



Universidad de la República  
Facultad de Ciencias Sociales  
DEPARTAMENTO DE ECONOMIA

## **Documentos de trabajo**

### **Estimating a Risky Term Structure of Uruguayan Sovereign Bonds**

Serafín Frache

Gabriel Katz

**Documento No. 03/04**  
Mayo, 2004

# Estimating a risky term structure of Uruguayan sovereign bonds

Serafín Frache

Gabriel Katz

May, 2004

Documento No. 03/04

## Abstract

Based on a joint three – factor affine model, we estimate the term structure of interest rates and default spreads for Uruguay using the reduced - form approach developed by Duffie and Singleton. We find that Uruguayan average term structure was negatively sloped between 1997 and 2003, as indicated by previous empirical evidence for low – quality debtors. Surprisingly, Uruguayan average yield curve was also negatively sloped between 1997 and 2001, when the country’s foreign – currency denominated debt was considered investment grade by the leading rating agencies. We also find that the estimated Uruguayan default spread is able to capture the behavior and dynamics of a more traditional country risk benchmark such as the “Uruguayan Bond Index” (UBI), with observations on a single Uruguayan bond. Finally, we find that regional, international and local financial crises cause parallel shifts in the Uruguayan yield curve, with higher increases in short – term rates, and that the banking and debt crises experienced by the country in 2002 had the biggest effects on the average Uruguayan term structure.

## Resumen

En base a un modelo lineal de tres factores, estimamos la estructura temporal de las tasas de interés y los spreads para Uruguay utilizando el enfoque de forma reducida desarrollado por Duffie y Singleton. Encontramos que la curva de rendimientos uruguaya fue en promedio descendente entre 1997 y 2003, como indica la evidencia empírica disponible para los deudores de baja calidad. Sorprendentemente, la curva de rendimientos promedio fue también descendente entre 1997 y 2001, cuando la deuda uruguaya en moneda extranjera era considerada con grado de inversión por parte de las principales agencias calificadoras. Encontramos asimismo que los spreads estimados capturan el comportamiento y la dinámica de una medida más tradicional de riesgo país como es el “Uruguayan Bond Index” (UBI), con observaciones de un único bono uruguayo. Finalmente, hallamos que las crisis financieras internacionales, regionales y locales provocan desplazamientos paralelos en la curva de rendimientos uruguaya, con mayores incrementos en los rendimientos de corto plazo, y que la crisis bancaria y de deuda que Uruguay atravesó en el año 2002 tuvo la mayor incidencia sobre la curva de rendimientos promedio.

**JEL Classification:** C1, C51, F34, G12, G15.

**Keywords:** default risk, term structure, reduced – form model, default spread.

# 1 Introduction<sup>1</sup>

In this paper we estimate the term structure of interest rates and default spreads for Uruguay. An accurate estimation of the term structure is important for the pricing of interest – rate contingent claims and fixed income derivative securities and for computing hedging and risk management strategies. Monetary policy is a second reason for studying the term structure: for a given state of the economy, a model of the yield curve reveals how transmission mechanisms between short and long term yields work, helping to understand both how central bank conducts policy and whether it is being effective [Frankel and Lown[26]; Piazzesi[45]]. Debt policy constitutes yet another reason: a government issuing new debt must decide about the maturity of the new bonds, and the shape of the yield curve embodies information about the optimal maturity in order to minimize the interest cost of issuing [Campbell[11]]. Besides, having precise estimates of the term structure allows investors to discount future cash flows at appropriate rates, reducing misestimation problems faced by prevailing valuation techniques in emerging markets [Alonso et al.[1]]. Finally, the current yield curve conveys valuable information about future interest rates, inflation rates and real activity [Bernard and Gerlach[5]; Kozicki [32]]

Originally, term structure literature focused on claims with certain payoffs (coupon and principal) and did not account for the possibility that debtors might fail to honor their debts. Recently, as a result of the expansion of markets for bonds exposed to default risk over the last twenty years, the rapid growth of credit derivatives and the changes in the regulatory framework, a new literature devoted to the pricing of defaultable bonds and to estimating defaultable term structure appeared.<sup>2</sup> Models for pricing credit risk are broadly divided into two categories: structural models and reduced - form models. The former draw on the approach by Black – Scholes[8] and Merton[39] and contain an underlying value process for the bond issuer that is used to generate default probabilities by solving for the probability that it will cross a

---

<sup>1</sup>The authors would like to thank Ernesto Mordecki and Juan Dubra for their invaluable guidance. They are also grateful to Mariano Pando for his helpful advice. Of course, the authors are responsible for any remaining error.

