces more risk-taking, it does not cause more financial fragility due to the fact that the planner can alleviate the effects of adverse financial shocks by using bailouts.

We then explore the importance of prudential policy to discourage excessive risk-taking by investigating financial stability in an economy with bailouts but no macro-prudential policy. In this case, bailouts create a tradeoff: bailouts ameliorate credit crunches ex post, while they cause too much risk-taking ex ante. One of our key findings is that if targeted bailouts are implemented only in a systemic financial crisis, there are still strictly positive welfare gains from conducting bailouts. In fact, the magnitudes of the welfare gains come quite close to the gains obtained by the fully optimal intervention that allows for both prudential policy and ex-post policy. We emphasize that the welfare gains from bailouts in this setting occur not only ex post but also ex ante. That is, even if bailouts generate excessive risk-taking, the benefits from better insurance outweigh the moral hazard effects. On the other hand, if bailouts are idiosyncratic and are fully determined by individual financial decisions, this causes a sharp increase in financial fragility and substantial ex-ante welfare losses.

Related Literature — This paper draws on the extensive literature on the macroeconomic effects of financial frictions, shaped by the work of Bernanke and Gertler (1989) and Kiyotaki and Moore (1997). In particular, our model shares with Jermann and Quadrini (2012) the emphasis on financial shocks and equity financing decisions, and with Mendoza

(2010) the emphasis on non-linear dynamics beyond the steady state. However, these papers do not address normative issues.

This paper is also related to a growing quantitative literature that studies the effects of credit policy during a credit crunch.[1] For reasons of tractability, most of this literature focuses on policy measures in response to unanticipated crises or on log-linear dynamics around the deterministic steady state, and does not address risk considerations and the moral hazard effects of credit policy. Instead, a distinctive feature of this paper is the consideration of how expectations of future bailouts affect ex-ante risk-taking. This is crucial to assessing the dynamic implications of credit intervention on financial stability and on social welfare.

This paper also builds on the theoretical literature that analyzes the effects of bailouts on risk-taking incentives and financial stability.[2] In particular, Farhi and Tirole (2012) show that bailouts generate incentives to correlate risks resulting in excessive financial fragility, and draw implications for ex-ante regulation to rule out bailouts in equilibrium. Our paper emphasizes the idea that bailouts can be welfare improving from an ex-ante point of view due to their insurance role. In this respect, it is more closely related to Keister (2012). He studies a Diamond-Dybvig economy and shows that commitment to a no-bailout policy induces banks to remain too liquid from a social point of view, and may increase the vulnerability to a bank -run. Our main contribution to this literature is to provide a quantitative framework to assess the effects bailouts over financial stability.

In recent work, Gertler, Kiyotaki, and Queralto (2011) develop a model in which banks have access to debt and equity financing and investigate the moral hazard effects of credit policy. They restrict attention to macro dynamics around a "risk-adjusted steady state" in which financial constraints are always binding. In contrast, we study full equilibrium dynamics in a stochastic steady state in which binding financial constraints only bind occasionally.

[1] See e.g. Gertler and Karadi (2011), Del Negro, Eggertsson, Ferrero, and Kiyotaki (2010), Kollmann, Ratto, Roeger, and in′t Veld (2012) for models of credit policy. See Guerrieri and Lorenzoni (2011), Bigio (2010), Midrigin and Philippon (2011), Shourideh and Zetlin-Jones (2012) for other recent examples of models of credit crunches.

[2] Examples include Burnside, Eichenbaum, and Rebelo (2001), Schneider and Tornell (2004) Farhi and Tirole (2012), Chari and Kehoe (2009), Diamond and Rajan (2009), Keister (2012), Keister and Narasiman (2011), Pastén (2011), and Nosal and Ordonez (2012). For empirical evidence on the anticipation of bailouts in the US financial crisis see Kelly, Lustig, and Van Nieuwerburgh (2011).

We also complement their work by characterizing and solving for the optimal bailout policy and prudential policy to avoid excessive risk-taking.

Finally, this paper is also related to a growing quantitative literature on how macro-prudential policy can be used to reduce the level of financial fragility.[3] The inefficiency in this literature relates to the effects of an inter-temporal reallocation of wealth of leveraged borrowers on prices affecting funding constraints. In contrast, the scope for policy here arises because of the effects of an intra-temporal reallocations of wealth between households and firms on future production capacity.

The remainder of the paper is organized as follows: Section 2 presents the analytical framework; Section 3 analyzes the optimal intervention; Sections 4 and 5 present the quantitative analysis; and Section 6 discusses the conclusions.

2 Analytical Framework

Our model economy is populated by firms and workers, who are also the firms' shareholders. Firms face an exogenous supply of funds and their capacity to finance investment is limited by a collateral constraint and an equity constraint. We begin by describing the decisions made by different agents in the economy, and then we discuss the general equilibrium.

2.1 Households

There is a continuum of identical households of measure one that maximize:

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t u(c_t - G(n_t)) \tag{1}$$

where c_t is consumption, n_t is labor supply, β is the discount factor, and $G(\cdot)$ is a twice-continuously differentiable, increasing, and convex function. The utility function $u(\cdot)$ has the constant-relative-risk-aversion (CRRA) form. The composite of the utility function has

[3] See e.g. Bianchi (2011), Bianchi and Mendoza (2010), Jeanne and Korinek (2010), and earlier theoretical work by Lorenzoni (2008) and Caballero and Krishnamurthy (2001). Benigno, Chen, Otrok, Rebucci, and Young (2012) also discuss ex-post policy measures to address a pecuniary externality (see also Jeanne and Korinek (2011) for a discussion of ex-ante versus ex-post policies).

the Greenwood-Hercowitz-Huffman (GHH) form, eliminating wealth effects on labor supply. The advantage of these preferences is that they deliver realistic responses of employment during a credit crunch without introducing frictions in labor markets that would complicate the analysis.

Households do not have access to bond markets, and they are the firms' shareholders. This yields the following budget constraint:

$$s_{t+1}p_t + c_t \leq w_t n_t + s_t(d_t + p_t), \tag{2}$$

where s_t represents the holdings of firm shares and p_t represents the price of firm shares.

The first-order conditions are given by:

$$w_t = G'(n_t), \tag{3}$$

$$p_t u'(t) = \beta \mathbb{E}_t u'(t+1)(d_t + p_{t+1}). \tag{4}$$

Iterating forward on (4) and imposing a no-bubble condition yields the result that in equilibrium, the price of shares must be equal to:

$$p_t = \mathbb{E}_t \sum_{j=1}^{\infty} \beta^j m_{t+j} d_{t+j}, \tag{5}$$

where $m_{t+j} \equiv (\beta^j u'(c_{t+j} - G'(n_{t+j})))/(u'(c_t - G'(n_t)))$ represents the stochastic discount factor.

2.2 Corporate Entities

There is a continuum of identical firms of measure one with technology given by $F(z_t, k_t, h_t)$ that combines capital denoted by k, and labor denoted by h to produce a final good. Productivity denoted by z_t follows a first-order Markov process. Consistent with the typical timing convention, k_t is chosen at time $t-1$ and is therefore predetermined at time t. Instead, the input of labor h_t can be flexibly changed in period t.

Firms have the following technology to transform final goods into investment goods.

$$k_{t+1} = k_t(1-\delta) + i_t, \tag{6}$$

where i_t is the level of investment and δ is the depreciation rate. Capital accumulation is subject to convex adjustment costs, given by $\psi(k_t, k_{t+1})$. Adjustment costs are introduced to improve the quantitative performance of the model in terms of the volatility of investment.

Firms pay dividends, denoted by d_t, and issue one-period non-state contingent debt, denoted by b_{t+1}. Firms finance investment, including capital adjustment costs $(i_t + \psi(k_t, k_{t+1}))$, debt repayments (b_t), dividend payments (d_t) with internal cash flows $(F(z_t, k_t, h_t) - w_t n_t)$, and new debt (b_{t+1}). The flow of funds constraint for firms is then given by:

$$b_t + d_t + i_t + \psi(k_t, k_{t+1}) \leq F(z_t, k_t, h_t) - w_t n_t + \frac{b_{t+1}}{R_t}, \tag{7}$$

where w_t is the wage rate, and R_t is the gross interest rate determined exogenously in international markets. R_t is stochastic and follows a first-order Markov process. Implicit in the flow of funds constraint is the fact that firms cannot issue new shares (we normalize the total number of shares to 1). However, they can adjust retained earnings by cutting dividend payments and servicing debt subject to the constraints we describe below.

Firms face two types of liquidity constraints on their ability to finance investment. Firms are subject to a collateral constraint that limits the amount of borrowing to a fraction of their capital holdings:

$$b_{t+1} \leq \kappa_t k_{t+1}, \tag{8}$$

This constraint is similar to those used in existing literature, and we interpret it as arising in an environment where creditors can only recover a fraction κ_t of the firms' assets.[4] As in Jermann and Quadrini (2012), κ_t represents a "financial shock" that hits exogenously the borrowing capacity of firms. For simplicity, this shock follows a two-state Markov chain with

[4]We implicitly assume that the liquidation value of capital is set at book value rather than market value, thereby turning off a fire-sale externality mechanism (see Bianchi and Mendoza (2010) for an analysis of this channel in the context of a representative firm-household model). We make this assumption to focus on the inefficiency that arises from reallocation of funds between households and firms.

values given by κ^H and κ^L. In our quantitative analysis, we set κ^H so that the collateral constraint never binds when the $\kappa = \kappa^H$. When κ switches from high to low, this may lead to a binding constraint and a credit crunch. Whether the economy enters a credit crunch depends endogenously on the degree of leverage in the economy.

Without any constraints on equity financing, the shadow value of external funds would be equal to one. We assume that there is a lower bound on dividend payments given by $\bar{d}$, i.e., in each period firms are required to satisfy:

$$d_t \geq \bar{d}. \tag{9}$$

A special case is the restriction that dividends need to be non-negative, which effectively implies that the issuance of new shares is not available. This constraint reflects the notion that dividend payments are required to reduce agency problems and information asymmetries between shareholders and managers.

