Economic Operation of Power Systems Optimal Pricing of Energy

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***Abstract*—In this work the equations that determine the short term optimal point of operation of a power system are obtained from two different perspectives. The first one, optimizing the system from a global point of view. The second one, takes into account the invididual agent's behaviour which buys and sells electricity at each of the power systems' busbars. From the comparison of the equations obtained from each case, the prices of active and reactive energy that optimize the system from the global perspective and from the individual agents' perspective at the same time, are deduced. This leads to the definition of the system marginal price and the nodal factors. An interpretation of these magnitudes is done and the current practices for nodal factor calculations is analysed, looking at possible inconviniencies and contradictions. Finally, a particular case, considering the uruguayan power system is studied.**

***Index Terms*— Electricity Pricing, Optimization, Market Place, Nodal Factors, System Marginal Price.**

I. Introduction

THE basic theory of real-time or spot market pricing of electricity was developed by Vickery [1] and Schweppe, et. al [2]. As set forth by Schweppe, et. al., the optimal price for electricity is differenciated in space and time and accounts for the variable costs of producing any electricity at the time it is used, any added requirements to compensate for whatever transmission losses accompany the supply and delivery of the electricity used, and any generation or transmission capacity limitations that might influence the availability of supply as a function of time.

Extensions to the basic theory of real-time pricing have been reported. The basic theory of real-time pricing has been extended to consider system security by Caramanis, Bohn, and Schweppe [3], Alvarado et al. [4], and Kaye, et. al. [5]. Use of real-time pricing to assist in load frequency control was addressed by Berger and Schweppe [6]. Real-time pricing of reactive power was the topic of Baughman and Siddiqi [7], while pricing of spinning reserve was discussed in Siddiqi and Baughman [8]. Also, extensions to the theory that includes constraints on power quality and environmental impact may be found in [9] and [10].

In this work, however, all network aspects have been ignored, except for somewhat crude representation of transmission and generation operation limits.

The approach used in this paper consists of looking for the optimal economic signals that generators and consumers must receive so that their behaviour, is consistent with the goal of a correct regulatory policy: the maximization of global net social benefit. This idea was presented by Pérez-Arriaga et al. in [11].

However, in this work, we go deeper in the interpretation of the system marginal price and we define the nodal factors. Nodal factors have been used in many regulation frameworks such as the argentine and chilean. We will see how they may be used for the economical dispatch and how nodal factor calculation may affect the optimal economic operation of the power system.

In Section II of this paper we will address the global optimization of a power system. In Section III we will consider the individual agents' behaviour in a competitive power system. In Section IV, by comparing the equations obtained in the previous cases, we will establish the prices for active and reactive energy that optimizes the system from the two perspectives (the global and the individual) at the same time. Finally, in Section V we will address the interpretation of the system marginal price, we will define and discuss the calculation of the nodal factors and how they may be used for the system economic dispatch.

II. GLOBAL POWER SYSTEM OPTIMIZATION

Let us consider the generic power system of Fig. 1 which is composed by $\boldsymbol{n_g}$ generation busbars and $\boldsymbol{n_e}$ demand busbars.

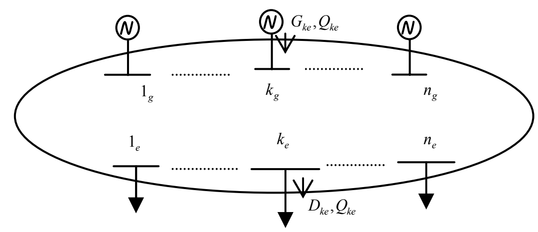

Fig. 1. Generation and demand busbars in the power system.

We define:

G_{k_g}, Q_{k_g} respectively, as the active and reactive power injected by generator k_g into busbar k_g.

D_{k_e}, Q_{k_e} respectively, as the active and reactive power consumed by demand k_e and extracted from busbar k_e.

In order to simplify the notation we asume that a busbar may only be a generating busbar or a demand busbar.
In addition, we also assume that all power injections and extractions are independent of each other.

