Separation of Powers with Ideological Parties

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Abstract

Separation of powers with checks and balances (SP) is usually regarded as a key institution complementing elections in the control of elected officials. However, some analysts and many politicians also warn that excessive checks on the executive in the presence of polarization may lead to political inaction. We present a political agency model with ideological parties where citizens and politicians care about rents (the valence issue) and policy (a positional issue). We show that SP unambiguously raises voters welfare in highly-polarized non-competitive political environments, because it strengthens both discipline and selection without causing political gridlock (voters can grant the executive the legislative majorities needed to enact reforms). SP also raises voters’ welfare if elections are very effective at disciplining first period incumbents. However, SP may reduce it if most rents go undetected and reform is not a first order issue.

Keywords: Political agency, separation of powers, checks and balances.

JEL Codes: D72, E690, P160

Resumen

La separación de poderes con controles y contrapesos (SP) es usualmente considerada como una institución clave que complementa las elecciones en el control de los políticos electos. Sin embargo, algunos analistas y muchos políticos también advierten que controles excesivos del ejecutivo en presencia de polarización, puede conducir a una inacción política. Presentamos un modelo de agencia política con partidos ideológicos donde los ciudadanos y los políticos se preocupan tanto por las rentas como por la política. A diferencia de las rentas que son valoradas de la misma forma por todos los votantes, distintos votantes tienen diferentes preferencias en cuanto a la política a implementar. Demostramos que la

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Elected officials who are expected to act in favor of the public interest may deviate in pursuit of their private benefit by exerting low effort, favoring narrow constituencies or simply extracting rents. This confronts citizens with one of the main challenges of democracies: reconciling delegation with control.

Modern representative democracies rely on two main institutions to deal with the tradeoff between delegation and control: elections and separation of powers. The literature on political agency has mostly focused on elections as the main institution to control elected officials (see Ashworth 2012; Besley 2005; Duggan and Martinelli 2017; Persson and Tabellini 2000, for surveys). However, it has long been known that elections are an imperfect mechanism of control. Classic writers were well aware of this limitation when they advocated for separation of powers with checks and balances as an additional accountability mechanism (Locke, Montesquieu, Tocqueville, the Federalist Papers).

Current literature has only recently incorporated both electoral accountability (EA) and separation of powers with checks and balances (SP) in formal models of political agency (Persson, Roland, et al. 1997, 2000; Stephenson and Nzelibe 2010). However, this literature abstracts from political parties and political selection, two crucial elements of our analysis. Firstly, with ideological parties voters are often faced with a key dilemma previously overlooked: if they want to vote down an incumbent who has proved dishonest, they must accept a policy change they may not want. This dilemma may weaken EA if voters’ preference for the incumbent’s policy is sufficiently strong, since in this case voters always reelect in order to keep the policy in place. If their concern for policies is not so strong compared to their concern for corruption, policy instability may arise, since voters choose a different party with a different agenda each time the incumbent extracts rents. In this environment, we argue that SP gives voters a tool to better balance delegation and control. Voters can either give the president strong majorities to advance his program or give the opposition the control of congress to strengthen checks and balances.

Secondly, previous analysis of the interplay of EA and SP focused on moral hazard abstracting from
selection issues (Persson, Roland, et al. 1997, 2000; Stephenson and Nzelibe 2010). In our model, SP forces the executive and the legislature to agree in order to implement policies and extract rents. As a result, comparing with a situation with only one body (and hence only EA), there will be less instances where voters learn about the politicians in power, and where they are able to make a good selection. Thus, selection becomes a key element to consider in order to evaluate the effects of SP.

The main goal of the paper is to study the effects of the inclusion of SP in a political agency model with ideological parties, moral hazard and adverse selection. As a benchmark, we consider a scenario with only one body (the executive) that rules in terms of policies and rents. In this environment, elections represent the only mechanism of control. We then analyze the effect of introducing a second elected body, the legislature. The introduction of SP has three main effects in our model.

First, it strengthens checks on the executive. This is particularly clear in the case of term limited executives in which elections totally fail at controlling rent extraction in the last period. In periods in which the president can run for reelection, SP may induce more or less discipline so the net effect can go either way. We analyze these effects in detail. We show in particular that in contexts of strong polarization where elections fail at controlling rents, SP unambiguously reduces rent extraction and raises voters’ welfare.

Second, in line with the usual concerns about the risk of political gridlock associated to SP, we find that SP tends to reduce the ability of the executive to enact reforms (it reduces the system’s “decisiveness“, to use McCubbins (2000) terminology), but it also strengthens the system capacity to maintain a reform once it has been enacted (it strengthens the system “resolutness” in McCubbins wording). So in our model SP is associated to policy stability. Notice however that reduced policy activism is not necessarily negative from the point of view of citizens, since it may prevent non-beneficial reforms from being adopted.

Third, SP may weaken the ability of voters to select politicians. Because SP reduces the opportunities that dishonest politicians have to extract rents, voters are less able to separate honest from dishonest politicians with than without SP. This would not matter if SP totally curbed rent extraction, but it matters as the mechanism is imperfect. Thus, SP may reduce welfare when political selection is a first order issue.

In our model, citizens and politicians care not only about rent extraction —or more generally a valence issue—, but also about an independent positional policy issue. There are three groups of citizens. Two of them (left- and right-wing citizens) have strong preferences but of opposite sign regarding the positional issue. The third group leans towards one or the other policy depending on the

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1Tim Besley and Hannes Mueller are currently working on a model of EA and SP that also incorporates moral hazard and adverse selection (Besley and Mueller 2018).

2Besley and Mueller (2017), for example, show that stricter constraints on the executive are associated with less volatile investment.
prevailing circumstances. We assume there are two ideologically motivated political parties controlled by left- and right-wing citizens. Parties’ policy preferences are publicly known. In both parties, there are honest politicians who dislike rent extraction, and dishonest politicians who like it and extract it if the circumstances are appropriate. Citizens do not directly observe whether the politicians in office are honest, but they imperfectly observe whether politicians extracted rents. We model the imperfect observation of rents assuming that there are observable and unobservable rents. Unobservable rents are intended to capture the costs that voters have because of poor political selection. The larger the unobservable rents, the greater the importance of political selection. We introduce these rents as a simple form of representing the imperfection of both elections and SP at controlling rents. In this modeling strategy, the imperfection of these mechanisms of control is linked to voters imperfect information about what politicians in office do.

We represent SP as a government with two branches, the executive and the legislature, elected directly by voters, so we focus on presidential systems. There is a decision procedure that requires the participation of both branches and exploits the opposition of interests to curb rent extraction. We assume that this mechanism à la Persson, Roland, et al. (1997, 2000) works well to impede the extraction of observable rents only if different parties control the executive and the legislature (there is a divided government).\(^3\) If the same party controls both branches (there is a unified government), the executive and the legislature may in principle agree to extract rents.\(^4\) The mechanism does not work regarding unobservable rents: a dishonest executive can always extract these rents.

Our analysis highlights a tradeoff that arises when ideological parties are included in an electoral accountability model: policy stability and political discipline/selection. The implicit threat of voting down an incumbent who extracted rents is now linked to a policy change. Therefore, the effectiveness of the discipline/selection mechanism implies policy instability. SP gives voters an additional tool to handle this tradeoff: the option of majority vs minority government. If a majority of voters are interested in a reform, they can give the president a strong legislative support in his first term, and vote for a minority or divided government in his last term. If the president extracts rents in the first term, voters can remove him without changing policy by choosing a divided government for the following period. In the first term, when the president can run for reelection, control rests on electoral accountability, and in the second, when the president is term limited, control rests on strong SP through a minority government.

We show in particular that the introduction of SP is specially valuable in contexts of high polarization. Although many scholars have emphasized the potential gridlock that strong checks may produce

\(^3\)Instead of the introduction of unobservable rents to model the imperfection of EA and SP to control rent extraction, we could capture this imperfect control by relaxing the assumption that the probability of rent extraction is zero with divided government in the SP environment.

\(^4\)In our model, divided and unified governments are the same as minority and majority governments, respectively. We discuss the connection between our model and the extensive literature on divided government in Section 7.
in this context, we show that voters unambiguously benefit from $SP$ when there is strong polarization because the alternative mechanism of control, the election, fails in this environment. This result challenges the identification of strong checks with political stalemate as $SP$ restores some political discipline and selection, at the same time that provides policy stability.

We think our model without $SP$ can also be seen as a stylized model of hyper-presidentialism. This is a common issue in Latin America and other developing regions (Chavez 2004; Linz 1990; O’Donnell 1998; Rose-Ackerman et al. 2011). Our model prediction of limited control of the executive with little discipline and poor selection in contexts of high polarization is resonant of numerous accounts of Latin American politics. Similarly, our model prediction of policy instability associated to corruption accounts for the numerous cases of policy switches that have followed corruption scandals. The recent “progresist era” followed a period of pro-market reforms under the Washington consensus in the nineties in which the privatization of state owned enterprises opened opportunities for the extraction of rents. Many citizens turned their back to these policies they had previously supported partly because of these corruption scandals (Argentina with Menem, Peru with Fujimori and Venezuela with Carlos Andrés Pérez are just three emblematic examples). More recently, voters have moved away from “progresist” parties and leaders partly because of the corruption scandals associated to public investment and the managing of state owned enterprises (Argentina with the Kirchner, Brazil with the labor party —the PT of Luis Inacio Lula Da Silva and Dilma Rouseff).

After this introduction, the paper proceeds as follows. We present a selective review of the related literature in the next section. Section 3 contains a description of the setting. In sections 4 and 5, we study the model without and with separation of powers, respectively. In Section 6 we compare policy outcomes and welfare in equilibrium without and with $SP$. Section 7 discusses the main assumptions of the model. The paper ends with a few concluding remarks in Section 8. We present the proofs of the propositions in the Appendix.

2 Related literature

It is commonly accepted that $SP$ plays at least two different roles in democracies, namely the control of public officials and the protection of minorities. In its former role, $SP$ is one of the two pillars of political accountability. It provides the “horizontal accountability” that complements the “vertical accountability” provided by elections. The latter role of $SP$ consists of protecting the minorities against

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5The term “horizontal accountability” was introduced by O’Donnell (1998) and is not free of controversies. In particular, Moreno et al. (2003) argue that there is no accountability between two elected branches of government and rather talk about “horizontal exchange”. Mainwaring and Welna (2000) use the term “intrastate accountability”. Besley and Mueller (2018) talk about “internal” and “external” control, to refer to the control by another branch of government and citizens, respectively.
the “tyranny of the majority”. While these two roles are intertwined in the real world, they can and have been analytically separated in order to better understand how SP impacts on these two different fronts. In the present paper, we focus on the first role of SP.

Formal political agency theory paid little attention to SP and focused almost exclusively on electoral accountability until the seminal papers of Persson, Roland, et al. (1997, 2000). They embedded SP in moral hazard models of electoral accountability with retrospective voting on the lines of Austen-Smith and Banks (1989), Barro (1973), and Ferejohn (1986) (see Persson and Tabellini 2000, for a complete survey of models of political agency with retrospective voting). Stephenson and Nzelibe (2010) extend the model to include ‘Opt-in Checks’, i.e. an institutional arrangement in which the president can seek congressional authorization or act unilaterally.

The pure moral hazard model has limitations for the analysis of situations in which the heterogeneity among politicians plays a prominent role in voters decisions. Furthermore, the equilibria in those models may not be robust to the introduction of heterogeneity (Besley 2005; Fearon 1999). Therefore, the literature has moved towards models that combine moral hazard and adverse selection (Ashworth 2012; Besley 2005). Research on the interaction of EA and SP in this richer framework is scarce, though. Besley (2005, section 3.4.8) presents a “multiple agents” model with all these ingredients, i.e. two or more branches of government, moral hazard and adverse selection. We build on this model, adding ideological parties. This allows us to discuss conditions and possible mechanisms through which SP may cause political gridlock.

At the center of our analysis lies the tradeoff between delegation and control in democracies. We are interested in exploring the possibility that mechanisms of control limit policy change. In this regard, our paper is related to the literature on veto players that followed the seminal paper of Tsebelis (1995). Similar concerns have motivated an empirical political science literature that analyzes hyper-presidentialism and the arising of strongmen politics as responses to the belief that excessive checks on the executive have caused political gridlocks (Carrión 2006; Fukuyama 2017; Howell and Moe 2016; Kenney 2000; O’Donnell 1994; Rose-Ackerman et al. 2011). In Latin America, Linz (1990) and Shugart and Mainwaring (1997) argue that political gridlocks caused by non-majority governments contributed to the breakdown of democracies.

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6This was one of the main concerns of classical advocates of SP (Locke, Montesquieu, Madison, Tocqueville) and is also present in modern papers like Aghion et al. (2004, specially section IV), Buchanan (1975), Buchanan and Tullock (1962), Hayek (1960), and Maskin and Tirole (2004).

7Stephenson and Nzelibe (2010) however present a model in which “adding a congressional veto does not change the ex ante probability of policy change.” Using a retrospective voting logic, they argue that the direct negative effect of SP on the probability of policy change will be compensated by an indirect effect arising from citizens’ change of their reelection strategy. Knowing the new institution reduces the likelihood of policy changes, citizens accommodate their reelection strategy to undo this effect. This “surprising result”, as Stephenson and Nzelibe put it, does not arise in models that incorporate political selection, like ours, since a retrospective voting strategy would not be credible in the presence of adverse selection (see however Snyder and Ting (2008) for a model that combines retrospective voting and selection). Tommasi et al. (2014) also challenge the widespread view that a higher number of veto players is necessarily associated to less policy activism.
Departing from most political agency models, we assume politicians have ideological preferences and belong to ideological parties à la Alesina (1987, 1988), i.e. parties that care about the performance of the society. This assumption has been very useful in the study of policy convergence (Alesina 1988; Alesina and Rosenthal 2000), political business cycles (Alesina 1987), and divided governments (Alesina and Rosenthal 1995, 1996), but it has played almost no role in political agency models. The only exception we are aware of is Besley’s model on polarization and political competition (Besley 2005, section 3.4.1). His model highlights the possibility that partisan preferences undermines electoral accountability, an issue we revisit in the present paper. But Besley’s model of polarization does not analyze SP and political gridlock, themes that are at the center of our analysis.

Persson, Roland, et al. (1997, 2000) and Stephenson and Nzelibe (2010) treat SP as an exogenously given institution. We do the same in the present paper. Acemoglu et al. (2013), Aghion et al. (2004), and Forteza and Pereyra (2018) endogenize SP assuming there is a referendum in which citizens vote on SP, but they do not analyze electoral accountability. Aghion et al. (2004) endogenize SP introducing an initial vote over the number of citizens that is needed to ex-post block the ruling politician’s action (be it a reform or expropriation). Their model does not have elections. Acemoglu et al. (2013) also endogenize SP through an initial vote, in this case between two discrete options. There are elections in their model, but they play no role in providing incentives to politicians. Forteza and Pereyra (2018) also present a model in which citizens vote over granting the executive special powers (thus weakening checks and balances), but have no elections. Clearly, none of these models are designed to analyze the interaction between elections and SP.

3 The setting

Consider a model of a democracy with two periods, $t = 1, 2$.

Elections. There are two elections, one at the beginning of each period. Because of term limits, only new entrants compete in the first election. Incumbents run for reelection in the second election.

Policies and rents. Citizens care about a position and a valence policy issue. Policy regarding the position issue can take two values $p_t \in \{0, 1\}$. For concreteness, we identify the valence issue with rents. There are observable and unobservable rents. The amount of observable rents extracted in the first period is observed by voters at the end of that period, before elections take place. Unobservable rents are observed only at the end of period 2, when payoffs are distributed. The maximum amount of rents that politicians can extract in each period — $r_t \in [0, \bar{r}]$ and $\rho_t \in [0, \bar{\rho}]$ for observable and unobservable

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Robinson and Torvik (2016) argue that in developing countries, unlike in the US, presidential regimes lack checks on the executive, so the real choice is between parliamentary and unchecked presidential regimes. They then model the choice between these two regimes.
of rents, respectively — are random and independently distributed with cumulative distribution functions \( G(r) \) and \( F(\rho) \). We identify the unconditional expected rents as 

\[
E[r] = \int_0^r r dG(r), \quad E[\rho] = \int_0^\rho \rho dF(\rho).
\]

Let \( r_{X,t} \in [0, r_1] \), \( r_{C,t} \in [0, r_1 - r_{X,t}] \) and \( \rho_{X,t} \in [0, \rho_1] \) represent the observable rents effectively extracted by the executive and the congress and the unobservable rents extracted by the executive, respectively.