We assume that firms maximize shareholder value as is standard in the corporate finance literature. Maximization of shareholder value implies that firms must discount profits at state $t+j$ at the rate m_{t+j} defined above. That is, their problem is to maximize $\mathbb{E}_t \sum_{j=0}^{\infty} m_{t+j} d_{t+j}$.[5]

2.3 Recursive Problem and Optimality Conditions

The aggregate state vector of the economy is given by $X = \{K, B, \kappa, z, R\}$. The optimization problem for firms can be written recursively as:

$$V(k, b, X) = \max_{d,h,k',b'} d + \mathbb{E}m'(X, X')V(k', b', X'), \tag{10}$$

$$s.t.$$

$$b + d + k' + \psi(k, k') \leq (1-\delta)k + F(z, k, h) - wn + \frac{b'}{R},$$

$$b' \leq \kappa k',$$

$$d \geq \bar{d}.$$

[5]We note that since the firm is representative, in equilibrium the firm never exposes itself to the possibility of not being able to satisfy the financial constraints. Hence, there is no voluntarily or involuntary default.

where $V(k, b, X)$ denotes the cum-dividend market value of the firm. The optimality condition for labor demand yields a standard static condition:

$$F_h(z_t, k_t, h_t) = w_t. \tag{11}$$

There are also two Euler intertemporal conditions that relate the marginal benefit from distributing one unit of dividends today with the marginal benefit of investing in the available assets and distributing the resulting dividends in the next period. Denoting by μ, the multiplier associated with the borrowing constraint, and by η the multiplier associated with the equity constraint, the Euler equations and associated complementary slackness conditions are given by:

$$1 + \eta_t = R_t \mathbb{E}_t m_{t+1}(1 + \eta_{t+1}) + R_t \mu_t, \tag{12}$$

$$(1 + \eta_t)(1 + \psi_{2,t}) = \mathbb{E}_t m_{t+1} \left[1 - \delta + F_{k,t+1} - \psi_{1,t+2}\right](1 + \eta_{t+1}) + \kappa_t \mu_t, \tag{13}$$

$$\mu_t(\kappa_t k_{t+1} - b_{t+1}) = 0, \quad \mu_t \geq 0, \tag{14}$$

$$\eta_t(d_t - \bar{d}) = 0, \quad \eta_t \geq 0. \tag{15}$$

In the absence of financial constraints on borrowing and dividend payments, the firm would be indifferent at the margin between equity and debt financing. However, when the collateral constraint binds, there is a wedge between the marginal benefit of borrowing one more unit and distributing it as dividends in the current period and the marginal cost of cutting dividends in the next period to repay the debt increase. In addition, when the equity constraint binds, a positive wedge arises between the marginal benefit from investing one more unit in capital or bonds and the marginal cost of cutting dividends today to finance the increase in capital and bonds.

In the model simulations, the collateral constraint and the equity constraint will often bind at the same time. Intuitively, both constraints impose a limit on a firm's funding ability. A binding equity constraint forces higher levels of borrowing for given investment choices. Similarly, a tighter constraint on borrowing puts pressure on the firms to finance to reduce dividend payments.

Discussion of Financial Frictions — Our normative analysis requires a model of incomplete markets that departs from Modigliani-Miller. We discuss now the specific assumptions that we have made to deviate from Modigiliani-Millers results.

The crucial friction in our setup is that firms face an equity constraint that imposes a lower bound on dividend payments. This is a relatively standard way of capturing agency problems and information asymmetries between a firm's shareholders and its managers in the corporate finance literature. It is also in line with an extensive literature documenting the importance of agency frictions between shareholders and corporate managers (see e.g. Shleifer and Vishny (1997) for a survey). Without this constraint on dividend payments, firms would be able to raise enough equity to finance desired investments and would fail to reproduce the evolution of real and financial variables in the data.[6]

Borrowing by firms is limited by imperfect enforceability of contracts. In particular, we assume that creditors require firms to hold collateral to back promised repayments according to (8). In order to enrich the model, we introduce shocks to how much collateral firms firms are required to pledge. A possible interpretation of such shocks relates to disruptions in financial intermediaries, which become more constrained on their ability to lend or they become more concerned about the riskiness of the corporate sector. We will show that when leverage is sufficiently high, a negative financial shock produces a credit crunch with similar features to the data. In fact, Jermann and Quadrini (2012) recently pointed out that financial shocks improve the quantitative performance of business cycle models.

We have also assumed that asset markets are restricted to one-period non-state contingent bonds, which is standard in the literature and represents a simplification of the firms' limited access to insurance. What is critical for our results is that firms cannot fully undo the equity

[6] More generally, what is necessary is to assume that equity becomes relatively costly to issue in bad states of nature. One motivation for frictions on equity financing is private information about investment opportunities. For example, in Myers and Majluf (1984), good firms may find it optimal not to issue equity when they are pooled with those of lower quality (see also Bigio (2011) for recent work linking adverse selection in credit markets with banks' equity financing).

constraints using contingent debt.[7] Finally, households in our model do not have access to credit markets, but they can smooth consumption through dividend payments.[8]

Overall, these assumptions allow us to formulate a parsimonious analysis of optimal bailouts. Moreover, these assumptions are important for the model to produces financial and real flows that are broadly consistent with key features of the data in terms of general co-movements and financial crises dynamics.

2.4 Competitive Equilibrium

The competitive equilibrium for a small open economy that borrows from abroad at an exogenous interest rate can be constructed in the usual form. Market clearing in the labor market requires:

$$h_t = n_t. \tag{16}$$

Market clearing in equity markets requires:

$$s_t = 1. \tag{17}$$

Using the two equations above and combining the household budget constraint and the firms' flow of funds constraint, we obtain the resource constraint for the economy:

$$b_t + c_t + k_{t+1} + \psi(k_t, k_{t+1}) = (1-\delta)k_t + F(z, k_t, h_t) + \frac{b_{t+1}}{R_t} \tag{18}$$

The recursive competitive equilibrium can be defined as follows:

[7] Standard motivations for restrictions on state-contingent liabilities are the lack of commitment by investors to inject funds to the firms in bad times or the inability to verify the realization of the shocks. We do not model these frictions, though.

[8] At the computational cost of introducing an additional state variable, this constraint can be relaxed to some extent. What is important is that households do not have access to perfect credit markets in order to guarantees that firms do not have an incentive to deleverage in the long run. Notice that when households have *unrestricted* access to international credit markets by borrowing and saving at the interest rate R_t, if the collateral constraint binds with strictly positive probability in the future, firms pay the minimum dividend. This can be seen by combining the household's first-order condition, ($R_t \mathbb{E}_t m_{t+1} = 1$), and the firm's first-order condition, which yields ($1 + \eta_t = 1\mathbb{E}_t m_{t+1}\eta_{t+1} + \mu = 1$). Moreover, if $\bar{d} = -\infty$, the competitive equilibrium would be unaffected by financial shocks. Therefore, the Modigiliani- Miller theorem would hold, and the model would become a standard RBC model.

Definition 1 . *A recursive competitive equilibrium is given by firms' policies* $\left\{\hat{d}(k,b,X), \hat{h}(k,b,X), \hat{k}(k,b,X), \hat{b}(k,b,X)\right\}$*; households's policies* $\hat{s}(s,X), \hat{c}(s,X), \hat{n}(s,X)$*; a stochastic discount factor* $m(X,X')$*; firm's value* $V(k,b,X)$*; prices* $w(X), p(X)$*; and a law of motion of aggregate variables* $X' = \Gamma(X)$*, such that: (i) households solve their optimization problem; (ii) firms' policies and firms' value solve (10); (iii) markets clear in equity market* ($\hat{s}(1,X) = 1$) *and the labor market* ($\hat{h}(K,B,X) = \hat{n}(1,X)$)*; (iv) the stochastic discount factor for firms is given by the household's marginal rate of substitution* $m(X,X') = \beta u'(\hat{c}(1,X) - G(\hat{n}(1,X)))/(u'(\hat{c}(1,X') - G(\hat{n}(1,X')))$*; and (v) the law of motion* $\Gamma(\cdot)$ *is consistent with individual policy functions and stochastic processes for* κ, R*, and* z*.*

Necessary and sufficient conditions for a competitive equilibrium can be established due to the fact that the optimization problem for households and firms are convex programs. In particular, given stochastic processes for R_t, z_t and κ_t, a set of stochastic sequences $\{c_t, k_{t+1}, i_t, b_{t+1}, d_t, h_t, n_t, w_t, p_t, \mu_t, \eta_t, s_t\}_{t\geq 0}$ is a competitive equilibrium if and only if equations (3)-(4) and (6)-(18) are satisfied.

In order to illustrate the properties of the model, it is useful to first analyze the case without uncertainty. In a deterministic steady state with $\beta R < 1$, (i) the collateral constraint is always binding, and (ii) there exists $\hat{d}$ such that the equity constraint binds if $\bar{d} > \hat{d}$. For (i), note that in a deterministic steady state, $m_t = 1$ and (12) is simplified to $1 = \beta R + \mu$. Since $\beta R < 1$, this implies that $\mu > 0$. For (ii), one can obtain the steady state values $[k^{ss}, h^{ss}, b^{ss}, \mu^{ss}]$ from (3),(8),(12), and (13). Substituting these expressions, (8) and (3) in the flow of funds constraint (7) yields the value of dividends at steady state $d^{ss} = F(z^{ss}, k^{ss}, h^{ss}) - k^{ss}(\delta + \bar{\kappa}(R-1)/R) - h^{ss}G'(h^{ss})$.

In general, in a stochastic steady state, these financial constraints may or may not bind depending primarily on the magnitudes of the shocks, the discount factor, the interest rate, and the tightness of the constraints.

3 Normative Analysis

The normative analysis begins by discussing the scope for policy in our model. Then we set up a constrained social planner's problem that can control risk-taking decisions and analyze possible decentralization to this problem. When we turn to a decentralized setting, we show that it is generally necessary to employ ex-ante and ex-post policy instruments. In the quantitative analysis, we analyze the case in which the planner is restricted to using ex-post policy instruments and analyze the moral hazard effects.

3.1 Scope for Policy

The key externality in the model is related to the undercapitalization of firms. When the equity constraint binds, funds are more valuable for the firms than compared to the households. The externality arises because households are not willing to unilaterally transfer funds to firms because they only incur costs. Instead, a social planner recognizes that transferring resources to the firm increases labor payments and dividend payments in future periods for all households in the economy.

This inefficiency is reminiscent of Holmström and Tiroles analysis of liquidity provision, where there is a rationale for the government to transfer resources from consumers to producers to expand production possibilities. In our setup, the ability of the government to improve welfare hinges on the ability to extract payments from households via taxes to address the free-rider externality. In fact, because households own firms, bailouts will lead to Pareto-improving interventions. The government, however, does not have a superior debt capacity than the private sector and the government does not use public debt as private liquidity (Woodford, 1990). To reflect the fact that transfers are costly in practice, we will assume that there is an iceberg cost φ proportional to the volume of transfers.

3.2 Constrained Social Planner's Solution

We consider a benevolent social planner who (a) chooses a sequence of transfers Υ_t between firms and households at a linear cost φ; (b) directly chooses the sequence of debt, capital, and equity payout subject to the liquidity constraints and the resource constraint; (c) lets labor markets, the stock market, and goods markets clear competitively. By making the planner subject to the same financial constraints as the decentralized equilibrium, the economy is also subject to the deleveraging effects of financial shocks. Notice also that while the social planner cannot directly affect labor market outcomes, it may affect the labor market indirectly through the choice of capital as it affects wages in the next period.