<sup>2</sup>See Basel Committee on Banking Supervision[14].

defined default threshold. Reduced - form models, on the other hand, assume that undefined economic mechanisms generate a stochastic process for default probabilities. While structural models are more “economically meaningful” [Keswani[31]] as they provide a causality for default, reduced - form models are more tractable mathematically and might be more useful in practical applications. Besides, reduced - form models may be more appropriate in situations where it is reasonable to assume that the value of the issuer is not observable, as is the case of sovereign debt.

In this paper we use prices of a sovereign bond to estimate term structures of interest rates in Uruguay from July 1997 to May 2003. We apply a multifactor reduced - form model in which the default event is defined as the realization of an exogenous intensity process. This is the first attempt to estimate Uruguayan term structure using a continuous - time reduced - form model. Starting from September 2003, the “Bolsa Electrónica de Valores del Uruguay” implements a spline – based technique to estimate Uruguayan term structure on a given date[9]. However, our modeling approach is less demanding in terms of data requirements and, contrary to the spline – based method, allows us to analyze the dynamics of the Uruguayan term structure in a consistent manner [Cortázar et al.[15]].

## **2 Recent developments concerning Uruguayan securities market and sovereign debt**

The Uruguayan securities market is characterized by few participants and a low trading volume, historically under 5% of GDP. Despite the fact that the market underwent institutional, legal and operational changes during the 90s aiming to promote its activity, it still presents an extremely low level of development, even when compared to other markets in the sub-region [Barbieri et al.[3]]. The Government is virtually the only issuer, as private securities represented in 2002 less than 2% of the volume traded in the domestic market. The fact that, up until a relatively short time ago, there were practically no issues denominated in currencies other than the US dollar, also contributed to the poor development of the market.

Government securities comprise mostly bonds and bills issued by the Central

Government, and, to a lesser extent, by Government – owned companies. The various government securities can be grouped in two basic categories: those issued in the domestic market (Treasury Bonds, Treasury Bills and Previsional Savings Bonds) and those issued for their trading in international markets (Euronotes, Eurobonds, Global Bonds, Samurai Bonds, Chilean peso-denominated Bonds).<sup>3</sup>

From the mid-80s to the late 90s, Uruguay’s consolidated public sector deficits were financed mostly through the issuance of US dollar-denominated Treasury Bills and Bonds placed in the domestic market. The first international issuance in the recent history of the country took place in 1992 (Euronote A). Until then, Uruguay had resorted to the international credit markets only through guaranteed loans. Since the restructuring of the foreign debt under the Brady Plan, Uruguay benefitted from the improvement of the access conditions to international financial markets [Steneri[51]] and diversified its debt profile, issuing international bonds of varying maturities. In the following years two more Euronotes were issued and in 1996 the Government issued the first 10 year – Eurobond with a spread over US Treasuries of 160 basis points (b.p.), even lower than those on bonds issued by many countries considered investment grade at the moment.

At the beginning of 1997, IBCA (currently Fitch – IBCA) and Duff & Phelps assigned investment grade credit rating to Uruguay and the two leading agencies, Moody’s and Standard & Poor’s, followed suit in the next months. The new credit rating paved the way to tap markets paying low yields and getting long maturities. At a time of massive surge of portfolio capital flows to emerging markets [Eichengreen and Mody[23]] and resumption of private capital flows to Latin America [Edwards[22]], world capital markets were extremely receptive to newly issued Uruguayan bonds: Uruguayan outstanding international debt securities grew by nearly 600% between 1997 and 2001.<sup>4</sup> International issues became particularly relevant starting in 2000 due to the reluctance of local investors to lend the Government in view of the recession that the country was enduring since the end of 1998. In fact,

---

<sup>3</sup>There are other Government securities which have some particularities that differentiate them from the aforementioned instruments and which have a relatively low market share.

<sup>4</sup>Not taking into account Uruguayan Brady bonds.

in April 2002, the Government suspended auctions of Treasury bills denominated in US dollars due to the unfavorable conditions of the domestic market. Taking advantage of the low spreads faced by the country until the end of 2001, almost 80% of new Government securities between 2000 and 2002 were issued in international markets.