**Citizens.** Citizens preferences regarding the policy \( p_t \) depend on the state of nature \( s_t \), which is uncertain and takes two values: \( s_t = 1 \) with probability \( q \) and \( s_t = 0 \) with probability \( (1 - q) \). Citizen \( i \) bliss point regarding this position issue is \( s_t + \delta_i \), with \( \delta_i \in \mathbb{R} \). Deviations of the policy from the bliss point cause utility losses proportional to the expected square deviations. Citizens utility is linear in rents. Citizen \( i \) expected utility is:

\[
V_t = v_{i,1} + \beta v_{i,2}
\]

\[
v_{i,t} = -aE_s [(p_t - s_t - \delta_i)^2] - r_{X,t} - r_{C,t} - \rho_t; \quad t \in \{1, 2\},
\]

where \( \beta \in [0, 1] \) is a discount factor, and \( a \geq 0 \) is a parameter that captures the weigh of the position relative to the valence issue.

Depending on their preferences regarding the positional issue, citizens can be classified in three different groups: left-wing citizens, with \( \delta_L = -1 \); center or swing citizens, with \( \delta_S = 0 \); and right-wing citizens, with \( \delta_R = 1 \). Left- and right-wing citizens prefer policies \( p_t = 0 \) and \( p_t = 1 \), respectively, no matter the state of nature. Swing citizens prefer policy \( p_t = s_t \). We assume that \( L \) and \( R \) citizens each represent less than \( 1/2 \) of voters, so swing citizens are decisive (for a formal proof, see Lemma 1 in the Appendix).

**Politicians.** Politicians are citizens so their preferences when they are out of office are described by Equation (1). When they are in office, politicians enjoy “ego rents” — they value being in office per se — and some politicians — the dishonest ones — derive utility from extracting monetary rents. All politicians in office dislike rents extracted by the politician in the other branch. Politician \( j \) expected utility is:

\[
U_j = u_{j,1} + \beta u_{j,2}
\]

\[
u_{j,t} = \begin{cases} 
- aE_s [(p_t - s_t - \delta_j)^2] + b_j (r_{X,t} + \rho_t) - r_{C,t} + E, & \text{if } j \text{ in } X \text{ office in } t \\
- aE_s [(p_t - s_t - \delta_j)^2] + b_j r_{C,t} - (r_{X,t} + \rho_t) + E, & \text{if } j \text{ in } C \text{ office in } t \\
- aE_s [(p_t - s_t - \delta_j)^2] - (r_{X,t} + r_{C,t} + \rho_t), & \text{if } j \text{ is out of office in } t
\end{cases}
\]

where \( E > 0 \) are the “ego rents”, and \( b_j = b_D > 1 \), if the politician is dishonest, and \( b_j = -1 \), if the politician is honest. Citizens observe the party affiliation of each politician, but not if he is honest or dishonest. Individuals with \( \delta_j < -1/2 \) and \( \delta_j > 1/2 \) prefer \( p_t = 0 \) and \( p_t = 1 \), respectively, rather than the alternative, no matter the state of nature. We chose \( \delta_L = -1 \) and \( \delta_R = 1 \) to simplify computations.
dishonest.\textsuperscript{10}

**Parties.** There are two political parties, \(L\) and \(R\), controlled by left- and right-wing citizens, respectively. Each politician belongs to one of these parties. Those from the \(L\) party, have \(\delta_j = \delta_L = -1\), and then prefer \(p_t = 0\) to \(p_t = 1\). Those from the \(R\) party, have \(\delta_j = \delta_R = 1\) and therefore have the opposite preferences. Because of the bipartisan political system, only four type of citizens can be political candidates: \(j \in \{HL, DL, HR, DR\}\), where \(HL\) stands for “honest” from party \(L\), \(DL\) for “dishonest” from party \(L\), \(HR\) for “honest” from party \(R\), and \(DR\) for “dishonest” from party \(R\). As indicated above, these four types differ in the values of the parameters \(\delta_j\) and \(b_j\): \(\delta_{HL} = \delta_{DL} = -1, \delta_{HR} = \delta_{DR} = 1\), \(b_{HL} = b_{HR} = -1\) and \(b_{DL} = b_{DL} = b_D > 1\). The unconditional probability that a politician is honest is \(q_H\), and it is the same in both parties.

**The structure of government.** There is a government that can be organized in one or two bodies. In the former case, we say there is no separation of powers and this unique body —the executive— chooses policies \(p_t, r_{Xt} \in [0, r_t]\) and \(\rho_{Xt} \in [0, \rho_t]\) by its own. In the latter case, there is separation of powers between the executive (\(X\)) and the legislature or congress (\(C\)). With separation of powers, the executive proposes \(\tilde{p}_X, t \in \{0, 1\}, \tilde{r}_{Xt} \in [0, r_t]\) and \(\tilde{\rho}_{Xt} \in [0, \rho_t]\) and the legislature proposes \(\tilde{p}_C, t \in \{0, 1\}\) and \(\tilde{r}_{Ct} \in [0, r_t]\). The outcome is determined by a system of checks and balances between the executive and the legislature.

**Checks and balances.** The institution of checks and balances works as follows. A policy \(p_t\) is implemented if the two branches of government agree on it. In case of disagreement, the status quo policy prevails. Therefore, there is no policy change if there is a divided government. Indeed, with a divided government, politicians at each office are from different parties, and will have different policy proposals.\textsuperscript{11} The effectiveness of the system in controlling rents depends on whether the government is divided or unified. With a divided government politicians cannot extract observable rents: \(r_t = 0\). In the presence of a unified government (politicians at both branches are from the same political party), politicians extract observable rents only when both the executive and the legislature agree on that. The system is not totally effective in controlling rents though, because the executive can extract unobservable rents even with a divided government. Thus, the outcome with separation of powers is determined as follows:\textsuperscript{12}

\textsuperscript{10}If there is no separation of powers, there is no legislature so the corresponding row in equation (2) does not apply and there are no legislature’s rents.

\textsuperscript{11}The value of \(\delta_j\) is observed by voters, so politicians do not have incentives to propose a policy different from their preferred one.

\textsuperscript{12}We borrow from Besley (2005, section 3.4.8) the modeling of the policy process with the “unanimity rule” (first line in (3)) and the status quo outcome in case of disagreement (second line in (3)). Alesina and Rosenthal (1995, 1996, 2000) assume that in case of disagreement an intermediate outcome arises, so divided government leads to policy moderation. In both versions, divided government reduces the ability of the executive to implement reforms. We chose Besley’s version
Timing without separation of powers. When there is no separation of powers events unfold as follows:

1. Nature chooses:
   (a) Period zero policy: \( p_0 = 0.13 \)
   (b) Two candidates: one candidate from each political party. Nature chooses candidates at random, and the probability that a candidate is honest is \( q_H \), the same for both parties.

2. At the beginning of period 1, voters elect one of the two candidates for the executive office. They observe candidates’ policy preference parameter \( \delta_j \) but ignore whether they are honest, i.e. they ignore their preference parameter \( b_j \).

3. Nature randomly chooses period 1 state of nature \( s_1 \), and the maximum amount of rents politicians can extract (\( r_1 \) and \( \rho_1 \)).

4. The executive chooses \( p_1, r_{X1} \) and \( \rho_{X1} \). He observes previous Nature’s moves, including period 1 state of nature.

5. Nature chooses a politician from the opposition party to run as a candidate in the second election. As in the first election, the probability that the candidate is honest is \( q_H \).

6. At the beginning of period 2, citizens vote again. They can reelect the incumbent or vote for the challenger. At the election time, citizens observe (i) \( s_1 \), (ii) \( p_1 \), (iii) \( r_{X1} \) and (iv) whether the because we wanted to allow for the strongest form of stalemate. This modeling aims at capturing the usual concern for the potential gridlock that is associated with separation of powers (Kenney 2000; O’Donnell 1994, 1998). The rent sharing rule is also inspired in Besley (2005, section 3.4.8), but in our framework observable rents can be extracted only with unified governments. We assume that a divided government does not extract observable rents, because of effective checks due to an opposition of interests à la Persson, Roland, et al. (1997). The assumption that effective rent control requires a divided government addresses the common concern about the limited effectiveness of \( SP \) to control the executive when the same party controls the executive and the legislature (Alt and Lassen 2008; Díaz-Cayeros and Magaloni 2003; Moreno et al. 2003).

\[ p_t = \begin{cases} \hat{p}_{X,t} & \text{if } \hat{p}_{X,t} = \hat{p}_{C,t} \\ p_{t-1} & \text{if } \hat{p}_{X,t} \neq \hat{p}_{C,t} \end{cases} \]

\[ r_{X,t} = r_{C,t} = \begin{cases} \hat{r}_{X,t} & \text{if there is a unified government and } \hat{r}_{X,t} = \hat{r}_{C,t} \in [0, \frac{r}{2}] \\ 0 & \text{otherwise} \end{cases} \]

\[ \rho_t = \hat{\rho}_{X,t} \]

\( ^{13} \)For concreteness, we assume \( p_0 = 0 \). The analysis is symmetric if \( p_0 = 1 \).
incumbent and challenger belong to party \( L \) or \( R \), i.e. they observe \( \delta_j \), but do not observe (i) \( \rho_1 \) and (ii) whether the incumbent and the challenger are honest or dishonest, i.e. \( b_j \).

7. Nature randomly chooses period 2 state of nature \( s_2 \), and the maximum amount of rents politicians can extract (\( r_2 \) and \( \rho_2 \)).

8. The executive chooses \( p_2, r_{X2} \), and \( \rho_{X2} \). He observes previous Nature’s moves, including period 2 state of nature.

9. The game ends and payoffs are computed.

**Timing with separation of powers.** When there is separation of powers events unfold as follows:

1. Nature chooses:
   
   (a) Period zero policy: \( p_0 = 0 \).

   (b) Four candidates: one candidate from each political party for each office. Nature chooses the candidates at random, and the probability that a candidate is honest is \( q_H \), the same for both parties and offices, and independent of other candidates.

2. At the beginning of period 1, voters elect one politician for each branch of government from the pool of candidates. They observe candidates’ policy preference parameter \( \delta_j \) but ignore whether they are honest, i.e. they ignore their preference parameter \( b_j \).

3. Nature randomly chooses period 1 state of nature \( s_1 \), and the maximum amount of rents politicians can extract (\( r_1 \) and \( \rho_1 \)).

4. The executive proposes \( \tilde{p}_{X,1}, \tilde{r}_{X,1} \) and \( \tilde{\rho}_1 \). He observes previous Nature’s moves, including period 1 state of nature.

5. The legislature proposes \( \tilde{p}_{C,1} \) and \( \tilde{r}_{C,1} \). He observes the same as the executive plus the executive’s move.

6. The first period outcomes are determined according to the institutional rules (3).

7. Nature randomly chooses candidates to run as challengers for the executive and the legislature from the party that is not in office in the corresponding branch of government. Each of them is honest with probability \( q_H \).

8. At the beginning of period 2, citizens vote again. They can reelect the incumbents or vote for the challengers in each branch of government (i.e. they can reelect one, both or none of the incumbents). At the election time, citizens observe (i) \( s_1 \), (ii) \( \tilde{p}_{X,1}, \tilde{p}_{C,1} \), (iii) \( \tilde{r}_{X,1}, \tilde{r}_{C,1} \) and (iv)
whether the incumbents and challengers belong to party $L$ or $R$, i.e. they observe $\delta_j$, but do not observe (i) $\hat{p}_1$ and (ii) whether the incumbents and the challengers are honest or dishonest, i.e. $b_j$.

9. Nature randomly chooses period 2 state of nature $s_2$, and the maximum amount of rents politicians can extract ($r_2$ and $\rho_2$).

10. The executive proposes $\tilde{p}_{X,2}$, $\tilde{r}_{X,2}$, and $\tilde{\rho}_2$. He observes previous Nature’s moves, including period 2 state of nature and the maximum amount of rents that can be extracted.

11. The legislature proposes $\tilde{p}_{C,2}$ and $\tilde{r}_{C,2}$. He observes the same as the executive plus the executive’s move.

12. The second period outcomes are determined according to the institutional rules (3). The game ends and payoffs are computed.

**Solution concept.** Both games, without and with $SP$, are incomplete information games so we look for perfect Bayesian equilibria (PBE).

## 4 No Separation of Powers (NSP)

The inclusion of ideological parties in a standard model of EA highlights a tradeoff between policy stability and discipline/selection. The model shows weak electoral accountability when voters care much for the positional issue. Voters reelect the incumbent irrespective of corruption if their concern for the positional issue is sufficiently strong. In this case there is neither discipline nor selection. If voters do not place so much weight on the positional issue, elections provide some discipline and selection. But in this case, policies become unstable and directly linked to corruption: the incumbent is removed from office, causing a policy switch each time he extracts rents.$^{14}$

We present in Proposition 1 a PBE in which voters reelect the first period incumbent iff they observe no rent extraction, and dishonest politicians “discipline”, i.e. decide not to extract observable rents, iff observable rents are sufficiently small. This equilibrium exists iff moderate voters’ expected gains from the implementation of their desired policy (relative to the undesired one) is not larger than the expected utility cost of rent extraction. This proposition embeds as a particular case the classic result of the standard model of electoral accountability.

Before presenting the first proposition, we need the following notation. Let $V(R)$ and $V(L)$ be the expected discounted utility of electing party $R$ and $L$, respectively, in the first election.$^{15}$

---

$^{14}$Consistent with this result, Besley and Mueller (2017) present evidence that countries with strong constraints on the executive have higher growth stability.

$^{15}$Clearly, $V(R)$ and $V(L)$ depend on the equilibrium which is played. To make notation lighter, we omit this argument.
Proposition 1. The electoral accountability equilibrium with NSP. If the following condition holds

\[ a(2q - 1) \in [-q_H(r^e + \rho^e), q_H(r^e + \rho^e)], \] (4)

then, there is a PBE in which:

1. Voters reelect an incumbent who did not extract rents in the first period and do not reelect an incumbent who did extract rents.

2. An honest incumbent never extracts rents: \(r_1 = r_2 = \rho_1 = \rho_2 = 0\).

3. A dishonest incumbent extracts the maximum amount of unobservable rents in both periods (\(\rho_1\) and \(\rho_2\)) and of observable rents in the second period (\(r_2\)).

4. A dishonest incumbent chooses a threshold \(r_{NSP}\) such that, in the first period, he does not extract observable rents, i.e. \(r_{X1} = 0\), if \(r_1 \leq r_{NSP}\) and extracts \(r_{X1} = r_1\), otherwise. The threshold is

\[ r_{NSP} = \begin{cases} r_L = \frac{\beta}{b_D} [a(3 - 2q) + (b_D + 1 - q_H)(r^e + \rho^e) + E], & \text{if } L \text{ incumbent, and} \\ r_R = \frac{\beta}{b_D} [a(2q + 1) + (b_D + 1 - q_H)(r^e + \rho^e) + E], & \text{if } R \text{ incumbent.} \end{cases} \] (5)

5. In the first election, voters vote for the R candidate if \(V(R) - V(L) > 0\), for the L candidate if \(V(R) - V(L) < 0\), and flip a coin if \(V(R) - V(L) = 0\), where

\[
V(R) - V(L) = a(2q - 1)[1 + \beta(P(R) + P(L) - 1)] + (1 - q_H) \int_{r_L}^{r_R} r dG(r)
+ \beta q_H (1 - q_H)(G(r_L) - G(r_R))(r^e + \rho^e),
\] (6)

and

\[
P(j) = q_H + (1 - q_H)G(r_j), j \in \{L, R\}\]

6. The L and R candidates implement \(p_1 = p_2 = 0\) and \(p_1 = p_2 = 1\), respectively.

Condition (4) compares swing voters expected gains from the adoption of the policy that is more likely to match the state of nature (\(a(2q - 1)\)), and the expected losses due to rent extraction associated with the reelection of a dishonest incumbent, \(q_H(r^e + \rho^e)\). The expected gains from implementing \(p = 1\) rather than \(p = 0\) are:\(-a(1 - q) - (-aq) = a(2q - 1)\). If the first period executive turns out to be dishonest, the expected cost (regarding rent extraction) of reelection him rather than voting for the challenger is: \((r^e + \rho^e) - (1 - q_H)(r^e + \rho^e) = q_H(r^e + \rho^e)\). When voters policy concerns are not stronger than the expected cost of rent extraction, the strategy profile described in the proposition conforms an equilibrium. Otherwise, voters are willing to pay rents in order to have their preferred policy implemented, so they always reelect the first period incumbent and this equilibrium breaks down.
Note that the condition (4) is fulfilled and the semi-separating equilibrium described in Proposition 1 exists if voters do not care about the positional issue, i.e. if $a = 0$. In this case, there is no substantive basis to distinguish between the two parties in our model and all dishonest politicians have the same discipline threshold $r_L = r_R$. Voters are indifferent between the two candidates competing in the first election $V(L) = V(R)$. This special case of our model is in essence the canonical electoral accountability model with moral hazard and adverse selection.\footnote{There are still some differences, like the existence of unobservable rents and a first election in our model, but these differences play no substantive role in the model without SP.}

Condition (4) is also fulfilled and the equilibrium in the proposition exists if the two states of nature have the same probability ($q = 1/2$). Furthermore, if this is the case, the dishonest politicians from the two parties set the same discipline threshold ($r_L = r_R$), and voters are indifferent between the left and the right party in the first election ($V(L) = V(R)$).

