

AN IMPLEMENTATION OF THE CONTINUATION METHOD FOR VOLTAGE STABILITY ANALYSIS INCLUDING REACTIVE POWER GENERATION AND TAP CHANGER LIMITS

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Abstract— This paper provides an overview of local bifurcation theory and its application to power system voltage collapse analysis. Two methods for finding turning points are described: a direct method and a continuation method. A computational program based on the continuation power flow method in Matlab environment, developed for ac systems including reactive power generation limits, transformer tap changer limits and an alternative speed up calculation process are also presented. The algorithm is tested on the uruguayan transmission network and an example is given. Some ideas about our future work are included

Key Words— Voltage stability, Continuation Method, Bifurcation Analysis.

1 Introduction

Today most power systems operate close to their transmission limits, so continuous increasing in the load may drive the system to an unstable situation in which bus voltage magnitudes decrease in a non-oscillatory and fast way, known as *voltage collapse*. This phenomenon has been reported frequently since the 80's and it has been captured researchers attention in the early 90's. Reliability, optimality and contingency strategies are very important facts in power networks operation. Essentially, voltage collapse can be characterized as the loss of stability related to the busbar voltages, i.e., the operation point becomes unstable in some sense. A complete overview of the voltage stability issue can be found in (Cañizares, 2001).

Bifurcation theory is widely accepted as the best way of modelling voltage collapse in large power systems. Based on the relationship between voltage collapse in power systems and saddle-node bifurcation of dynamical systems, an algorithm implementing the continuation power flow method is developed. This program finds the voltage collapse point under a pre-defined load increase for a system with reactive power generation limits and on load tap changer automatic regulation. Good results have been obtained with simulations on the uruguayan electrical network.

Section 2 introduces the standard electrical power system model and the saddle-node bifurcation. In Sections 3 and 4 the direct method and the continuation method for finding voltage collapse points are explained. Section 5 describes the continuation power flow algorithm and its main features. Finally, an example and some conclusions and fu-

ture work directions are presented.

2 Voltage collapse and bifurcations

A suitable model for electrical power networks is needed in order to understand the voltage stability phenomenon and to deal with it and the relationships with bifurcation theory.

2.1 Standard model

The standard model for power systems is a set of ordinary nonlinear differential equations with an algebraic constraint (DAE):

$$\begin{aligned}\dot{x} &= f(x, y, \lambda) \\ 0 &= g(x, y, \lambda)\end{aligned}\tag{1}$$

where vector $x \in \mathcal{R}^n$ stands for state variables describing generators angles and their velocities, $y \in \mathcal{R}^m$ represents voltages and angles at load buses and $\lambda \in \mathcal{R}$ is a real parameter modelling usually some slow varying load (Kundur, 1994). Function f captures the dynamics of the generators and their relationships with the load, whose accurate modelling is a very important fact. Function g represents the interconnection of the power network and the appropriated active and reactive power balance at the loads.

A standard hypothesis is that the Jacobian

$$D_y g(x, y, \lambda)\tag{2}$$

is nonsingular for all (x, y, λ) under consideration, so system (1) can be locally reduced to

$$\dot{x} = f[x, h(x, \lambda), \lambda]\tag{3}$$

where h comes from the Implicit Function Theorem (Khalil, 1996). When $D_y g$ is singular, the

situation is very complex and the very quasistationary assumption of phasor dynamic model loses its validity beyond the singular points (Venkatasubramanian et al., 1995).

2.2 Bifurcation analysis

It is accepted that saddle node bifurcation is an accurate way of modelling voltage collapse in power systems (*Proceedings of the IEEE: Special Issue on Nonlinear Phenomena in Power Systems*, 1995). Characterization of this particular kind of bifurcation in nonlinear dynamical systems with scalar parameter and in absence of algebraic constraints was given in (Sotomayor, 1973; Perko, 1990). Essentially, the system

$$\dot{x} = f(x, \lambda) \quad (4)$$

undergoes a saddle node bifurcation at point (x_*, λ_*) if the following conditions are satisfied

