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Changes in social welfare in an intertemporal economy

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Changes in social welfare in an intertemporal economy [1]

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Abstract

General Equilibrium Theory (GE) scrutinizes the ability of markets to achieve efficient allocation of resources. The main purpose of this work is, in the framework to the GE to analyze, the possibility of design a mechanism enabling agents to make independent decisions compatible with social welfare. More precisely, we address the problem of the possibility of decentralization mechanism to sustain efficiency and social welfare at the same time. Specifically, introducing a social utility function, we argue on the possibility of improving the social welfare transferring resources between periods of the economy. This mechanism introduces a Rawlsian solution improving the welfare of the individuals worst positioned.

Keywords: Welfare, efficiency, decentralization.

Resumen

La Teoría del Equilibrio General examina la capacidad de los mercados para lograr una asignación eficiente de los recursos. Nuestro principal objetivo en este trabajo es, el de analizar, en el marco de la referida teoría, la posibilidad de implementar un mecanismo que permita a los agentes tomar decisiones independientes compatibles con el bienestar social. Más precisamente, abordar el problema de la existencia de un mecanismo que, de manera descentralizada, permita lograr la eficiencia y el bienestar social al mismo tiempo. En concreto, se muestra como si se permite en una economía de dos períodos, transferir libremente recursos entre ambos, la acción independiente de los agentes económicos en el mercado, es capaz de lograr la distribución de recursos preferida por la sociedad.

Palabras clave: Bienestar social, eficiencia, descentralización.

JEL: D6, D7

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1 Introduction

To consider plans for improving the welfare of an economy is necessary to consider intertemporal economies over several periods. We will consider in this work a simple dynamic model having two periods. In particular, we will specialize the two-period model even more by analyzing a two-period exchange economy model, inhabited by agents who live two periods. Maximization is over feasible consumption program $\{c^1, c^2\} : c^t \in R_+^{ln}$ and $t \in \{1,2\}$. Where c_1 is an allocation for time $t = 1$ and c^2 one allocation for period $t = 2$, n is the number of agents, leaving in two periods, and l the number of different goods. Goods are available in positive quantities in both periods. We consider two possible points of view, the point of view of individual and perfectly informed agents, and on the other hand the point of view of a social planner interested in the society's aggregate welfare. We will analyze the possibility of defining a mechanism able to unify these two different and some times antagonistic points of view.

We are particularly interested in comparing the possible levels of social welfare that can be attained from individual decisions in an economy when consumers have possibilities to transfer wealth between periods, with the levels of welfare that can be attained when this possibility does not exist. At the same time, we relate these levels with the social welfare attained for an economy when the action of a benevolent central planner is considered. We focus on the intertemporal consumption/saving as a individual decision and then we compare this decision with the decisions of a central planner looking for the social welfare. We find that the result of the participation of a benevolent central planner can be substituted by individual decisions of agents if those with better opportunities have a high degree of social commitment. But if this is not the case, then the central planner can do more than lump-sum redistributions to improve the social welfare.

The paper is organized as follows: In section (2) we introduce the model, for simplicity, this paper considers an economy lasting only two periods. There is also complete certainty, as well as perfect and complete information. Each agent is assumed to have a feasible set of net trades which is separable into two history dependent feasible sets, one for each period. Agents have preferences which can be represented by the sum of two separate history dependent single period utility functions, each defined on the relevant set of feasible net trades. Moreover, a Negishi social welfare function in the form of a sum over all agents' utilities is postulated. In section (3) we consider a two-period economy such that consumers can not do transferences from one period to another, and we look for the efficient allocations attainable. In section (4) we allow transferences between periods. This kind of transferences changes the endowments disposable in each period for each agent, but not the total of resources disposable for each agent of the economy in aggregating in both periods. We look for a distribution of resources between periods allowing to attain a Pareto optimal allocation maximizing the social utility function. In section (5) we introduce the central planner. The main role of the central planner is to implement an incentive policy to attain an optimal transference of resources between periods, considering as optimal a policy that gives the possibility to attain the maximal social welfare. We compare the social welfare level possible to be attained following the central planner policy, with the optimal level of welfare attained by a policy of lending and saving developed by the individual agents of the economy. Finally as a conclusion, in section (6) we consider the possibilities to attain a maximal social welfare, in a decentralized way.

2 The model

We consider a two period neoclassical economy symbolized by

$$E = \left\{R_+^l, u_i, w_i, i \in I\right\}$$

where $I = \{1,...n\}$ is an index set, one for each agent. The economy have l goods in each period. Assume that the n agents of the economy derive utilities $u_i(c_i)$ from the two periods consumption bundle $c_i = (c_i^1, c_i^2), c_i^t \in R^l, i = 1,...,n; t = 1,2.$ By $c_i^t = (c_{i1}^t,...,c_{il}^t)$ we symbolize a bundle set for the $i-th$ agent available in time t.