Denote by $\Upsilon_t \geq 0$ the transfer from households to firms, and by $\tilde{w}_t(k,z)$ and $\tilde{h}_t(k,z)$, the market clearing wage and labor allocations. Since labor is chosen by households and firms in competitive markets, $\tilde{w}_t(k,z)$ and $\tilde{h}_t(k,z)$ satisfy (3) and (11), i.e., $\tilde{w}_t(k_t,z_t) = G'(\tilde{h}_t(k_t,z_t)) = F_L(z_t,k_t,\tilde{h}_t(k_t,z_t))$. Therefore, the problem of the social planner can be written as follows:

$$\max_{\{k_{t+1},b_{t+1},c_t,p_t,\Upsilon_t\geq 0\}} \mathbb{E}\sum_{t=0}^{\infty}\beta^t u(c_t - G(\tilde{h}_t(k_t,z_t))) \tag{19}$$

$$(1-\delta)k_t + F(z_t,k_t,\tilde{h}_t(k_t,z_t)) + \frac{b_{t+1}}{R_t} - b_t - k_{t+1} - \psi(k_t,k_{t+1}) - \varphi\Upsilon_t = c_t,$$

$$(1-\delta)k_t + F(z,k_t,\tilde{h}_t(k_t,z_t)) - \tilde{w}_t(k_t,z_t)\tilde{h}_t(k_t,z_t) + \frac{b_{t+1}}{R_t} + \Upsilon_t - b_t - k_{t+1} - \psi(k_t,k_{t+1}) \geq \bar{d},$$

$$b_{t+1} \leq \kappa_t k_{t+1},$$

$$\beta\mathbb{E}_t u'(t+1)(d_{t+1}+p_{t+1}) = p_t u'(t).$$

We attach $\beta^t\eta_t$ and $\beta^t\mu_t$ to the financial constraints. Notice that the last condition is irrelevant for the planner, as the price of shares do not affect the set of feasible allocations. The resulting planner's problem is time-consistent: a future government that is free to choose a bailout policy does not have incentives to deviate from the path of bailouts chosen by its predecessors.

First-order condition with respect to Υ_t yields:

$$\varphi u'(c_t - G(h_t)) \geq \eta_t \text{ with equality if } \Upsilon_t > 0 \tag{20}$$

Condition (20) is crucial to identifying the tradeoffs involved in the bailout policy. This condition establishes that the planner will transfer resources from households to firms until the marginal cost given by $\varphi u'(t)$ equals the marginal benefits, given by η_t, the shadow value from relaxing the equity constraint. It also follows that $\Upsilon_t = 0$ if the equity constraint is not binding or if the shadow value from relaxing the equity constraint is small enough. Note that it is not optimal to fully relax the equity constraint, i.e., if $\Upsilon_t > 0$ for some t, it also follows that $\eta_t > 0$. We also have the following two results:

Corollary 1 *If $\varphi = 0$, the equity constraint does not bind for the social planner.*

Proof: Setting $\Upsilon_t > \bar{d} + b_t + i_t + \psi(k_t, k_{t+1}) - F(z_t, k_t, n_t) - w_t h_t + \frac{b_{t+1}}{R_t}$, the planner can completely relax the equity constraint without affecting the objective function or the rest of the constraints. Intuitively, if taxes are not distortive, the planner can use cost-free transfers as a substitute for lower dividend payments when the equity constraint becomes binding.

Corollary 2 *If $\bar{d} = -\infty$, the competitive equilibrium and the social planner's solution coincide.*

Proof: The proof notes that $\bar{d} = -\infty$ implies $\Upsilon_t = 0$ and $\eta_t = 0$, which yields that the conditions characterizing the competitive equilibrium are identical to those characterizing the social planner. Since firms have unrestricted access to equity, implementing a transfer from households to firms has no benefits.

Taking first-order condition with respect to capital and normalizing the Lagrange multipliers by the marginal utility of consumption yields:

$$(1+\eta_t)(1+\psi_{2,t}) = \mathbb{E}_t m_{t+1}(1-\delta+F_{k,t+1}-\psi_{1,t+2})(1+\eta_{t+1}(1-\tilde{h}_{t+1}(\partial\tilde{w}_{t+1}/\partial k_{t+1}))+\kappa_t\mu_t \quad (21)$$

An important difference between (21) and the analogous condition for firms (13) is that the planner internalizes how next period capital stock affects next period wages, which in turn affects the tightness of the equity constraint. In particular, firms do not internalize that one more unit of capital tightens the constraint by $\tilde{h}_{t+1}(\partial\tilde{w}_{t+1}/\partial k_{t+1})$, which has a marginal utility cost of η_{t+1}.

3.3 Decentralization

This section analyzes possible decentralization of the social planner's allocations. As we will see, the decentralization requires in general both ex-ante prudential measures and ex-post policy measures.

Debt Relief — We first analyze the role of debt relief. We consider a policy in which the government pays a fraction γ_t of private debts and finances this transfer of funds and its iceberg cost with lump sum taxes T_t to households. In addition, the government sets taxes on borrowing and capital income τ_t^b and τ_t^k that are rebated by a lump-sum transfer to firms T_t^f. With these policies, the households' budget constraint and the firms flow of funds constraint become respectively:

$$s_{t+1}p_t + c_t \leq w_t n_t + s_t(d_t + p_t) - T_t, \tag{22}$$

$$(1-\gamma_t)b_t + d_t + i_t + \psi(k_t, k_{t+1}) \leq (F(z_t, k_t, h_t) - w_t n_t)(1-\tau_t^k) + \frac{b_{t+1}}{R_t}(1-\tau_t^b) + T_t^f. \tag{23}$$

First-order condition with respect to b_{t+1} yields:

$$1 + \eta_t = R_t(1+\tau_t^b)\mathbb{E}_t m_{t+1}(1+\eta_{t+1})(1-\gamma_{t+1}) + R_t(1+\tau_t^b)\mu_t. \tag{24}$$

The rest of the optimality conditions remain the same. Note that from (24), the private costs of borrowing at time t are reduced by a factor of $(1-\gamma_{t+1})$ in a state t+1 in which the government provides debt relief. An examination of these first-order conditions leads to the following proposition:

Proposition 1 *The government can implement the constrained-efficient allocations with an appropriate combination of state contingent debt relief, taxes on debt and capital, and lump-sum taxes. These polices are given by:*

$$\gamma_t = \frac{\Upsilon_t}{b_t}, \quad T_t = \Upsilon_t(1+\varphi), \qquad T_t^f = \frac{b_{t+1}}{R_t}\tau_t^b + \tau_t^k(F(z_t, k_t, h_t) - G'(h_t)h_t).$$

$$\tau_t^b = \frac{\mathbb{E}_t m_{t+1}(1+\eta_{t+1}) + \mu_t}{\mathbb{E}_t m_{t+1}(1+\eta_{t+1})(1-\gamma_{t+1}) + \mu_t} - 1, \quad \tau_t^k = \frac{\mathbb{E}_t m_{t+1}(1-\delta+F_{k,t+1}-\psi_{1,t+2})\tilde{h}_{t+1}\partial(\tilde{w}_{t+1})/\partial k_{t+1}}{\mathbb{E}_t m_{t+1}(1+\eta_{t+1})F_{k,t+1}},$$

where all variables are evaluated at the constrained-optimal allocations.

Proof: The proof follows from noting that with the specified policy instruments the conditions characterizing the regulated competitive equilibrium are identical to those of the constrained optimal allocations.

The role of the taxes on debt and capital is to correct ex-ante financial decisions. The tax on debt aims to correct the private cost of borrowing, which is distorted by debt relief policies. The tax on capital aims to make firms internalize how a larger amount of capital increases the next period wages which tightens the equity constraint when it becomes binding. Notice that both taxes are strictly positive only when debt relief is implemented with strictly positive probability in the next period, an event that occurs only when the equity constraint becomes binding in the economy. Hence, both taxes are prudential.

Equity Injections — Another policy that can deliver the constrained efficient allocations is that of equity injections (see the appendix for a formal derivation). Unlike debt relief, equity injections involve a cost for shareholders because they perceive a reduction in their ownership of the firm. However, there is still a need for a prudential tax on debt if $\varphi > 0$. Intuitively, because firms do not internalize the social costs of the bailout, they take too much debt relative to the social optimum.

Lump-sum transfers — The final policy instrument we consider is a lump-sum transfer that is independent of any individual choice made by the firms. Because firms perceive the benefits from the bailout as entirely exogenous from their financial decisions, their borrowing decisions are not affected at the margin. Hence, there is no need for a prudential tax on debt. We note that lump-sum transfers are impractical as they involve a transfer which is completely independent of firms' balance sheet positions. In this respect, we see the implementation of lump-sum transfers as mostly illustrative.

Financial Intermediaries — In practice, central banks implement a variety of policies with the aim of facilitating the corporate sector's access to credit. For example, under the Commercial Paper Funding Facility (CPFF), the Federal Reserve expanded eligible collateral to include commercial paper, directly targeting the corporate sector, as in our model. Other policies included in the TARP involved equity injections to financial institutions. To simplify the analysis, we do not model financial intermediaries and consider only direct bailouts to firms. It is possible, however, to map our setup to a model in which financial intermediaries face the financial frictions that firms face in our model and lend to firms subject to no agency frictions.[9] The crucial factor for our analysis is that this intervention relaxes balance sheets across the economy and mitigates the fall in credit and investment that occurs during crises.

4 Quantitative Analysis

4.1 Quantitative Policy Experiments

In our quantitative analysis, we will start by exploring the properties of the optimal policy instruments and its effects over macroeconomic dynamics. We are also interested in examining the importance of the complementarity between bailouts and prudential policy. For this purpose, we will analyze two additional policy experiments. One experiment consists of imposing the optimal debt relief policy computed, but without the use of prudential taxes on debt and capital income. We call this policy "systemic bailout policy". Second, we study an "idiosyncratic bailout policy". In this scenario, bailouts now depend entirely on firm-specific choices and are independent of aggregate states. In particular, the government uses the debt relief policy solved above, but now the bailout is given by $\Upsilon(b, k, z, \kappa, R)$, i.e., there is no subscript in Υ associated with macro variables.

These two additional experiments allows us to analyze the trade-off between the ex-post benefits of bailouts and the ex-ante moral hazard effects. Ex post, bailouts can address the undercapitalization of firms. Ex-ante, there is too much risk-taking relative to the social optimum. Hence, we will study whether it is possible to increase welfare using bailouts without prudential policy.

4.2 Numerical Solution

The model is solved using a version of the policy function iteration algorithm modified to handle the two financial constraints. Our procedure computes the value of all policy functions over a discrete grid $B \times K \times z \times \kappa \times R$. These functions are not restricted to follow a specific parametric function; for values outside the grid, we use bilinear interpolation. Using an

[9] It is also possible to map the tax on debt on firms to capital requirements and margin requirements on financial intermediaries (see Bianchi, 2011).

iterative procedure, we compute the policy functions satisfying the competitive equilibrium conditions at all grid points. Given the policy functions and the stochastic processes, it is possible to simulate the model and compute the joint stationary distribution. This procedure allows us to deal with the well-known complications of non-linearities that arise in incomplete markets. In particular, occasionally binding financial constraints create kinks in the policy functions, which leads to a different behavior of the model depending on how close is the economy to the constraints and to a stationary distribution for state variables that are not confined to a narrow region of the state space.

4.3 Calibration

We calibrate the model to an annual frequency using data from the U.S economy. To focus on post-financial globalization period, our reference period is 1984:Q1-2010:Q2.