When voters care for the policy ($a > 0$), and the states of nature have different probabilities ($q \neq 1/2$), but condition (4) still holds, then the equilibrium in Proposition 1 exists, but now dishonest politicians from the two parties set different discipline thresholds ($r_L \neq r_R$) and citizens are not indifferent between the two candidates in the first election ($V(L) \neq V(R)$). In this equilibrium, the left has more discipline than the right incumbent —i.e. $r_L > r_R$— iff $q < 1/2$ (see Equation (5)). In Appendix 9.1.1 we present an explanation of why dishonest politicians from different parties have in general different thresholds.

We present in Proposition 2 the case in which condition (4) does not hold and hence the semi-separating equilibrium identified in Proposition 1 does not exist. In this case, there is a PBE in which voters always reelect the first period incumbent and dishonest politicians do not discipline. Because of partisan preferences, in our model voters may not “reward” an incumbent who did not extract observable rents with reelection and “punish” an incumbent who did extract observable rents voting him out of office. Citizens may be willing to reelect a dishonest incumbent if they agree with the policy he favors even if he proves to be dishonest. If this happens, elections only work as an instrument to choose policies —left- or right-wing policies—, but they do not provide incentives to discipline or select out bad types. Therefore, good and bad types fully separate in equilibrium, and yet there is no political selection. In this context, the high polarization and lack of political competition due to partisan preferences undermine electoral accountability.

**Proposition 2. The always reelect equilibrium with NSP.** Iff the following condition holds

$$a(2q - 1) \notin [-q_H(r^e + \rho^e), q_H(r^e + \rho^e)],$$

there is a PBE in which:

1. Voters always reelect the incumbent.
2. An honest incumbent never extracts rents: \( r_1 = r_2 = \rho_1 = \rho_2 = 0 \).

3. A dishonest incumbent always extracts the maximum amount of observable and unobservable rents in both periods, i.e. he chooses \( r_{NSP} = 0 \).

4. In the first election, voters vote for the L candidate if \( q < \frac{1}{2} \), and for the R candidate otherwise.

5. The L and R candidates implement \( p_1 = p_2 = 0 \) and \( p_1 = p_2 = 1 \), respectively.

Inequalities (7) are the converse of (4) and provide the key conditions that separate the equilibria in propositions 1 and 2. Voters always reelect the R incumbent, even if he extracted rents, if their net expected policy gains from having an R rather than an L government, \( a(2q - 1) \), are larger than the expected losses from rents due to the reelection of a dishonest incumbent rather than the election of a potentially honest challenger, \( q_H(r^e + \rho^e) \). Analogously, voters always reelect an L incumbent if the net expected policy gains from voting for L rather than R, \( a(1 - 2q) \), are larger than the expected losses from rents due to the reelection of a dishonest incumbent.\(^{17}\)

5 Separation of powers (SP)

With no separation of powers, the election at the end of period one is the only tool voters have to control the extraction of observable rents. SP strengthens voters control in two ways. First, the extraction of observable rents is now only possible if there are dishonest politicians in both branches. Second, voters have an additional device to curb rent extraction: electing a divided government. Nevertheless, there are two potential costs associated with a divided government. One is political inaction: there is no policy change. The other is political selection: with a divided government voters never learn whether the executive is honest or not. This has a direct impact on the non-observable rents extracted in the second period.

Note that neither of the two previous costs is relevant in the second period. First, a policy that is expected to be beneficial is adopted in the first period and maintained in the second, so inaction in the last term is not an issue in our framework (remember that \( q \) is constant). Second, given that there is no next period after \( t = 2 \), learning is not valuable in this period. Hence, in our model voters always choose a divided government for the last term.\(^{18}\)

\(^{17}\)The expression \( a|2q - 1| \) is the difference of moderate voters expected utility with policies 0 and 1, so it measures the distance between these two policies in terms of moderate voters preferences. When this distance is sufficiently large, so (4) do not hold, even moderate voters stop conditioning their vote on first period performance and always reelect the incumbent. This is an environment of high polarization and lack of political competition à la Besley (2005, section 3.4.1).

\(^{18}\)Schelker (2018) finds that the probability of a divided government is indeed higher when governors cannot run for reelection. While this evidence is roughly consistent with our result, it is obviously not true that voters always choose divided governments when the politician in the executive office is about to begin his last term in office. This stark prediction of our model can be softened adding a random popularity shock in the tradition of probabilistic voting models (Lindbeck and Weibull 1987; Persson and Tabellini 2000).
Remark 1. With SP voters always elect a divided government for the second period.

In the first election, voters may choose a unified government. They do it if the expected utility losses due to rent extraction are countervailed by the benefits from reform and improved political selection. Proposition 3 presents an equilibrium where voters choose a unified government for the first period.

Proposition 3. The unified government equilibrium with SP. Iff the following condition holds:

$$E[r | r \geq r_u] = \int_{r_u}^{\bar{r}} \frac{r}{1 - G(r_u)} dG(r) \leq \max \left\{ 0, \frac{(1 + \beta)a(2q - 1)}{(1 - q_H)^2(1 - G(r_u))} \right\} + \beta q_H \rho^e, \quad (8)$$

where the threshold $r_u$ is defined as follows

$$r_u = \frac{2\beta}{b_D - 1} \left[ (1 - q_H + b_D)\rho^e + E \right], \quad (9)$$

then there is a PBE in which:

1. In the first election, voters choose a party R unified government if $q > 1/2$, a party L unified government if $q < 1/2$, and toss a coin otherwise.

2. In the second election, voters reelect only the executive if observable rents are zero and only the legislature otherwise.

3. Policies are $p_1 = p_2 = 1$, if voters choose a right-wing first-period government, and $p_1 = p_2 = 0$ if they choose a left-wing first-period government.

4. The first period incumbents extract observable rents in the amount $r_1$ iff both the executive and the legislature are dishonest and $r_1 > r_u$. They do not extract observable rents otherwise.

5. In the second period, the government does not extract observable rents. The executive extracts unobservable rents in the amounts $\rho_1$ and $\rho_2$ iff he is dishonest.

The key condition for this equilibrium to exist is inequality (8). It says that voters choose a unified government in the first election iff the expected first period observable rents that arise with a unified government are not “too” large. More specifically, the expected rents must be smaller than the expected gains from policy and selection.

There is a positive policy gain associated to a unified government if and only if the elected unified government implements a reform. Under the maintained assumption that the period zero policy is $p_0 = 0$, this happens iff $q > 1/2$. In this case, voters prefer the right-wing policy. If instead $q \leq 1/2$, voters prefer the status quo policy $p_1 = p_0 = 0$ and obtain no policy gains from choosing a non-reformist unified government (the first term in the right hand side of inequality (8) is zero when $q \leq 1/2$).

There is a gain associated to improved political selection when voters choose a first period unified government, because with a unified government there is a possibility that dishonest incumbents reveal
their type. With a divided government instead voters never learn the politicians type. If this effect is large enough, voters may choose a non-reformist first period government. Policy gains are zero in this case, and there is an increase in expected first period observable rents, but voters still choose this government if the gains from improved selection (lower second period unobservable rents) outweigh the expected increase in first period rents.

In this equilibrium, dishonest incumbents extract observable rents in the first period iff the incumbents in the two offices are dishonest and potential rents surpass the discipline threshold \( r_u \), i.e. if rents are too tempting to let them go.

When expected observable rents are “too” large, voters choose a divided government in both periods.

**Proposition 4. The divided government equilibrium with SP.** Iff the following condition holds:

\[
\mathbb{E}(r | r \geq r_u) = \int_{r_u}^{\bar{r}} \frac{r}{1 - G(r_u)} dG(r) > \max \left\{ 0, \frac{(1 + \beta)(2q - 1)}{(1 - q_H)^2(1 - G(r_u))} \right\} + \beta q_H \rho^e, \tag{10}
\]

there is a PBE in which:

1. Voters vote for a divided government in both periods.

2. Policies are \( p_1 = p_2 = 0 \).

3. Dishonest incumbents set the discipline threshold at \( \bar{r} \) and do not extract observable rents \( (r_1 = r_2 = 0) \), and extract unobservable rents in the amounts \( \rho_1 \) and \( \rho_2 \).

4. Honest incumbents do not extract rents.

Inequality (10) is the converse of (8), and summarizes the basic tradeoff voters face in this institutional environment. They can choose a unified government making reform possible \( (p_1 \neq p_0) \) and allowing political selection that reduces unobservable rents in the second period (first and second terms in the right hand side of (8), respectively), but at the cost of positive expected observable rents in the first period. Alternatively, they can choose a divided government that blocks the extraction of observable rents in the first period, but then there is neither reform \( (p_1 = p_0) \) nor political selection.

The inability to enact a reform with a divided government is of course not a problem for swing voters if they do not want a reform. But if the inherited policy is \( p_0 = 0 \), as we assumed just for the sake of concreteness in these propositions, then voters benefit from a reform in expected terms if the probability that the state of nature is \( s = 1 \) is larger than one half, i.e. if \( q > 1/2 \). In this case, if the expected rents are not too large —i.e. condition (8) holds— they choose a party R unified government that implements the policy switch from \( p_0 = 0 \) to \( p_1 = 1 \).
In the equilibria identified in propositions 3 and 4, the swing voter “splits the ticket”, i.e. the same individual votes differently for the executive and the legislature. Split ticket voting is a well documented phenomenon, but there seems to be no widely accepted explanation (see Burden and Hahnke 2009, and other papers in the 2009 special number of Electoral Studies devoted to split-ticket voting and divided government). Alesina and Rosenthal (1996) and Fiorina (1992), among others, focus on the policy moderating effect of a divided government, i.e. divided governments may reduce policy polarization. We emphasize its rent control effect. We could introduce the policy moderating effect in our model, but given our political agency focus and our interest in highlighting the possibility of political gridlock, we prefer to shut down this mechanism.

6 The effects of the separation of powers

We assess the effects of $SP$ comparing the equilibrium outcomes and welfare without and with $SP$. We consider the four possible combinations of equilibria, namely (i) the “electoral accountability” (semi-separating) equilibrium without and the “unified government” equilibrium with $SP$, (ii) the “electoral accountability” without and “divided government” equilibrium with $SP$, (iii) the “always reelect” (separating) without and “unified government” equilibrium with $SP$, and (iv) the “always reelect” without and the “divided government” equilibrium with $SP$. It is immediate to check that there are mutually exclusive sets of parameter values such that these four possible combinations of equilibria exist.\(^\text{19}\)

6.1 The effects of $SP$ on equilibrium outcomes

Our model highlights several channels through which $SP$ may contribute to a better control of politicians in office, but it also shows that it can sometimes backfire, undermining the effectiveness of elections to discipline and select politicians, and causing political inaction. We first discuss the impact of $SP$ on the tradeoff between discipline and selection, and then analyze the general equilibrium effects of introducing $SP$ on each outcome.

6.1.1 The tradeoff between discipline and selection with $SP$

$SP$ has direct and indirect effects on rent extraction. With $SP$ there needs to be an agreement between the two branches of government to extract observable rents, something that is not needed without $SP$. This direct discipline effect reduces the probability that politicians extract observable rents. In our model, this probability drops down to zero if voters choose a divided government, but even with a unified government there is extraction of observable rents only if both the executive and the legislature

\(^{19}\)These equilibria exist if conditions (i) (4) and (8) hold, (ii) (4) and (10) hold, (iii) (7) and (8) holds, and (iv) (7) and (10) holds, respectively.
are dishonest and agree on rent extraction. Without \( SP \) all that is required is that the executive wants to extract rents.\(^{20}\)

\( SP \) has two **indirect** effects of opposite sign on electorally induced discipline. On one hand, \( SP \) reduces the reward to discipline, or equivalently increases the incentives to extract rents in the first period, because reelection is less luring with than without \( SP \). Indeed, politicians know there is less at stake with \( SP \), because voters block the extraction of observable rents and reform in the second period choosing a divided government. On the other hand, \( SP \) decreases the temptation to extract rents by forcing the executive to share rents with the legislature.

The effects of \( SP \) on selection are just the counterpart of its direct and indirect effects on discipline. \( SP \) reduces the effectiveness of elections to select good politicians for the second period, if it raises the probability that a dishonest first period incumbent disciplines. With more discipline, voters are less able to distinguish honest from dishonest incumbents, and hence are less able to select dishonest incumbents out. Conversely, \( SP \) strengthens selection if it weakens discipline in the first period. Therefore, \( SP \) does not eliminate the trade-off between incentives and selection in electoral accountability, but it shifts the effectiveness of elections to discipline first period and to select second period incumbents.

### 6.1.2 Observable rents

\( SP \) unambiguously reduces observable rents in the second (and last) term. Because of the indirect effects discussed above, \( SP \) may not contribute to the reduction of observable rents in the first term when the incumbents face the possibility of reelection. But in the last term, only the direct effect remains, so \( SP \) unambiguously reduces the probability that the government extracts observable rents forcing the executive to agree with the legislature. In fact, this probability drops down to zero as voters choose a divided government in the second election. Therefore, the difference between the expected observable rents with and without \( SP \) is unambiguously negative:

\[
E[r_2|SP] - E[r_2|NSP] = -E[r_2|NSP]
\]

where

\[
E[r_2|NSP] = \begin{cases} (1 - q_H)[1 - q_H(1 - G(r_{NSP}))]r^e > 0 & \text{if “EA” equilibrium (Prop. 1)} \\ (1 - q_H)r^e > 0 & \text{if “always reelect” eq (Prop. 2)} \end{cases}
\]

In the first period, the results can go either way. Propositions 1 to 4 imply that the difference between the expected observable rents with and without \( SP \) is as follows:

\[
E[r_1|SP] - E[r_1|NSP] = (1 - q_H)^2 \int_{r_{SP}}^{\tilde{r}} rdG(r) - (1 - q_H)\int_{r_{NSP}}^{\tilde{r}} rdG(r)
\]

where:

\[
r_{SP} = \begin{cases} r_u, & \text{if unified government} \\ \tilde{r}, & \text{if divided government} \end{cases}
\]

\(^{20}\)Stephenson and Nzelibe (2010) make a similar point.
SP reduces the probability of a dishonest government in the first period \((1 - q_H)^2 \text{ vs } (1 - q_H))\), but has ambiguous effects on the expected rents once a dishonest government is in place. Nevertheless, the model predicts an unambiguous reduction of first period expected observable rents when SP are introduced if one of the conditions described in the next corollary holds.

**Corollary 1.** SP reduces the expected amount of observable rents extracted in the first period \(\mathbb{E}[r_1|SP] < \mathbb{E}[r_1|NSP]\) if any of the following conditions holds:

1. With SP, voters choose a divided government in the first period \(r_{SP} = \bar{r}\).

2. Voters choose a unified government in the first period and there is an “always reelect” equilibrium without SP \(r_{NSP} = 0\).

3. The discipline threshold is larger than or equal to with than without SP \(r_{SP} \geq r_{NSP}\).

Conversely, SP induces an increase in expected rents in the first period if it induces a sufficiently large drop in discipline to compensate for the reduced probability of a dishonest government. Formally, let \(r^*\) be defined by \((1 - q_H) \int_{\bar{r}}^r rdG(r) = \int_{r_{NSP}}^\bar{r} rdG(r)\), then \(r_{SP} < r^* \iff \mathbb{E}[r_1|SP] > \mathbb{E}[r_1|NSP]\).