The agents welfare is defined as the present value of the sum of current and future utility discounted at a rate θ:

$$u_i(c_i^1, c_i^2) = U_i(c_i^1) + e^{-\theta} U_i(c_i^2). \tag{1}$$

In each period utilities are strictly concave, increasing and C^1 in interior of R_+^l i.e: $\frac{\partial U_{it}}{\partial x_j} > 0 \; \forall j \in \{1,...,l\}, i \in \{1,..,n\}$ and $t \in \{1,2\}$ and satisfies the boundary condition is a sequence of vectors c_i converges to a vector with some coordinate equal to zero, then

$\lim_{c_{ij}^t \to 0} \dfrac{\partial U_i}{\partial x_j}(c_i) = \infty\ \forall i \in \{1,...,n\}$ *and* $j \in \{1,...l\}$. Because utilities are normalized to take nonnegative values, $U_i \geq 0$.

The endowments are denoted by $w_i = (w_i^1, w_i^2)$ where $w_i^t \in R_+^l$, $i = 1,...,n$, and $t = 1,2$. The total resources distribution is given by $\Omega = (\Omega^1, \Omega^2)$, where the coordinates represent respectively the stocks of goods available in the economy in each period $t = 1,2$; i.e: $\Omega^t = \sum_i w_i^t$, $t = 1,2$.

Following [Negishi, T.], we consider a social welfare function as a weighted sum of individual utility functions. Denote this social utility function by the time-additive separable function:

$$W_\lambda(c_1,...,c_n) = \sum_i \lambda_i u_i(c_i^1, c_i^2) = \sum_i \lambda_i \left(U_i(c_i^1) + e^{-\theta} U_i(c_i^2)\right) \tag{2}$$

where $\lambda = (\lambda_1,...,\lambda_n) \in \Delta^{n-1}$. We symbolize by Δ^{n-1} $n-1$ simplex, in R^n i.e; the set of $\lambda \in R_+^n : \lambda_1 + ... + \lambda_n = 1$.

Definition 1 *An allocation* $c = (c_1,...,c_n)$ *is a specification of a consumption vector* $c_i = (c_i^1, c_i^2) \in R_+^l \times R_+^l$ *for each consumer* $i = 1,...,n$ *an period* $t = 1,2$. *An allocation is feasible if* $\sum_{i=1}^n c_i^t \leq \Omega_t\ \forall t = 1,2$. *We denote the set of feasible allocations by*

$$F = \left\{ c = ((c_1^1, c_1^2),...,(c_n^1, c_n^2)) \in (R_+^l \times R_+^l)^n : \sum_{i=1}^n c_i^t \leq \Omega^t,\ t = 1,2 \right\}.$$

The utility possibility set for this economy is given by de subset of R_+^n:

$$U = \left\{ u \in R_+^n ; \exists c \in F : u_i \leq u_i(c_i^1, c_i^2)\ \forall\ i = 1,...n \right\}$$

For an allocation c, we introduce the notation $u(c)$ to denote the utility vector $u(c) = (u_1(c_1),...,u_n(c_n))$, where

$$u_i(c_i) = u_i(c_i^1, c_i^2) = U_i(c_i^1) + e^{-\theta} U_i(c_i^2)$$

By UP we symbolize the boundary of the utility possibility set. It is easy to see that if c is a Pareto optimal allocation, then $u(c) \in UP$.

Proposition 1 *The simplex* $\Delta^{n-1} = \left\{\lambda \in R_+^n : \sum_{i=1}^n \lambda_i = 1\right\}$ *is homeomorphic to the boundary of the utility possibility set.*

Proof: Consider the function: $\xi : UP \to R^{n-1}$ defined by $\xi(u) = u \frac{1}{\sum_i u_i}$ is continuous and establishes a bijection between de subsets Δ^{n-1} and UP. •

3 A two-period economy without transferences

In this section we focuss on the set of Pareto optimal allocations of a two periods economy. The Pareto optimal allocations corresponding to a neoclassical economy, are determined by the total amounts of wealth existing in the economy and not by the distribution of this wealth among the agents. We consider that the consumers faces two constraints one en each period. These constraints simply say that in each period, the aggregate consumption of the economy can not exceed the existing wealth. This constraint are written as

$$\sum_{i=1}^n c_i^1 \leq \Omega_1$$

$$\sum_{i=1}^n c_i^2 \leq \Omega_2$$

So the set of feasible allocations are given by definition (1).

The following proposition holds:

Proposition 2 *For every Pareto allocation* $c = (c_1, ..., c_n)$ *there is vector* $\bar{\lambda} \in \Delta^{n-1}$ *such that* c *is a solution to problem:*

$$\max_c W_{\bar{\lambda}}(c) = \sum_{i=1}^n \bar{\lambda}_i u_i(c_i)$$

$$s.t. \qquad \sum_{i=1}^n c_i^1 \leq \Omega_1 \tag{3}$$

$$\sum_{i=1}^n c_i^2 \leq \Omega_2$$

and reciprocally, for each $\bar{\lambda} \in \Delta^{n-1}$ every solution to a problem (3) is a Pareto optimal allocation.