Functional Forms— We make the following assumptions regarding functional forms for preferences and technology:

$$
\begin{aligned}
u(c - G(n)) &= \frac{\left(c - \chi \frac{n^{1+\frac{1}{\omega}}}{1+\frac{1}{\omega}}\right)^{1-\sigma} - 1}{1-\sigma}, \\
F(z, k, h) &= z k^{\alpha} h^{1-\alpha}, \\
\psi(k_t, k_{t+1}) &= \frac{\phi_k}{2} \left(\frac{k_{t+1} - k_t}{k_t} \right)^2 k_t.
\end{aligned}
$$

Stochastic Processes— We model the shocks to the interest rate and productivity as a first-order bivariate autoregressive process:

$$
\begin{pmatrix} \hat{z}_t \\ \hat{R}_t \end{pmatrix} + \boldsymbol{\rho} \begin{pmatrix} \hat{z}_{t-1} \\ \hat{R}_{t-1} \end{pmatrix} + \begin{pmatrix} \epsilon_{z,t} \\ \epsilon_{R,t} \end{pmatrix},
$$

where $\varepsilon_t = [\epsilon_{z,t}\epsilon_{R,t}]'$ follows a bivariate normal distribution with zero mean and contemporaneous variance-covariance matrix V. To construct these series, we take the ex-post real interest rate on the 3 month US -Treasury Bills for the interest rate, and we follow the standard Solow residuals approach to construct the series for productivity. Our OLS estimation yields the following values:

$$\boldsymbol{\rho}= \begin{bmatrix} 0.755972 & -0.030037 \\ -0.074327 & 0.743032 \end{bmatrix}, \qquad \mathbf{V} = \begin{bmatrix} 0.0000580 & -0.0000107 \\ -0.0000107 & 0.0001439 \end{bmatrix}.$$

We discretize the VAR(1) process for productivity and interest rate shocks using the Tauchen-Hussey quadrature based procedure with 9 values. The mean values for the productivity and interest rate process are denoted by $\bar{z}$ and $\bar{R}$.

Financial shocks are modeled as an independent process following a two-state Markov chain with values given by $\{\kappa^L, \kappa^H\}$ and transition matrix

$$P = \begin{bmatrix} P_{L,L} & 1 - P_{L,L} \\ 1 - P_{H,H} & P_{H,H} \end{bmatrix},$$

with values to be determined below.

Parameter Values— Parameter values are summarized in Table 1. We need to assign values to 14 parameters that we classify in two sets. The first subset includes parameters that are chosen independently of equilibrium conditions or are calibrated using steady state targets, most of which are typical in the business cycle literature. This subset is given by $\{\alpha, \delta, \omega, \beta, \varphi, \chi, \overline{z}, \overline{R}\}$. The capital share α is set to 0.34; the depreciation rate is set at 11 percent; the risk aversion σ is set to 2; $\bar{R} - 1$ is set to 1.015 percent; the Frisch elasticity of labor supply in the GHH preference specification ω is set to 2. We normalize the labor disutility coefficient χ and the average value of productivity $\bar{z}$ so that employment and output equal one in the deterministic steady state. The value of β is pinned down by setting the capital-output ratio equal to 2.5 in a deterministic steady state with $\kappa = 0$, which results in a value of 0.97.[10]

The efficiency cost φ is more specific to our framework. For this parameter, we choose a benchmark value of 50 bps. Considering that financial intermediation represents about 5 percentage points of GDP, this implies that cost of the public supply of credit is 10 percent higher than the private one.[11]

The remaining six parameters are $\{\phi_k, \kappa^L, \kappa^H, \bar{d}, P_{L,L}\, P_{H,H}\}$. As mentioned above, we set the value of κ^H high enough so that the collateral constraint never binds when κ takes this

[10]Due to precautionary savings, average capital is 2.6 in our simulations, which is still within the range of empirical estimations.

[11]As a robustness check, we have also experimented with financing the bailout with a labor tax, finding similar results.

value. The remaining parameters are set to jointly match a set of five long-run moments for the no-bailout-policy economy. These moments are: (1) a standard deviation of investment of 13 percent; (2) an average leverage ratio of 45 percent; (3) four credit crunches occurring every 100 years; (4) an average duration of a credit crunch of 3 years; (5) a probability of a binding dividend constraint equal to the probability of a binding collateral constraint. While all these parameters affect all the target moments, each parameter has a more significant impact on one particular moment, as we explain below.

The adjustment cost on capital is calibrated to match the standard deviation of investment, which yields $\phi_k = 2.2$. The value of κ^L is set to target an average leverage of 45 percent. The choice of a leverage ratio of 45 percent corresponds to the ratio of credit market instruments to net worth in the years preceding the 2007 financial crisis (see Table B102 in the Flow of Funds database).

We calibrate the transition matrix for the financial shock to target the frequency and the duration of financial crises. We define a financial crisis as an episode in which credit falls below two standard deviations. The financial crisis begins in the period in which credit falls below one standard deviation, providing that at some point within the next two years, the level of credit falls at least two standard deviations below its mean. The crisis ends when the level of credit exceeds one standard deviation below its mean. Consistent with the empirical literature (e.g. Reinhart and Rogoff, 2009), we target an incidence of crises of 4 every 100 years and an average duration of 3 years. This procedure yields that $P_{L,L}$, which mostly affects the duration of crises, equals 0.15 and $P_{H,H}$, which primarily affects the long-run probability of a crisis, equals 0.93. With these values, the economy spends 8 percent of the time with negative financial shocks.

We set the dividend threshold $\bar{d}$, so that the borrowing constraint and the equity constraint bind with the same probability in the long run. We follow this route because it is difficult to pin down from the data whether constraints on equity financing or on borrowing are more pervasive. This yields $\bar{d} = 0.035$ and probabilities of binding constraints equal to 8 percent.

Table 1: Calibration

	Value	
Mean Interest rate	$R - 1 = 0.015$	
Discount factor	$\beta = 0.97$	
Depreciation rate	$\delta = 0.11$	
Share of capital	$\alpha = 0.34$	
Labor disutility coefficient	$\chi = 0.66$	
Risk aversion	$\sigma = 2$	
Frisch elasticity parameter	$\omega = 2.0$	
Efficiency cost	$\varphi = 50bps$	
Parameters set by simulation	**Value**	**Target**
Financial shock		
	$\kappa_L = 0.43$	Average leverage =45 percent
	$\kappa_H = 0.54$	Non-binding collateral constraint
	$P_{HH} = 0.93$	Probability of credit crunch = 4 percent
	$P_{LL} = 0.15$	Duration of credit crunch = 3 years
Adjustment cost	$\phi_k = 2.2$	SD of investment = 13 percent
Dividend threshold	$\bar{d} = 0.035$	Equalize prob. binding constraints

5 Results of the Quantitative Analysis

5.1 Dynamics of Financial Crises

We begin by analyzing the dynamics of financial crises in the competitive equilibrium without bailouts. We will later show that crises in the model are consistent with several features of financial crises in the data, and analyze how bailouts affect the incidence and the severity of crises.

We construct a crisis event for the no-bailout economy, following these steps. First, we run a long time-series simulation of the model by feeding a sequence of shocks, drawn from the distribution of the stochastic processes for (κ, z, R), to the policy functions computed. Second, we identify financial crisis events. As explained above, these events are defined as periods in which credit falls by more than two standard deviations. Third, we compute averages of the main macro variables of the model centered around those episodes. The results of this experiment are illustrated in Figure 1 (period t denotes the financial crisis event).

The top left panel of Figure 1 shows the role of the different shocks around financial crises. A first key result is that financial crises are always triggered by negative financial shocks while productivity shocks and interest rate shocks play a minor role. In fact, all crises episodes coincide with a negative financial shock while productivity and the interest rate are close to the mean. Moreover, crises are preceded by favorable credit conditions with $\kappa = \kappa^H$ and interest rates below the mean. Because the financial shock is persistent, the average value of κ is below κ^H following financial crises. In line with the evolution of κ, the top right panel shows that financial constraints are slack preceding financial crises, then they become largely binding during crises, and are reduced considerably following crises.

The medium panels of Figure 1 show the evolution of leverage and investment measured as percentage deviations from the mean values in the simulations. Leverage is above trend when crises hit and then there is significant deleveraging following the crises. Investment collapses during a crisis and then it recovers as the economy becomes relatively unconstrained again in period $t + 1$. The stock of capital, however, remains relatively depressed as adjustment costs make it relatively unattractive to rebuild the capital stock.

The bottom panels of Figure 1 show the evolution of output and employment. In line with the evolution of investment, output and employment drops significantly following a crisis. Notice that when the financial shock hits at time t, output does not drop on impact. This occurs because the absence of wealth effects on labor supply implies that the level of output at each point in time depends only on the level of capital and the productivity shock. Finally, crises are quite persistent as output and employment remain significantly depressed two years after crises.[12]

[12] Overall, these dynamics are consistent with Mendoza (2010). An important difference, however, is that in our model a crisis is caused by financial shock that triggers binding equity and collateral constraints.

5.2 Financial Crises Comparison

We now compare the magnitudes of financial crises in the no-bailout economy to the crises in the constrained-efficient allocations.— In addition, we also compare these magnitudes with the Great Recession.

Following the methodology described above, we simulate the constrained-efficient economy and select from the resulting distribution of crises a subset of those events. In particular, we extract events in which the fall in GDP following the crisis is close to 7 percent, which is the drop in GDP during the US Great Recession for the 2009:Q2 and 2010:Q1 period, and the level of productivity and interest rate are close to the mean. We then compute averages of macro variables and financial flows during these episodes. Next, we conduct a counterfactual policy that consists of analyzing what would be the response of the economy if there were no bailout. That is, given the initial state X_t for the subset of crises identified above and using the policy rules of the unregulated economy, we simulate what would be the response of the economy. The first two columns of Table 2 illustrate the results of the counterfactual experiments.

Table 2 shows that without intervention the credit crunch is substantially more severe. The fall in output and employment are -8.6 percent and -5.5 percent for the no-bailout economy versus -7.7 and -5.0 percent with the optimal bailout policy. Key to understanding the differences in the depth of the recession is how bailouts contribute to mitigating the fall in investment. Without the bailout, firms are forced to cut investment by about 60 percent. A bailout of 1.5 percentage point of GDP, however, provide extra resources for firms to invest, which in turn allows them to borrow more and reduce the contraction in investment even further. By maintaining a higher level of capital, firms demand more labor following the crises and the recession becomes less protracted. In fact, the effects of the bailout are even larger if one considers that crises have persistent effects over the real economy. Assuming that the negative financial shock lasts only for one period and the productivity and interest rate shocks remain the same, the cumulative output gain due to the bailout, measured as the difference of the sum of future output with and without bailouts, is 8 percentage points of GDP. This yields a cumulative bailout-multiplier of 5.3.

Table 2: Financial Crises Comparison

	Optimal Policy [b]	No-Bailout Policy [c]	Great Recession [d]
Output	−7.7	−8.6	−7.7
Hours	−5.0	−5.5	−9.8
Investment	−47.0	−58.5	−38.8
Consumption	−7.7	−5.6	−6.4
Debt-Repurchases	13.9	15.4	7.1
Equity-Payouts	−0.7	−0.7	−1.0
Bailout	1.5		
Cumulative Output Gain	8.0		

Note: The table reports magnitudes of financial crises for the optimal bailout policy economy, the no-bailout economy, and the US Great Recession. For the model simulations, all variables with the exception of bailouts, are expressed in terms of deviation with respect to the mean of the optimal policy economy (output, hours and investment are percentage deviations). Output and employment correspond to the period following the crisis.

[b] Magnitudes for the Optimal Policy correspond to the average macro responses conditional on an output drop of 7.7 percent following a financial crisis, and shocks to productivity and interest rate close to the mean.

[c] Magnitudes for No-Bailout Policy correspond to average responses of an economy that starts with the same states for capital, bonds and shocks as in the Optimal Policy (see note b), but where no bailouts are implemented.