Towards the end of 2001, Uruguayan Global bonds were priced above par (101.2%), and the country risk averaged 227 b.p., one of the lowest among emerging countries and only higher than Chile's in Latin America. However, the protracted recession, the difficult fiscal stance, the increasing public indebtedness and the worsening of the regional economic performance owing to the collapse of the Argentinean economy impinged on the prices and ratings of Uruguayan debt securities.<sup>5</sup> In February 2002 Standard & Poor's cut Uruguay's long – term foreign – currency denominated debt to BB+ from BBB-, and in the following months both Moody's and Fitch – IBCA also downgraded Uruguayan bonds several notches below investment grade. As a result, the prices of Uruguayan bonds plunged and the spreads over US Treasury rates reached unprecedented levels, while at the same time real GDP contracted 10.8% and the country's banking system confronted its worst crisis since 1982 – 83, leading to the need to float the currency to protect the scarce Central Bank reserves.<sup>6</sup>

On March 11, 2003 the Government announced its decision to initiate a debt re – profiling. The underlying causes of such decision were Uruguay's high foreign – currency indebtedness, the heavy short – term amortization schedule, the reduced ability of the Central Government to access the voluntary capital markets and the depletion of Central Bank international reserve assets. The initiative consisted of a massive voluntary swap of Uruguay's public debt involving USD 5,300 millions of foreign-currency-denominated public debt, including USD 3,700 millions of international debt securities. The proposal aimed at extending the average maturity of the debt, with no reductions on the principal. Additionally, the proposal included “regulatory incentives” to encourage participation of bondholders (e.g., old bonds

---

<sup>5</sup>Fiscal deficit reached 4,2% in 2002 and Debt/GDP ratio rose from 54% in 2001 to 92% in 2002.

<sup>6</sup>A more detailed account of 2002 developments can be found in the Prospectus elaborated by the Central Bank of Uruguay[4].

would become non - tradable securities due to the suspension of stock market quotations).<sup>7</sup> On May 29, the Government successfully completed the voluntary swap covering 93% of foreign-currency-denominated public securities in circulation.

### 3 The Model

We jointly model the instantaneous risk – free interest rate and Uruguayan spread with a three factor continuous – time reduced - form model. The model relies on the credit framework developed by Duffie and Singleton[21] and is analogous to the one implemented by Pando[43] for estimating Argentinean term structure.

As documented by Litterman and Scheinkman[36], two factors explain over 90% of the variability of the US riskless term structure.<sup>8</sup> They also show that these two factors can be associated with the “level” and the “slope” of the yield curve. Therefore, we assume that the instantaneous riskless interest rate is an affine function of two latent variables,  $x_1$  and  $x_2$ :

$$r_t = x_{1t} + x_{2t}$$

The instantaneous Uruguayan spread is defined as:

$$s_t = \gamma r_t + x_{3t}$$

According to the empirical evidence presented by Pagès[42] and Duffie et al.[20], we assume a direct correlation between the Uruguayan spread and the instantaneous risk – free interest rate, captured by the parameter  $\gamma$  . As documented by Keswani[31], allowing for correlation between factors driving the riskless and the risky term structure enhances the explanatory power of the model in a statistically significant way. The third factor,  $x_3$ , accounts for the idiosyncratic risk, i.e., the issuer - specific, unsystematic component of credit risk [Wilson[55]; Jarrow et al.[28]].

---

<sup>7</sup>See Stenery [52] for a brief description of the conditions of the offer.

<sup>8</sup>As is usual in this literature, we denote by “risk free” securities that are free of default risk, but not necessarily free of interest – rate risk.



We assume that the three latent variables are independent and that each of them follows an Ornstein – Uhlenbeck process under the real probability measure  $\mathcal{P}$ :

$$dx_{it} = \kappa_i(\theta_i - x_{it})dt + \sigma_i dw_{it} \quad i = 1, 2, 3$$

where  $\theta_i$  is the long - run mean of the  $i$  – th factor,  $\kappa_i$  is the speed of mean reversion,  $\sigma_i$  is the factor volatility and  $w_{it}$  is a standard Brownian motion under  $\mathcal{P}$ . It is a well documented stylized fact [see Oldfield and Rogalski[41], Moreno[40] and Cortázar et al.[15], amongst others] that interest rates are mean reverting, while Fons[25] and Kao[30] find that spreads also have a tendency to revert to the mean.<sup>9</sup> Vasicek [54] was one of the first authors who introduced Ornstein – Uhlenbeck processes in term structure models. His original one factor model was later extended to  $n$  – dimensional state vectors by Langetieg[33], Dai and Singleton[16] and Yang[57], amongst others.<sup>10</sup>