### 6.1.3 Unobservable rents

SP does not impact on the extraction of unobservable rents in the first period because it has no impact on the probability that the first period executive is dishonest. In the second period, the effect can go either way, depending on whether SP weakens or strengthens voters’ ability to select in the second election. The effect of SP on expected unobservable rents can be computed as the change in the probability that the executive is dishonest \(\Delta Pr(D_1)\) times the expected unobservable rents conditional on the executive being dishonest \(\rho^e\). Using propositions 1 to 4 we compute the difference between the expected unobservable rents with and without SP as follows:

\[
\mathbb{E}[\rho_1|SP] - \mathbb{E}[\rho_1|NSP] = \Delta Pr(D_1)\rho^e
\]

where:

\[
\Delta Pr(D_1) = 0
\]

\[
\Delta Pr(D_2) = \begin{cases} 
(1 - q_H)q_H [q_H (1 - G(r_{SP})) + G(r_{SP}) - G(r_{NSP})] & \text{if EA equilibrium} \\
-(1 - q_H)^2 q_H (1 - G(r_{SP})) & \text{if “always reelect” equilibrium}
\end{cases}
\]

The possibility that SP weakens electoral accountability is reminiscent of Powell and Whitenn (1993), but the mechanism operating in our model is different from theirs. Powell and Whitenn argue that SP, among other institutional features that limit the discretion of the executive, reduces voters ability to identify responsibilities and hold politicians accountable. In our model, SP does not reduce voters’ ability to observe rent extraction and, more importantly, it does not diffuse responsibilities in rent extraction. Nevertheless, SP may reduce the effectiveness of elections to discipline politicians.
The effects of \( SP \) on the second period expected unobservable rents depend on its effects on political selection. \( SP \) raises unobservable rents in the second period if it weakens the effectiveness of elections to select honest politicians. This may happen in the “electoral accountability” (semi separating) equilibrium, but not in the “always reelect” (separating) equilibrium, because in this latter case elections totally fail to select politicians in the no \( SP \) environment. The next corollary presents this result.

**Corollary 2.** Suppose that the EA equilibrium is played without \( SP \). Then, \( SP \) decreases the extraction of observable rents in the first period (strengths discipline) iff it increases the extraction of unobservable rents in the second period (weakens selection).

### 6.1.4 Reform

In the no \( SP \) environment, and given we have assumed that \( p_0 = 0 \), there is a reform in the first period if voters choose a party \( R \) government in the first election.\(^{22}\) This choice depends on the expected policy fit and extraction of rents with each party in office.\(^{23}\) If \( q > 1/2 \), voting for \( R \) rather than \( L \), swing voters gain from a better expected policy fit and more discipline in the first period, but lose from weaker selection.\(^{24}\)

With \( SP \), voters choose a party \( R \) unified government only if rents are not “too large”, and a divided government otherwise. While the unified party \( R \) government implements a reform, the divided government cannot agree on any policy change and the status quo policy prevails. Hence choosing a divided government voters control rents but at the cost of political inaction. Therefore, in our model, depending on parameter values, \( SP \) may reduce the political system’s level of “decisiveness (i.e., the ability to enact and implement policy change).” (McCubbins 2000, p3).

Voters always choose a divided government for the second period with \( SP \). In this environment, there is no policy change between periods one and two. With no \( SP \) and under the “electoral accountability” equilibrium, voters remove the first period incumbent if the government extracts rents. In this institutional framework, the only option voters have to get rid off a dishonest incumbent is to vote for the challenger, who belongs to the opposition party, and this implies a policy switch. Thus, policies may change between periods one and two without but not with \( SP \). The policies become more stable. Using McCubbins’ terminology, in our model \( SP \) confers the political system more “resoluteness (i.e., the ability to maintain and commit to a policy once established).” (McCubbins 2000, p3).

\(^{22}\)We focus on this case, but of course the analysis is symmetric if \( p_0 = 1 \).

\(^{23}\)The three terms in the right hand side of equation (6) compute moderate voters net gains from policy, first period rents and second period rents, respectively, when they vote for \( R \) rather than \( L \).

\(^{24}\)R disciplines more than \( L \) when \( q > 1/2 \), as the discipline thresholds in equation (5) show; because of a larger cost of not being reelected. This larger cost comes from the special inadequacy of \( L \) policies when \( q > 1/2 \). Second period rents are larger with an \( R \) than an \( L \) first period government because of worse selection due to better discipline.
6.2 The effects of SP on welfare

We show in the appendix that $SP$ increases the welfare of the majority of voters iff it does it for the swing voters (Lemma 4 in the Appendix).

$SP$ raises swing voters and at least one other group of voters welfare if the first period incumbent is always reelected in the no $SP$ regime (that is, if the always reelect equilibrium is played). If voters always reelect without $SP$, elections fail at both disciplining and selecting out dishonest incumbents. The introduction of $SP$ reduces rent extraction in this case. In turn, the policy is the same with and without $SP$ if either voters choose (i) a unified government with $SP$ or (ii) a divided government with $SP$ and a pro status quo government without $SP$. In this case $SP$ contributes to both more discipline and better selection without causing political gridlock, so all voters benefit from $SP$.\(^{25}\) The case where a divided government with $SP$ and a pro reform government without $SP$ are elected deserves more analysis.

$SP$ blocks reform if voters choose a reformist government without and a divided government with $SP$.\(^{26}\) The reduction of rents increases but the blocking of reform reduces swing voters welfare. Nevertheless, as we show in Corollary 3, swing voters welfare loss due to policy inaction is fully compensated by gains from lower rents. Intuitively, swing voters choose a divided government already in the first election iff they know that expected rents with a unified government are too costly compared to the benefits of reform. So if the $SP$ equilibrium involves a divided first period government, then the swing voters welfare losses associated to policy inaction are smaller than their utility gains from lower rents. If the policy switch that $SP$ is blocking is the one favored by the $R$ citizens, then the $L$ citizens necessarily benefit as well, and conversely the $R$ citizens benefit from $SP$ if the reform that is being blocked is the one favored by the $L$ citizens. Therefore, a majority of voters benefit from $SP$ in this environment.

**Corollary 3.** If the always reelect equilibrium is played with NSP, the introduction of $SP$ raises a majority of voters welfare.

Considering the usual concerns for political gridlock, it may look paradoxical that $SP$ unambiguously raises a majority of voters welfare precisely when voters value policies more (remember that the always reelect equilibrium requires voters to have a sufficiently high valuation of policy relative to rents). In our model, this happens because both a high valuation of the positional issue severely undermines electoral accountability in the no $SP$ environment and in the $SP$ environment swing voters can avoid political gridlock choosing a unified government.

In Corollary 4 we show that voters also benefit from $SP$ when the electoral incentives to discipline

\(^{25}\) An always reelect equilibrium without $SP$ and a unified government with $SP$ arise if conditions (4) do not hold and (8) holds. A status quo always reelect equilibrium without $SP$ and a divided government equilibrium with $SP$ arise if neither conditions (4) nor (8) hold and $q < 1/2$.

\(^{26}\) These equilibria arise if neither conditions (4) nor (8) hold and $q > 1/2$. 

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are so high that dishonest politicians never extract observable rents in the first period. This result could look counterintuitive because it says that SP raises voters welfare precisely when electoral accountability without SP works very well in the no SP environment. But the explanation is simple. SP raises welfare in this environment because it eliminates the extraction of observable rents in the second period, without bringing other associated costs. With “full discipline” voters cannot select politicians and expected unobservable rents are the same with and without SP. Policies are also the same in the two regimes.

**Corollary 4.** The introduction of SP raises voters welfare if elections induce a sufficiently high level of discipline in the first period without SP. This occurs if (i) $\rho^* \to \infty$, or (ii) swing voters hold the first period incumbent accountable in the no SP environment —i.e. (4) holds— and $E \to \infty$.

Voters expected utility gains from SP are $\beta(1 - q_H)r^e \geq 0$ if these conditions hold.

In Corollary 5, we consider a case in which SP reduces voters welfare.

**Corollary 5.** The introduction of SP reduces voters welfare if $a = 0$, $\bar{r} \to 0$ and $0 < \rho^e < \rho^*$ where the threshold $\rho^*$ is defined by

$$\bar{r} = r_{NSP}(\rho^*) = \frac{\beta}{b_D}(1 - q_H + b_D)\rho^* + \frac{\beta E}{b_D}.$$  \hfill (14)

Corollary 5 describes a case in which most rents go undetected ($\bar{r} \to 0$) and voters do not care for policies ($a = 0$). SP reduces voters welfare in this case because it weakens selection raising the extraction of unobservable rents in the second period. Potential gains from policies are zero —voters do not care about policies ($a = 0$)— and observable rents tend to zero ($\bar{r} \to 0$). SP can strengthen discipline and undermine selection only if dishonest politicians do not fully discipline in the no SP environment, and this requires that $\rho^e < \rho^*$.  

7 Assumptions: discussion and rationale

In this section we discuss some of the main assumptions of the model.

Divided government. In our modeling of SP, voters decide not only the party in the executive office but also whether the executive will enjoy strong support in congress. In our highly stylized framework, voters can split the ticket, i.e. they can vote for a candidate from one party for the executive and for the other in the legislature. While ticket splitting is a well documented fact in the US, it is not even allowed in many countries. Nevertheless, we do not think ticket splitting is a crucial characteristic of our model, but just a simplifying modeling device. A divided or minority government can arise without ticket splitting, for example, in mid term elections or in multiparty systems, especially with two-round electoral systems. In this context, when moderate voters think that a reform is not necessary or at least not crucial, they can vote for small parties that contribute to exert control on the executive. Without
being too sophisticated or strategic, they are choosing a minority government to strengthen control. In contrast, when moderate voters think that the country desperately needs a reform, they can vote for the reformist party, increasing the president support in congress and weakening control. All that is needed for our argument is that voters use elections to decide the support the executive will have in congress, which we think is a mild and realistic assumption.

**Political Parties.** In the tradition of Alesina and Rosenthal (1995, 2000) we assume there are only two distinct and ideologically homogeneous political parties and a presidential system. This framework naturally leads to the identification of minority and majority governments with divided and unified governments, respectively. Our modeling strategy is meant to represent in a simple form the fundamental fact that, in presidential systems, SP gives voters the possibility of granting or denying the president the majorities in congress they need to enact significant reforms. We use the ideologically homogeneous parties assumption just as a simple form of representing this option voters have. But in the real world whether a clear mandate to reform is achieved through a unified party government or something more complex—like a vote for party factions favoring reform—may vary much depending on the specific characteristics of the party and electoral systems, and also of the form of government. We do not see our model as a particularly useful tool to contribute to the much more context-specific debate about unified versus divided government. Readers should keep this in mind in interpreting our results regarding when voters are more likely to choose a divided government and which are the consequences of this choice in terms of reform.

**Divided vs Unified governments, and the capacity for enacting reforms.** We have assumed that only unified governments can enact reforms. Divided governments do not reform in our model. These assumptions are meant to put at the center of the analysis the tradeoff between delegation and control. Voters maximize delegation when they choose a unified government and control when they choose a divided government. We make these stark assumptions in order to have a tractable model and make our point clearly. We conjecture that these assumptions can be relaxed and the qualitative results remain the same provided we make the milder assumption that the president’s capacity for enacting reforms is higher with unified than divided governments, even if the probability that the reform will be passed is less than one and higher than zero with unified and divided governments, respectively. But admittedly, even this milder assumption has been challenged in the empirical literature, especially in the US case.

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27For the sake of concreteness, our model focuses on presidential systems. We agree with Elgie (2001) in that a divided government in a presidential system “is the equivalent of minority government in parliamentary regimes”. Nevertheless, the working of electoral accountability and interbranch control is different in presidential and parliamentary systems. Political gridlock seems to be more prevalent in presidential than parliamentary systems, because in the latter “irreconcilable conflict usually leads to the dismissal of the government or the dissolution of parliament rather than the ongoing stalemate” (Elgie 2001).
Indeed, there has been controversy in the American politics literature over the alleged impact of divided government on political gridlock. Mayhew (1991) challenged the conventional wisdom that divided governments caused policy stalemates in the United States showing that, after the second world war, the number of landmark laws passed in Congress did not depend on whether the government was unified or divided. “Divided We Govern” is the suggestive title of his highly influential book. Mayhew’s work sparked an interesting literature on divided government in the US and the controversy does not seem to be settled.

Some scholars have pointed out that the ideological distance between Democrats and Republicans was not large during most of the period analyzed by Mayhew. Frymer (1994) argues that American voters split their vote in most elections after the second world war because they did not see much ideological difference between the two parties. Therefore, the argument goes, they did not expect that a divided government would block reform or even moderate policies. Other scholars have challenged Mayhew’s findings refining measurement of political gridlock. Binder (2003, p44) questions Mayhew’s measure of gridlock on the grounds that it looks at the number of laws enacted without considering the number of issues in the agenda (on a similar argument see Fiorina 1992).

The empirical evidence on the impact of divided government out of the US is sparser, but it is commonly argued that presidential regimes tend to fall into stalemate and crisis when different parties or even factions control the executive and the legislature. Ackerman (2000) argues that the most obvious problem of presidentialism is impasse: “house and president may be dominated by different parties (or different factions of the same party). How to govern until the next election?” According to Linz (1990), this impasse often led to political crisis and the breakdown of democracy in Latin America.

Binder (2003) also shows that party polarization was small during most of the period covered in Mayhew study and increased dramatically afterwards. Krause (2000) argues that fiscal deficits in the US are larger when there is more ideological fragmentation, independently of divided party governments. Saeki (2009) finds no significant effect of divided government and a significant effect of veto congressmen preferences on policy change. These studies point out that party polarization was relatively low during most of the period analyzed by Mayhew and without party polarization divided government had little or no influence on political gridlock. In terms of our formal model, this situation can be represented by a relatively small difference between the two policies in moderate voters utility metric, which can arise when either \(a\) is sufficiently small or \(q\) is sufficiently close to one half (see footnote 17). In this context, our model predicts divided government and policy stability.

Binder (2003) builds an index of political gridlock that aims at measuring the proportion of issues at consideration that were favorably solved in the legislative period. Using this index, she concludes that there is a statistically significantly higher probability of gridlock in periods of divided than unified government. Nevertheless, she also stresses that divided government was not the only or even the most important determinant of stalemate in the US during the period she studied. Also on measurement, some scholars have looked at delay rather than the number of laws passed in congress. There is some recent compelling evidence that divided government causes delay in important legislation (Hughes and Carlson 2015) and, more specifically, in the budget process at the state level (Kirkland and Phillips 2018) in the US. Looking specifically at welfare reforms, Bernecker (2016) finds that divided US state governments are more likely to implement reforms than unified governments. He conjectures that this surprising result is due to stronger political competition in divided than in unified governments.

Ackerman (2000) also argues that divided government caused stalemate in Japan after 1989, when the Liberal Democratic Party lost control of the upper house. Elgie (2001) provides only limited support for the hypothesis that links divided government and stalemate in several countries. More importantly, Elgie and collaborators provide a nuanced view of how divided governments arise and operate in different institutional contexts.
Divided vs Unified government and the control of rent extraction. The working of SP to curb rent extraction in our model is inspired in Persson, Roland, et al. (1997, 2000) but with a twist: we add political parties and assume that the mechanism of control is more effective with divided than with unified government. This is consistent with Alt and Lassen (2008) findings that the control of rents in the US is more effective with divided than unified governments and Kriner and Schwartz (2008) finding that divided governments tend to significantly increase congress’ investigatory activity of the president.

A unique vs a multi-agent legislature. The structure of government we have assumed is indeed highly stylized. Following previous literature in agency theory (Besley 2005; Persson, Roland, et al. 1997; Stephenson and Nzelibe 2010), we model the legislature as a unique agent. Of course, in the real world legislatures are multi-agent bodies and often have more than one chamber.31 These and similar traits can potentially have a significant impact on the outcomes. Also we could expect that the results depend on whether executive and legislative elections are synchronized, voters are allowed to split the ticket, the systems are majoritarian or proportional, there are open or closed party lists, etc. We think all these factors should be incorporated in future extensions of our basic model. In our view, our basic point that positional issues and party politics play a key role in political agency will remain in these extensions.