Let's B_{k_e} be the valued production function or total revenue determined by the use of the electricity at demand busbar k_e.
We can write:
$B_{k_e} = B_{k_e}(D_{k_e}, Q_{k_e})$
Let's C_{k_g} be the total cost produced when (G_{k_g}, Q_{k_g}) is injected into busbar k_g. In the same way, we may write,
$C_{k_g} = C_{k_g}(G_{k_g}, Q_{k_g})$

The maximization of the global net social benefit consists in the following problem: to find $G_{k_g}, Q_{k_g}, D_{k_e}, Q_{k_e} \quad \forall k_g, k_e$ so that,

$B^{glob} = \sum_{k_e=1}^{n_e} B_{k_e}(D_{k_e}, Q_{k_e}) - \sum_{k_g=1}^{n_g} C_{k_g}(G_{k_g}, Q_{k_g})$ is maximum.

The following constraints apply,

1. The power system must operate in steady state, so that,

Active power generation= Active power demand + Losses

Let us consider the losses function,

$$Loss = Loss\left[\underbrace{(G_{1g},...,G_{ng})}_{G};\underbrace{(D_{1e},...,D_{ne})}_{D};\underbrace{(Q_{1g},...,Q_{ng})}_{Q_g};\underbrace{(Q_{1e},..Q_{ne})}_{Q_e}\right]$$

$$Loss = Loss(G, D, Q_g, Q_e)$$

Then the equality constraint may be written as,

$$\sum_{k_g=1}^{n_g} G_{k_g} - \sum_{k_e=1}^{n_e} D_{k_e} = Loss(G, D, Q).$$

2. The transmission network may present a set of ***p*** operating limits which may be expressed as inequality constraints involving magnitudes *G, D* and *Q*.

$$R\_R_i(G, D, Q) \le 0 \quad \forall i \in [1, p].$$

3. Similar to the case of transmission, we establish operating constraints for the generators which consider the generators' load curves and also constraints for the loads,

$$\begin{cases} R\_G_{k_g}(G_{k_g}, Q_{k_g}) \le 0 & \forall k_g \le n_g \\ R\_D_{k_e}(D_{k_e}, Q_{k_e}) \le 0 & \forall k_e \le n_e \end{cases}$$

To sum up, the problem of optimal global dispatch of the power system may be expressed as follows,

$$\text{Max } B^{glob} = \sum_{k_e=1}^{n_e} B_{k_e}(D_{k_e}, Q_{k_e}) - \sum_{k_g=1}^{n_g} C_{k_g}(G_{k_g}, Q_{k_g})$$

subject to,

1) $Loss(G, D, Q) - \sum_{k_g=1}^{n_g} G_{k_g} + \sum_{k_e=1}^{n_e} D_{k_e} = 0$ (electric balance)

2) $R\_R_i(G, D, Q) \le 0 \quad \forall i \in [1, p]$ (network constraints).

3) $\begin{cases} R\_G_{k_g}(G_{k_g}, Q_{k_g}) \le 0 & \forall k_g \le n_g \text{ (generators' constraints)} \\ R\_D_{k_e}(D_{k_e}, Q_{k_e}) \le 0 & \forall k_e \le n_e \text{ (loads' constraints)} \end{cases}$

The Lagrangian of this problem is,

$$L(G, D, Q, \lambda, \mu, \eta, \xi) = \left\{ \left[\sum_{k_g=1}^{n_g} C_{k_g}(G_{k_g}, Q_{k_g}) - \sum_{k_e=1}^{n_e} B_{k_e}(D_{k_e}, Q_{k_e}) \right] + \right.$$

$$\left. + \lambda \left[Loss - \sum_{k_g=1}^{n_g} G_{k_g} + \sum_{k_e=1}^{n_e} D_{k_e} \right] \right\} + \quad \text{Contribution from the power system without constraints}$$

$$+ \left\{ \sum_{k=1}^{p} \mu_k R\_R_k(G, D, Q) + \sum_{k_g=1}^{n_g} \eta_{k_g} R\_G_{k_g}(G_{k_g}, Q_{k_g}) + \right.$$

$$\left. + \sum_{k_e=1}^{n_e} \xi_{k_e} R\_D_{k_e}(D_{k_e}, Q_{k_e}) \right\} \quad \text{Contribution from physical constraints}$$