[d] Data: The period corresponds to Q2 2009-Q1 2010. Series for output, hours, consumption and investment are in real terms and the log value is linearly detrended. Series for equity payouts and debt repurchases are constructed as in Jermann and Quadrini (2012).

On the financial side, while both financial constraints are binding during the crisis event, there are significant differences in the magnitude of the deleveraging. Equity payouts are the same as firms pay the minimum dividend, though.

It is important to note that this counterfactual experiment does not take into account the anticipation effects of bailouts, as both economies start with the same initial state variables. In particular, the economy without bailout would reduce leverage due to the lack of bailouts.

As a result, this experiment represents an upper bound of how bailouts can mitigate financial crises. We will show in the non-linear impulse responses below that the anticipation of future bailouts leads to higher leveraging, which in turn contributes to making crises more severe.

Finally, the results in Table 2 also show that the model performs relatively well when compared to the Great Recession. On the real side, there is an important contraction in employment, consumption, and investment. On the financial side, there is both significant deleveraging as well as a contraction in equity payouts.

5.3 Optimal Bailout Policy

What does the optimal bailout policy look like? Figure 2 shows the optimal bailout policy $\Upsilon(X)$ when a negative financial shock hits the economy. In particular, it displays the optimal bailout for different values of capital and debt when productivity and the interest rate are equal to their mean values.

Figure 2 shows that bailouts are increasing in the level of debt and decreasing in the level of capital, i.e., they are increasing in leverage. Moreover, bailouts are zero in a wide range of the state space with low leverage. That is, in this region, even if a negative financial shock hits the economy, the planner does not conduct bailouts. Recall that condition (20) prescribes that bailouts are conducted only when the equity constraint is sufficiently binding.

There is also a region with large bailouts in the order of 15 percentage points of GDP, but the economy never reaches these states with strictly positive probability. In fact, the largest bailout observed in the simulation is about 3 percentage points of GDP, as shown by Figure 3. This figure displays a scatter diagram with the drop in investment on the x-axis and the bailout on the y-axis for a long simulation. Notice that only when investment falls more than 10 percent, is it possible to observe bailouts taking place in the model simulations. Conditional on a crisis, the average bailout is 0.9 percentage points of GDP. If the economy is not in a crisis, the average bailout is zero.

5.4 Policy Functions and Laws of Motion

In order to illustrate the workings of the model, we now analyze the equilibrium policy functions over the state space for the unregulated economy without intervention and in the economy with the optimal bailout policy, i.e., the bailout policy that implements the constrained-efficient allocations.

Figure 4 shows the policy functions for debt, capital, and leverage in the economy as a function of the current level of debt. This plot corresponds to an economy where the current level of capital is approximately equal to the mean value and productivity and interest rate shocks are also equal to their mean values. Since average output is approximately one, all variables can be interpreted as a fraction of GDP. The left (right) panel corresponds to a positive (adverse) financial shock. The straight lines correspond to the competitive equilibrium with a no-bailout policy, and the dashed lines correspond to the economy with the optimal bailout policy. Let us first describe the behavior of the economy without intervention.

An important feature of the model is that the occasionally binding collateral constraint produces a non-monotonic law of motion for debt, which is apparent in the top right panel of Figure 4. For low values of current debt, the collateral constraint is not binding. In this region, next period level of debt is increasing in the current period debt. For $b > 1.18$, the collateral constraint become binding and next period debt is decreasing at the current debt levels. This shift in the slope occurs because as firms need to cut down on investment, the borrowing capacity shrinks. We also note that for this law of motion, the equity constraint and the collateral constraint becomes binding at about the same value of debt.

The intermediate panels show the book-value leverage (b_{t+1}/k_{t+1}), for different values of current debt. In the right panel, the leverage ratio is bounded by κ^L, which becomes binding at $b = 1.18$, as analyzed above. The law of motion for capital, which correspond to the lower panels, is decreasing in the amount of debt. This occurs because higher level of debt increases risk premium and reduces the willingness to invest in capital, which is a risky asset. This negative relationship between the current level of debt and next period capital becomes more pronounced when the collateral constraint becomes binding, as it can be seen in the right panel.

We turn attention now to the effects of bailouts over these laws of motion. We can distinguish the "ex-post effects", i.e., the effects once the bailout is implemented, which is illustrated with the shaded region, and the "ex-ante effects", the effects before the bailout is implemented. In the terms of the ex-post effects, by providing extra resources to invest in capital, bailouts allow firms to borrow more. The increase in the ability to borrow is apparent in the shaded region, which denotes the region where the planner conducts bailouts.

A central aspect of our analysis is how these ex-post responses modify the ex-ante financial decisions. In the region where the financial constraints are not binding, firms also borrow more in the competitive equilibrium with bailouts because there is a lower incentive to accumulate precautionary savings during normal times since crises become less severe. This effect is markedly stronger when the economy has a positive financial shock and has a relatively large amount of debt, so that a future financial crisis is relatively likely.

The effects of bailouts in capital accumulation when the collateral constraint is not binding are generally ambiguous due to two opposing forces. On one hand, bailouts lead to lower precautionary savings which in turn generates a lower capital accumulation. On the other hand, the the demand for risky assets increases. Overall, the lower panels illustrate that the second effect dominates for high values of debt.

5.5 Non-Linear Impulse Responses

We now conduct a non-linear impulse response to show how the anticipation of bailouts induces more risk-taking and how this affects the severity of financial crises.

We simulate the economy using a long sequence of $\kappa = \kappa^H$. At $t = T^\star$, the value of κ drops to κ^L and then reverts to κ^H. Productivity and interest rates are fixed at their average values along the simulations. The initial values for capital and debt are the values at which the competitive equilibrium would converge after a long sequence of κ^H. The negative financial shock hits at $T^\star = 60$ so that the optimal bailout policy economy would remain approximately constant in the absence of any other shocks.

We feed this sequence of shocks to the policy functions of the economy without intervention and for the constrained-efficient allocations. In these experiments, agents form rational expectations based on the objective probability distribution. We also compute a third economy where government intervention is unanticipated. That is, the economy behaves from $t = 0$ to $t = 60$, as if bailouts have a zero probability of occurring, and at $T^\star$ agents are "surprised". For $t > T^\star$, we assume that all economies become no-bailout policy economies so that all economies converge to the same values.

The result of the non-linear impulse response is illustrated in Figures 5. This figure shows the evolution of leverage, output, employment, investment, and the exogenous shocks. The evolution of output, employment, investment are expressed as percentage deviations of the no-bailout policy economy. The economy with the unanticipated bailout policy behaves like the no-bailout policy economy up to period $T^\star$. When κ^L hits, the bailout mitigates the contraction in investment and recovers faster the long-run level of output. Notice, however, that 5 years after the shock, the economy is still about 25 percent below trend.

When the bailout policy is anticipated, there is a clear increase in leverage in the run-up to the crisis as the economy perceives that the government will intervene ex post in case the economy enters a crisis. In this case, the overall effects during a crisis become more ambiguous: The increase in the amount of debt makes the economy more vulnerable to a negative shock, while bailouts relax balance sheet constraints ex post. Overall, output and employment contracts as sharply as in the no-bailout policy economy.

5.6 Moral Hazard

We now turn to a central aspect of our normative analysis: How critical are policies to prevent excessive risk-taking?

As we explain above, the increase in leverage and risk-taking described in the previous section is efficient in the sense that this is the optimal response to the higher level of insurance provided by bailouts. To see the importance of the prudential policy, we consider now the systemic bailout and idiosyncratic bailout policy. We reconstruct the non-linear impulse response from Figure 5, but now we compare the optimal policy economy, which includes debt relief policy and prudential policy, with the systemic bailout policy and the idiosyncratic bailout policy economies that do not include prudential policy. Figure 6 shows the results of this experiment. This figure also includes the taxes on debt and capital income that implement the optimal policy in the top right panel.

Figure 6 shows that the two economies without prudential policy experience an increase in borrowing relative to the optimal bailout policy and experience a larger crash in period $T^{\star}$. Moreover, the increase in leverage results in larger bailouts when the crisis hits. An important finding that Figure 6 illustrates is that the systemic bailout policy has a financial crash which is only slightly larger than the economy with the prudential policy.

On the other hand, the economy with the idiosyncratic bailout policy causes the financial crisis to be several times deeper than the optimal bailout policy. Moreover, this occurs despite the government implementing ex post a bailout which is 5 times larger than in the economy with the optimal bailout policy.

5.7 Welfare

Next, we compute the welfare gains from policy intervention. We consider the welfare effects of the three cases analyzed above: (a) optimal bailout policy; (b) systemic bailout, and (c) idiosyncratic bailout. We compute the percentage increase in consumption ζ_0 that leaves a household indifferent between living in an economy with the corresponding government policy and remaining in a no-bailout policy economy. That is for every possible initial state, we compute:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^{\star} - G(n_t^{\star})) = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^{NBP}(1+\zeta_0) - G(n_t^{NBP})) \tag{25}$$

where $c_t^{NBP}, G(n_t^{NBP})$ correspond to the policies in the no-bailout policy economy, and the superscript * correspond to the economy with a specific bailout policy.

The results from the calculations for the constrained-efficient allocations are shown in Figure 8 for different values of bonds and capital and for average productivity and interest rate shocks and an adverse financial shock. Welfare gains are strictly positive in all states and reach the maximum levels when the economy is highly leveraged. On average, welfare gains are close to 0.1 percentage point of permanent consumption. [13]

[13] For graphical purposes, we attach a value of zero for those states that are non- feasible, i.e., those with very high levels of debt and low values of capital such that firms cannot satisfy the financial constraints. The benefits of bailouts would be infinite in this region, but the economy never reaches those states.

Figure 9 shows the comparison of the welfare gains for the three economies for the same shock realizations and taking an initial value of capital equal to the mean. This figure shows that the systemic bailout policy delivers welfare gains which are quite close to the optimal policy. This is consistent with the non-linear impulse response that showed that the absence of prudential policy does not significantly increase financial fragility when bailouts are contingent on a systemic financial crisis. On the other hand, idiosyncratic bailouts deliver substantial welfare losses in most of the state space. In particular, welfare losses reach its minimum when the economy has a high value of debt but has not reached a crisis state. There are welfare gains, however, when the economy is in a crisis state with a sufficiently high level of debt, so that the benefits from relaxing balance sheet constraints ex post compensate for future effects of risk-taking incentives.

6 Conclusion

This paper developed a quantitative framework to examine the macroeconomic and welfare effects of bailouts. In our setup, bailouts are desirable because they can address the undercapitalization of firms during financial crises. In particular, households fail to internalize that transferring funds to firms allows all households to obtain higher dividends and higher labor income in the future. Implementing the constrained-efficient allocations in a decentralized setting requires both *ex-post* intervention and *ex-ante* prudential policy. Ex-post intervention involves a transfer from households to firms while ex-ante prudential policy involves taxes on debt and capital income to correct risk-taking decisions.

We have also used our framework to analyze the "moral hazard-financial stability trade-off". In particular, we analyzed a decentralized setting where the government uses bailouts but no prudential policy. Our central finding in this dimension is that the moral hazard effects of bailouts are significantly mitigated by making bailouts contingent on the occurrence of a systemic financial crisis. Hence, the financial stability effects of bailouts *ex post* can outweigh its moral hazard effects *ex ante*. On the other hand, an idiosyncratic bailout policy causes an important increase in financial fragility.