Applying Girsanov Theorem one can show that the state variables also follow Ornstein - Uhlenbeck processes under a risk – neutral probability measure  $Q$ :

$$dx_{it} = \kappa_i \left[ \left( \theta_i - \frac{\lambda_i \sigma_i}{\kappa_i} \right) - x_{it} \right] dt + \sigma_i dw_{it}^Q$$

where  $w_{it}^Q$  is a standard Brownian motion under  $Q$  and  $\lambda_i$  is the “market price of risk” of the  $i$  – th factor. Following Vasicek[54],  $\lambda_i, i = 1, 2, 3$ , are assumed to be constant.

From standard no arbitrage arguments, the price at time  $t$  of a default free zero - coupon bond with maturity  $T$  whose principal is \$ 1 can be written as follows:

$$P(t, T) = E_t^Q \left[ e^{-\int_t^T r_s ds} \right] \quad (1)$$

---

<sup>9</sup>The main drawback of assuming that the state variables follow Ornstein –Uhlenbeck processes is that they allow for a positive probability of negative factor values (and consequently, negative interest rates and spreads may occur). However, authors like Rogers[47], Babbs [2] and Papageorgiou[44] find that such probability is extremely low for “reasonable” parameter values.

<sup>10</sup>Our specification of the parameters affecting the factor dynamics is more restrictive than the one presented by these authors, since factor correlation is excluded in our model. Many authors have resorted to independent factors when estimating term structure models, including Longstaff and Schwartz[37] and Chen and Scott[12].

Duffie and Singleton[21] extend equation (1) to account for the pricing of defaultable claims such as risky zero - coupon bonds. In this framework, default is an unpredictable stopping time modeled by the first occurrence of a point process with stochastic intensity  $h_t$  under  $Q$ , and losses at default are parametrized in terms of the fractional reduction in market value that occurs at default, defined as  $L_t$ .<sup>11</sup> The instantaneous spread  $s_t = h_t L_t$  is the instantaneous expected loss rate in the value of the bond in case of default under  $Q$ .<sup>12</sup> Duffie and Singleton prove that, provided  $h_t$  and  $L_t$  can be taken to be exogenous, i. e., not dependent on the value of the defaultable claim itself, risky bonds can be priced as if they were risk - free by replacing the instantaneous interest rate  $r_t$  with the default adjusted rate  $R_t$ :

$$V(t, T) = E_t^Q \left[ e^{-\int_t^T R_s ds} \right] \quad (2)$$

where<sup>13</sup>

$$R_t = r_t + s_t = r_t + h_t L_t$$

Solving for (1) and (2) we obtain  $P(t, T)$  and  $V(t, T)$ :

$$\log P(t, T) = A(\tau) - B_1(\tau)x_{1t} - B_2(\tau)x_{2t} \quad (3)$$

$$\log V(t, T) = \tilde{A}(\tau) - \tilde{B}_1(\tau)x_{1t} - \tilde{B}_2(\tau)x_{2t} - B_3(\tau)x_{3t} \quad (4)$$

where  $\tau = T - t$  and the  $A, \tilde{A}, B_i, \tilde{B}_i, i = 1, 2, 3$  coefficients are nonlinear functions of the parameters driving  $x_i$  under  $Q$ . As we assumed that the  $x_i, i = 1, 2, 3$ , follow Ornstein - Uhlenbeck, we obtain closed - form solutions for these coefficients.

From (4), the yield at time  $t$  of an Uruguayan zero - coupon bond with maturity  $T$  is:

$$y_{URU}(t, T) = -\frac{\log V(t, T)}{T - t} = -\frac{\tilde{A}(\tau) - \tilde{B}_1(\tau)x_{1t} - \tilde{B}_2(\tau)x_{2t} - B_3(\tau)x_{3t}}{\tau} \quad (5)$$

---

<sup>11</sup>Duffie and Singleton assume that  $\{L_t\}_{t \geq 0}$  is bounded by 1 and predictable, so the information determining  $L_t$  is available before  $t$ . It is possible to extend their main results to the case when  $L_t$  cannot be determined based on the information available until  $t$ . See Schönbucher[49].