The Karush-Khun-Tucker conditions are,

$$\begin{cases} \dfrac{\partial L}{\partial G_{k_g}} = 0 \quad \forall k_g \le n_g \quad ; \quad \dfrac{\partial L}{\partial Q_{k_g}} = 0 \quad \forall k_g \le n_g \\ \dfrac{\partial L}{\partial D_{k_e}} = 0 \quad \forall k_e \le n_e \quad ; \quad \dfrac{\partial L}{\partial Q_{k_e}} = 0 \quad \forall k_e \le n_e \end{cases} \quad (1.1)$$

$$\frac{\partial L}{\partial \lambda} = 0 \quad (1.2)$$

$$\begin{cases} R\_R_i(G, D, Q) \le 0 & \forall i \in [1, p] \\ R\_G_{k_g}(G_{k_g}, Q_{k_g}) \le 0 & \forall k_g \le n_g \\ R\_D_{k_e}(D_{k_e}, Q_{k_e}) \le 0 & \forall k_e \le n_e \end{cases} \quad (1.3)$$

$$\begin{cases} \mu_i(R\_R_i(G, D, Q)) = 0 ; \mu_i \ge 0 & \forall i \in [1, p] \\ \eta_{k_g}(R\_G_{k_g}(G_{k_g}, Q_{k_g})) = 0 ; \eta_{k_g} \ge 0 & \forall k_g \le n_g \\ \xi_{k_e}(R\_D_{k_e}(D_{k_e}, Q_{k_e})) = 0 ; \xi_{k_e} \ge 0 & \forall k_e \le n_e \end{cases} \quad (1.4)$$

As a result, we have found that in order to determine the power system operating point which maximizes the global net social benefit, we must find the value of $(3(ng + ne) + 1 + p)$ unknown variables. The number of

unknown variables equals three times the number of generating and demand busbars, plus ***p*** (the number of physical constraints imposed by the electric network that links the busbars), plus one (the λ, from the Lagrangian, which characterizes the whole power system).
On the other hand, we have obtained the same number of equations plus some inequations that must be satisfied.

Equations (1.1) and (1.2) may be also expressed as,

$$\frac{\partial L}{\partial G_{k_g}} = \frac{\partial C_{k_g}}{\partial G_{k_g}} + \lambda\left(\frac{\partial Loss}{\partial G_{k_g}} - 1\right) + \sum_{k=1}^{p}\mu_k \frac{\partial R\_R_k}{\partial G_{k_g}} + \eta_{k_g}\frac{\partial R\_G_{k_g}}{\partial G_{k_g}} = 0$$

n_g equations Type 1.1A

$$\frac{\partial L}{\partial D_{k_e}} = -\frac{\partial B_{k_e}}{\partial D_{k_e}} + \lambda\left(1 + \frac{\partial Loss}{\partial D_{k_e}}\right) + \sum_{k=1}^{p}\mu_k \frac{\partial R\_R_k}{\partial D_{k_e}} + \xi_{k_e}\frac{\partial R\_D_{k_e}}{\partial D_{k_e}} = 0$$

n_e equations Type 1.1B

$$\frac{\partial L}{\partial Q_{k_g}} = \frac{\partial C_{k_g}}{\partial Q_{k_g}} + \lambda\frac{\partial Loss}{\partial Q_{k_g}} + \sum_{k=1}^{p}\mu_k \frac{\partial R\_R_k}{\partial Q_{k_g}} + \eta_{k_g}\frac{\partial R\_G_{k_g}}{\partial Q_{k_g}} = 0$$

n_g equations Type 1.1C

$$\frac{\partial L}{\partial Q_{k_e}} = -\frac{\partial B_{k_e}}{\partial Q_{k_e}} + \lambda\frac{\partial Loss}{\partial Q_{k_e}} + \sum_{k=1}^{p}\mu_k \frac{\partial R\_R_k}{\partial Q_{k_e}} + \xi_{k_e}\frac{\partial R\_D_{k_e}}{\partial Q_{k_e}} = 0$$

n_e equations Type 1.1D

$$\frac{\partial L}{\partial \lambda} = Loss - \sum_{k_g=1}^{n_g} G_{k_g} + \sum_{k_e=1}^{n_e} D_{k_e} = 0$$

1 equations Type 1.2

III. OPTIMAL AGENTS' BEHAVIOUR

Let us study the behaviour of an individual agent that plays in a competitive electricity market. This agent must find the values of G_{k_g}, Q_{k_g} (if generator) or D_{k_e}, Q_{k_e} (if demand), at busbar $\boldsymbol{k_g}$ or $\boldsymbol{k_e}$ in the power system.