Our analysis abstracted from several important features. In order to maintain our focus on how bailouts affect the real economy, we have modelled direct bailouts to the corporate sector. An interesting extension would be to model how bailouts affect the banking channel. Moreover, it is possible that government bailouts affect the financial sector's incentives to supply insurance. These issues are left for future research.

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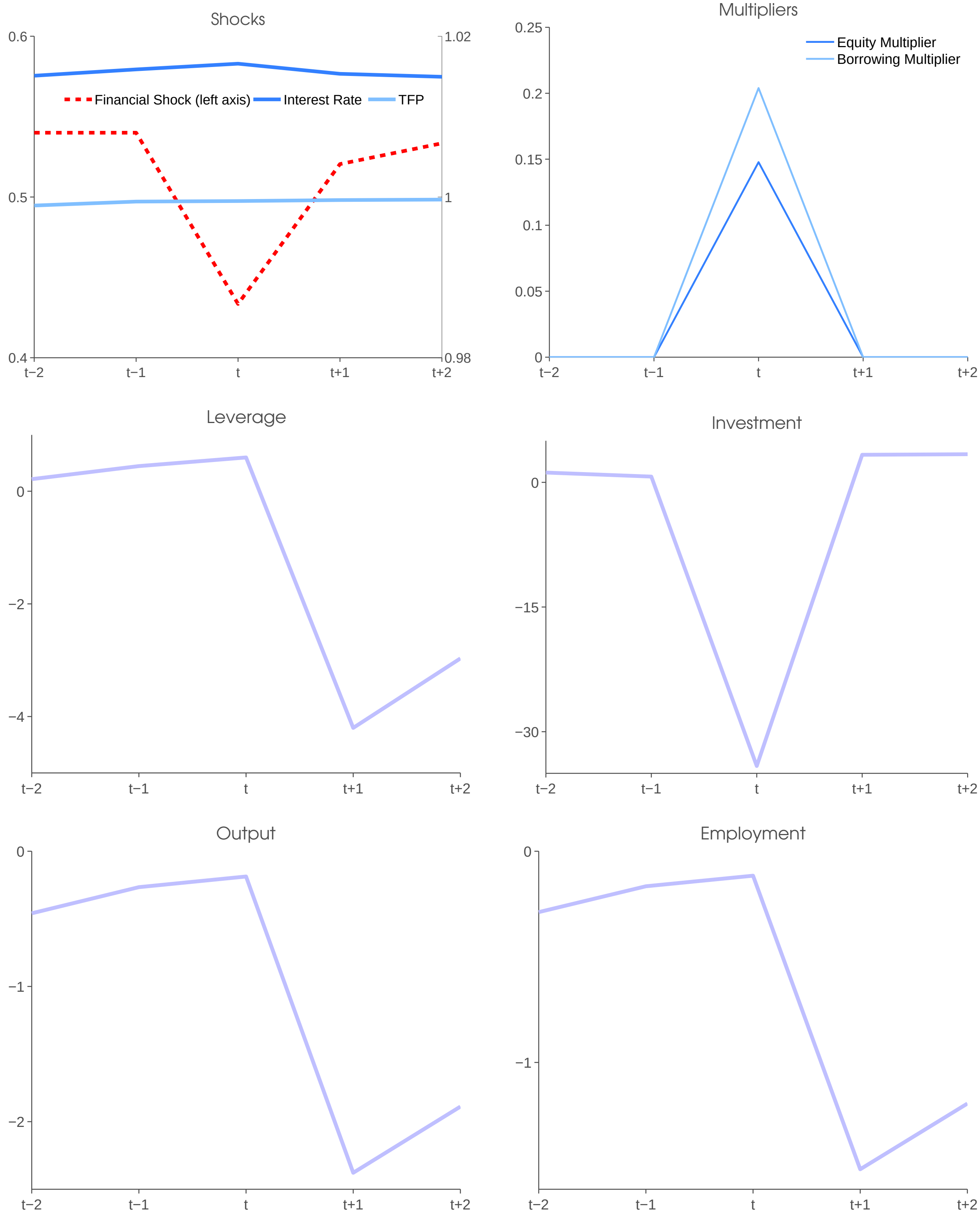

Figure 1: Dynamics of Financial Crises. Values correspond to averages of the main macro variables of the model centered around financial crises events. Financial crises are defined as periods in which credit falls by more than two standard deviations.

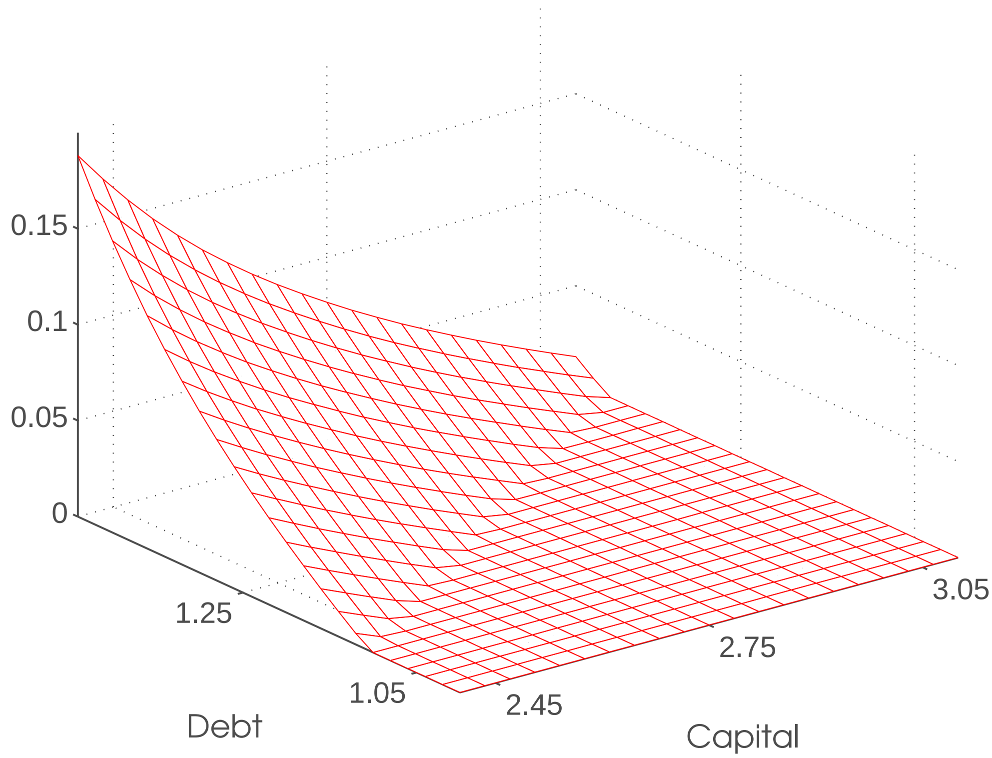

Figure 2: Optimal Bailout Policy $\Upsilon(b, k, \bar{z}, \kappa^L, \bar{R})$ for all values of debt and capital. Shocks to TFP and interest rate are equal to the averages, and $\kappa = \kappa^L$.

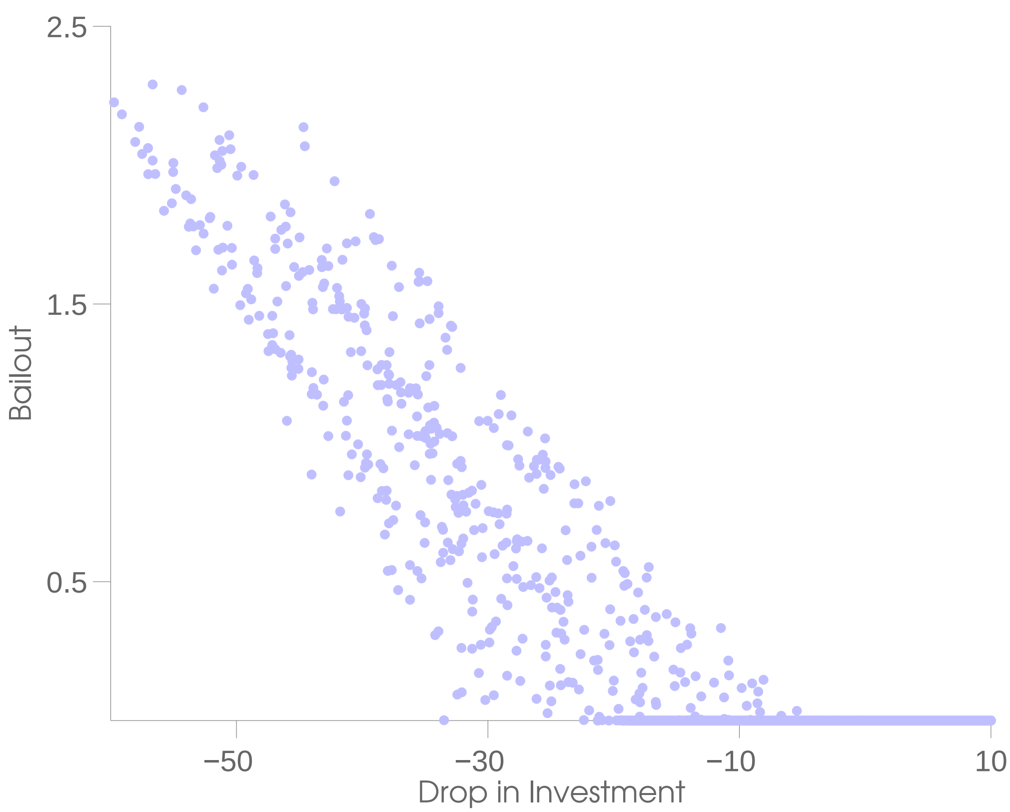

Figure 3: Model Simulations: Relationship between bailouts and investment. Values correspond to investment and bailouts in a long-period simulation.

