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$\mathcal{H}_\infty$ and $\mathcal{H}_2$ Robust Design Techniques for Static Prefilters

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Abstract

The problem studied in this paper is to obtain the optimal robust performance achievable by a static prefilter in the presence of structured uncertainty. The performance indexes we adopt are the $\mathcal{H}_\infty$ and $\mathcal{H}_2$ norms, and we consider either Linear Time-Invariant (LTI) as Linear Time Varying (LTV) uncertainties. It is shown that the optimal solutions for the LTV cases can be obtained by solving finite dimensional convex problems. The LTI cases can be posed as (infinite dimensional) convex problems, for which several algorithms are available that dispense with the classical $\mathcal{D}-\mathcal{K}$ iteration.

1 Introduction

In this paper we consider the uncertain control system of Figure 1 where w denotes the disturbances, u the control action, s the performance output, r the reference inputs, and v the measured outputs. The uncertainty Δ is supposed to be structured.

The plant G_p is controlled by a two-degrees-of-freedom controller K which is composed by the feedback part F and the prefilter P such that $u = Fv + Pr$.

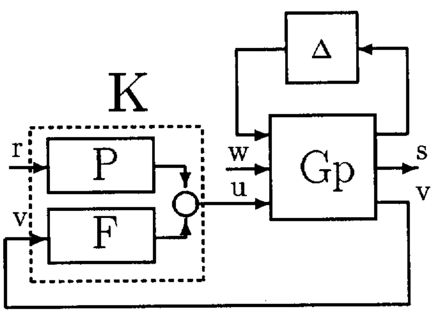

Figure 1: *Two-Degrees-of-Freedom Controller*

We will suppose that a direct measure of the external inputs r is available. Instead, we have no access to the disturbances w which must be compensed by the feedback action.

The feedback part F is classically designed from specifications like stability and disturbance rejection. The prefilter P is introduced to improve the command response, i.e. the transfer between r and s, in aspects as constraints on the step response, decoupling between input channels, steady state response, etc..

It was proven [1] for the nominal case that if the unique constraint on P and F is their realizability, the stability and disturbance attenuation on the one hand and command response on the other are independent requirements. A more precise statement of that independence may be formulated: let us suppose that a given transfer function T_{sr} between r and s is achieved by a couple of a prefilter P_1 and a closed loop stabilizing feedback controller F_1. Then, for all stabilizing F_2 there exists a prefilter P_2 such that the desired transfer function can be reached.

The potential cost of that independence may be an excessive dynamic order. However, this result is a valuable guide in designing controllers with two degrees of freedom.

For the uncertain case the robust stability and disturbance rejection exclusively depend on the feedback controller F. The robust command tracking also depends on F, but a strong dependence on P is present.

The joint synthesis of P and F for robust performance is technically viable, see e.g. [2]. The resulting controller has a dynamic order equal to the generalized plant plus the order of the corresponding scaling matrices. However, this approach has some drawbacks.

First, we must use the same norm for the characterization of the disturbance rejection and the command tracking. If this is not possible, the joint synthesis of P and F may not be convenient due to the lack, at our knowledge, of an efficient technique for the design for robust mixed $\mathcal{H}_2/\mathcal{H}_\infty$ performance. The use of the $\mathcal{H}_2$ norm as performance index for the command tracking is natural since it is habitual working with the references belonging to a set of test inputs as steps or ramps. Also it is not difficult to conceive that there can be cases in

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which the disturbances w can be deterministic signals that can concentrate its energy in a relatively narrow uncertain band of frequency. If such is the case, the $\mathcal{H}_\infty$ norm seems to be adequate for the characterization of the disturbance rejection.

In second place, we are not able to obtain the global optimum for the robust design of controllers with two degrees of freedom. In the general case we have no alternative to suboptimal design techniques as the classical $\mathcal{D}-\mathcal{K}$ iteration [8].

The approach we adopt in this work is the independent design of each degree of freedom. The feedback action must be firstly designed by a suitable method to assure robust stability and a good disturbance rejection. In a second step the prefilter must be designed to improve the robust command tracking. However, once the feedback action is incorporated to the plant, the use of classical full order design techniques may easily lead to relatively high dynamic orders.

We will develop design methodologies for prefilters in the presence of LTI or LTV uncertainties. We will consider either $\mathcal{H}_\infty$ or $\mathcal{H}_2$ norms as performance index. We will center the study on static prefilters, but some comments on extensions to the general case will be made.