The judiciary. We have not incorporated the judiciary in our model, but it could be argued that it implicitly plays a role in the working of the institutional mechanisms mapping policy proposals into actual policies assumed in our model. In particular, there is suggestive evidence that the judiciary is more independent and exert more control on the executive with divided than unified governments (Chávez et al. 2011). Therefore, voters choice of a divided government may not only strengthen the control of the legislature on the executive but also the judiciary checks, limiting both the government ability to reform and to extract rents.

8 Summary and Conclusions

We present a model of political agency with ideological parties, and argue that partisan issues can dramatically impact on the working of electoral accountability and separation of powers.

With no SP, we identify two type of equilibria: (i) an equilibrium in which voters hold the first period incumbent accountable in the sense that they reelect if and only if the incumbent did not extract rents; and (ii) a “polarization” equilibrium in which voters reelect the incumbent even if he extracted rents.

31 Binder (2003) argues that in the case of the US bicameralism play a key role in legislative gridlock.
We also identify two type of equilibria in the separation of powers environment: (i) a “unified government” equilibrium in which voters grant the executive the legislative majorities needed to enact reforms; and (ii) a “divided government” equilibrium in which voters choose one party for the executive office and another to control the legislature.

Separation of powers benefits a majority of voters in the case of high polarization. Paradoxically, it is precisely in this type of environment of high polarization in which more concerns are usually voiced regarding checks and balances on the grounds that strict control of the executive can lead to stalemate. In our model, elections totally fail at disciplining and selecting politicians in the no separation of powers environment with high polarization, so separation of powers improves both discipline and selection. It does not cause political gridlock because voters can choose a unified government to facilitate reform.

Separation of powers also improves voters welfare if elections are extremely effective at disciplining first period incumbents in the no separation of powers environment. This result is driven by the elimination of second period observable rents. In turn, because of high first period discipline, voters fail at selecting politicians. Expected unobservable rents and policies are the same with and without separation of powers in both periods.

Separation of powers can reduce voters welfare if most rents go undetected and the partisan issue is not of first order importance. In these conditions, separation of powers may reduce welfare because it weakens selection and raises the extraction of unobservable rents in the second period. If most rents are unobservable and there are no significant gains from reform, then other potential effects of separation of powers are swallowed by the negative effect of higher expected unobservable rents.

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9 Appendix

Lemma 1. Swing voters are decisive: the candidates they vote win the election.

Proof of lemma 1. Let $\Delta v_{R,t} - \Delta v_{S,t} = v_{R,t}(p_t = 1) - v_{R,t}(p_t = 0) - [v_{S,t}(p_t = 1) - v_{S,t}(p_t = 0)]$ be the difference between voters $R$ and $S$ expected net utility gains from choosing candidates that implement policy $p_t = 1$ rather than $p_t = 0$. Since they have the same valuation of the valence issue, these differences depend only on policies in the positional issue. From equation (1), this is $\Delta v_{R,t} - \Delta v_{S,t} = \Delta v_{S,t} - \Delta v_{L,t} = 2a > 0$. Therefore, if $\Delta v_{S,t} > 0$, then $\Delta v_{R,t} > 0$ and if $\Delta v_{S,t} < 0$, then $\Delta v_{L,t} < 0$. Swing and right wing voters conform a majority when $\Delta v_{S,t} > 0$ and swing and left wing voters do when $\Delta v_{S,t} < 0$.

Based on this result, in the following propositions we focus on the behavior of swing voters.

9.1 No separation of powers (proofs)

In this subsection we solve the game with no separation of powers.

In the second election, citizens can condition their vote not only on the party’s preferences regarding reform but also on rent extraction in the first period. The second election has the potential to discipline and select out dishonest incumbents if the probability of reelection when the incumbent does not extract rents is higher than when he does. This is always the case in the canonical model, because rent extraction is a signal of a bad type and voters simply want to elect a good politician. But in our model citizens may be willing to reelect a dishonest incumbent if they agree with the policy he favors even if he proves to be dishonest. If this happens, elections only work as an instrument to choose policies —left- or right-wing policies—, but they do not provide incentives to discipline or select out bad types. In lemmas 2 and 3 we identify necessary and sufficient conditions for citizens to condition reelection on rent extraction.

Lemma 2. Voters do not elect in the first election a candidate they would not reelect even if he did not extract rents.

Proof of lemma 2. Swing voters expected net utility gains from reelecting a party $R$ incumbent over voting for a party $L$ challenger in the second election are

$$\Delta v_2(r_1) = a(2q - 1) - [1 - P(H|r_1) - (1 - q_H)](r^e + p^e)$$
They never reelect the incumbent if this expression is negative even if the incumbent did not extract rents. If they never reelect, there are no incentives to discipline so dishonest incumbents extract rents in the first period. This means that $P(H|r_1 = 0) = 1$ so $\Delta v_2(0) = a(2q-1) + (1-q_H)(r^e + \rho^e) < 0$. But this implies that voters expected utility gains in the first period from electing the party $R$ over the party $L$ candidate are also negative. Indeed, even if voters had the most favorable belief about $R$, their expected first period utility gains from electing $R$ would be negative: $\Delta v_1 = a(2q-1) + (1-q_H)(r^e + \rho^e) < 0$. Therefore, if $\Delta v_2(r_1) < 0$ then $V(R) < V(L)$, and voters do not elect $R$ in the first election. The argument is similar for party $L$, substituting $a(1-2q)$ for $a(2q-1)$.

**Lemma 3.** In the second election, voters reelect the incumbent when he does not extract rents and vote for the challenger when he does iff

$$a(2q-1) \in [-q_H(r^e + \rho^e), q_H(r^e + \rho^e)]$$

(15)

Otherwise, voters always reelect the first period incumbent.

**Proof of lemma 3.** Voters do not reelect an $R$ incumbent after observing $r_1 > 0$ iff their expected utility gains from reelecting him are negative, i.e. iff

$$a(2q-1) - q_H(r^e + \rho^e) < 0$$

(16)

By virtue of lemma 2, voters reelect after observing $r_1 = 0$, if they do not reelect after observing $r_1 > 0$. Suppose to the contrary they did not reelect despite of $r_1 = 0$. Then voters would not reelect a party $R$ incumbent irrespective of rents, but then lemma 2 implies they would have not elected the party $R$ candidate in the first election.

Suppose now that inequality (16) does not hold and hence voters reelect a party $R$ incumbent after observing $r_1 > 0$. This implies that they will also reelect after observing $r_1 = 0$. Indeed, voters expected gains from reelecting the $R$ incumbent conditional on $r_1$ are $a(2q-1) + [P(H|r_1) - q_H](r^e + \rho^e)$. We know that $P(H|r_1 > 0) = 0 < P(H|r_1 = 0)$. Hence, if voters reelect after observing $r_1 > 0$, i.e. if $a(2q-1) - q_H(r^e + \rho^e) > 0$, then they also reelect after observing $r_1 = 0$:

$$a(2q-1) - q_H(r^e + \rho^e) > 0 \Rightarrow a(2q-1) + [P(H|r_1 = 0) - q_H](r^e + \rho^e) > 0$$

In turn, voters do not reelect an $L$ incumbent after observing $r_1 > 0$ iff

$$-a(2q-1) - q_H(r^e + \rho^e) < 0$$

(17)

For analogous reasons as with party $R$, voters must reelect a party $L$ incumbent after observing that there was no rent extraction in the first period.
Suppose now that inequality (17) does not hold and hence voters reelect a party L incumbent after observing \( r_1 > 0 \). This implies they will also reelect the L incumbent after observing \( r_1 = 0 \), since \( a(2q - 1) - q_H(r^e + \rho^e) \geq 0 \Rightarrow a(2q - 1) + [P(H|r_1 = 0) - q_H](r^e + \rho^e) \geq 0 \).

**Proof of proposition 1.** We first describe the strategy profile and beliefs and then prove that no one has incentives to deviate.

(i) Strategy profile.

1. Voters play in the two elections. In the first election, swing voters vote for the R candidate if \( V(R) - V(L) > 0 \), for the L candidate if \( V(R) - V(L) < 0 \), and flip a coin if \( V(R) - V(L) = 0 \), where \( V(R) - V(L) \) is defined in equation (6). In the second election, they vote for the challenger if the incumbent extracted rents and reelect the incumbent otherwise.

2. Politicians play after the two elections.

   (a) An honest politician does not extract rents in either period.

   (b) A dishonest politician extracts unobservable rents in both periods and observable rents in the second period. In the first period, he chooses a threshold \( r_j, j \in \{L, R\} \), with \( r_j \) defined in equations (5), such that he extracts rents iff the realization of the stochastic process \( r \) surpasses the threshold, i.e. if \( r > r_j \).

(ii) Beliefs. Voters use Bayes rule to update their prior belief that the incumbent is honest. Their posteriors are:

\[
P(H|r_1, j) = \begin{cases} 
0, & \text{if } r_1 > 0, \\
\frac{q_H}{q_H + (1 - q_H)G(r_j)}, & \text{if } r_1 = 0, 
\end{cases}
\]

(iii) No deviations.

1. Voters. Lemma 3 proves that voters do not deviate in the second election. Swing voters net gains from voting for the R candidate in the first election are \( V(R) - V(L) = v_1(R) - v_1(L) + \beta[v_2(R) - v_2(L)] \), where

\[
v_1(R) - v_1(L) = a(2q - 1) - (1 - q_H) \int_{r_L}^{r_R} rg(r)dr,
\]

and

\[
v_2(R) - v_2(L) = -a[\mathbb{E}[(p_2 - s_2)^2|R] - \mathbb{E}[(p_2 - s_2)^2|L]] - (P(D|R) - P(D|L)(r^e + \rho^e)),
\]

where \( P(D|R) \) stands for the probability of having a dishonest incumbent in the second period conditional on electing an R politician in the first election. Operating

\[
-a[\mathbb{E}[(p_2 - s_2)^2|R] - \mathbb{E}[(p_2 - s_2)^2|L]] = -a(2q - 1)(1 - 2q_H - (1 - q_H)(G(r_R) + G(r_L))
\]

31
and

\[(P(D|L) - P(D|R))(r^e + \rho^e) = (1 - q_H)(G(r_L) - G(r_R))q_H(r^e + \rho^e)\]  

Equation (6) follows from equations (19) to (22).

2. An honest politician has no incentives to deviate. Rent extraction reduces his utility directly and indirectly, by causing that voters do not reelect him.

3. A dishonest incumbent knows he will be removed from office if he extracts rents and will be reelected otherwise. Hence, his expected net gains from discipline are \(-r_1 + \beta[a(3 - 2q) + (2 - q_H)(r^e + \rho^e) + E]\), if he belongs to the left party, and \(-r_1 + \beta[a(2q + 1) + (2 - q_H)(r^e + \rho^e) + E]\), if he belongs to the right party. Therefore, given voters behavior, the first period incumbent disciplines iff \(r \leq r_j\), with \(r_j\) defined in equations (5).

(iv) Expected utilities. Voters’ expected utility from electing a \(R\) or \(L\) politician in the first election is as follows. Let \(v_i(j)\) denote voters’ expected utility in period \(i \in \{1, 2\}\) from electing politician \(j \in \{R, L\}\).

Then:

\[v_1(R) = -a[0q + 1(1 - q)] - (1 - q_H)((1 - G(r_R))E[r \geq r_R] + \rho^e].\]

That is, a \(R\) politician always implements \(p = 1\), with probability \((1 - q_H)\) the politician is dishonest, extracts non-observable rents \(\rho^e\), and if in addition the realization of rents is higher than \(r_R\) he also extracts observable rents \(E[r \geq r_R]\).

In the second period, with probability \(q_H + (1 - q_H)G(r_R)\) the first period politician is reelected (with probability \(q_H\) he is honest, and with probability \((1 - q_H)G(r_R)\) he is dishonest but decides not to extract rents), and the expected utility from the implemented policy is \(-a(1 - q)\). With probability \((1 - q_H)(1 - G(r_R))\) he is not reelected, and a \(L\) politician is in power in the second period with an expected utility from the implemented policy equal to \(-a q\). As before, with probability \((1 - q_H)G(r_R)\), the first period executive is dishonest, does not extract rents, and is reelected, so there is rent extraction in the second period (both observable and non-observable rents). Finally, there is also rent extraction if the first period executive is dishonest, extracts rents, and the challenger who is elected is dishonest, something that happens with probability \((1 - q_H)^2(1 - G(r_R))\). Then:

\[v_2(R) = q_H(-a(1 - q)) + (1 - q_H)[G(r_R)(-a(1 - q) - r^e - \rho^e) + (1 - G(r_R))((1 - q_H)(-r^e - \rho^e) - aq)].\]

Thus,

\[V(R) = -a(1 - q) - a\beta[\frac{q_H(1 - q) + (1 - q_H)[G(r_R)(1 - 2q) + q]}{E[r \geq r_R] + \rho^e}]
- (1 - q_H)(1 - G(r_R))E[r \geq r_R] + \rho^e]
- \beta(1 - q_H)(r^e + \rho^e)[G(r_R) + (1 - G(r_R))(1 - q_H)].\]  

\[\text{(23)}\]
Equivalently, voters’ expected utility from electing a $L$ politician in the first period is:

$$v_1(L) = -a[1q + 0(1 - q)] - (1 - q_H)[(1 - G(r_L))E(r|r \geq r_L) + \rho^e],$$

and

$$v_2(L) = q_H(-aq) + (1 - q_H)[G(r_L)(-aq - r^e - \rho^e) + (1 - G(r_L))((1 - q_H)(-r^e - \rho^e) - a(1 - q))].$$

Thus,

$$V(L) = -aq - a\beta[q_Hq + (1 - q_H)[G(r_L)(2q - 1) + (1 - q)]] - (1 - q_H)((1 - G(r_L))E(r|\geq r_L) + \rho^e] - \beta(1 - q_H)(r^e + \rho^e)[G(r_L) + (1 - G(r_L))(1 - q_H)].$$

(24)

Proof of proposition 2. By virtue of lemma 3, voters always reelect the first period incumbent if conditions (4) do not hold. If voters always reelect, dishonest incumbents have no incentives to discipline, so they always extract rents, no matter how small they are. Thus they choose thresholds $r_R = r_L = 0$.

Voters’ expected utility from electing a $R$ politician in the first election under this equilibrium is:

$$V(R) = -(1 + \beta)(a(1 - q) + (1 - q_H)(r^e + \rho^e)),$$

and from electing a $L$ politician:

$$V(L) = -(1 + \beta)(aq + (1 - q_H)(r^e + \rho^e)),$$

Thus, $V(R) - V(L) = a(2q - 1)(1 + \beta)$, which is positive if $q > 1/2$, negative if $q < 1/2$ and zero if $q = 1/2$. 

9.1.1 Analysis of dishonest politicians thresholds $r_R$ and $r_L$ in Proposition 1.

Remember that when individuals care for the policy ($a > 0$), and the states of nature have different probabilities ($q \neq 1/2$), but condition (4) still holds, then the equilibrium in Proposition 1 exists, but now dishonest politicians from the two parties set different discipline thresholds ($r_L \neq r_R$) and citizens are not indifferent between the two candidates in the first election ($V(L) \neq V(R)$). In this equilibrium, the left has more discipline than the right incumbent —i.e. $r_L > r_R$— iff $q < 1/2$ (see equation (5)). In what follows we explain why these two thresholds are different.

With $q \neq 1/2$, politicians from the two parties have different utility costs of not being reelected and hence face different incentives to discipline. The costs in terms of foregone monetary and ego rents are the same for the two parties, but the costs associated to the policy shift are not the same if $q \neq 1/2$ (first
term in the right hand side of equation (5)). The policy shift is symmetric—from $p_2 = 0$ to $p_2 = 1$, if an $L$ incumbent extracts rents, and from $p_2 = 1$ to $p_2 = 0$, if an $R$ incumbent does—, so both an $L$ and an $R$ incumbent suffer a similar policy shift away from their preferred policy and by the same amount if they are not reelected. But the utility cost of this policy shift is not the same if $q \neq 1/2$.