$$0 = f(x_*, \lambda_*) \quad (5)$$

$$w^T D_x f(x_*, \lambda_*) = 0 \quad (6)$$

$$w^T \left. \frac{\partial f}{\partial \lambda} \right|_{x_*, \lambda_*} \neq 0 \quad (7)$$

$$w^T [D_{xx}^2 f(x_*, \lambda_*) v] v \neq 0 \quad (8)$$

Equation (5) means that (x_*, λ_*) is a fixed point; equation (6) indicates the existence of a null eigenvalue of the Jacobian at (x_*, λ_*) with associated left and right (nonzero) eigenvectors w and v . Expressions (7) and (8) imply transversality conditions and determine the *generic aspect of saddle node bifurcation* implying that this kind of bifurcation will exist even under small perturbations of the original system (i.e. it is a robust phenomenon).

So, voltage collapse can be seen as the disappearance of the stable operation point as long as the parameter reaches some critical value. The system is then driven away from acceptable zones in the state space. The point (x_*, λ_*) is usually referred as *turning point* (Seydel, 1988).

For the case of DAE's, a similar condition is inferred in (Cañizares, 1991), leading to the related expressions:

$$0 = F(z_*, \lambda_*) \quad (9)$$

$$w^T D_z F(z_*, \lambda_*) = 0 \quad (10)$$

$$w^T \left. \frac{\partial F}{\partial \lambda} \right|_{z_*, \lambda_*} \neq 0 \quad (11)$$

$$w^T [D_{zz}^2 F(z_*, \lambda_*) v] v \neq 0 \quad (12)$$

where

$$z = \begin{bmatrix} x \\ y \end{bmatrix}, \quad F(z, \lambda) = \begin{bmatrix} f(x, y, \lambda) \\ g(x, y, \lambda) \end{bmatrix}$$

There are standard methods for solving equations (9-12), i.e. for finding turning points. Other types of bifurcation, like pitchfork and transcritical, are not generic in the sense that they are not expected to occur in general. Hopf bifurcation, another generic kind of bifurcation, is not considered here.

3 Direct method

This method was initially proposed by (Seydel, 1988) for finding saddle-node bifurcations on nonlinear systems without constraint manifolds. An extended version of this method is applied in (Cañizares, 1991) to the complete set of power equations, including algebraic constraints, to detect voltage stability problems in ac power networks. The method for finding the bifurcation point is to impose conditions for equilibrium with constraints ensuring a zero eigenvalue at the point of interest. Hence, the equations for x , y , v and λ take the following form:

$$\begin{bmatrix} f(x, y, \lambda) \\ g(x, y, \lambda) \end{bmatrix} = 0$$

$$\begin{bmatrix} D_x f & D_y f \\ D_x g & D_y g \end{bmatrix} \begin{bmatrix} v_x \\ v_y \end{bmatrix} = 0$$

$$\begin{bmatrix} v_x \\ v_y \end{bmatrix} \neq 0$$

The nonzero condition for the eigenvector can be guaranteed by requiring a particular component of v_x to be different from zero. This method also gives the eigenvector at the bifurcation point.

4 Continuation method

Voltage profiles of power systems have been typically obtained by calculating a series of equilibrium points of equation (9) with successive power flow simulations. These profiles correspond to parts of the bifurcation diagram. By solving $F(z, \lambda) = 0$ for increasing values of λ , one can trace relevant parts of the bifurcation diagram; however, since the system Jacobian becomes singular at the saddle-node bifurcation (z_*, λ_*) and no equilibrium points exist for $\lambda > \lambda_*$, one cannot accurately compute λ_* using this technique.

The *continuation method* (Cañizares, 1991) overcomes the difficulties of the successive power flow solutions method, allowing the user to trace the complete bifurcation diagram by automatically changing the value of λ . The strategy used in this method is illustrated in figure 1. The curve of the figure 1 represents a branch of the system equilibria as the parameter changes. The technique consists of a three step approach to tracing the equilibrium points as one parameter in the system changes.