The design methodologies developed here allow us to obtain the global optimum for the stated problems when we restrict ourselves to the static case. The apparent poorer performance due to the lack of dynamics in the prefilter may be misleading. There are no known methodology that allows us to obtain the optimum full order prefilter. In addition, if the resulting order is excessive, a model reduction for the prefilter can be necessary. The performance of the static prefilter begins to be comparable to the dynamic alternative prefilter. See [3] for an example involving the nominal case.

It is shown that when LTV uncertainties are present, either the $\mathcal{H}_2$ as the $\mathcal{H}_\infty$ problem can be solved through an optimization procedure involving Linear Matrix Inequalities (LMI) [10]. For those cases where the uncertainties are LTI, the optimum static prefilters may be obtained through an infinite dimensional convex problem. This can be adequately solved, from an engineering point of view, e.g. by griding the frequency domain and solving a set of LMIs. The novel aspect is that is not necessary any iteration between the controller and the corresponding scaling matrices.

The design problems are formulated in the next section. Section 3 presents the main results of this work. Section 4 contains some remarks on computational issues and the fifth section finishes the work with some concluding remarks.

Notation: The transfer function

$$G(s) = C(sI-A)^{-1}B + D$$

will be denoted as $G(s) = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$.

For a partitioned operator

$$M \triangleq \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$$

and two operators P and Q with compatible dimensions, the lower and upper linear fractional transformations (LFT) are defined as

$$F_l(M,Q) \triangleq M_{11} + M_{12}Q(I-M_{22}Q)^{-1}M_{21}$$

$$F_u(M,P) \triangleq M_{22} + M_{21}P(I-M_{11}P)^{-1}M_{12}$$

2 Problem statement

Let us assume that an adequate feedback controller F was already designed that assures closed loop stability. In order to design the prefilter, a model following interconnection diagram can be constructed (see e.g. [2]). The resulting system, including the controller F and all the introduced weights, may be represented as in Figure 2.

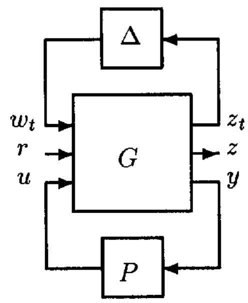

Figure 2: Uncertain system

The plant G is given by

$$G(s) = \left[\begin{array}{c|ccc} A & B & E & M \\ \hline C_1 & L_1 & H_1 & N_1 \\ C_2 & L_2 & H_2 & N_2 \\ 0 & 0 & I & 0 \end{array}\right] \qquad (1)$$

where the partition is according to Figure 2. The dimensions of the signals w_t, r, u, z_t, z and y are respectively dw, dr, du, dw, dr and dy. The dimension of the state vector is n. The prefilter P is assumed to be static, i.e. $P \in R^{du \times dr}$.

We are assuming that the reference input is measured and no feedback action is done by the static prefilter P, i.e. $y = r$. The nominal plant G is supposed to be stable.

The uncertainty Δ is assumed to have the structure

$$\mathbf{\Delta} = diag[\delta_1 I_{r1}, \ldots, \delta_L I_{rL}, \Delta_{L+1}, \ldots, \Delta_{L+F}]$$

where the blocks in Δ represent dynamical perturbations. Let us consider the normalized set $\mathbf{B\Delta}$ defined as $\mathbf{B\Delta} \triangleq \{\Delta \in \mathbf{\Delta} : ||\Delta|| \leq 1\}$ in some operator norm.

The transfer function between $[w_t, r]'$ and $[z_t, z]'$ is

$$T_P(s) \triangleq \mathcal{F}_l(G(s), P)$$

In the presence of LTI uncertainties Δ, the system is said to have (normalized) **robust $\mathcal{H}_\infty$ performance** if

$$\sup_{\Delta \in \mathbf{B\Delta}} ||F_u(T_P(s), \Delta)||_\infty < 1$$

Analogously, the system has normalized **robust $\mathcal{H}_2$ performance** if

$$\sup_{\Delta \in \mathbf{B\Delta}} ||F_u(T_P(s), \Delta)||_2 < 1$$

The definitions above can be generalized to the LTV cases [5, 7, 9]. In order to obtain the respective conditions for robust performance we introduce scaling matrices of the form

$$X = diag[X_1, \ldots, X_L, x_{L+1} I_{m1}, \ldots, x_{L+F} I_{mF}] \quad (2)$$

which commute with the elements in $\mathbf{\Delta}$. Let us denote by $\mathbf{X}$ the set of positive definite, continuous scaling $X(\omega)$ with the structure (2).