<sup>12</sup>As usual within the reduced - form approach, we assume that the value of the spread is the same under  $\mathcal{P}$  and  $Q$ . See Jarrow et al.[28] for a relaxation of this assumption.

<sup>13</sup>Since  $h$  and  $L$  enter the adjustment for default in the discount rate  $R$  in the product form  $hL$ , the knowledge of defaultable bond prices before default alone is not sufficient to separately identify  $h$  and  $L$ . See Duffie and Singleton[21].

Equation (5) allows us to obtain the Uruguayan yield curve for each date in our sample period using risk - free bonds and a single Uruguayan bond.

## 4 Data Description

In order to estimate the riskless term structure, we use Constant Maturity Treasury (CMT) rates for maturities of two, three, five and ten years published by the Federal Reserve[24]. The CMT rates represent the coupon rate that a US Treasury bond should pay to be priced at par. The main advantage of CMT rates is that they are available on a constant – maturity basis, which simplifies the estimation procedure. On the other hand, since estimates of CMT rates are based on newly issued bonds, they could introduce bias to the estimation: Sarig and Warga[48] suggest that younger bonds are more frequently traded and so exhibit lower spreads resulting from their greater liquidity.<sup>14</sup> However, as the difference between Uruguayan and Treasury yields is quite big, this bias should not be very important.

The Uruguayan data consists of average clean prices of 30 year USD denominated Global Bond 2027 with a semiannual coupon rate of 7.875%, provided by the Central Bank of Uruguay.<sup>15</sup> The reasons that led us to choose this particular bond were:

- it was the most liquid Uruguayan international bond before the voluntary swap of Uruguay’s public debt, with an amount issued of USD 510 million;
- this bond provides one of the longest price series among Uruguayan international bonds for the period considered;
- unlike Brady Bonds, its coupon and principal payments are not collateralized.<sup>16</sup>

---

<sup>14</sup>On the other hand, Stigum[53] indicates that the government securities issued some time in the past tend to be less actively traded and the quoted prices may not be reasonable approximations of equilibrium prices.

<sup>15</sup>Using prices that are averages of actual prices within a certain period (thursdays, in our case) may introduce spurious autocorrelations in the data. Working[56] was the first to point out this time – averaging problem. Unfortunately, we did not have other data to avoid this problem.

<sup>16</sup>Since Brady bonds are collateralized, there is less uncertainty about the size of the write – down when bond pricing models are estimated with them.

The sample period ranges from 07/17/1997, when Global Bond 2027 was issued, to 05/08/2003, the week prior to the suspension of stock market quotations. We use weekly observations in order to mitigate problems derived from missing observations, “day of the week” effect and other microstructure anomalies. The data that we use consist of observations sampled every Thursday, as this is the day of the week for which we have the most observations. When there is a Thursday observation missing, the preceding day’s observation is used. We have 304 observations for the entire sample period.

## 5 Estimation Method

The estimation problem consists of two parts: the estimation of the state vector  $X_t = (x_{1t}, x_{2t}, x_{3t})^T$  at each point in time and the estimation of the constant parameters.

The unobserved factors are nonlinearly related to the observed data (the CMT rates and the price of the Uruguayan bond). In the case of a CMT rate, the mapping relating the factors and the data is given by the following equation:<sup>17</sup>

$$CMT(t, T) = \frac{2[1 - P(t, T)]}{\sum_{i=1}^{2T} P(t, \frac{i}{2})} \quad (6)$$

Hence, from equations (3) and (6), the CMT rate is a function of factors  $x_1$  and  $x_2$ . Similarly, the price at time  $t$  of the Global Bond 2027 with maturity  $T$ , denoted  $Z(t, T)$ , satisfies the following equation:

$$Z(t, T) = \sum_{i=1}^{2T} \frac{c}{2} V(t, \frac{i}{2} - t) + V(t, \tau) \quad (7)$$

where  $\tau = T - t$ ,  $V(t, \tau)$  is the time  $-t$  price of an Uruguayan zero – coupon bond with maturity  $\tau$ . From equations (4) and (7),  $Z(t, T)$  depends on  $x_1$ ,  $x_2$  and  $x_3$ .