In our model, due to the concavity of the utility function, a similar move away from the preferred policy is more painful the further is the initial from the preferred policy. Policy $p = 0$ is further from the left politician bliss point than policy $p = 1$ is from the right politician bliss point if $q < 1/2$. Consider, for example, the case in which $q = 0$ and hence the state of nature is always $s = 0$. The $L$ and $R$ politicians bliss points are -1 and 1, respectively. Hence, while the policy of an $L$ incumbent $p = 0$ is already one point away from his bliss point, the policy of an $R$ incumbent $p = 1$ is his bliss point. Not being reelected means a move away from a policy that for the left incumbent is already different from his preferred one and for the right incumbent is his preferred policy. More generally, when $s = 0$ is more likely than $s = 1$ the $R$ policy is more painful for the $L$ than the $L$ policy is for the $R$ incumbent, so the $L$ disciplines more than the $R$ incumbent.\footnote{It can be shown that these results do not hinge on the simplifying assumption that the biases of the $L$ and $R$ parties are -1 and +1, respectively, but on the assumption that the biases are symmetric. With general biases, the difference between the two thresholds is $r_R - r_L = \frac{2ab}{qz} (\delta_L + \delta_R + 2q - 1)$, so all results follow if the biases are symmetric in the sense that $\delta_L = -\delta_R$. The concavity of the utility function in the policy deviations from the individuals bliss points is also crucial. It determines that the deviations from the bliss point causes larger utility losses the farther are the initial from the bliss point. This effect is not present and the two parties have the same discipline if their expected losses from a policy mismatch are $-aE_s (p_t - s_t - \delta_j)$, rather than $-aE_s [(p_t - s_t - \delta_j)^2]$, as we have assumed.}

Intuitively, while a left candidate would never support an $R$ policy no matter the state of nature, he thinks that the $R$ policy is particularly damaging when the state of nature more strongly claims for an $L$ policy (i.e. when $s = 0$). Conversely, a right candidate thinks that the $L$ policy is particularly damaging when $s = 1$. As an example, suppose the $L$ party always favor expansive fiscal policies to reduce unemployment and the $R$ party always favor fiscal adjustment to reduce fiscal deficit, debt and inflation. If the economy receives an expansionary shock (which in this example corresponds to $s = 1$) the $L$ party continues thinking that fiscal adjustment ($p = 1$) is not the best option, but it recognizes that the $R$ contractionary policy is not so damaging in the upturn as it would have been in the downturn of the business cycle. The $R$ party thinks that the expansionary policy pushed by the left party ($p = 0$) is always wrong, but it is particularly misleading when the economy is already in a boom. So, as our model shows, with an expansionary shock, the $R$ party loses more than the $L$ party from the choice of what they think is the wrong policy. In expected terms, when a boom is more likely than a recession, the $R$ party has more to lose than the $L$ party from the enactment of the policy supported by the other party. Hence $R$ has stronger incentives to discipline than $L$. The converse is true if a recession is more likely than a boom.

In the first election, the party whose policy is more likely to match the state of nature — $R$ if $q > 1/2$ —
and \( L \) if \( q < 1/2 \)— has two strong and one weak points in the eyes of voters. It strength lies in that swing voters prefer their policy and know they have stronger incentives to discipline (first and second terms in the right hand side of equation (6)). It weakness lies in that selection is weaker (the last term in the right hand side of equation (6)). Weaker selection is just the counterpart of better discipline.

9.2 Separation of powers (definitions and proofs)

9.2.1 A transformed game

In the original game, Nature chooses the honesty type of candidates before the first election. However, since voters do not observe this move, we can represent the game as if Nature chose the type after the first election. In this transformed game, four subgames begin after the first election, corresponding to voters choice of an L and R unified government (LL and RR, respectively), and a divided government with party L and R controlling the executive (LR and RL, respectively). Nature moves first in each subgame, choosing the type of the executive and the legislature, and the maximum amount of rents to be extracted. Then the executive proposes the amount of rents to extract, knowing previous moves by Nature. The legislature follows proposing rents, knowing previous moves by Nature and the executive. Finally, voters decide whether to reelect one branch, two branches or none.

9.2.2 Strategies, beliefs and payoffs in the subgames that begin after the first election

We only discuss in detail the decisions concerning the extraction of observable rents. Policy choices in the positional issue dimension are straightforward as the policy bias of each politician is observable. \( R \) and \( L \) politicians propose \( p = 1 \) and \( p = 0 \) policies, respectively. Also the extraction of unobservable rents is straightforward: dishonest executives extract and honest executives do not extract unobservable rents.

We focus on strategies in which the executive proposes to equally share observable rents with the congress iff the opportunity to extract rents surpass a threshold. More formally, politicians in the executive office choose \( r_{sp} \) and propose \( \tilde{r}_{X} = r/2 \) iff \( r > r_{sp} \), where \( r \) is the realization of the process of rents. The legislature can propose any \( \tilde{r}_{C} \in [0, r] \). However, according to rule (3), both the executive and the legislature obtain \( r/2 \) if the proposals coincide and zero otherwise, so we focus on cases in which the congress proposes \( \tilde{r}_{C} \in \{0, \tilde{r}_{X}\} \). In practice, this means that the congress either accepts or rejects the executive proposal.

Strategies:

1. The executive has two information sets, each containing only one node, following Nature’s choice of honest and dishonest politicians for congress. A strategy of the executive consists of two thresholds, one for each information set. In all the strategy profiles we analyze, only a dishonest executive sharing government with a dishonest congress in a unified government chooses a threshold.
\(r_{SP} = r_u \leq \bar{r}\) and proposes to extract rents \(\tilde{r}_X = r/2\) iff \(r > r_u\). In all other cases, executives choose a threshold \(r_{SP} = \bar{r}\) and hence propose to extract no rents no matter the realization of \(r\).

2. The congress can accept or reject the executive proposal to share observable rents. It has four information sets, each containing only one node, following Nature’s choice of politicians for the executive (honest or dishonest) and the executive proposal regarding the extraction of rents (zero or positive rents).

3. Voters have four information sets, each containing four nodes, following the executive and congress proposals of rent extraction: \((\tilde{r}_X, \tilde{r}_C) \in \{(0, 0), (0, \tilde{r}_C \neq 0), (r/2, \tilde{r}_C \neq r/2), (r/2, r/2)\}\). Each information set has 4 nodes, in which either both politicians, only the executive, only the congress, or none are honest. At each information set, voters have four possible actions: reelect both politicians, reelect only the executive, reelect only the congress, and reelect none. We restrict the analysis to voters strategies such that, no matter what congress does, voters reelect:

(a) only the congress if there is a unified government in the first period and the executive proposed the extraction of rents,

(b) only the executive if there is a unified government in the first period and the executive did not propose the extraction of rents,

(c) both if there is a divided government in the first period.

When voters use this type of strategies, the dishonest legislature always proposes the extraction of rents after the executive did it. Therefore, when the executive and the legislature are dishonest, the probability with which both politicians propose rents is equal to the probability with which the executive proposes rents, which is \(1 - G(r_{SP})\).

Voters beliefs:

We define voters posterior beliefs as follows:

\[
\begin{align*}
P_{HH}(\tilde{r}_X, \tilde{r}_C) &= \text{Prob}(X_H, C_H | \tilde{r}_X, \tilde{r}_C) \\
P_{HD}(\tilde{r}_X, \tilde{r}_C) &= \text{Prob}(X_H, C_D | \tilde{r}_X, \tilde{r}_C) \\
P_{DH}(\tilde{r}_X, \tilde{r}_C) &= \text{Prob}(X_D, C_H | \tilde{r}_X, \tilde{r}_C) \\
P_{DD}(\tilde{r}_X, \tilde{r}_C) &= \text{Prob}(X_D, C_D | \tilde{r}_X, \tilde{r}_C)
\end{align*}
\]

It follows that the posterior probabilities of a dishonest executive \(P_{DH}(\tilde{r}_X, \tilde{r}_C)\) and congress \(P_{DD}(\tilde{r}_X, \tilde{r}_C)\)
are:

\[ P_{\Delta}(\tilde{r}_X, \tilde{r}_C) = \text{Prob}(X_D | \tilde{r}_X, \tilde{r}_C) = P_{\Delta H}(\tilde{r}_X, \tilde{r}_C) + P_{\Delta D}(\tilde{r}_X, \tilde{r}_C) \]

\[ P_{\Delta}(\tilde{r}_X, \tilde{r}_C) = \text{Prob}(C_D | \tilde{r}_X, \tilde{r}_C) = P_{\Delta D}(\tilde{r}_X, \tilde{r}_C) + P_{\Delta D}(\tilde{r}_X, \tilde{r}_C) \]

(27)

Beliefs off the equilibrium path are not restricted, but on the equilibrium path beliefs are updated using Bayes rule.

There are two voters’ information sets that lie on the equilibrium path, namely (0, 0) and (r/2, r/2). The posterior beliefs on these information sets are computed using Bayes rule as follows:

\[ P_{HH}(0,0) = \frac{q_H}{q_H + 2q_H(1-q_H)+(1-q_H)^2G(r_{SP})} \]

\[ P_{HD}(0,0) = \frac{q_H(1-q_H)}{q_H + 2q_H(1-q_H)+(1-q_H)^2G(r_{SP})} \]

\[ P_{DD}(0,0) = \frac{(1-q_H)^2G(r_{SP})}{q_H + 2q_H(1-q_H)+(1-q_H)^2G(r_{SP})} \]

(28)

\[ P_{HH}(r/2,r/2) = P_{HD}(r/2,r/2) = P_{DD}(r/2,r/2) = 0, r > 0 \]

\[ P_{DD}(r/2,r/2) = 1, r > 0 \]

Voters expected payoffs

At the second election time, forward looking voters look at their expected second period utility to choose their vote. The expected utility associated to votes casted in the election at the end of period one depends on who the first period incumbents are and which policy results. The identity of the first period incumbent matters because the expected utility of voting for a given party depends on the information citizens have about the candidate and this depends on whether the candidate is an incumbent running for reelection or a challenger. The first period policy matters because it conditions the second period policy outcome if voters choose a divided government for the second period. Based on lemma 1, in what follows we focus on swing voters. To simplify notation, we drop the S subindex.

Voters second period expected utilities at the election time in the information sets (\( \tilde{r}_X, \tilde{r}_C \)) are indicated in Table 1. We focus on the case \( p_0 = 0 \).
<table>
<thead>
<tr>
<th>Incumbents in $t = 1$</th>
<th>Incumbents in $t = 2$</th>
<th>Payoffs</th>
</tr>
</thead>
<tbody>
<tr>
<td>$RR$</td>
<td>$RR$</td>
<td>$-a(1-q) - \rho^e P_D(\tilde{r}_X, \tilde{r}<em>C) - r^e P</em>{DD}(\tilde{r}_X, \tilde{r}_C)$</td>
</tr>
<tr>
<td>$RR$</td>
<td>$RL$</td>
<td>$-a(1-q) - \rho^e P_D(\tilde{r}_X, \tilde{r}_C)$</td>
</tr>
<tr>
<td>$RR$</td>
<td>$LR$</td>
<td>$-a(1-q) - \rho^e(1-q_H)$</td>
</tr>
<tr>
<td>$RR$</td>
<td>$LL$</td>
<td>$-aq - \rho^e(1-q_H) - r^e(1-q_H)^2$</td>
</tr>
<tr>
<td>$LL$</td>
<td>$RR$</td>
<td>$-a(1-q) - \rho^e(1-q_H) - r^e(1-q_H)^2$</td>
</tr>
<tr>
<td>$LL$</td>
<td>$RL$</td>
<td>$-aq - \rho^e P_D(\tilde{r}_X, \tilde{r}_C)$</td>
</tr>
<tr>
<td>$LL$</td>
<td>$LR$</td>
<td>$-aq - \rho^e P_D(\tilde{r}_X, \tilde{r}<em>C) - r^e P</em>{DD}(\tilde{r}_X, \tilde{r}_C)$</td>
</tr>
<tr>
<td>$RL$</td>
<td>$RR$</td>
<td>$-a(1-q) - \rho^e P_D(\tilde{r}_X, \tilde{r}<em>C) - r^e P</em>{DD}(\tilde{r}_X, \tilde{r}_C)(1-q_H)$</td>
</tr>
<tr>
<td>$RL$</td>
<td>$RL$</td>
<td>$-aq - \rho^e P_D(\tilde{r}_X, \tilde{r}_C)$</td>
</tr>
<tr>
<td>$RL$</td>
<td>$LR$</td>
<td>$-aq - \rho^e(1-q_H) - r^e(1-q_H)P_D(\tilde{r}_X, \tilde{r}_C)$</td>
</tr>
<tr>
<td>$RL$</td>
<td>$LL$</td>
<td>$-aq - \rho^e(1-q_H) - r^e(1-q_H)^2P_{DD}(\tilde{r}_X, \tilde{r}_C)$</td>
</tr>
<tr>
<td>$LR$</td>
<td>$RR$</td>
<td>$-a(1-q) - \rho^e(1-q_H) - r^e(1-q_H)P_D(\tilde{r}_X, \tilde{r}_C)$</td>
</tr>
<tr>
<td>$LR$</td>
<td>$RL$</td>
<td>$-aq - \rho^e(1-q_H)$</td>
</tr>
<tr>
<td>$LR$</td>
<td>$LR$</td>
<td>$-aq - \rho^e P_D(\tilde{r}_X, \tilde{r}_C)$</td>
</tr>
<tr>
<td>$LR$</td>
<td>$LL$</td>
<td>$-aq - \rho^e P_D(\tilde{r}_X, \tilde{r}<em>C) - r^e P</em>{DD}(\tilde{r}_X, \tilde{r}_C)(1-q_H)$</td>
</tr>
</tbody>
</table>

Table 1: Voters expected utility at each information set. Ass: $p_0 = 0$.

Inspection of table 1 shows that some actions are dominated:

- If $RR$ in $t = 1$, then $RL$ dominates $RR$ in $t = 2$ in all the information sets and for any possible belief (on and off the equilibrium path). Intuitively, the status quo is $p_1 = 1$ in this scenario, and hence the policy outcome is the same with $RR$ and $RL$. In turn, $RL$ is preferred to $RR$ because the latter opens the door to the extraction of observable rents.

- If $LL$ in $t = 1$, then $LR$ dominates $LL$ in $t = 2$ in all the information sets and for any possible belief (on and off the equilibrium path). Intuitively, the status quo is $p_1 = 0$ in this scenario, and hence the policy outcome is the same with $LL$ and $LR$. In turn, $LR$ is preferred to $LL$ because the latter opens the door to the extraction of observable rents.

- If $LR$ or $RL$ in $t = 1$, then $LR$ dominates $LL$ in $t = 2$. 


9.2.3 Auxiliary propositions: equilibria of the subgames that begin after the first election

The subgames that begin when voters choose a unified government at the beginning of period 1 have perfect Bayesian equilibria in which a dishonest executive proposes to extract observable rents iff (i) the rents to be extracted surpass a threshold, and (ii) the legislature is dishonest. Voters reelect only the executive if the executive proposed no rents and only the legislature if the executive proposed positive rents. We present equilibria for party R and L unified governments in propositions 5 and 6, respectively.

**Proposition 5. Equilibrium after a first period party R unified government (RR).**

Suppose the first period government is RR. Consider the following strategy profile:

1. The executive proposes to extract rents iff the realization of the stochastic process of rents surpass a threshold and the legislature is dishonest. In the first period, an honest executive, and a dishonest executive sharing power with a honest congress, propose to extract no rents, i.e. they choose the threshold \( \bar{r} \). A dishonest executive sharing government with a dishonest congress proposes \( \tilde{r}_X = r/2 \) iff \( r > r_u \equiv \frac{2q}{b_d - 1}[(1 - q_H + b_D)\rho^e + E] \). In the second period, both types of executives use the threshold \( \bar{r} \).

2. An honest legislature proposes \( \tilde{r}_C = 0 \), and a dishonest legislature proposes \( \tilde{r}_C = \tilde{r}_X \).