We will denote by $\mathbf{\Phi}$ the set of matrix functions $\Phi(\omega) \in C^{dr \times dr}$ such that $\Phi(\omega) = \Phi^*(\omega) > 0$.

An index γ for robust performance will be introduced. With the help of several known results on analysis of uncertain systems [4, 5, 6, 8, 9, 12] we can state computable conditions for robust performance.

Condition 1 **[$\mathcal{H}_\infty$, LTI]** : *There exists a scaling function $X(\omega) \in \mathbf{X}$ and $P \in R^{du \times dr}$ such that for all ω it is satisfied*

$$T_P(j\omega)^* \begin{bmatrix} X(\omega) & 0 \\ 0 & I \end{bmatrix} T_P(j\omega) - \begin{bmatrix} X(\omega) & 0 \\ 0 & \gamma^2 I \end{bmatrix} < 0 \quad (3)$$

Condition 2 **[$\mathcal{H}_\infty$, LTV]**: *There exists a constant $X \in \mathbf{X}$ and $P \in R^{du \times dr}$ such that*

$$\left\| \begin{bmatrix} X & 0 \\ 0 & I \end{bmatrix} T_P(s) \begin{bmatrix} X^{-1} & 0 \\ 0 & \gamma^{-1} I \end{bmatrix} \right\|_\infty < 1 \quad (4)$$

Condition 3 **[$\mathcal{H}_2$, LTI]**: *There exists a scaling function $X(\omega) \in \mathbf{X}$, $P \in R^{du \times dr}$ and a matrix function $\Phi(\omega) \in \mathbf{\Phi}$ such that for all ω it is satisfied*

$$T_P(j\omega)^* \begin{bmatrix} X(\omega) & 0 \\ 0 & I \end{bmatrix} T_P(j\omega) - \begin{bmatrix} X(\omega) & 0 \\ 0 & \Phi(\omega) \end{bmatrix} < 0$$

$$\int_{-\infty}^{+\infty} trace(\Phi(\omega)) \tfrac{d\omega}{2\pi} < \gamma^2 \quad (5)$$

Condition 4 **[$\mathcal{H}_2$, LTV]**: *There exists a constant $X \in \mathbf{X}$, $P \in R^{du \times dr}$ and a matrix function $\Phi(\omega) \in \mathbf{\Phi}$ such that for all ω it is satisfied*

$$T_P(j\omega)^* \begin{bmatrix} X & 0 \\ 0 & I \end{bmatrix} T_P(j\omega) - \begin{bmatrix} X & 0 \\ 0 & \Phi(\omega) \end{bmatrix} < 0$$

$$\int_{-\infty}^{+\infty} trace(\Phi(\omega)) \tfrac{d\omega}{2\pi} < \gamma^2 \quad (6)$$

It is worth to note that conditions 1 and 3 are only sufficient to guarantee robust performance.

Given the uncertain system of Figure 2, we will consider the problem of obtaining the static prefilter P that assures the optimal robust performance against either LTI or LTV structured uncertainties Δ. This objective is achieved in the LTV cases with an arbitrary degree of approximation through a convex optimization problem as it is treated in sections 3.1 and 3.2.

Despite of the fact that there are no available computable conditions that exactly characterize robust performance in the LTI case, we can synthesize a reasonably good approximation by minimizing the index γ subject to the satisfaction of conditions 1 or 3. This is the topic we discuss in sections 3.3 and 3.4.

3 Main Results

The transfer function $T_P(s)$ is given by

$$T_P(s) = \left[\begin{array}{c|cc} A & B & E + MP \\ \hline C & L & H + NP \end{array} \right] \quad (7)$$

where the matrices C, L, H, N result of stacking the corresponding matrices in (1). When we will consider the $\mathcal{H}_2$ performance we will restrict ourselves to a $G(s)$ strictly proper, i.e $L = 0$, $H = 0$ and $N = 0$. These assumptions can be relaxed in some extent at the cost of a higher complexity in the formulae.