We define the following variables,
pa_{k_e}, the price that a demand type agent will pay for one unit of active energy at busbar $\boldsymbol{k_e}$.
$B_{k_e}^{ind}$, the total revenue (or benefit) of the demand type agent corresponding to the use of the active energy at busbar $\boldsymbol{k_e}$.
$R\_D_{k_e}^{ind}(D_{k_e}, Q_{k_e})$, the electrical constraint imposed by the demand type agent's equipment at busbar $\boldsymbol{k_e}$.
pa_{k_g}, the price that a generating type agent will offer for one unit of active energy at busbar $\boldsymbol{k_g}$.
$C_{k_g}^{ind}$, the individual cost for the generating type agent to produce active energy at busbar $\boldsymbol{k_g}$.
$R\_G_{k_g}^{ind}(D_{k_g}, Q_{k_g})$, the electrical constraint imposed by the generator at busbar $\boldsymbol{k_g}$.
pr_{k_g}, pr_{k_e}, similar definitions but for the reactive energy.

Each agent will try to maximize its net benefit. We will establish the equations which dictate the agent's behaviour.

A. Demand type agent

The problem we have to solve is,

$$\text{Max}\left[B_{k_e}^{ind}(D_{k_e}, Q_{k_e}) - (pa_{k_e}D_{k_e} + pr_{k_e}Q_{k_e})\right]$$

subject to:

$$R\_D_{k_e}^{ind}(D_{ke}, Q_{ke}) \le 0 \qquad k_e = 1,2......n_e$$

The Lagrangian of this problem is,

$$L_{k_e}(D_{k_e}, Q_{k_e}, \xi_{k_e}^{ind}) = pa_{k_e}D_{k_e} + pr_{k_e}Q_{k_e} - B_{k_e}^{ind}(D_{k_e}, Q_{k_e}) + \xi_{k_e}^{ind}(R\_D_{k_e}^{ind})$$

The Karush-Kuhn-Tucker conditions are,

$$\frac{\partial L_{k_e}}{\partial D_{k_e}} = 0 \quad ; \quad \frac{\partial L_{k_e}}{\partial Q_{k_e}} = 0 \qquad (2.1.1)$$

$$R\_D_{k_e}^{ind}(D_{k_e}, Q_{k_e}) \le 0 \qquad (2.1.2)$$

$$\xi_{k_e}^{ind}(R\_D_{k_e}^{ind}(D_{k_e}, Q_{k_e})) = 0, \quad \xi_{k_e}^{ind} \ge 0 \qquad (2.1.3)$$

Consequently, for each demand busbar we have a system of three equations with three unknown variables. Then, it is possible to determine the values of that variables that maximizes the agents' net benfit.

Equations (2.1) may also be written as,

$$\frac{\partial L_{k_e}}{\partial D_{k_e}} = pa_{k_e} - \frac{\partial B_{k_e}^{ind}}{\partial D_{k_e}} + \xi_{k_e}^{ind}\frac{\partial R\_D_{k_e}^{ind}}{\partial D_{k_e}} = 0 \qquad (2.1.1B)$$

$$\frac{\partial L_{k_e}}{\partial Q_{k_e}} = pr_{k_e} - \frac{\partial B_{k_e}^{ind}}{\partial Q_{k_e}} + \xi_{k_e}^{ind}\frac{\partial R\_D_{k_e}^{ind}}{\partial Q_{k_e}} = 0 \qquad (2.1.1D)$$

B. Generator type agent

For this agent, the optimization problem may be expressed as follows,

$$\text{Max}\left[(pa_{k_g}G_{k_g} + pr_{k_g}Q_{k_g}) - C_{k_g}^{ind}\right]$$

subject to,

$$R\_G_{k_g}^{ind}(G_{k_g}, Q_{k_g}) \le 0 \qquad k_g = 1,2......n_g$$

The Lagrangian of this problem is,

$$L_{k_g}(G_{k_g}, Q_{k_g}, \eta_{k_g}^{ind}) = C_{k_g}^{ind}(G_{k_g}, Q_{k_g}) - pa_{k_g}G_{k_g} - pr_{k_g}Q_{k_g} + \eta_{k_g}^{ind}(R\_G_{k_g}^{ind})$$