Although only two CMT rates and the price of the Uruguayan bond are needed to solve equations (6) and (7) and recover  $X_t$  for every  $t$ , we decided to use two

---

<sup>17</sup>We assume that the coupon payments of the US Treasury bond are paid semiannually and that its principal is \$1.

additional CMT rates to incorporate the cross - sectional variation from these yields and improve the accuracy of the estimation. However, this means that the number of yields exceeds the number of state variables, so the theoretical model cannot explain all variation in the data. To overcome this problem, we adopt the methodology proposed by Chen and Scott[12] and assume that the 2 - year and 10 - year CMT rates as well as the price of the Uruguayan bond are observed without any error in each moment, whereas the 3 - year and 5 - year CMT rates are contaminated with “measurement errors” originating from quoting and data - entry errors, rounding of prices, non - synchronous trading, bid - ask spreads, etc.<sup>18</sup>

Let  $Y_t = [CMT(t, 2), CMT(t, 10), Z(t, T)]^T$  be the data observed without error at time  $t$ ,  $\psi$  the parameter vector and  $\tilde{Y}_t = [CMT(t, 3), CMT(t, 5)]^T$  the vector of imperfectly observed yields. Using the data observed without error, we obtain  $X_t^\psi$  by a standard Newton method, given an initial parameter vector  $\psi^0$ . Secondly, we estimate the parameters of the model by the maximum likelihood method. The joint density of  $Y_2, \dots, Y_T$  (where  $Y_k = Y(t_k)$ ,  $k = 2, \dots, T$ ) is:

$$f(Y_2, Y_3, \dots, Y_T | Y_1; \psi) = \prod_{k=2}^T f(Y_k | Y_{k-1}; \psi)$$

where the term on the right hand side of the above equation follows from the fact that  $\{X_t\}_{t \geq 0}$  is a Markov process.<sup>19</sup> The conditional density of  $Y$  is the product of the conditional density of  $X$  and the determinant of the Jacobian:

$$f(Y_k | Y_{k-1}; \psi) = f(X_k | X_{k-1}; \psi) \left| \frac{\partial X_k}{\partial Y_k} \right|$$

Since we are estimating a continuous - time model using discretely sampled observations,  $f(X_k | X_{k-1}; \psi)$  must be obtained from the exact discrete - time distribution

---

<sup>18</sup>While these assumptions regarding the measurement errors are obviously ad - hoc, they provide a convenient computational framework. Alternatively, we could have assumed that all the data are observed with measurement errors and estimate the model with the Kalman filter algorithm. Li[35] finds that the Chen and Scott method is a reasonable alternative to the Kalman filter method when weekly data are used.

<sup>19</sup>Since  $\{X_t\}_{t \geq 0}$  is an asymptotic independent process, the effect of  $f(Y_1; \psi)$  is asymptotically negligible and can be ignored. See Spanos[50].

of the state variables, which can be shown to be a VAR(1) model with Gaussian innovations. Thus, the transition densities for the unobserved variables are normally distributed, and the conditional log – likelihood function for  $X_k | X_{k-1}$  is:

$$\frac{\left[ X_k^\psi - M_{k-1}(\psi) \right]^T \Sigma(\psi)^{-1} \left[ X_k^\psi - M_{k-1}(\psi) \right] + \log |\Sigma(\psi)| + 2 \log |J(\psi)|}{2} \quad (8)$$

where

$$|J| = \left| \frac{\partial Y_k}{\partial X_k^T} \right|$$

$\Sigma$  is a diagonal variance – covariance matrix:

$$\begin{bmatrix} \frac{\sigma_1^2}{2\kappa_1} (1 - e^{-2\kappa_1(t_k - t_{k-1})}) & 0 & 0 \\ 0 & \frac{\sigma_2^2}{2\kappa_2} (1 - e^{-2\kappa_2(t_k - t_{k-1})}) & 0 \\ 0 & 0 & \frac{\sigma_3^2}{2\kappa_3} (1 - e^{-2\kappa_3(t_k - t_{k-1})}) \end{bmatrix}$$

and  $M_{k-1}(\psi)$  is a 3 x1 vector with elements:

$$M_{k-1}(\psi) = \begin{bmatrix} e^{-\kappa_1(t_k - t_{k-1})} x_{1,k-1} + \theta_1 (1 - e^{-\kappa_1(t_k - t_{k-1})}) \\ e^{-\kappa_2(t_k - t_{k-1})} x_{2,k-1} + \theta_2 (1 - e^{-\kappa_2(t_k - t_{k-1})}) \\ e^{-\kappa_3(t_k - t_{k-1})} x_{3,k-1} + \theta_3 (1 - e^{-\kappa_3(t_k - t_{k-1})}) \end{bmatrix}$$