It can be shown that

$$T_P(s) = [\ T_0(s) \quad T_1(s) + T_2(s)P\ ] \quad (8)$$

where

$$T_0(s) = \left[\begin{array}{c|c} A & B \\ \hline C & L \end{array} \right]; \; T_1(s) = \left[\begin{array}{c|c} A & E \\ \hline C & H \end{array} \right]$$

$$T_2(s) = \left[\begin{array}{c|c} A & M \\ \hline C & N \end{array}\right]$$

3.1 $\mathcal{H}_\infty$, LTV case

Following proposition gives us a convex formulation for condition 2.

Proposition 1 *: Given $\gamma > 0$, condition 2 is satisfied if and only if there exists a symmetric $W \in R^{n\times n}$, $Z \in \mathbf{X}$ and $P \in R^{du\times dr}$ such that $W > 0$ and*

$$\begin{bmatrix} AW+WA' & BZ & E+MP & WC' \\ ZB' & -Z & 0 & ZL' \\ (E+MP)' & 0 & -\gamma^2 I & (H+NP)' \\ CW & LZ & H+NP & -\begin{bmatrix} Z & 0 \\ 0 & I \end{bmatrix} \end{bmatrix} < 0 \tag{9}$$

Proof :

Let us denote $Q \triangleq \begin{bmatrix} X & 0 \\ 0 & I \end{bmatrix}$.

By applying the Bounded Real Lemma (see [13]) condition 2 is equivalent to the existence of $P \in R^{du\times dr}$ and symetric matrices $X_l > 0$, $X \in \mathbf{X}$ such that:

$$\begin{bmatrix} X_lA + A'X_l & X_l[\ B \quad \gamma^{-1}(E+MP)\ ]Q^{-1} \\ Q^{-1}[\ B \quad \gamma^{-1}(E+MP)\ ]'X_l & -I \\ QC & Q[\ L \quad \gamma^{-1}(H+NP)\ ]Q^{-1} \end{bmatrix}$$

$$\left.\begin{matrix} C'Q \\ \ldots\ Q^{-1}[\ L \quad \gamma^{-1}(H+NP)\ ]'Q \\ -I \end{matrix}\right] < 0$$

By left and right multiplying by

$$\begin{bmatrix} X_l^{-1} & 0 & 0 \\ 0 & Q^{-1} & 0 \\ 0 & 0 & Q^{-1} \end{bmatrix}$$

and by denoting

$$W \triangleq X_l^{-1}$$

we have

$$\begin{bmatrix} AW + WA' & [\ B \quad \gamma^{-1}(E+MP)\ ]Q^{-2} \\ Q^{-2}[\ B \quad \gamma^{-1}(E+MP)\ ]' & -Q^{-2} \\ CW & [\ L \quad \gamma^{-1}(H+NP)\ ]Q^{-2} \end{bmatrix}$$

$$\left.\begin{matrix} WC' \\ \ldots\ Q^{-2}[\ L \quad \gamma^{-1}(H+NP)\ ]' \\ -Q^{-2} \end{matrix}\right] < 0$$

By denoting $Z \triangleq X^{-2}$,

$$Q^{-2} = \begin{bmatrix} Z & 0 \\ 0 & I \end{bmatrix}$$

and expression (9) follows in a direct manner. □

Note that expression (9) represents an LMI constraint on γ^2, W, Z and P. The optimum prefilter can be obtained with the help of a standard algorithm for the minimization of a linear objective subject to an LMI constraint.

3.2 $\mathcal{H}_2$, LTV case

An LMI condition for robust $\mathcal{H}_2$ performance in the presence of LTV uncertainties can be found in [6]. The proposition presented in this section is a straightforward application of this result. The proof is omitted.

Proposition 2 *: Given $\gamma > 0$, condition 4 is satisfied if and only if there exists $P \in R^{du\times dr}$, $X \in \mathbf{X}$, hermitian $n\times n$ matrices P_-, P_+, and $Z \in R^{dr\times dr}$ such that*

$$trace(Z) < \gamma^2$$

$$P_- > 0$$

$$\begin{bmatrix} AP_- + P_-A' + BXB' & P_-C' \\ CP_- & -\begin{bmatrix} X & 0 \\ 0 & I \end{bmatrix} \end{bmatrix} < 0$$

$$\begin{bmatrix} AP_+ + P_+A' + BXB' & P_+C' \\ CP_+ & -\begin{bmatrix} X & 0 \\ 0 & I \end{bmatrix} \end{bmatrix} < 0$$

$$\begin{bmatrix} Z & (E+MP)' \\ E+MP & P_+ - P_- \end{bmatrix} > 0 \tag{10}$$

The optimum static prefilter can be obtained as the solution of a finite dimensional convex problem with the help of proposition 2.