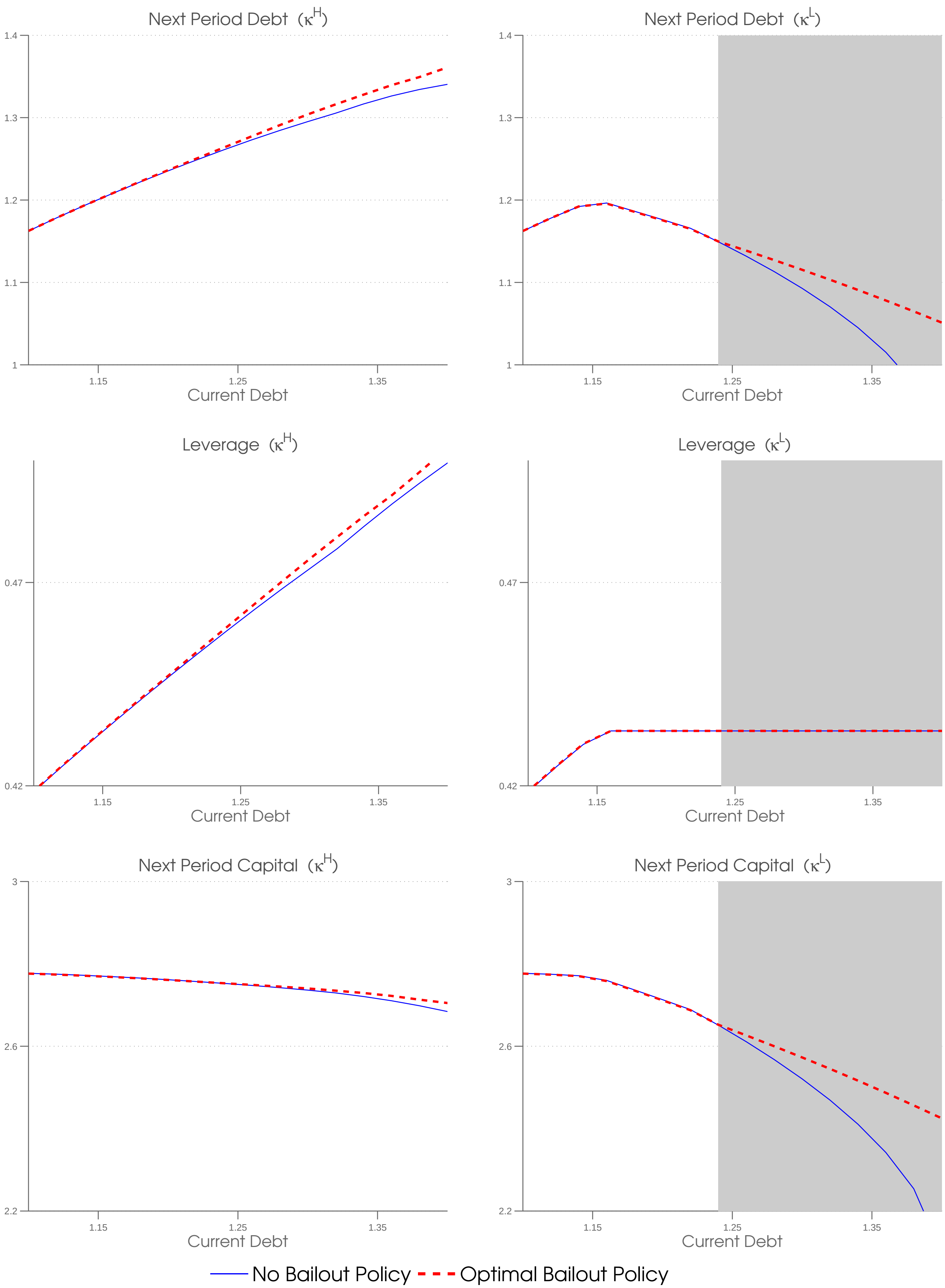

Figure 4: Policy functions for debt, leverage and capital. The plots correspond to values for current capital, TFP and interest rate at their mean values. The left (right) column corresponds to $\kappa^H(\kappa^L)$. The shaded region represents the area where bailouts are strictly positive in the allocations with the Optimal Bailout Policy.

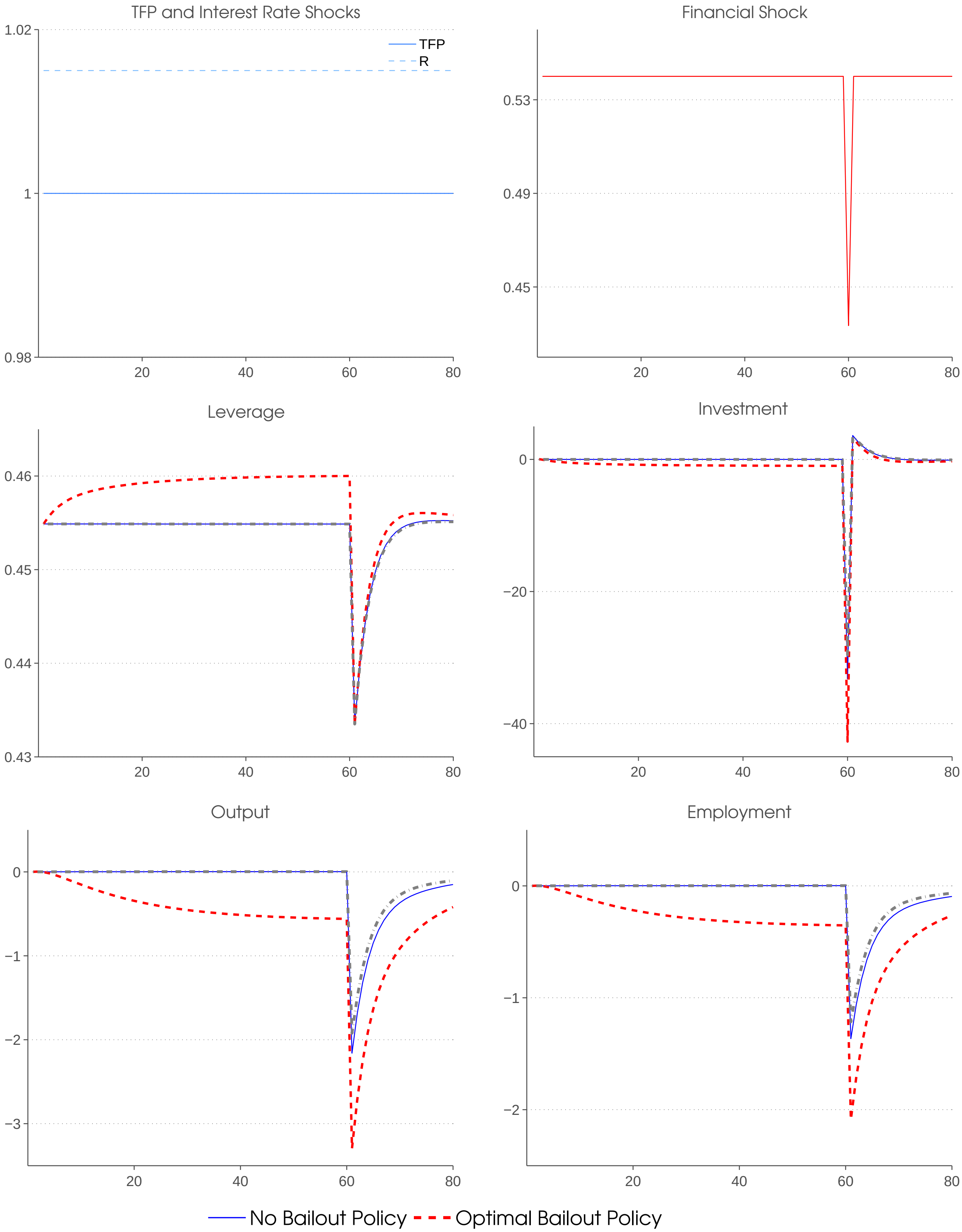

Figure 5: Non-Linear Impulse Response. Output, employment and investment are expressed as percentage deviation from pre-crisis values of the no-bailout economy.

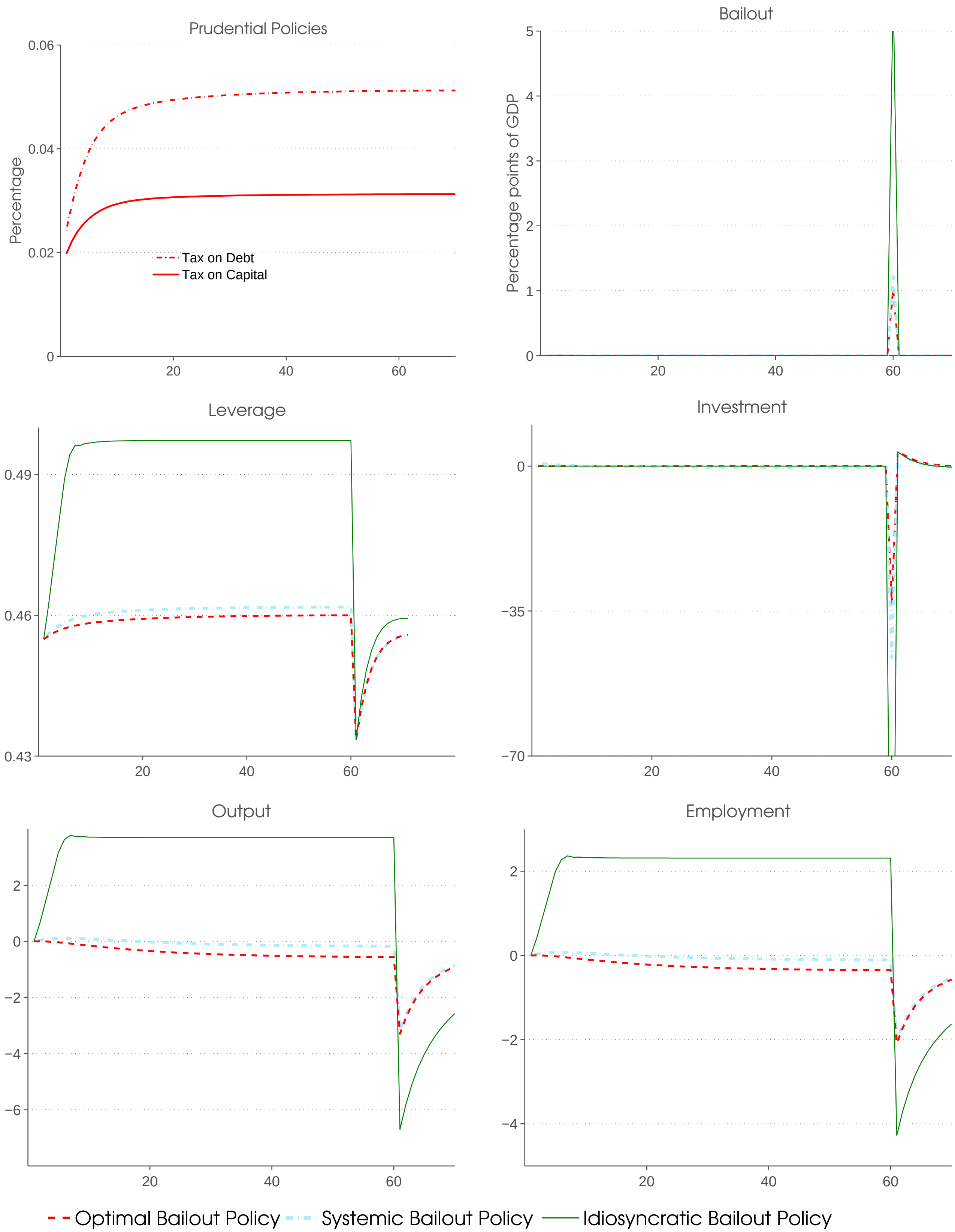

Figure 6: Non-Linear Impulse Response. Output, employment and investment are expressed as percentage deviation from pre-crisis values in the no-bailout economy. Prudential Policies correspond to the implementation of the constrained-efficient allocations using debt relief policy.