The Karush-Khun-Tucker conditions are,

$$\frac{\partial L_{k_g}}{\partial G_{k_g}} = 0 \quad ; \quad \frac{\partial L_{k_g}}{\partial Q_{k_g}} = 0 \qquad (2.2.1)$$

$$R\_G_{k_g}^{ind}(G_{k_g}, Q_{k_g}) \le 0 \qquad (2.2.2)$$

$$\eta_{k_g}^{ind}(R\_G_{k_g}^{ind}(G_{k_g}, Q_{k_g})) = 0$$
$$\eta_{k_g}^{ind} \ge 0 \qquad (2.2.3)$$

Consequently, for each generator busbar we have a system of three equations with three unknown variables. Then, it is possible to determine the values of the variables that maximizes the agents' net benefit.

Equations (2.2) may also be written as,

$$\frac{\partial L_{k_g}}{\partial G_{k_g}} = \frac{\partial C_{k_g}^{ind}}{\partial G_{k_g}} - pa_{k_g} + \eta_{k_g}^{ind} \frac{\partial R\_G_{k_g}^{ind}}{\partial G_{k_g}} = 0 \qquad (2.2.1A)$$

$$\frac{\partial L_{k_g}}{\partial Q_{k_g}} = \frac{\partial C_{k_g}^{ind}}{\partial Q_{k_g}} - pr_{k_g} + \eta_{k_g}^{ind} \frac{\partial R\_G_{k_g}^{ind}}{\partial Q_{k_g}} = 0 \qquad (2.2.1C)$$

IV. COMPARISON BETWEEN II AND III

Let us compare the magnitudes and equations obtained in Section II and III.

A. Magnitudes

The magnitudes that appear in both cases are: revenue, total cost and constraints.

In the equations for the global system optimization, the magnitudes that appear are: $B_{k_e}, C_{k_g}, R\_G_{k_g}, R\_D_{k_e}$. We may observe that this magnitudes were defined for each busbar independently.

On the other hand, in the equations for the individual optimization the magnitudes $B_{k_e}^{ind}$, $C_{k_g}^{ind}$, $R\_G_{k_g}^{ind}$, $R\_D_{k_e}^{ind}$, correspond to each busbar. Then,

$$B_{k_e}^{ind} = B_{k_e};\ C_{k_g}^{ind} = C_{k_g};\ R\_G_{k_g}^{ind} = R\_G_{k_g};\ R\_D_{k_e}^{ind} = R\_D_{k_e}; \qquad (3.1)$$

B. Equations

For the global system optimization we have $\boldsymbol{n_g}$ equations of type 1.1A ,

$$\frac{\partial C_{k_g}}{\partial G_{k_g}} + \lambda\left(\frac{\partial Loss}{\partial G_{k_g}} - 1\right) + \sum_{k=1}^{p} \mu_k \frac{\partial R\_R_k}{\partial G_{k_g}} + \eta_{k_g} \frac{\partial R\_G_{k_g}}{\partial G_{k_g}} = 0 \quad (1.1A)$$

On the other hand, for each of the $\boldsymbol{n_g}$ generator busbars we have the equations 2.2.1A,

$$\frac{\partial C_{k_g}^{ind}}{\partial G_{k_g}} - pa_{k_g} + \eta_{k_g}^{ind} \frac{\partial R\_G_{k_g}^{ind}}{\partial G_{k_g}} = 0 \qquad (2.2.1A)$$

But, if we take into account 3.1 and we choose pa_{k_g} as,

$$pa_{k_g} = \lambda\left(1 - \frac{\partial Loss}{\partial G_{k_g}}\right) - \sum_{k=1}^{p} \mu_k \frac{\partial R\_R_k}{\partial G_{k_g}} \qquad (3.2)$$

then, equations (1.1A) and (2.2.1A) result the same.

Consequently, if we assign the price pa_{k_g} to the active energy of generator $\boldsymbol{k_g}$, then we will be optimizing the global system and the individual agents' behaviour at the same time.

In the same way, if we apply the same procedure to the equations from the global optimization 1.1B, 1.1C and 1.1D, and from the individual optimizacion 2.1.1B, 2.1.1C and 2.2.1D, we may obtain the values of $pa_{k_e}, pr_{k_g}, pr_{k_e}$.