Proof: If the allocation $c=(c_1,...,c_n)$, where $c_i=(c_i^1,c_i^2)$ is Pareto optimal, then the vector of utilities $u=u(c)\in UP$. For each allocation c and $\lambda\in\Delta^{n-1}$ we can write:

$$\sum_i \lambda_i u_i(c_i^1,c_i^2)=\sum_{i=1}^{n}\lambda_i\left[U_i(c_i^1)+e^{\theta}U_i(c_i^2)\right]=\sum_i \lambda_i u_i$$

where $u_i=u_i(c_i^1,c_i^2)=U_i(c_i^1)+e^{\theta}U_i(c_i^2)$ and for each $u=(u_1,...,u_n)$ in the boundary of the utility possibility set there exists $\bar{\lambda}\in\Delta^{n-1}$: u solves the problem

$$\max_u \sum_i \bar{\lambda}_i u_i=\bar{\lambda}\bar{u} \qquad (4)$$

and such $\bar{\lambda}=u\dfrac{1}{\sum_i u_i}$. •

Remark 1 Notation:

• If $U^1=(U_1^1,...,U_n^1)$, and $U^2=(U_1^2,...,U_n^2)$ by $u=U^1+e^{-\theta}U^2$ we denote the vector

$$u=(u_1,...,u_n)=(U_1^1+e^{-\theta}U_1^2,...,U_n^1+e^{-\theta}U_n^2)$$

• Let $c=(c^1,c^2)$ a feasible allocation, where $c^t=(c_1^t,...,c_n^t)$ symbolize a feasible consumption bundle for period $t=1,2$.

• $u(c)=(U(c^1)+e^{-\theta}U(c_2))=\left(U_1(c_1^1)+e^{-\theta}U_1(c_1^2),...,U_n(c_n^1)+e^{-\theta}U_n(c_n^2)\right)$

• By F^t we denote the feasible allocation set for period $t=1,2$,

$$F^t=\left\{c^t\in R^{ln}:\ \sum_{i=1}^{n}c_i^t\le\Omega_t\right\}.$$

Note that, if $u=U^1+e^{-\theta}U^2\in UP$ then $U^1\in UP^1$ and $U^2\in UP^2$ where by UP^t, $t=1,2$ we denote the boundary of the utility possibility set given by:

$$U^t=\left\{U\in R^n:\exists\, c^t\in F^t:U_i\le U_i(c_i^t)\ \forall\, i\in I\right\}.$$

To explore the relationships of this social weights given by λ with the first and second welfare theorems, let us consider the following intuition for the social weights. We consider, an economy with locally non satiable preferences, and such that every $u_i(\cdot)$ is a differentiable and concave function. Note that pair $(c^*, p) \in (R_+^l \times R_+^l)^n \times R_{++}^l \times R_{++}^l$ is Walrasian equilibrium without transference if and only if the first order conditions of the n budget constraint utility maximization problems (one for each consumer)

$$\max_{c_i \in R_+^l \times R_+^l} u_i(c_i^1, c_i^2)$$

$$s.t.\ \ p^1 c_i^1 + p^2 c_i^2 \le p^1 w_i^1 + p^2 w_i^2$$

$$\sum_{i=1}^n c_i^t \le \sum_{i=1}^n w_i^t,\ t = 1,2$$

are satisfied. Denoting by γ_i^t the respective multipliers, we obtain:

$$\frac{\partial u_i}{\partial c_{ij}^t}(c_i^*) - \gamma_i^t p_j^t = 0\ \ \forall i, j\ \ and\ \ t = 1,2 \tag{5}$$

If now we consider the maximization problem (3) we obtain the first order conditions:

$$\lambda_i \frac{\partial u_i}{\partial c_{ij}^t} = \psi_j^t\ \ \forall\ i, j, t \tag{6}$$

Where $\psi^t, t = 1,2$ are the multipliers corresponding to the maximization problem (3).

So, once that p_j^t represent the marginal social utility of the good j in time t then we can get $p_l^t = \psi_l^t$ then the following relationship are verified

$$\lambda_i = 1/\gamma_i^1 = e^\theta/\gamma_i^2$$

i.e: the weight λ_i of the utility of the $i-th$ consumer equals the reciprocal utility (discounted) evaluated at the supporting prices.

4 A two-period economy with transferences

It is natural to assume that any economy living for at least two periods, can trade current consumption for future consumption. This means that consumers face a trade-off between present and future consumption.

Consider that the agents living in a two period economy E can transfer resources from one period to another. After transference the endowments of the $i-th$ consumer, in times $t = 1,2,$ are given by: $w_i^1(\alpha_i) = w_i + \alpha_i \ge 0$ then $w_i^2(\alpha_i) = w_i^2 - \alpha_i \ge 0,$ where $\alpha_i = (\alpha_{i1},...,\alpha_{il}) \in R^l$ is the vector of transference between periods. These inequalities mean that, after transference, the endowments have no negative components. This means that if $\alpha_{ij} < 0$ the $i-th$ agent is transferring the amount α_{ij} of $j-$ good from the period 1 to 2, and then for the commodity j the inequality $w_{ij}^1 + \alpha_{ij} > 0$ must be verified. Reciprocally, if $\alpha_{ij} > 0$ then the agent is transferring the amount α_{ij} of $j-$ good from the period 2 to 1 and for this commodity the inequality $w_{ij}^2 - \alpha_{ij} > 0$ must be verified. To denote the new economy (i.e: the after transference economy) we will use the symbolism E_α.