Assuming that the system is initially at the state (z_1, λ_1) , the first step is known as the **predictor**

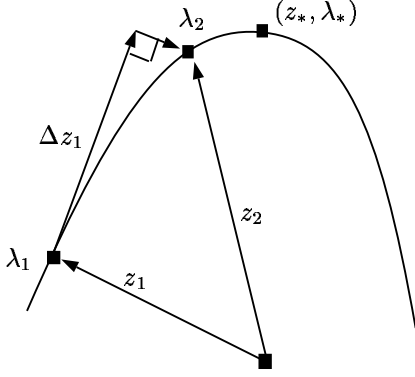


Figure 1. Predictor-corrector diagram

step, since it generates an initial guess

$$(z_1 + \Delta z_1, \lambda_1 + \Delta \lambda_1)$$

by calculating direction Δz_1 using the tangent vector to the curve at (z_1, λ_1) . Since $F(z_1, \lambda_1) = 0$ then

$$\frac{dF}{d\lambda}(z_1, \lambda_1) = D_z F(z_1, \lambda_1) \frac{dz}{d\lambda} \Big|_1 + \frac{\partial F}{\partial \lambda} \Big|_1 = 0$$

$$D_z F \Big|_1 \frac{dz}{d\lambda} \Big|_1 = - \frac{\partial F}{\partial \lambda} \Big|_1 \quad (13)$$

Hence

$$\Delta z_1 = \Delta \lambda_1 \cdot \frac{dz}{d\lambda} \Big|_1 \quad (14)$$

where the parameter increment $\Delta \lambda_1$ can be defined as a function of a scaling k_{sc} to vary the speed at which the equilibrium branch is traced, i.e.

$$\Delta \lambda_1 = \frac{k_{sc}}{\left\| \frac{dz}{d\lambda} \Big|_1 \right\|} \quad (15)$$

The initial guess $(z_1 + \Delta z_1, \lambda_1 + \Delta \lambda_1)$ is then used in the **corrector** step to compute a new equilibrium point (z_2, λ_2) on the bifurcation diagram. The new equilibrium must be calculated by solving the following set of equations for z and λ :

$$\begin{cases} F(z, \lambda) = 0 \\ \Delta \lambda \cdot (\lambda - \lambda_1 - \Delta \lambda_1) + \Delta z^T \cdot (z - z_1 - \Delta z_1) = 0 \end{cases} \quad (16)$$

The first equation of (16) corresponds to the steady-state system equation, which has a singular Jacobian $D_z F|_*$ at the saddle-node bifurcation point (z_*, λ_*) . The second scalar equation consists in a hyperplane orthogonal to the tangent vector $[\Delta^T z_1, \Delta \lambda_1]^T$, passing through $(z_1 + \Delta z_1, \lambda_1 + \Delta \lambda_1)$.

By initially setting z to $z_1 + \Delta z_1$ and λ to $\lambda_1 + \Delta \lambda_1$, solving this set of equations usually takes one or two iterations of the standard Newton-Raphson Method (Cañizares, 1991). If the process fails

to converge, the steps $\Delta z_1, \Delta \lambda_1$, are halved until convergence is attained. Another appropriated surface can be used instead the proposed hyperplane (Kundur, 1994).

However, this method has difficulties when the equilibrium point is close to the bifurcation point, since the Jacobian of system equations (13) becomes ill-conditioned. To avoid this problem a third step can be used. This step, known as **parameterization**, consists on interchanging, close to the bifurcation, the parameter λ with the component z_i of vector z that has the largest normalized entry in the tangent vector, so that λ becomes part of the equation variables, whereas z_i becomes the new parameter p , i.e. for $N = n + m$,

$$p \leftarrow \max \left\{ \left| \frac{\Delta z_1}{z_1} \right|, \left| \frac{\Delta z_2}{z_2} \right|, \dots, \left| \frac{\Delta z_N}{z_N} \right|, \left| \frac{\Delta \lambda}{\lambda} \right| \right\}$$

The global method goes around the bifurcation point, allowing the user to trace the *unstable* side of the equilibrium branch. However, it is necessary to find this turning point, in order to change the sign of $\Delta \lambda$ in equation (14).