3. Voters choose RL if \( \tilde{r}_X = 0 \) and LR otherwise.

Consider the following beliefs: 
\[
P_D.(0, 0) = \frac{q_H(1 - q_H) + (1 - q_H)^2G(r_u)}{q_H + 2q_H(1 - q_H) + (1 - q_H)^2G(r_u)}, \quad P_D.(0, \tilde{r}_C > 0) \leq 1 - q_H, \quad P_D.(\tilde{r}_X > 0, 0) \geq 1 - q_H \text{ and } P_D.(r/2, r/2) = 1.
\]

This strategy profile and beliefs are a PBE of the subgame that begins after voters choose an RR government in the first election iff
\[
a(1 - 2q) \leq r^e(1 - q_H)^2
\]
(29)

Voters expected utility in this equilibrium is
\[
V(RR) = -a(1 - q)(1 + \beta) - (1 - q_H)^2(1 - G(r_u))E[r|r > r_u] \quad (30)
\]
\[- (1 - q_H)\rho^e - \beta(1 - q_H)[q_H + (1 - q_H)(G(r_u) + (1 - q_H)(1 - G(r_u)))]\rho^e
\]

**Proof of proposition 5.** We check that no one has incentives to deviate at any information set.

1. Honest executive \( (X_H) \)

---

\(^{34}\)Second period outcomes do not depend on politicians' proposals about observable rents for voters choose a divided government in this period (see voters strategies below). In turn, politicians no longer care about revealing their type. Because of this, it is irrelevant the actions politicians take in the second period regarding observable rents. For completeness, we assume they choose the threshold \( \bar{r} \).
(a) After Nature chooses an honest congress ($C_H$). The utility of $X_H$ is (i) $\beta(-aq + E)$, if he proposes zero rent extraction (i.e. if $\tilde{r}_X = 0$), and (ii) $\beta[-aq - (1 - q_H)\rho^e]$, if he proposes $\tilde{r}_X > 0$. Thus, $X_H$ never proposes rents in this information set, which means that he chooses the threshold $\tilde{r}$.

(b) After Nature chooses a dishonest congress ($C_D$). The utility of $X_H$ is (i) $\beta(-aq + E)$, if he proposes $\tilde{r}_X = 0$, (ii) $-\tilde{r}_X + \beta[-aq - (1 - q_H)\rho^e]$, if he proposes $\tilde{r}_X = (0, r/2]$ —i.e. positive rents that can be equally shared with congress—, and (iii) $\beta[-aq - (1 - q_H)\rho^e]$, if he proposes $\tilde{r}_X > r/2$ —i.e. positive rents that cannot be equally shared with congress. Thus, $X_H$ optimal threshold is $\tilde{r}$ and he never proposes rents.

2. Dishonest executive ($X_D$)

(a) After $C_H$. With a honest legislature, the utility of $X_D$ is (i) $\beta[-aq + b_D\rho^e + E]$, if he proposes $\tilde{r}_X = 0$, and (ii) $\beta[-aq - (1 - q_H)\rho^e]$, if he proposes $\tilde{r}_X > 0$. The dishonest executive never extracts rents if the legislature is honest, so he chooses the threshold $\tilde{r}$.

(b) After $C_D$. With a dishonest legislature, the utility of $X_D$ is (i) $\beta[-aq + b_D\rho^e + E]$, if he proposes $\tilde{r}_X = 0$, (ii) $(b_D - 1)\frac{r}{2} + \beta[-aq - (1 - q_H)\rho^e]$, if he proposes $\tilde{r}_X = (0, r/2]$, and (iii) $\beta[-aq - \rho^e(1 - q_H)]$, if he proposes $\tilde{r}_X > r/2$. So, after observing that the legislature is dishonest, a dishonest executive chooses the threshold $r_u = \frac{2\beta}{b_D - 1}[(1 - q_H + b_D)\rho^e + E]$.

3. Honest legislature

(a) After ($X_H, 0$), i.e. after Nature chooses a honest executive and the executive proposes zero rents. The strategy profile indicates that the honest legislature then proposes $\tilde{r}_C = 0$. A deviation to $\tilde{r}_C > 0$ has no consequences: the legislature cannot extract rents if the executive does not propose the same, and the proposal has no impact on reelection probabilities, for the legislature is not reelected if $\tilde{r}_X = 0$, no matter what the legislature proposes.

(b) After ($X_H, r/2$), i.e. after Nature chooses a honest executive and the executive proposes $\tilde{r}_X = r/2 > 0$. The path is $X_H, C_H, r/2, 0, LR$, if the legislature does not deviate, and $X_H, C_H, r/2, \tilde{r}_C > 0, LR$, if it does. The honest legislature does not deviate since the utility gains from deviating are $-r < 0$, if $\tilde{r}_C = r/2$, and 0, if $\tilde{r}_C \neq r/2$.\(^{35}\)

(c) After ($X_D, 0$), i.e. after Nature chooses a dishonest executive and the executive proposes zero rents $\tilde{r}_X = 0$. The path is $X_D, C_H, 0, 0, RL$, if the legislature does not deviate and $X_D, C_H, 0, \tilde{r}_C > 0, RL$ if it does. The legislature does not deviate since the utility gains from deviating are zero.

\(^{35}\)The path $X_H, C_H, r/2, 0, LR$ reads as follows: Nature chooses honest executive and congress ($X_H, C_H$), the executive proposes $\tilde{r}_X = r/2$, the congress proposes $\tilde{r}_C = 0$, and voters chooses an LR government for the second period. Other paths should be read accordingly.
5. Voters. From inspection of table 1, and recalling that the current first period government is RR, is dominated by reelecting both the executive and congress, and so having a party R unified government again in the second period RR, is dominated by reelecting the executive and changing the congress, and so having an RL government in the second period. Other orders of preferences vary across information sets.

(a) Consider first the information set \((0, 0)\) which is on the equilibrium path. Computing \(P_D(0, 0)\) from equations (27) and (28), we note that \(P_D(0, 0) < 1 - q_H\) and conclude that voters prefer RL also to LR. So, we need to compare RL and LL. Voters do not have incentives to deviate from RL to LL iff \(a(1 - 2q) < r^c(1 - q_H)^2 + \rho^c(1 - q_H - P_D(00))\), but this inequality is not binding since (i) \(1 - q_H > P_D(00)\) and (ii) \(a(1 - 2q) < r^c(1 - q_H)^2\).

(b) At the information set \((0, r/2)\), voters do not deviate from RL to LR, if \(P_D(0, r/2) \leq 1 - q_H\), or to LL, if \(P_D(0, r/2) \leq \frac{1}{\rho^c} [-a(1 - 2q) + \rho^c(1 - q_H + r^c(1 - q_H)^2)]\), but this latter inequality is not binding since \(P_D(0, r/2) \leq 1 - q_H\) and \(a(1 - 2q) \leq r^c(1 - q_H)^2\).

(c) At the information set \((r/2, 0)\), voters do not deviate from LR either to RL, if \(P_D(r/2, 0) \geq 1 - q_H\), or to LL, if \(a(1 - 2q) \leq r^c(1 - q_H)^2\), i.e. if inequality (29) holds.

4. Dishonest legislature

(a) After \((X_H, 0)\). The path is \(X_H, C_D, 0, 0, RL\) if the legislature does not deviate and \(X_H, C_D, 0, \tilde{r}_C > 0, RL\) if it does. The legislature does not deviate since the utility gains from deviating are zero.

(b) After \((X_H, r/2)\). The path is \(X_H, C_D, r/2, r/2, LR\) if the legislature does not deviate and \(X_H, C_D, r/2, \tilde{r}_C, LR\) with \(\tilde{r}_C \neq r/2\), if it does. The legislature does not deviate since the utility gains from deviating are \(-(b_D - 1)^2 < 0\).

(c) After \((X_D, 0)\). The path is \(X_D, C_D, 0, 0, RL\) if the legislature does not deviate and \(X_D, C_D, 0, \tilde{r}_C > 0, RL\) if it does. The legislature does not deviate since the utility gains from deviating are zero.

(d) After \((X_D, r/2)\). The path is \(X_D, C_D, r/2, r/2, LR\) if the legislature does not deviate and \(X_D, C_D, r/2, \tilde{r}_C, LR\) with \(\tilde{r}_C \neq r/2\), if it does. The utility gains of the dishonest legislature from deviating are: \(-(b_D - 1)^2 < 0\) so the dishonest legislature does not deviate.
At the information set \((r/2, r/2)\), voters do not deviate from \(LR\) either to \(RL\), since \(P_D(r/2, r/2) = 1 \geq 1 - q_H\), or to \(LL\), if \(a(1 - 2q) \leq r^e(1 - q_H)^2\).

\[\text{Proposition 6. Equilibrium after a first period party } L \text{ unified government (LL).}\]

Suppose the first period government is \(LL\). Consider the following strategy profile:

1. The executive proposes to extract rents iff the realization of the stochastic process of rents surpass a threshold and the legislature is dishonest. In the first period, an honest executive, and a dishonest executive sharing power with a honest congress, propose to extract no rents, i.e. they choose the threshold \(\bar{r}\). A dishonest executive sharing government with a dishonest congress proposes \(\tilde{r}_X = r/2\) iff \(r > r_u \equiv \frac{2\beta}{b_D - 1}[(1 - q_H + b_D)\rho^e + E]\). In the second period, both types of executives use the threshold \(\bar{r}\), i.e. they do not extract rents.

2. An honest legislature proposes \(\tilde{r}_C = 0\), and a dishonest legislature proposes \(\tilde{r}_C = \tilde{r}_X\).

3. Voters choose \(LR\) if \(\tilde{r}_X = 0\) and \(RL\) otherwise.

Consider the following beliefs: \(P_D(0, 0) = \frac{aq(1 - q_H) + (1 - q_H)^2G(r_u)}{q_H + 2q_H(1 - q_H) + (1 - q_H)^2G(r_u)}\), \(P_D(0, \tilde{r}_C > 0) \leq 1 - q_H\), \(P_D(r/2, 0) \geq 1 - q_H\) and \(P_D(r/2, r/2) = 1\).

This strategy profile and beliefs are a PBE of the subgame that begins after voters choose an \(LL\) government in the first election iff

\[a(2q - 1) \leq r^e(1 - q_H)^2\] \hspace{1cm} (31)

Voters expected utility in this equilibrium is

\[V(LL) = -aq(1 + \beta) - (1 - q_H)^2(1 - G(r_u))\mathbb{E}[r > r_u] - (1 - q_H)\rho^e - \beta(1 - q_H)(q_H + (1 - q_H)(G(r_u) + (1 - q_H)(1 - G(r_u))))\rho^e\] \hspace{1cm} (32)

\textbf{Proof of proposition 6.} The proof is analogous to the one of Proposition 5. \hfill \qed

The subgames that begin when voters choose a divided government in the first election have perfect Bayesian equilibria in which the executive never proposes to extract observable rents — or, more formally, use a discipline threshold \(\bar{r}\)—, a dishonest legislature proposes zero rents if the executive proposes zero rents and \(r\) otherwise, an honest legislature proposes no rents, and voters reelect the executive and the legislature iff extracted observable rents are zero. The policy outcome and the existence conditions in the divided government equilibria depend on the initial status quo \(p_0\). For the sake of brevity, we focus exclusively on the case \(p_0 = 0\). The other case is analogous.

Suppose that $p_0 = 0$ and the first period government is RL. Consider the following strategy profile:

1. The executive never proposes to extract rents. This means that both honest and dishonest executives use the threshold $\bar{r}$ in both periods.

2. An honest legislature proposes $\tilde{r}_C = 0$, and a dishonest legislature proposes $\tilde{r}_C = \tilde{r}_X$.

3. Voters choose RL if $\tilde{r}_X = 0$ and LR otherwise.

Consider the following beliefs: $P_D.(0,0) = 1 - q_H$, $P_D.(0,\tilde{r}_C > 0) \leq 1 - q_H$, $P_D.(r/2,0) \geq 1 - q_H$ and $P_D.(r/2,r/2) \geq 1 - q_H$.

This strategy profile and beliefs are a PBE of the subgame that begins after voters choose an RL government in the first election iff

$$a(2q - 1) \leq r^e(1 - q_H)P_D.(0,r/2)$$

(33)

Voters expected utility in this equilibrium is

$$V(RL) = (1 + \beta)[-aq - (1 - q_H)r^e].$$

(34)

Proof of proposition 7. We check that no one has incentives to deviate at any information set.

1. Honest executive ($X_H$)

   (a) After Nature chooses an honest congress ($C_H$). The utility of $X_H$ is (i) $\beta(-a(3q + 1) + E)$, if he proposes zero rent extraction (i.e. if $\tilde{r}_X = 0$), and (ii) $\beta[-a(3q + 1) - (1 - q_H)r^e]$, if he proposes $\tilde{r}_X > 0$. Thus, $X_H$ never proposes rents in this information set, which means that he chooses the threshold $\bar{r}$.

   (b) After Nature chooses a dishonest congress ($C_D$). The same as with an honest congress.

2. Dishonest executive

   (a) After $C_H$. With a honest congress, the utility of $X_D$ is (i) $\beta[-a(3q + 1) + b_Dr^e + E]$, if he proposes $\tilde{r}_X = 0$, and (ii) $\beta[-a(3q + 1) - (1 - q_H)r^e]$, if he proposes $\tilde{r}_X > 0$. Hence the dishonest executive proposes $\tilde{r}_X = 0$.

   (b) After $C_D$. The same as with an honest congress.

3. Honest legislature
(a) After \((X_H, 0)\), i.e. after Nature chooses a honest executive and the executive proposes zero rent extraction. The path is \(X_H, C_H, 0, 0, RL\) if the legislature does not deviate and \(X_H, C_H, 0, \tilde{r}_C > 0, RL\) if it does. The legislature does not deviate since the utility gains from deviating are zero.

(b) After \((X_H, r/2)\), i.e. after Nature chooses a honest executive and the executive proposes \(\tilde{r}_X = r/2 > 0\). The path is \(X_H, C_H, r/2, 0, LR\) if the legislature does not deviate and \(X_H, C_H, r/2, \tilde{r}_C > 0, LR\) if it does. The honest legislature does not deviate since the utility gains from deviating are zero.

(c) After \((X_D, 0)\), i.e. after Nature chooses a dishonest executive and the executive proposes \(\tilde{r}_X = 0\). The path is \(X_D, C_H, 0, 0, RL\) if the legislature does not deviate and \(X_D, C_H, 0, \tilde{r}_C > 0, RL\) if it does. The legislature does not deviate since the utility gains from deviating are zero.

(d) After \((X_D, r/2)\), i.e. after Nature chooses a dishonest executive and the executive proposes \(\tilde{r}_X = r/2 > 0\). The path is \(X_D, C_H, r/2, r/2, LR\) if the congress does not deviate and \(X_D, C_H, r/2, \tilde{r}_C > 0, LR\) if it does. The utility gains of the honest legislature from deviating are zero, so the honest legislature does not deviate.

4. Dishonest legislature

(a) After \((X_H, 0)\). The path is \(X_H, C_D, 0, 0, RL\) if the congress does not deviate and \(X_H, C_D, 0, \tilde{r}_C > 0, RL\) if it does. The legislature does not deviate since the utility gains from deviating are zero.

(b) After \((X_H, r/2)\). The path is \(X_H, C_D, r/2, r/2, LR\) if the legislature does not deviate and \(X_H, C_D, r/2, \tilde{r}_C \neq r/2, LR\) if it does. The legislature does not deviate since the utility gains from deviating are zero.

(c) After \((X_D, 0)\). The path is \(X_D, C_D, 0, 0, RL\) if the legislature does not deviate and \(X_D, C_D, 0, \tilde{r}_C > 0, RL\) if it does. The legislature does not deviate since the utility gains from deviating are zero.

(d) After \((X_D, r/2)\). The path is \(X_D, C_D, r/2, r/2, LR\) if the legislature does not deviate and \(X_D, C_D, r/2, \tilde{r}_C \neq r/2, LR\) if it does. The legislature does not deviate since the utility gains from deviating are zero.

5. Voters. From inspection of table 1, and recalling that the current first period government is RL, we conclude that LL is dominated by LR. Other orders of preferences vary across information sets.
(a) Consider first the information set \((0, 0)\), which is on the equilibrium path. We note that
\[ P_D(0, 0) = 1 - q_H \] and conclude that voters are indifferent between \(RL\) and \(LR\). Voters do not have incentives to deviate from \(RL\) to \(RR\) iff
\[ a(2q - 1) \leq r^e(1 - q_H)^2, \] i.e. if condition (33) holds.

(b) At the information set \((0, r/2)\), voters do not deviate from \(RL\) to \(LR\), if \(P_D(0, r/2) \leq 1 - q_H\), or to \(RR\), if \(a(2q - 1) \leq r^e(1 - q_H)P_D(0, r/2)\).