By the symbol T we denote the subset of R^{nl} of the feasible transference vectors, i.e:

$$T = \left\{ (c^1, c^2) \in R^{nl} \times R^{nl} : \sum_{i=1}^{n} c_i^1 \leq \Omega^1 + \alpha \geq 0, \sum_{i=1}^{n} c_i^2 \leq \Omega^2 - \alpha \geq 0 \right\}$$

Let

$$E_\alpha = \left\{ R_+^l, u_i, (w_i, \alpha_i), I \right\}$$

be the economy after transference $\alpha \in T$, where by (w_i, α_i) we denote the endowments after transference i.e: $(w_i, \alpha_i) = (w_i^1 + \alpha_i, w_i^2 - \alpha_i)\ \forall\ i \in I$.

Theorem 1 *For each $\lambda \in \Delta^{n-1}$ and $\alpha \in T$ the solution, $c(\lambda, \alpha)$ of the maximization problem*

$$\begin{aligned} &\max_c \sum_i \lambda_i u_i(c_i) \\ &s.t. \sum_i c_i^1 = \Omega^1 + \sum_i \alpha_i, \\ &\qquad \sum_i c_i^2 = \Omega^2 - \sum_i \alpha_i. \end{aligned} \tag{7}$$

is a Pareto optimal allocation, for the economy E_α.

So, if there exists the possibility of transference between periods then the set of Pareto efficient allocations is not smaller than the set of Pareto optimal allocations of a two periods economy where transference between periods are not allowed. Note that the Pareto optimal allocations corresponding to an economy without transference correspond to the particular case where $\sum_i \alpha_i = 0$. This means that the set of Walrasian allocations corresponding to an economy with transference is not smaller than the set of Walrasian allocations if transference are allowed. Recall that the set of Pareto optimal allocations of an economy E does not depend on the distribution of the resources between the agents, but on the total amount available in each period.

The next theorem is a reciprocal of theorem (1):

Theorem 2 *For each Pareto optimal allocation c of the economy E_α there exists a set of social weights $\bar{\lambda} \in \Delta^{n-1}$ such that it solves*

$$\bar{c} \in argmax \sum_i \bar{\lambda}_i u_i(c_i)$$

$$s.t. \sum_i c_i^1 = \Omega^1 + \sum_i \alpha_i, \qquad (8)$$

$$\sum_i c_i^2 = \Omega^2 - \sum_i \alpha_i.$$

The intuition behind this theorem, is that each Pareto optimal allocation corresponds a vector of social social weights representing the relative weights of the agent in the economy. As we shown in section (3), it follows that being each Walrasian allocation a Pareto optimal allocation, each Walrasian equilibrium determines in an univocal way a set of social weights.

For each $\bar{\lambda} \in \Delta^{n-1}$ fixed, let us consider the problem to maximize the social welfare, function based on an individual choice of an optimal $\alpha^* \in T$. This problem can be written as follows: To find a feasible allocation verifying the inequalities:

$$W_{\bar{\lambda}}(c(\bar{\lambda}, \alpha^*)) \geq W_{\bar{\lambda}}(c(\bar{\lambda}, \alpha)) \quad \forall\ \alpha \in T$$

The function $c : \Delta^{n-1} \times T \to R^{nl} \times R^{nl}$ defined as $c(\lambda, \alpha)$, solves for each $\lambda \in \Delta^{n-1}$ and $\alpha \in T$ the maximization problem:

$$W_\lambda(c(\lambda, \alpha)) = max_c \sum_{i=1}^{n} \lambda_i u_i(c_i)$$

$$\sum_i c_{i1} = \Omega_1 + \sum_i \alpha_i \qquad (9)$$

$$\sum_i c_{i2} = \Omega_2 - \sum_i \alpha_i$$

The real number $W_\lambda(c(\lambda,\alpha))$ is the social welfare level corresponding to the maximization problem (7).

Theorem 3 *For fixed* $\bar{\lambda} \in \Delta^{n-1}$ *the function* $c(\bar{\lambda},\cdot): T \to R^{nl} \times R^{nl}$ *is continuous.*

Proof: Since T is a compact subset of R^{nl} and the utilities are continuous functions, then the function $c(\bar{\lambda},\cdot): T \to R^{nl} \times R^{nl}$. is well defined. From the strict concavity of $u_i, \forall i \in I$, it follows that, for fixed $\lambda = \bar{\lambda}$ the function $W(\bar{\lambda},c) = \sum_{i=1}^{n} \bar{\lambda}_i u_i(c_i)$ is strictly concave function in c. Then if $c(\bar{\lambda},\alpha_n)$ solves the corresponding problem (9) and if $\alpha_n \to \alpha$ for $n \to \infty$ then $c(\bar{\lambda},\alpha_n) \to c(\bar{\lambda},\alpha)$. •