The results are summarized below,

$$pa_{k_g} = \lambda\left(1 - \frac{\partial Loss}{\partial G_{k_g}}\right) - \sum_{k=1}^{p} \mu_k \frac{\partial R\_R_k}{\partial G_{k_g}} \qquad (3.2)$$

$$pr_{k_g} = -\lambda\left(\frac{\partial Loss}{\partial Q_{k_g}}\right) - \sum_{k=1}^{p} \mu_k \frac{\partial R\_R_k}{\partial Q_{k_g}} \qquad (3.3)$$

$$pa_{k_e} = \lambda\left(1 + \frac{\partial Loss}{\partial D_{k_e}}\right) + \sum_{k=1}^{p} \mu_k \frac{\partial R\_R_k}{\partial D_{k_e}} \qquad (3.4)$$

$$pr_{k_e} = \lambda\left(\frac{\partial Loss}{\partial Q_{k_e}}\right) + \sum_{k=1}^{p} \mu_k \frac{\partial R\_R_k}{\partial Q_{k_e}} \qquad (3.5)$$

V. INTERPRETATION OF RESULTS

A. System Marginal Price

As seen in the previous Section the energy prices are basically composed by two terms. The first one, is the product between λ and a real number. The second one, depends on the operating constraints imposed by the network.

As also observed before, λ is the (unique) lagrangian variable associated to the whole power system. In addition, the dimensional units for λ are the same as for the prices. Therefore, λ must be a price and due to the first consideration must be regarded as the system price, as it is related to the whole power system.

In addition, from 2.1.1B, 2.1.1D, 2.2.1A and 2.2.1C, results that the energy prices represent, either the generator marginal cost or the demand marginal benefit. As a result we can say that λ is a marginal price.

In sum, we can consider λ as the System Marginal Price.

This interpretation may be obtained in a more formal way.

Let us consider the power system optimization problem without constraints. The problem to solve is then,

$$\text{Max } \pi(G, D, Q) = \sum_{k_e=1}^{n_e} B_{k_e}(D_{k_e}, Q_{k_e}) - \sum_{k_g=1}^{n_g} C_{k_g}(G_{k_g}, Q_{k_g})$$

subject to:

$$Loss(G, D, Q) - \sum_{k_g=1}^{n_g} G_{k_g} + \sum_{k_e=1}^{n_e} D_{k_e} = 0$$

Let us define the system balance funcion as follows,

$$A(G,D,Q) = Loss(G,D,Q) - \sum_{k_g=1}^{n_g} G_{k_g} + \sum_{k_e=1}^{n_e} D_{k_e}$$

If we consider virtual displacements in $\pi(G,D,Q)$ and $A(G,D,Q)$ then,

$$d\pi = \sum_{k_e=1}^{n_e} \frac{\partial B_{k_e}}{\partial D_{k_e}} dD_{k_e} + \sum_{k_e=1}^{n_e} \frac{\partial B_{k_e}}{\partial Q_{k_e}} dQ_{k_e} - \qquad (4.1.1)$$
$$- \sum_{k_g=1}^{n_g} \frac{\partial C_{k_g}}{\partial G_{k_g}} dG_{k_g} - \sum_{k_g=1}^{n_g} \frac{\partial C_{k_g}}{\partial Q_{k_g}} dQ_{k_g}$$

$$dA = \sum_{k_g=1}^{n_g} \frac{\partial Loss}{\partial G_{k_g}} dG_{k_g} + \sum_{k_e=1}^{n_e} \frac{\partial Loss}{\partial D_{k_e}} dD_{k_e} + \sum_{k_g=1}^{n_g} \frac{\partial Loss}{\partial Q_{k_g}} dQ_{k_g} +$$
$$\sum_{k_e=1}^{n_e} \frac{\partial Loss}{\partial Q_{k_e}} dQ_{k_e} - \sum_{k_g=1}^{n_g} dG_{k_g} + \sum_{k_e=1}^{n_e} dD_{k_e} \qquad (4.1.2)$$

In addition, for the maximum we have,

$$\frac{\partial B_{k_e}}{\partial D_{k_e}} = \lambda\left(1 + \frac{\partial Loss}{\partial D_{k_e}}\right) \qquad \frac{\partial B_{k_e}}{\partial Q_{k_e}} = \lambda \frac{\partial Loss}{\partial Q_{k_e}}$$

$$\frac{\partial C_{k_g}}{\partial G_{k_g}} = \lambda\left(1 - \frac{\partial Loss}{\partial G_{k_g}}\right) \qquad \frac{\partial C_{k_g}}{\partial Q_{k_g}} = -\lambda \frac{\partial Loss}{\partial Q_{k_g}}$$