(c) At the information set \((r/2, 0)\), voters do not deviate from \(LR\) either to \(RL\), if \(P_D(r/2, 0) \geq 1 - q_H\), or to \(RR\), if \(a(2q - 1) \leq \rho^e(P_D(r/2, 0) - (1 - q_H)) + r^e(1 - q_H)P_D(r/2, 0)\).

(d) At the information set \((r/2, r/2)\), voters do not deviate from \(LR\) either to \(RL\), if \(P_D(r/2, r/2) \geq 1 - q_H\), or to \(RR\), if \(a(2q - 1) \leq \rho^e(P_D(r/2, r/2) - (1 - q_H)) + r^e(1 - q_H)P_D(r/2, r/2)\).

Note that, given the restrictions on voters beliefs, conditions (i) \(a(2q - 1) \leq \rho^e(P_D(r/2, 0) - (1 - q_H)) + r^e(1 - q_H)P_D(r/2, 0)\) and (ii) \(a(2q - 1) \leq \rho^e(P_D(r/2, r/2) - (1 - q_H)) + r^e(1 - q_H)P_D(r/2, r/2)\), are not biding. Thus, apart from the restrictions on the beliefs, \(a(2q - 1) \leq r^e(1 - q_H)P_D(0, r/2)\) is the only binding condition.

\[ \square \]

**Proposition 8.** *Equilibrium after a divided government with an L executive and an R congress (LR).*

Suppose that \(p_0 = 0\) and the first period government is \(LR\). Consider the following strategy profile:

1. The executive never proposes to extract rents, so he chooses a threshold \(\tilde{r}\) in both periods.
2. An honest legislature proposes \(\tilde{r}_C = 0\), and a dishonest legislature proposes \(\tilde{r}_C = \tilde{r}_X\).
3. Voters choose \(LR\) if \(\tilde{r}_X = 0\) and \(RL\) otherwise.

Consider the following beliefs: \(P_D(0, 0) = 1 - q_H\), \(P_D(0, \tilde{r}_C > 0) \leq 1 - q_H\), \(P_D(r/2, 0) \geq 1 - q_H\) and \(P_D(r/2, r/2) \geq 1 - q_H\).

This strategy profile and beliefs are a PBE of the subgame that begins after voters choose an \(LR\) government in the first election iff

\[ a(2q - 1) \leq r^e(1 - q_H)P_D(0, r/2) \] (35)

Voters expected utility in this equilibrium is

\[ V(LR) = (1 + \beta)[-aq - (1 - q_H)\rho^e]. \] (36)

**Proof of proposition 8.** The proof is analogous to the one of Proposition 7. \[ \square \]
9.2.4 Proof of propositions 3 and 4

We now prove the main propositions in section 4. For simplicity, we have omitted in propositions 3 and 4 the conditions on the off-the-equilibrium beliefs and the Bayesian updating of the on-the-equilibrium beliefs detailed in the auxiliary propositions 5 to 8.

Proof of proposition 3. Suppose the conditions for the existence of the equilibria identified in the auxiliary propositions 5 to 8 hold. From equations (30), (32), (34) and (36) we conclude that voters prefer a unified government in the first period if condition (8) holds.

From equations (30) and (32) we conclude that voters prefer a party R over a party L unified government in the first period if \( q \geq \frac{1}{2} \), and a party L over a party R unified government if \( q \leq \frac{1}{2} \).

Suppose first that \( q \leq \frac{1}{2} \). From equations (32) and (34) we conclude that voters prefer an LL over a divided government iff

\[ \mathbb{E}[r|r \geq r_u] \leq \beta q_H \rho^e \]

Suppose now that \( q \geq \frac{1}{2} \). From equations (30) and (34) we conclude that voters prefer an RR over a divided government iff

\[ \mathbb{E}[r|r \geq r_u] \leq \frac{(1 + \beta)a(2q - 1)}{(1 - q_H)^2(1 - G(r_u))} + \beta q_H \rho^e, \]

Combining these two results, we have condition (8).

A unified government extracts observable rents in the first period only if both incumbents are dishonest and the realization of \( r \) is above the threshold \( r_u \). The probability of this event is \( (1 - q_H)^2(1 - G(r_u)) \). The government does not extract observable rents in the second period because voters choose a divided government (see propositions 5 to 8). A dishonest executive incumbent extracts unobservable rents. The probability that the first period incumbent executive is dishonest is \( 1 - q_H \).

The second period executive is dishonest if the first period executive is dishonest and (i) the first period legislature is honest (which occurs with probability \( q_H(1 - q_H) \)), or (ii) the first period legislature is dishonest but rents lied below the threshold (which occurs with probability \( (1 - q_H)^2G(r_u) \)), or (iii) the first period legislature is dishonest, rents lied above the threshold and the challenger for the executive is also dishonest (which occurs with probability \( (1 - q_H)^3(1 - G(r_u)) \)).

Proof of proposition 4. Suppose the conditions for the existence of the equilibria identified in the auxiliary propositions 5 to 8 hold. From equations (30), (32), (34) and (36) we conclude that voters prefer a divided over a unified government in the first period if condition (8) does not hold.

We have the following auxiliary result:

Lemma 4. SP increases the welfare of the majority of voters iff it does it for the swing voters.
Proof. All voters share the same preferences regarding rent extraction. They only differ in how they value policies \( p_t \). Left, swing and right voters expected utility from policies are

\[
\begin{align*}
  w_L &= -aE_s[(p_t - s_t + 1)^2], \\
  w_S &= -aE_s[(p_t - s_t)^2], \\
  w_R &= -aE_s[(p_t - s_t - 1)^2],
\end{align*}
\]

respectively.

Thus the expected utilities from implementing each of the two possible policies are:

\[
\begin{array}{|c|c|c|}
\hline
\text{Voter} & w^0_i & w^1_i & \Delta^1_{i} - \Delta^0_{i} = w^1_i - w^0_i \\
\hline
\text{Left} & -a(1 - q) & -a(4 - 3q) & a(2q - 3) \\
\text{Swing} & -aq & -a(1 - q) & a(2q - 1) \\
\text{Right} & -a(3q + 1) & -aq & a(2q + 1) \\
\hline
\end{array}
\]

Table 2: Voters expected utility associated with policies \( p \in \{0, 1\} \)

where \( w^0_i \) and \( w^1_i \) are the expected payoffs from \( p = 0 \) and \( p = 1 \), respectively, \( i \in \{\text{Left, Swing, Right}\} \), and \( \Delta^j_{i - k} \) is the gain from implementing \( j \) rather than \( k \).

Note that \( \Delta^1_{L} - \Delta^0_{L} \leq \Delta^1_{S} - \Delta^0_{S} \leq \Delta^1_{R} - \Delta^0_{R} \), and thus \( \Delta^0_{R} - \Delta^1_{R} \leq \Delta^0_{S} - \Delta^1_{S} \leq \Delta^0_{L} - \Delta^1_{L} \).

Consider first the case where only the second period policy changes when we include SP. If the change is from 0 to 1 then, \( \Delta^1_{L} - \Delta^0_{L} \leq \Delta^1_{S} - \Delta^0_{S} \leq \Delta^1_{R} - \Delta^0_{R} \), and if it is from 1 to 0 then \( \Delta^0_{R} - \Delta^1_{R} \leq \Delta^0_{S} - \Delta^1_{S} \leq \Delta^0_{L} - \Delta^1_{L} \).

Now consider the case where both periods policy change. If the change at \( t = 1 \) is from 0 to 1 then, the expected gains from SP are:

\[
\Delta^1_{i} - \Delta^0_{i} + \beta[P_R \Delta^1_{L} - \Delta^0_{L} + (1 - P_R)0],
\]

where \( P_R \) is the probability of reelection with no SP (note that with probability \( (1 - P_R) \) the challenger is elected with no SP, so there is a change in the implemented policy in the second period, and then it coincides with the policy implemented with SP).\(^{36}\)

Then,

\[
\Delta^1_{L} - \Delta^0_{L} + \beta P_R \Delta^1_{L} - \Delta^0_{L} \leq \Delta^1_{S} - \Delta^0_{S} + \beta P_R \Delta^1_{S} - \Delta^0_{S} \leq \Delta^1_{R} - \Delta^0_{R} + \beta P_R \Delta^1_{R} - \Delta^0_{R}.
\]

If the change at \( t = 1 \) is from 1 to 0 a similar reasoning shows that:

\[
\Delta^0_{R} - \Delta^1_{R} + \beta P_R \Delta^0_{R} - \Delta^1_{R} \leq \Delta^0_{S} - \Delta^1_{S} + \beta P_R \Delta^0_{S} - \Delta^1_{S} \leq \Delta^0_{L} - \Delta^1_{L} + \beta P_R \Delta^0_{L} - \Delta^1_{L}.
\]

Finally, \( SP \) has the same impact on the expected utility of all voters if it does not induce any change in the positional policies.

\(^{36}\)\( P_R = q_H + (1 - q_H)G(r_i), \) where \( i \in \{R, L\} \).
9.2.5 Proof of corollaries 3 to 5

Proof of corollary 3. We first prove that $SP$ increases swing voters welfare if the always reelect equilibrium arises in the no $SP$ environment. We then appeal to lemma 4 to conclude that at least other group of voters also benefits from $SP$.

The parameter values that are consistent with an always reelect equilibrium in the no $SP$ environment are also consistent with four different equilibria in the $SP$ environment: (i) first period unified government with $q \leq 1/2$; (ii) first period unified government with $q > 1/2$; (iii) first period divided government with $q \leq 1/2$; and (iv) first period divided government with $q > 1/2$. We prove the claim in each of these cases.

1. Unified government, $q \leq 1/2$.

According to propositions 2 and 3, when $q \leq 1/2$ voters elect $L$ and $LL$ governments in the without and with $SP$ environments, respectively, so we compute the gains from introducing separation of powers using equations (26) and (32):

$$V(LL) - V(L) = (1 - q_H)\{r^e - (1 - q_H)(1 - G(r_u))E[r > r_u] + \beta [r^e + (1 - q_H)q_H(1 - G(r_u))\rho^e]\}.$$

$SP$ increases voters’ second period welfare —the second term in the right hand side of the above equation— because it eliminates the extraction of observable rents and brings some political selection to an otherwise total failure of accountability in the no $SP$ environment. $SP$ also increases first period welfare —the first term in the right hand side of the equation— since $r^e - (1 - q_H)(1 - G(r_u))E[r > r_u] = \int_0^\infty r f(r)dr - (1 - q_H) \int_{r_u}^\infty r f(r)dr > 0$.

2. Unified government, $q > 1/2$.

According to propositions 2 and 3, when $q > 1/2$ voters elect $R$ and $RR$ governments in the without and with $SP$ environments, respectively, so we compute the gains from introducing separation of powers using equations (25) and (30):

$$V(RR) - V(R) = (1 - q_H)\{r^e - (1 - q_H)(1 - G(r_u))E[r > r_u] + \beta [r^e + (1 - q_H)q_H(1 - G(r_u))\rho^e]\}.$$

For the same reasons as in the previous case, the welfare gains from introducing $SP$ are positive.

3. Divided government, $q \leq 1/2$.

According to proposition 2, when $q \leq 1/2$ voters elect an $L$ government in the without $SP$ environment, so we compute the gains from introducing separation of powers using equations (26) and (34):

$$V(RL) - V(L) = (1 + \beta)(1 - q_H)r^e > 0$$
4. Divided government, \( q > 1/2 \).

According to proposition 2, when \( q > 1/2 \) voters elect an \( R \) government in the without \( SP \) environment, so we compute the gains from introducing separation of powers using equations (25) and (34):

\[
V(RL) - V(R) = (1 + \beta)[-a(2q - 1) + (1 - q_H)r^e].
\]

Therefore, \( SP \) raises swing voters welfare iff \( (1 - q_H)r^e > a(2q - 1) \). We now show that this is the case.

Voters choose a first period divided government iff inequality (8) does not hold, i.e. iff:

\[
\int_{r_u}^{\bar{r}} rdG(r) > \frac{(1 + \beta)a(2q - 1)}{(1 - q_H)^2} + \beta q_H(1 - G(r_u))r^e.
\]

In turn,

\[
r^e = \int_0^{\bar{r}} rdG(r) \geq \int_{r_u}^{\bar{r}} rdG(r) > \frac{(1 + \beta)a(2q - 1)}{(1 - q_H)^2} + \beta q_H(1 - G(r_u))r^e.
\]

Hence, if voters choose a divided first period government then:

\[
(1 - q_H)r^e > \frac{(1 + \beta)a(2q - 1)}{(1 - q_H)} + \beta q_H(1 - G(r_u))r^e \geq a(2q - 1).
\]

\( \square \)

**Proof of corollary 4.** (i) \( G(r_i) = 1 \iff r_i \geq \bar{r}, i \in \{NSP, SP\} \). (ii) Voters hold the incumbent accountable in the no \( SP \) environment if \( \rho^e \to \infty \) (inequality (4) holds). (iii) By virtue of (5) and (9), the dishonest incumbents fully discipline, i.e. \( G(r_{SP}) = G(r_{NSP}) = 1 \), if (i) \( \rho^e \to \infty \) or (ii) (4) hold and \( E \to \infty \). (iv) By virtue of equation (6), \( G(r_{NSP}) = G(r_L) = G(r_R) = 1 \) implies that \( V(R) > V(L) \iff q > 1/2 \). (v) In the \( SP \) environment full discipline implies that voters choose a unified first period government. Indeed, \( \mathbb{E}[r|r \geq r_u] = 0 \) with full discipline and hence condition (8) holds. (vi) In a unified government equilibrium, voters choose a party L or party R government if \( q < 1/2 \) or \( q > 1/2 \), respectively, and flip a coin if \( q = 1/2 \). (vi) Points (i) to (v) imply that swing voters expected gains from \( SP \) are:

\[
V(jj) - V(j) = a\beta(1 - q_H)|2q - 1|(1 - G(r_{NSP})
+ \int_{\min(r_{NSP},\bar{r})}^{\bar{r}} rg(r)dr - (1 - q_H)\int_{\min(r_{SP},\bar{r})}^{\bar{r}} rg(r)dr
+ \beta(1 - q_H)r^e[G(r_{NSP}) + (1 - G(r_{NSP}))(1 - q_H)]
+ \beta(1 - q_H)q_H[G(r_{NSP}) - G(r_{SP}) - q_H(1 - G(r_{SP}))]r^e, \ j \in \{L, R\}
\]

Equations (5) and (9) imply that \( r_{NSP} \) and \( r_{SP} \) are increasing in \( E \) and \( \rho^e \). Therefore, there are two thresholds \( \rho^a \) and \( \rho^b \), defined by the conditions \( r_{NSP}(\rho^a) = r_{SP}(\rho^b) = \bar{r} \) such that \( G(r_{SP}) = G(r_{NSP}) = 1 \) if \( \rho^e \geq \max\{\rho^a, \rho^b\} \). Similarly, two thresholds, \( E^a \) and \( E^b \) can be determined for \( E \). This implies that \( V(jj) - V(j) = \beta(1 - q_H)r^e \geq 0 \) if \( \rho^e \geq \max\{\rho^a, \rho^b\} \) or \( E \geq \max\{E^a, E^b\} \). \( \square \)
Proof of corollary 5. \( a = 0 \) implies that conditions (4) hold and hence an electoral accountability equilibrium arises in the no SP environment. Inequality (8) also holds as \( \bar{r} \to 0 \) if \( \rho^e > 0 \), and hence a unified government equilibrium arises in the SP environment. In these conditions,

\[
\begin{align*}
    r_{NSP} &= \frac{\beta}{b_D} (1 - q_H + b_D) \rho^e + \frac{\beta E}{b_D} \\
    r_{SP} &= 2 \frac{\beta}{b_D - 1} (1 - q_H + b_D) \rho^e + \frac{2\beta E}{b_D - 1} 
\end{align*}
\]

The thresholds \( r_{NSP} \) and \( r_{SP} \) are linear functions of \( \rho^e \), with positive intercepts and slopes, and both the intercept and slope are strictly smaller for \( r_{NSP} \) than for \( r_{SP} \). Then, it is immediately clear that \( r_{NSP} < r_{SP} \) if \( 0 \leq \rho^e < \rho^* \). Equation (37) computes voters gains from SP when an EA and a unified government equilibria arise in the no SP and SP environments respectively. The first three terms in the right hand side of (37) are or tend to zero in the case considered in this corollary and the last term is negative since \( r_{NSP} < r_{SP} \).