Let us define the set of transference vectors, VT such that

$$VT = \left\{ A \in R^l : \textit{there exists } \alpha \in T \textit{ such that } A = \sum_{i=1}^{n} \alpha_i \right\}$$

So for each $\alpha \in T$ we can write the equality:

$$W_\lambda(c(\lambda,\alpha)) = W_\lambda(c(\lambda,A)) \ \ if \ \ A = \sum_{i=1}^{n} \alpha_i$$

Let $k \in I = \{1,...,n\}$ and $m \in \{1,...,l\}$, considering $\bar{\lambda}$ fixed and assuming the differentiability of $c(\bar{\lambda},\cdot)$, taking derivatives with respect to each α_{km} in $W_{\bar{\lambda}}(c(\bar{\lambda},\alpha)$ we obtain:

$$\frac{\partial W_{\bar{\lambda}}}{\partial \alpha_{km}}(c(\bar{\lambda},A)) = \sum_{i=1}^{n} \bar{\lambda}_i \left\{ \sum_{j=1}^{l} \left[\frac{\partial U_i(c_i^1(\bar{\lambda},A))}{\partial c_{ij}^1} \frac{\partial c_{ij}^1}{\partial A_m}(\bar{\lambda},A) + e^{-\theta} \frac{\partial U_i(c_i^2(\bar{\lambda},A))}{\partial c_{ij}^2} \frac{\partial c_{ij}^2}{\partial A_m}(\bar{\lambda},A) \right] \right\} \quad (10)$$

where $A_j = \sum_{i=1}^{n} \alpha_{ij} \quad j \in \{1,...,l\}$ and $A = (A_1,...,A_n)$. Note that

$$\frac{\partial A_j}{\partial \alpha_{km}} = \begin{cases} 1 & if : j = m \\ 0 & in \ other \ case \end{cases}$$

So from a social point of view is preferable that the $k-th$ consumer transfer good j from the first period to the second if and only if

$$\frac{\partial W_{\bar{\lambda}}}{\partial \alpha_{kj}}(c(\bar{\lambda},\alpha)) > 0 \quad (11)$$

From the individual point of view of the $k-th$ consumer, considering as given

$$\alpha^*_{-k} = (\alpha^*_1,...\alpha^*_{k-1},\alpha^*_{k+1},...,\alpha^*_n)$$

he looks for α_i such that maximize

$$u_k(c_k(\bar{\lambda},\alpha^*_{-k},\alpha_k)) = U_k(c_{k1}(\bar{\lambda},\alpha^*_{-k},\alpha_k)) + e^{-\theta}U_k(c_{k2}(\bar{\lambda},\alpha^*_{-k},\alpha_k))$$

So if,

$$\frac{\partial u_k}{\partial \alpha_{km}}(c_k(\bar{\lambda},\alpha^*_{-k},\alpha_k)) = \sum_{j=1}^{l}\left[\frac{\partial U_i(c_i^1(\bar{\lambda},A))}{\partial c_{ij}^1}\frac{\partial c_{k1j}}{\partial A_m}(\bar{\lambda},A) + e^{-\theta}\frac{\partial U_i(c_i^2(\bar{\lambda},A))}{\partial c_{ij}^2}\frac{\partial c_{kj}^2}{\partial A_m}(\bar{\lambda},A)\right] > 0 \quad (12)$$

then the $i-th$ consumer prefer to transfer the $j-th$ good from the first period to the second, and reciprocally for the reciprocal inequality. So, it is possible to consider the optimal vector α^* as a cooperative solution (a Nash equilibrium) of a non cooperative game, where the set of pure strategies corresponds to the possible elections of lending and saving given by the vectors $\alpha \in T$.

From inequalities (11) and (12), it follows that if agents with social commitment, can freely transfer resources between periods, then the individual solution to the problem of optimize the saving/consumption problem, match with the problem of maximize the social welfare. In the next section we look for a benevolent policy maker, we will see that his interest match with the interest of social committed individuals

5 The central planer problem's

Let $PO(A)$ be the set of Pareto optimal allocations corresponding to an economy where the transfer are given by $A \in VT$. Suppose that the central planner looks for the total amounts of transference between periods, i.e. The central planner looks for $A = \sum_{i=1}^{n}\alpha_i$ since he is looking only for Pareto optimal allocations, does no mater how is the initial distribution of endowments, however does matter on the distribution of the endowments between periods.