5 Implementation of the continuation power flow

We have developed a program based on the continuation power flow method in a *Matlab* environment, with the following main features:

5.1 System modelling and data files

The input data are similar to those of conventional load flow, with the following additional particularities:

- The loads of the PQ busbars are modelled with a voltage dependency of the type

$$P = P_0 + P_1 \cdot V^{\alpha_1} + P_2 \cdot V^{\alpha_2}$$

and a similar model for the reactive load.

- Each PQ busbar has a binary flag, in order to identify whether the busbar load will be increased or not during the process.

- If the load in busbar j will be increased, a constant ΔP_j is assigned in the data file in order to identify the load increase direction of the system. The total active load in busbar j will be increased according to the law

$$P_j = P_{0j} + P_{1j} \cdot V_j^{\alpha_{1j}} + P_{2j} \cdot V_j^{\alpha_{2j}} + \lambda \cdot \Delta P_j$$

where λ is the load increase parameter. A similar law is used for the reactive load.

- Each PV generation busbar has a second binary flag, in order to identify whether the machine in this busbar will share or not the increase of load in the system during the process.

5.2 Predictor and corrector steps

Beginning with an initial steady state of the system, solved with a conventional Newton-Raphson load flow, the program increases step by step the load according to the predefined load increase direction and solves for each step the new steady state of the system applying the predictor-corrector method.

Constant k_{sc} in equation (15) is defined by *trial and error* in order to get acceptable total calculation time. We have checked on several tests made on the uruguayan transmission network that the total calculation time is improved by using a relative small k_{sc} , in order to get a system load increases of the order of 0.2% to 0.5% of the total system load when the $z-p$ curve is solved still far from the collapse point. If we use higher k_{sc} values, we have found that, although the predictor step advances faster towards the collapse point, the corrector step takes much more time to be solved.

5.3 Reactive generation limits

Each time a new steady state has been found at the end of the corrector step, the algorithm checks that the reactive generation of each machine lies within the predefined range. If at least one machine has violated its reactive generation limit, the new steady state is *rejected* and we have to go back to the previous state at the beginning of the last predictor step. The process predictor-corrector is then repeated, but this time with a reduced (halved) k_{sc} . This is repeated till we find a state where all the reactive generation limits are respected and at least one machine is generating *exactly* its reactive limit (within a predefined tolerance). At this point, the reactive generation of this machine is *frozen* in its limit, its voltage reference is released (the busbar type changes from PV to PQ) and k_{sc} is restored to its original value in order not to lose computation speed in the future predictor-corrector steps.

5.4 Tap changer limits

Similar with the reactive generation limits, each time a new steady state has been found at the end of a corrector step, the program checks that the tap changer position of each regulating transformer lies within the predefined range, using a method similar to the one described above. When a steady state with all regulating transformers with their tap changer positions within the acceptable range and at least one transformer working on a limit tap is obtained, the tap of this last transformer is *frozen* in its limit, its voltage reference is released (the busbar voltage substitutes the tap

position as a state variable) and k_{sc} is reset to its original value.

5.5 Parameterization

Before beginning a new predictor-corrector step from a valid steady state, the algorithm selects as the new parameter the PQ busbar voltage or load parameter which has had the largest relative absolute increase during the previous predictor-corrector step, as was explained in Section 4. For simplicity, the new parameter is sought between the busbar voltages only (Cañizares, 1991).

5.6 Collapse point detection

The algorithm assumes that the system will finally collapse when finding a saddle-node bifurcation, i.e. the system Jacobian will have a null eigenvalue at the collapse point. The collapse state is then found by detecting a change of sign of the determinant of the system Jacobian between two consecutive calculated states.

5.7 Collapse point evaluation

The program calculates the eigenvectors associated with the null eigenvalue of the system Jacobian at the collapse point. The greater absolute entries in the right eigenvector identify the busbars most *involved* in the collapse (Dobson, 1992b) and according to this criterion, the P-V and Q-V curves of these *worst* busbars are displayed and information about the total system load at the collapse point is given.

5.8 Alternative calculation

In order to speed up the calculation process, it is possible to select in the program an alternative method which uses conventional load flows with predefined load increases to calculate the first points in the $z-p$ curve. Then, automatical switching to the continuation power flow method is performed when a conventional load flow fails to converge after few iterations.