3.3 $\mathcal{H}_\infty$, LTI case

We will derive an auxiliary lemma that we will use later. Let us define the matrix function
Ψ: $R \times \mathbf{X} \times \mathbf{\Phi} \times R^{du\times dr} \rightarrow C^{(2dr+dw)\times(2dr+dw)}$

$$\Psi(\omega, Y(\omega), \Phi(\omega), P) \triangleq$$

$$\begin{bmatrix} T_0(j\omega)Y(\omega)T_0(j\omega)^* - \begin{bmatrix} Y(\omega) & 0 \\ 0 & I \end{bmatrix} & T_1(j\omega) + T_2(j\omega)P \\ (T_1(j\omega) + T_2(j\omega)P)^* & -\Phi(\omega) \end{bmatrix} \tag{11}$$

Lemma 1 *: The following sentences are equivalent:*
i) There exist $X(\omega) \in \mathbf{X}$, $P \in R^{du\times dr}$ and $\Phi(\omega) \in \mathbf{\Phi}$ such that for all ω

$$T_P(j\omega)^* \begin{bmatrix} X(\omega) & 0 \\ 0 & I \end{bmatrix} T_P(j\omega) - \begin{bmatrix} X(\omega) & 0 \\ 0 & \Phi(\omega) \end{bmatrix} < 0 \tag{12}$$

ii) There exist $Y(\omega) \in \mathbf{X}$, $\Phi(\omega) \in \mathbf{\Phi}$ and $P \in R^{du\times dr}$ such that for all ω it is satisfied

$$\Psi(\omega, Y(\omega), \Phi(\omega), P) < 0 \tag{13}$$

Proof:
For clarity, all dependences on frequency will be dropped.

By left and right multiplying condition i) by the matrix

$$\begin{bmatrix} X^{-\frac{1}{2}} & 0 \\ 0 & \Phi^{-\frac{1}{2}} \end{bmatrix}$$

we have

$$\begin{bmatrix} X^{-\frac{1}{2}} & 0 \\ 0 & \Phi^{-\frac{1}{2}} \end{bmatrix} T_P^* \begin{bmatrix} X & 0 \\ 0 & I \end{bmatrix} T_P \begin{bmatrix} X^{-\frac{1}{2}} & 0 \\ 0 & \Phi^{-\frac{1}{2}} \end{bmatrix} - I < 0$$

Thus

$$\bar{\sigma}\left\{\begin{bmatrix} X^{\frac{1}{2}} & 0 \\ 0 & I \end{bmatrix} T_P \begin{bmatrix} X^{-\frac{1}{2}} & 0 \\ 0 & \Phi^{-\frac{1}{2}} \end{bmatrix}\right\} < 1$$

Equivalently

$$T_P \begin{bmatrix} X^{-1} & 0 \\ 0 & \Phi^{-1} \end{bmatrix} T_P^* - \begin{bmatrix} X^{-1} & 0 \\ 0 & I \end{bmatrix} < 0 \qquad (14)$$

By defining $Y(\omega) \triangleq X(\omega)^{-1}$ and by using expression (8) in (14):

$$T_0 Y T_0^* + [T_1 + T_2 P]\Phi^{-1}[T_1 + T_2 P]^* - \begin{bmatrix} Y & 0 \\ 0 & I \end{bmatrix} < 0$$

Then, condition ii) follows from a direct application of the Schur's complement (see [13]). □

Note that expression (13) define a convex constraint on $Y(\omega)$, $\Phi(\omega)$ and P. This constraint is an infinite dimensional (parameterized by ω) LMI.