The set of feasible Pareto optimal allocations supported for a set of λ is independent of the election α_i of each consumer, but depends on the aggregate value $A = \sum_i \alpha_i$ of these elections. However, if the amount of transference A between periods, change then, then the solution of the problem (7) can change. The solution c of this problem can be consider as a function $c:\Delta\times K\to R_+^l$ defined by: $c=c(\lambda,A)$ where $K=\left[-\min\{\Omega^1,\Omega^2\},\min\{\Omega_1,\Omega_2\}\right]\subset R_+^l$ such that, the $i-$th coordinate is given by the corresponding minimum coordinate between the $i-th$ coordinates of Ω_1 and Ω_2. Consider $\lambda=\bar{\lambda}$ fixed, then changing in the amounts of A modify the boundary of the utility possibility. So for each $\lambda\in\Delta^{n-1}$ the Pareto optimal allocation solving the maximization problem (7). Then, for each $\bar{\lambda}\in\Delta^{n-1}$ and $A\in K$ it make sense to consider the following problem:

$$W_{\bar{\lambda}}(c(\bar{\lambda},A))=\max_{c\in R_+^{nl}\times R_+^{nl}}\sum_{i=1}^{n}\bar{\lambda}_i u_i(c_i)$$

$$s.t.\qquad \sum_i c_i^1=\Omega^1+A \tag{13}$$

$$\sum_i c_i^2=\Omega^2-A$$

Let $c(\bar{\lambda},A)$ be the solution of this problem, where $\bar{\lambda}$ and A are fixed. Let us write the problem of maximization given in (13), in the form

$$W_{\bar{\lambda}}(c(\bar{\lambda},A))=\max_{c\in\phi(A)}W_{\bar{\lambda}}(A,c).$$

Where $\phi:VT\to 2^{R_+^{nl}\times R_+^{nl}}$ defined by

$$\phi(A)=\left\{c=(c^1,c^2)\in R_+^{ln}\times R_+^{ln}:c^1=\Omega^1+A,\,c^2=\Omega^2-A\right\}$$

and $W_{\bar{\lambda}}(A,\cdot):\phi(A)\to R$, defined by

$$W_{\bar{\lambda}}(A,c)=\sum_{i=1}^{n}\bar{\lambda}_i u_i(c_i).$$

Let us define the function $W_{\bar{\lambda}}:K\to R$ given by $W_{\bar{\lambda}}(A)=W_{\bar{\lambda}}(c(\bar{\lambda},A))$, where $W_{\bar{\lambda}}(c(\bar{\lambda},A))\in R$ is the solution of the problem (13) for A fixed. For each $\bar{\lambda}\in\Delta^{n-1}$ it make sense to consider the maximization problem:

$$\max_{A\in K}W_{\bar{\lambda}}(A)$$

Theorem 4 *The function $W_{\bar{\lambda}}(\cdot):K\to R$ is a continuous function and reach its maximum value in the rectangle K.*

Proof: From the strict concavity of the utility functions, it follows that $W_{\bar{\lambda}}(\cdot): K \to R$ is a function. From the maximum theorem we know that this function is continuous. Finally, from de Weierstrass theorem it follows that, for each $\bar{\lambda} \in \Delta$, the function $W_{\bar{\lambda}}(A)$ reach its maximum value in the rectangle K. •

Consider A^* such that $W_{\bar{\lambda}}(A^*) \geq W_{\bar{\lambda}}(A)\ \forall A \in K$ where $K = \left[-\min\{\Omega^1, \Omega^2\}, \min\{\Omega^1, \Omega^2\}\right]$. We symbolize by $\frac{dU_i}{dc_i}(c_i(\bar{\lambda}, A^*))$ the gradient vector of u_i evaluated at $c_i(\bar{\lambda}, A^*)$ and by

$$\frac{\partial c_i^t}{\partial A_j} = \left(\frac{\partial c_{i1}^t}{\partial A_j}, \dots, \frac{\partial c_{il}^t}{\partial A_j}\right) \quad t = 1,2.$$

Taking derivatives with respect to each A_j it follows

$$\frac{\partial W_{\bar{\lambda}}}{\partial A_j}(A) = \sum_{i=1}^{n} \bar{\lambda}_i \left[\frac{du_i(c_i(\bar{\lambda}, \bar{A}))}{dc_i} \frac{\partial c_i}{\partial A_j}(\bar{\lambda}, A)\right] =$$
$$= \sum_{i=1}^{n} \bar{\lambda}_i \left[\frac{dU_i(c_i^1(\bar{\lambda}, A))}{dc_{i1}} \frac{\partial c_i^1}{\partial A_j}(\bar{\lambda}, A) + e^{-\theta} \frac{dU_i(c_i^2(\bar{\lambda}, A))}{dc_i^2} \frac{\partial c_i^2}{\partial A_j}(\bar{\lambda}, A)\right] \tag{14}$$

Assuming the differentiability of $W_{\bar{\lambda}}(A)$, then from theorem (4) it follows that:

Corolary 1 *For each λ here exists and optimal $\alpha^* \in T$ maximizing the social welfare and for $\sum_i \alpha_i^* = A^*$ it follows that*

$$\frac{\partial W_{\bar{\lambda}}}{\partial A_j}(A^*) \leq 0\ \forall\ j = 1, \dots, l.$$

The allocation $c(\bar{\lambda}, A^*))$ is the efficient allocation of greater social value, possible to be reached by means of transference of resources in an economy, that is supported by a particular social structure represented by $\bar{\lambda}$.

Looking at equation (12) and this corollary it follows that, the total amount of the optimal transfer from an individual point of view, given by $\sum_{i=1}^{n} \alpha_i^*$, is the same that solves the problem of the central planner, that is given by A^*.