Then, substituting this expressions in 4.1.1, we have,

$$d\pi = \sum_{k_e=1}^{n_e} \lambda\left(1 + \frac{\partial Loss}{\partial D_{k_e}}\right) dD_{k_e} + \sum_{k_e=1}^{n_e} \lambda \frac{\partial Loss}{\partial Q_{k_e}} dQ_{k_e} - \qquad (4.1.3)$$
$$- \sum_{k_g=1}^{n_g} \lambda\left(1 - \frac{\partial Loss}{\partial G_{k_g}}\right) dG_{k_g} + \sum_{k_g=1}^{n_g} \lambda \frac{\partial Loss}{\partial Q_{k_g}} dQ_{k_g}$$

Moreover, 4.1.2 may be written as,

$$dA = \sum_{k_e=1}^{n_e} \left(1 + \frac{\partial Loss}{\partial D_{k_e}}\right) dD_{k_e} + \sum_{k_e=1}^{n_e} \frac{\partial Loss}{\partial Q_{k_e}} dQ_{k_e}$$
$$- \sum_{k_g=1}^{n_g} \left(1 - \frac{\partial Loss}{\partial G_{k_g}}\right) dG_{k_g} + \sum_{k_g=1}^{n_g} \frac{\partial Loss}{\partial Q_{k_g}} dQ_{k_g}$$

Then, comparing the last two equations we have,

$$d\pi = \lambda dA \quad \Rightarrow \quad \lambda = \frac{d\pi}{dA}$$

Consequently, λ represents the system benefit (cost) marginal change when there is a balance displacement.

B. Nodal Factors

As seen before, the active energy marginal prices result (without regarding the constraints) from the product of λ by the factor,

- $(1 - \frac{\partial Loss}{\partial G_{k_g}})$ in the case of a generator busbar.
- $(1 + \frac{\partial Loss}{\partial D_{k_e}})$ in the case of a demand busbar.

If we make the following change of variables,

$P_k = D_{k_e}$; $P_k = -G_{k_g}$

then, it results, $pa_k = \lambda(1 + \frac{\partial Loss}{\partial P_k})$.

Therefore, we define $fn_k = (1 + \frac{\partial Loss}{\partial P_k})$ as the Nodal Factor corresponding to busbar *k.*

We observe that the partial derivative of the power system losses with respect to the extracted power at busbar *k* must be evaluated at the values of the electrical variables that correspond to the steady state equilibrium point for a given optimal dispatch.

C. Optimal Dispatch

Taking into account the previous results and definitions, we can say that if the power system is operated at the economic optimum from both the global persective and the individual agents' perspective, then the energy marginal cost at each busbar *k* is given by,

$$pa_k = \lambda . fn_k + \sum_{i=1}^{p} \mu_i \frac{\partial R\_R_i}{\partial P_k}$$

In particular, this must be valid for the marginal generator connected at busbar *m*. Thus,

$$\lambda = \frac{pmar_m - \sum_{i=1}^{p} \mu_i \frac{\partial RR_i}{\partial P_m}}{fn_m}$$

Let us suppose, that there are not network constraints. Then if we want that the power system moves in a process of continuous optimum economic states, the dispatch must be done ordering the generators in accordance to the ratio of the marginal cost to the nodal factor, from the smallest to the biggest.

D. Nodal Factor Calculation

As we have seen, nodal factors are defined as the partial derivatives of the total system losses with respect to the extracted power at the considered node.

As total system losses depend on all variables $G_{k_g}, Q_{k_g}, D_{k_e}, Q_{k_e}$, then nodal factors will necessarily depend on the same variables. Consequently, nodal factors depend on the particular load-generation state.