6 An application example

The algorithm has been tested on the uruguayan transmission network, which has around 100 nodes. Different load increase directions are assumed. In order to test full capability of the software, we have tested several load voltage dependency models, although none of them has been yet validated by field tests in the network.

The P-V and Q-V curves obtained for two of the most stressed busbars of the network during one of the simulated cases are shown in figures 2 and 3.

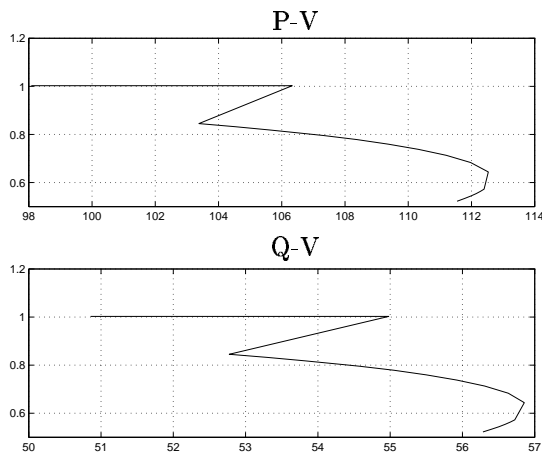


Figure 2. P - V and P - Q curves for busbar *MONTF315* of Montevideo.

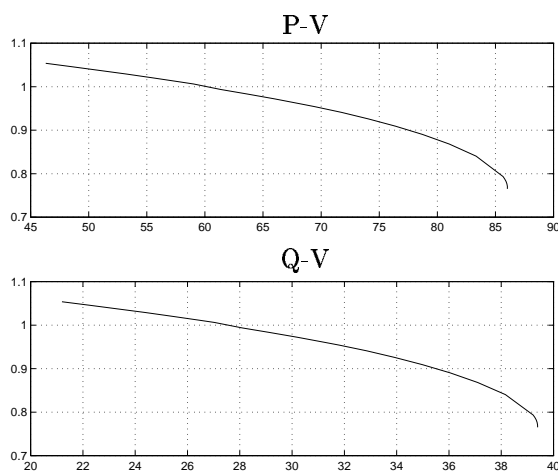


Figure 3. P - V and P - Q curves for busbar *MONTJ150* of Montevideo.

The busbar voltage jump in the *MONTF315* busbar is due to a regulating transformer connected to this busbar hitting a tap changer limit. The corresponding decrease in the active power load is due to the particular voltage dependency of the load model assumed to this busbar.

7 Conclusions and future work

The basic theory of voltage collapse as part of the bifurcation theory of non linear systems and one of the most widely used methods for finding the collapse state in electric power systems (the continuation power flow) have been reviewed. A software tool based on the continuation power flow method has been developed, taking into account the main factors which influence the voltage state of power systems when submitted to a continuous increase of load, i.e. limits of reactive power generation, on load tap changer automatic regulation and voltage dependency of the loads. The software has been tested in the uruguayan transmission network with satisfactory results, confirming that the

selected method is particularly useful for *tracking* the voltage evolution when the power system is subjected to discrete changes as a consequence of state variables (reactive power of generators, tap position of on load tap changers) hitting limits.

Our future work in this field will be concentrated in two main directions. One of these is the finding of the *closest bifurcation*: as far as is not always possible to preview a suitable load increase direction in the system, the method should be extended in order to be able to find the closest (in some sense) collapse state when all the load increase directions in the space of active and reactive loads are equally probable. In this way, the power system could be planned or operated in a conservative manner taking into account this worst load increase scenario. The method described in (Dobson, 1992a) for finding a closest bifurcation in a multidimensional parameter space is being implemented and will be tested in the near future on the uruguayan network. The other direction is the calculation of *mitigation measures*: having the ability of calculating the voltage collapse state is only the first step towards the implementation of planning or operating procedures which take into account the voltage collapse phenomenon in power systems. The next step is to develop calculation methods in order to be able to propose mitigation measures (reactive power reinforcements in selected busbars, load shedding, machines control voltage adjustment and regulating transformers) against the collapse in an optimal way (Cañizares, 1998). A research work in this direction will be developed in the near future.

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