However it should be noted that although the solution of the central planner and the individuals could match, individuals with better opportunities, must be willing to pursue a policy of transfers that ultimately aims to maximize social welfare. The degree of commitment of these individuals to society, will be the determining factor in the level of participation of central planner.

Note that if transference between periods are free, the set of Pareto optimal allocations corresponding to the problems (7) changing $A \in K$ is the same that the Pareto optimal allocations corresponding to the following problem:

$$\max_{c \in (R_+^l \times R_+^l)^n} \sum_i \lambda_i u_i(c_i^1, c_i^2)$$
$$s.t: \sum_i (c_i^1 + c_i^2) = \overline{\Omega}, \tag{15}$$

where $\overline{\Omega} = \Omega^1 + \Omega^2$, $c = (c_1, ...c_n)$ and $c_i = (c_i^1, c_i^2), c_i^t \in R_+^l, i = 1,...,n$; $t = 1,2$.

Let

$$U = \{u \in R_+^n : \exists c \in F \text{ such that } u_i \leq u_i(c_i) \; \forall i \in I\}$$

be the utility possibility set, where

$$F = \left\{ c \in (R_+^l \times R_+^l)^n : \sum_{i=1}^n (c_i^1 + c_i^2) \leq \Omega^1 + \Omega^2 \right\}.$$

By UP we symbolize the border of the utility possibility set.

Let PO be the set of Pareto optimal allocations corresponding to this problem, then

$$PO = \cup_{A \in VT} PO(A)$$

If $PO(\Omega^1, \Omega^2)$ is the set of feasible Pareto optimal allocation set for a two periods economy without transfer, then:

$$PO(\Omega^1, \Omega^2) \subset PO$$

Let $c_i^* = (c_{i1}^*, c_{i2}^*)$ be the bundle set for both periods for the $i-$th consumer, corresponding with the optimal transference problem α^* given in corollary (1), i.e: $c_i^* = c_i(\overline{\lambda}, \alpha^*)$. Since the allocation $c^* = (c_1^*, ..., c_n^*)$ is Pareto efficient, then it solves:

$$max_{c \in F} W_{\overline{\lambda}}(c) = \sum_{i=1}^n \overline{\lambda}_i u_i(c_i), \;\; s.t.: \;\sum_i (c_i^1 + c_i^2) = (\Omega_1 + \Omega_2). \tag{16}$$

6 Welfare and decentralization: a preliminary conclusion

Similarly to what was done in section (6), let us consider the relationships of the social weights λ with the first and second welfare theorems, in a two periods economy where transference are allowed. Given an economy E_α where $\alpha \in T$, a pair $(c_\alpha, p_\alpha) \in (R^l_+ \times R^l_+)^n \times (R^l_{++} \times^l_{++})$ is Walrasian equilibrium with transference if and only if the first order conditions of the n budget constraint utility maximization problems (one for each consumer):

$$\max_{c_i \in R^l_+ \times R^l_+} u_i(c_i^1, c_i^2)$$

$$s.t.\;\; p^1 c_i^1 + p^2 c_i^2 \leq p^1(w_i^1 + \alpha_i) + p^2(w_i^2 - \alpha_i)$$

$$\sum_{i=1}^{n} c_i^1 \leq \sum_{i=1}^{n} (w_i^1 + \alpha_i)$$

$$\sum_{i=1}^{n} c_i^1 \leq \sum_{i=1}^{n} (w_i^1 - \alpha_i)$$

are satisfied, for each $i \in I$. Denoting by $\gamma^t_{\alpha i}$ the respective multipliers, we obtain:

$$\frac{\partial u_i}{\partial c^t_{\alpha ij}}(c_{\alpha i}) - \gamma^t_{\alpha i} p^t_{\alpha j} = 0 \;\; \forall i \in \{1,...,n\},\; j \in \{1,...,l\},\; and\;\; t = 1,2. \tag{17}$$

If we now consider the maximization problem (9) we obtain the first order conditions:

$$\lambda_i \frac{\partial u_i}{\partial c^t_{\alpha ij}} = \psi^t_{\alpha j} \;\; \forall i \in \{1,...,n\},\; j \in \{1,...,l\},\; and\;\; t \in \{1,2\}. \tag{18}$$

Where $\psi^t_\alpha, t = 1,2$ are the multipliers corresponding to the maximization problem (9).

So, once that $p^t_{\alpha j}$ represent the marginal social utility of the good j in time t then we can get $p^t_{\alpha l} = \psi^t_{\alpha l}$ then the following relationship are verified

$$\lambda_{\alpha i} = 1/\gamma^1_{\alpha i} = e^\theta / \gamma^2_{\alpha 2}$$

i.e: the weight λ_i of the utility of the $i-th$ consumer equals the reciprocal utility (discounted) evaluated at the supporting prices.