A particular case of our interest is comprised in the following corollary:

Corollary 1 : *Given $\gamma > 0$, there exists a matrix function $X(\omega) \in \mathbf{X}$ and $P \in R^{du \times dr}$ such that for all ω it is satisfied*

$$T_P(j\omega)^* \begin{bmatrix} X(\omega) & 0 \\ 0 & I \end{bmatrix} T_P(j\omega) - \begin{bmatrix} X(\omega) & 0 \\ 0 & \gamma^2 I \end{bmatrix} < 0 \qquad (15)$$

if and only if there exist $Y(\omega) \in \mathbf{X}$ and $P \in R^{du \times dr}$ such that

$$\Psi(\omega, Y(\omega), \gamma^2 I, P) < 0 \qquad (16)$$

In order to improve the robust $\mathcal{H}_\infty$ performance against LTI uncertainties, we can minimize γ subject to condition 1 holds. Corollary 1 gives us a convex formulation for condition 1 which will be useful for solving this problem. So, the optimum static feedforward action P may be obtained by solving the following optimization problem:

Problem 1:

$$\min_{Y(\omega),\gamma,P} \gamma$$

subject to

$$\Psi(\omega, Y(\omega), \gamma^2 I, P) < 0$$

for all ω.

Comments on the computational characteristics of this problem will be made in section 4.

3.4 $\mathcal{H}_2$, LTI case

An analogous discussion involving condition 3 and lemma 1 leads us to formulate the following problem in order to design for robust $\mathcal{H}_2$ performance against LTI uncertainties:

Problem 2:

$$\min_{Y(\omega),\Phi(\omega),P} \int_{-\infty}^{+\infty} trace(\Phi(\omega)) \frac{d\omega}{2\pi}$$

subject to

$$\Psi(\omega, Y(\omega), \Phi(\omega), P) < 0 \qquad (17)$$

for all ω.

4 Computational issues and possible extensions

Two optimization problems were formulated in the previous section which allow us to compute the solution for the robust synthesis against LTI uncertainties. Both are convex, infinite dimensional problems. In order to solve these problems it is necessary to turn them into finite dimensional LMIs for which efficient algorithms are available [10].

It is always possible to select a finite set of basis functions and restrict our search to the corresponding finite dimensional subspace generated by the span of these functions, see e.g. [11].

Other alternative approach is based on the frequency domain and it consists of gridding the frequency axis by considering the points $\omega_0 \dots \omega_N$. A finite dimensional approximation to Problem 1 is

$$\min_{Y_0,\dots,Y_N,\gamma,P} \gamma$$

subject to

$$\Psi(\omega_i, Y_i, \gamma^2 I, P) < 0 \quad i = 0, \dots, N$$

where $Y_i \in \mathbf{X}$ are structured, constant matrices. This problem is convex and it can be efficiently solved.

Some special considerations must be made about the cost function of Problem 2. It is necessary to consider a support interval $[\omega_0, \omega_N]$ such that $trace[\Phi(\omega)]$

is neglected for all ω out of this interval. The support interval can be estimated from engineering considerations, or as a result of a brief iterative process. Then, the integral in Problem 2 may be discretized and this problem may be approximated as

$$\min_{Y_0,\ldots,Y_N,\Phi_0,\ldots,\Phi_N,P} \Sigma_1^N trace[\Phi_i](\omega_i - \omega_{i-1})$$

subject to

$$\Psi(\omega_i, Y_i, \Phi_i, P) < 0 \quad i = 0, \ldots, N$$

where $Y_i \in \mathbf{X}$ and $\Phi_i \in \mathbf{\Phi}$ are constant matrices.

These approaches offer no hard guarantees since that a "water bed" behavior is possible if an unadequate set of points is considered. However, it seems a valuable design method when it is accompanied with engineering judgement in selecting the grid points.

If we do not restrict our attention to static prefilters and we consider the class of dynamic prefilters $P(s)$ with arbitrary order, the extension of the results is possible. The convexity of problems 1 and 2 holds for a prefilter $P = P(j\omega)$. Similar conditions for the LTV cases can be obtained by restricting $X(\omega)$ in problems 1 and 2 to be constant. The optimum $P(s)$ can be recovered through an interpolation procedure in the frequency domain taking care to preserve the stability properties of the prefilter itself. The approach of the basis functions also provide a viable synthesis methodology. Current research efforts are directed to compare static vs. dynamic prefilters, and these techniques with the classical robust synthesis methods.

5 Conclusions

An extensive collection of robust synthesis problems for static prefilters was presented. These allows us to design the optimal static prefilter for robust performance against LTI or LTV structured uncertainties when either the $\mathcal{H}_2$ or $\mathcal{H}_\infty$ norm are adopted as performance index. All of these problems were shown to be convex ones and they fall into the scope of well known optimization algorithms. All of them dispense with the $\mathcal{D} - \mathcal{K}$ iteration which seems to be the general rule for robust synthesis problems.

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