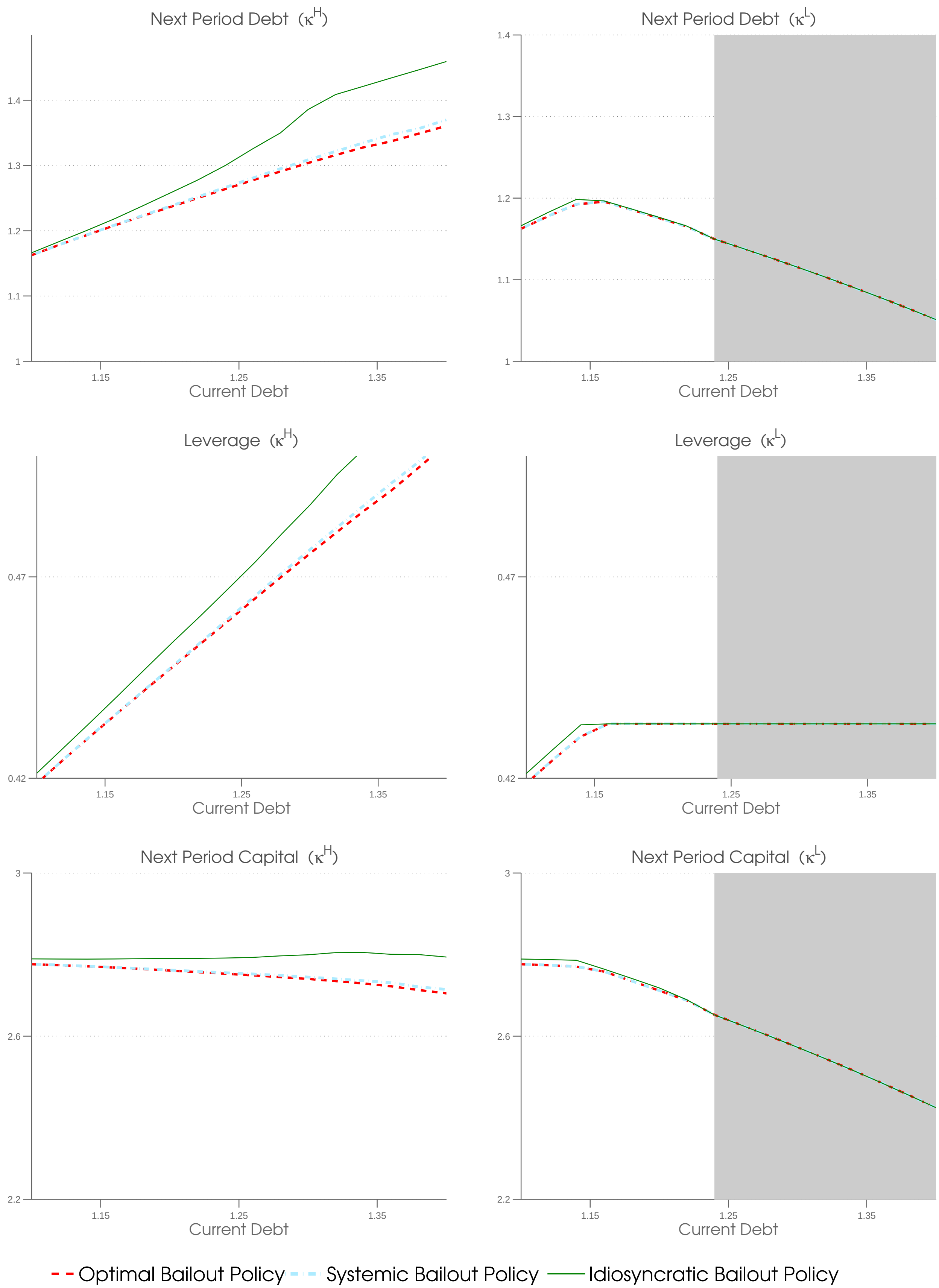

Figure 7: Policy functions for debt, leverage and capital. The plots correspond to values for current capital, TFP and interest rate at their mean values. The left (right) column corresponds to $\kappa^H(\kappa^L)$. The shaded region represents the area where bailouts are strictly positive.

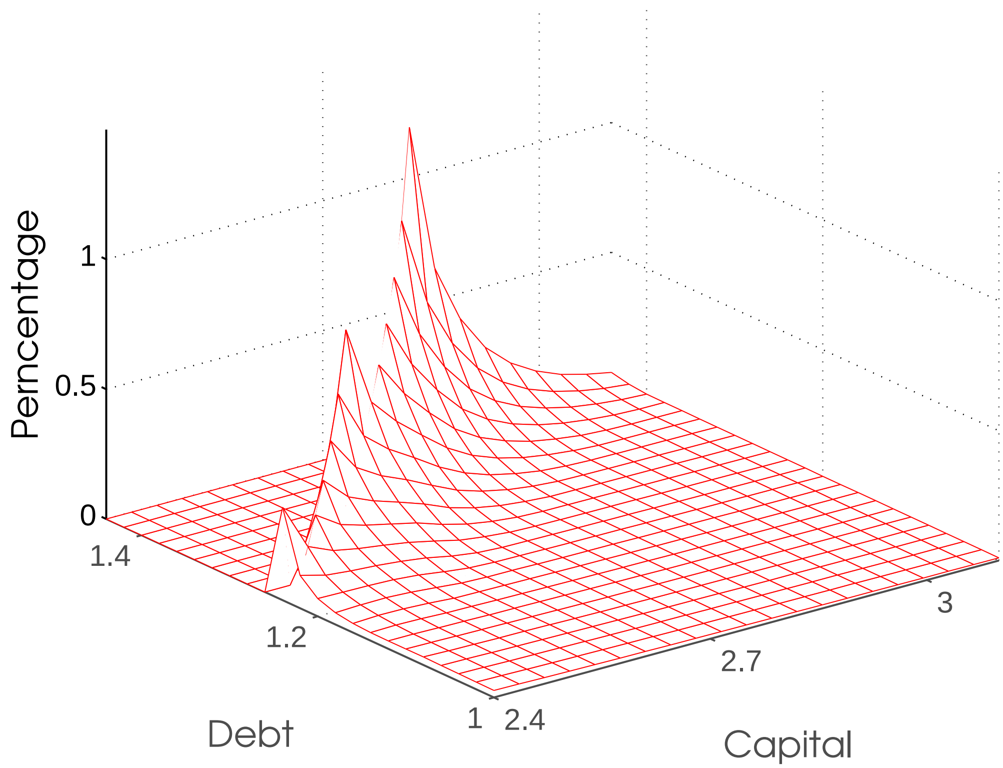

Figure 8: Welfare Gains from optimal policy for $\kappa = \kappa^L$ and average TFP shock

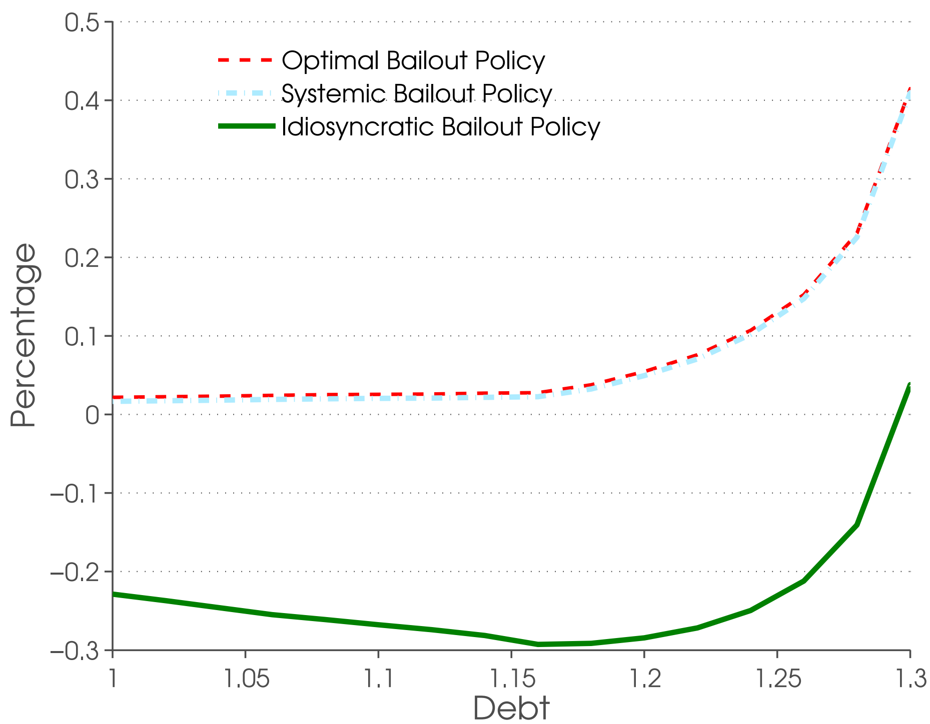

Figure 9: Welfare Gains for different policies for an initial value of capital equal to the mean in the simulation, $\kappa = \kappa^L$ and an average TFP shock

A Decentralization based of planner's problem with equity injections.

An alternative policy to the debt relief policy analyzed in the text is the injection of equity. In order to facilitate a comparison with debt relief policies, we consider an equity injection that is a fraction of the amount of individual debt held by the firm, i.e., the value of new shares issued is such that the value of equity injections equals $e_t b_t$. In particular, the government mandates firms to issue new shares and transfer those shares to the households, which then receive the future dividend payments.

The firms' objective can be expressed as maximize $\mathbb{E}\sum_{t=0}^{\infty} m_t(d_t - e_t b_t)$ subject to the flow of funds constraint $b_t + d_t + i_t + \psi(k_t, k_{t+1}) \leq (F(z_t, k_t, h_t) - w_t n_t)(1-\tau_t^k) + \frac{b_{t+1}}{R_t}(1-\tau_t^b) + e_t b_t + T_t^f$, (8) and (9). The first-order condition with respect to debt yields:

$$1 + \eta_t = R_t \mathbb{E}_t m_{t+1}(1 + \eta_{t+1}(1 - e_{t+1})) + R_t \mu_t. \tag{26}$$

At the beginning of each period, the total number of shares can be renormalized to one. Hence, the rest of the equilibrium conditions remain the same yielding a similar the following proposition.

Proposition 2 *The government can implement the constrained optimal allocations with an appropriate combination of equity injections, taxes on debt and capital, and lump sum taxes. In particular:*

$$e_t = \frac{\Upsilon_t}{b_t}, \quad T_t = \varphi e_t b_t, \qquad T_t^f = \frac{b_{t+1}}{R_t}\tau_t + \tau_t^k(F(z_t, k_t, h_t) - G'(h_t)h_t),$$

$$\tau_t^b = \frac{\mathbb{E}_t m_{t+1}(1+\eta_{t+1}) + \mu_t}{\mathbb{E}_t m_{t+1}(1 + \eta_{t+1}(1 - e_{t+1})) + \mu_t} - 1, \quad \tau_t^k = \frac{\mathbb{E}_t m_{t+1}(1 - \delta + F_{k,t+1} - \psi_{1,t+2})\tilde{h}_{t+1}\partial(\tilde{w}_{t+1})/\partial k_{t+1}}{\mathbb{E}_t m_{t+1}(1+\eta_{t+1})F_{k,t+1}}.$$

where all variables are evaluated at the constrained optimal allocations.

The proof follows the same steps as Proposition 1. Note from (26) that the shadow cost of tightening the equity constraint in a state $t+1$, in which the government is recapitalizing firms, is reduced by a factor of $(1 - e_{t+1})$. Comparing this condition with (24) yields that the tax on debt is smaller than the one required for debt relief. Intuitively, equity injections also involve a cost for shareholders because they perceive a reduction in their ownership of the firm. In addition, the tax on debt is strictly positive only if $\varphi > 0$.