In accordance with the first welfare theorem, every Walrasian allocation[5] is a Pareto optimal allocation. [Debreu, G.]. Thus, to each Walrasian equilibrium (c, p)

[5]A Walrasian allocation, is an allocation belonging to a Walrasian equilibrium.

corresponds a set of social weights $\bar{\lambda}$. However, by itself, this first welfare theorem is not at all satisfying from an ethical point of view. For instance the Walrasian allocation does not necessarily maximize de social utility finction $W_{\bar{\lambda}}(c)$. This theorem says only that perfect markets produce Pareto efficient outcomes, without any mention at all of distributive or social justice. Indeed, dictatorships and extreme inequality [Bergstrom, T.] even slavery or starvation [Coles, J.L.; Hammond, P.] can all be Pareto efficient.

The second welfare theorem is ethically much more satisfying, since it characterizes (virtually) all Pareto efficient allocations, both just and unjust. This theorem identifies conditions under which any Pareto optimal allocation can be able to be supported by a set of prices, in our case it will have the form, $p_j = (p_j^1, p_j^2)\ j = 1,...,l.$ However most of the works consider only static or one period economies. Yet Fisher (1907, 1930) and Hicks (1939) were able to describe intertemporal allocations of resources by means of bundles of dated commodities. In [Willmann, G.] is shown that, in a two-period general equilibrium model with heterogeneous agents, Pareto gains from trade may be unreachable if the government uses lump-sum redistribution after trade liberalization without being able to commit to a particular redistributive policy beforehand. However, focussing in to develop a economic policy committed to achieve an optimal vector of transference between periods A^*, , it is possible to obtain Pareto gains for a policy of saving and lending. This result can be obtained by the action of a central planer or by means the action of the individual agents with some degree of social commitment. In both cases is necessary to play an strategy whose result should be the vector of transference A^* corresponding with the solution of $\max_{A \in VT} W_\lambda(c(\lambda, A))$, i.e:

$$W_\lambda(c(\lambda, A^*) \geq W_\lambda(c(\lambda, A)) \ \ \forall\ A \in VT$$

The same result can be obtained by the central planner if is able to implement a economic policy whose result is A^*, or by individual agents choosing some $\alpha^* \in T$ verifying $A^* = \sum_{i=1}^n \alpha_i^*$, according with theorem (1).

Let $(\tilde{c}, \tilde{p})$ be a Walrasian equilibrium for an economy E and let $\tilde{\lambda}$ be the social weights corresponding to this equilibrium. Recall that for every Pareto optimal allocation c there exists a vector of social weights $\tilde{\lambda} \in \Delta^{n-1}$ such that $\tilde{c} \in argmax \sum_{i=1}^n \tilde{\lambda}_i u_i(c_i)$. We say that the economy E_{α^*} is at least as good as the economy E under $\tilde{\lambda}$, if and only if in

E_{α^*} the allocation $c(\bar{\lambda}, \alpha^*)$ can be attained as an allocation of equilibrium. Symbolically we write:

$$E_{\alpha^*} \pm_{\tilde{\lambda}} E$$

On the other side according with corollary (1) such $\alpha_i^* \ \forall\, i \in I$ represents the individual optimal saving/consumption policy. Or from the point of view of a benevolent policy maker, his goal will be to implement a two periods policy, such that in the first one be able to convince the agents to interchange $\alpha : \sum_{i=1}^{n} \alpha_i = A^*$ between periods, and in the second one, to implement a resource transference among the agents in such way that the allocation c^* can be attained as an equilibrium allocation. If agents of the economy E looks for maximize the social welfare utility, then they prefer $E_{\alpha*}$ and the social planner can be dispensed.

Notice that the maximum level of social welfare feasible to be obtained for a benevolent policy maker, corresponds to an economy E_{A^*} where $\sum_{i=1}^{n} \alpha_i = A^*$. This level is the same that the maximum social welfare feasible to be attained by the action of individual agents in the economy E_{α^*}. In this sense we can say that: from a social point of view these economies are equivalents, i.e.:

$$E_{A^*} \approx_{\bar{\lambda}} E_{\alpha^*}$$

However the central planner must be careful, because the action of the central planner implementing a policy to attain the allocation maximizing the welfare, make that the economy change from E to E_{A^*}. In this process, the distribution of the endowments between periods, change (even if in aggregate does not change), if this happen then, along this way changes in prices and allocations of equilibrium occur, prices change from p_j to p_{A^*j}, $j = 1,2,...,l$ and social weights change from λ to λ_{A^*}. At the end of this process the allocation $c(\lambda, A^*)$ can be attained as an equilibrium allocation with the new system of prices p_{A^*}. The second welfare theorem ensure this possibility. But this target can be attained following a smooth path, if and only if the changes in endowments give place only to regular economies, but if along this path there is a singular economy, then changes will be sudden and unforeseeable see [Accinelli, E.].

In conclusion, the participation of the central planner can be dispensed if the agents of the economy have a degree of commitment with the social welfare, and if they are able to play a non-cooperative game, consisting in lending and borrowing according with the strategic vector α^*. However, the non uniqueness of the Walrasian equilibrium, can be a problem to obtain the allocation maximizing the social welfare in a decentralized way in the new economies, either in the economy E_{α^*} determined by the social committed individuals, as in the economy E_{A^*} result of the policy of the benevolent central planner.

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