It is not difficult to find examples where a given busbar changes from being exporting power to be importing power when there is a change in the load-generation pattern. In this case, the nodal factor of that particular busbar will change from having a value less than one, to have a value greater than one. This has economic implications as we have seen that the active energy price in a busbar results from the

product of the system marginal price and the nodal factor. If we assume that the system marginal price remains unchanged, a change in the busbar nodal factor will produce a change in the active energy price at that busbar.
However, in some regulations such as the argentine, the dynamic characteristic of nodal factors is averaged through seasonal nodal factors [12] that take into account just an averaged seasonal load-generation pattern. This may lead to considerable errors in the determination of marginal prices, particularly in a system with dispersed hydro-thermic generation, where the power flux may reverse in some transmission lines from one generation-load pattern to another.

The other important simplification that may be observed in some regulation practices is related to the calculation of nodal factors. Normally, nodal factors are determined from solving a power flow.
To calculate the nodal factor of a given busbar, a small power variation is assumed at the busbar, calculating then the change in power system losses. The additional power to satisfy the new condition from the initial state is provided by the slack busbar. This busbar is choosen to be the "market busbar" which is established to be unchanged. Although the economic conceptual convenience of having a market place, where the energy is traded, the calculation of nodal factors considering that busbar may lead to inaccurate results.
As it results from the optimal economic dispatch methodology, the generator which must balance the power changes is the marginal generator. This means that the "market busbar" is necessarily changing as the marginal generator changes. If we assume an unchanged system slack busbar for nodal factor calculation, there will appear inaccuracies because of not taking the real active and reactive network power fluxes.

In order to show this situation we will present a simple example taken from the uruguayan system.
Let us consider the small part of the uruguayan system shown in Fig. 2.

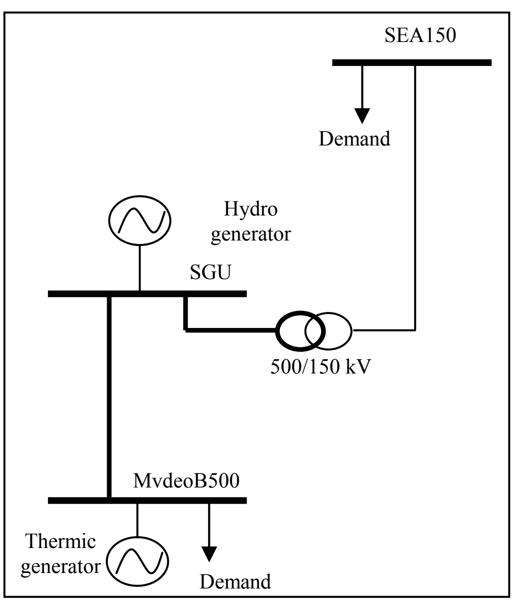

Fig. 2. Simplified uruguayan network.

In the figure it is shown substation Artigas 150 kV (SEA150), the busbar corresponding to Salto Grande Uruguay (SGU) and the busbar Montevideo B 500 kV (MvdeoB500). In addition, the transmission network between those busbars is represented.
We would like to calculate the nodal factor of busbar SEA150. If we take SGU as the slack busbar, then when we simulate an increment in the power consumed by SEA150, that increment will be provided by SGU. The power flux through the transmission line to MvdeoB500 will remain unchanged, the total system losses will increase (because of the increase in power flux in the transmission line from SGU to SEA150) and the nodal factor for busbar SEA150 will be greater than one.
On the other hand, if we take MvdeoB500 as the slack busbar, then an increment in the power demanded by SEA150 will be provided by MvdeoB500. Thus, the power flux through the transmission line to MvdeoB500 will decrease and the total system losses will also decrease (the increment of power flux in the transmission line from SGU to SEA150 is less than the decrease of power flux in the transmission line from SGU to MvdeoB500). As a result, the Nodal Factor will be less than one.

VI. CONCLUSIONS

In this work we have determined the regulated prices for a wholesale electricity market that optimize at the same time the global power system and the individual agents' behaviour.
Moreover, we have defined the system marginal price and the nodal factors making an in deep interpretation of both magnitudes.
In addition, we have discussed the influence of nodal factor calculation on the optimization of the power system, analysing a simple example taken from the ururguayan system. It results that inaccuracies may occur if averaging nodal factors, particularly for systems with high transmission losses.
In further publications, we will present the detailed simulation of various power systems comparing the results for the following cases:

- Dispatch and pricing with exact nodal factor calculation.
- Dispatch and pricing with seasonal nodal factor calculation.
- Dispatch and pricing neglecting losses (unity nodal factors).

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VIII. BIOGRAPHIES

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