We assume that  $\varepsilon_k$ , the vector of differences between  $\tilde{Y}_k$  and the values implied by the model, has a normal distribution with zero mean and a diagonal variance – covariance matrix  $\Sigma_\varepsilon$ , with generic elements  $\sigma_{\varepsilon_i}^2$ ,  $i = 1, 2$ .<sup>20</sup> The log – likelihood function for  $\varepsilon_k$  is:

$$-\frac{1}{2} \left[ \varepsilon_k^T \Sigma_k^{-1} \varepsilon_k + \log |\Sigma_k| \right] \quad (9)$$

The log – likelihood function  $L$  results from adding (8) and (9) over all the observations:

$$L(\psi) = \sum_{k=2}^T -\frac{1}{2} \left\{ \left[ X_k^\psi - M_{k-1}(\psi) \right]^T \Sigma(\psi)^{-1} \left[ X_k^\psi - M_{k-1}(\psi) \right] + \log |\Sigma(\psi)| + 2 \log |J(\psi)| + \varepsilon_k^T \Sigma_k^{-1} \varepsilon_k + \log |\Sigma_k| \right\}$$

As the riskless term structure can be estimated independently of the default spread component, we carry out the estimation in two steps: first we obtain estimates

<sup>20</sup>Alternative distributional assumptions for the errors could have been used without further difficulty. For example, Pagès[42] and Yang[57] assume that the errors follow AR(1) processes.

of the parameters corresponding to the US term structure, and then the remaining parameters affecting the Uruguayan term structure.<sup>21</sup>

## 6 Empirical results

### 6.1 Parameter estimates, estimated term structures and fitted spreads

Table 1 provides the estimated parameter values as well as their asymptotic standard errors for the entire sample.<sup>22</sup>

**Table 1**

Parameter	Estimate	Standard error
$\kappa_1$	0.3623	0.0006
$\kappa_2$	0.0063	0.0001
$\theta_1$	0.0192	0.0040
$\theta_2$	0.0350	0.0126
$\sigma_1$	0.0141	0.0000
$\sigma_2$	0.0114	0.0000
$\lambda_1$	-0.8613	0.0240
$\lambda_2$	-0.0716	0.0062
$\sigma_{\varepsilon_1}$	0.0006	0.0000
$\sigma_{\varepsilon_2}$	0.0009	0.0000
$\kappa_3$	0.8625	0.0106
$\theta_3$	0.3140	0.0120
$\sigma_3$	0.2020	0.0024
$\lambda_3$	0.8447	0.0562
$\gamma$	-1.2172	0.0065

The estimates are significant at usual confidence levels.<sup>23</sup> Since the initial parameter vector,  $\psi^0$ , was arbitrarily chosen, we implemented the estimation procedure

---

<sup>21</sup>As the 2027 Global Bond is denominated in USD, the risk – free term structure coincides with the US term structure.

<sup>22</sup>The asymptotic variance – covariance matrix is the inverse of the Hessian matrix, which consists of the second derivatives of the log - likelihood function with respect to the parameters.

<sup>23</sup>The value of the t – statistic for  $\kappa_i$  must be interpreted with some caution, since the non – stationarity under the null implies that its asymptotic distribution is not standard normal [Dickey and Fuller[18]].

in several opportunities starting from different initial values in order to check the robustness of the method with respect to the initial values of parameters; the estimates obtained in the different attempts did not differ significantly from those reported in Table 1. Besides, we tried to replicate the results of Pando[43], using observations on CMT rates for the period analyzed by him, ranging from 12/17/1993 to 12/21/2001. Our estimates of the parameters corresponding to the US term structure were similar to those of Pando.

With regard to the US term structure, we observe that  $\lambda_1$  and  $\lambda_2$  are negative, indicating that the term premiums are positive as maturity increases. According to the evidence presented by Litterman and Scheinkman[36] and Berardi et al.[6], we find that the high mean reverting variable,  $x_1$ , is negatively correlated with the spread between 30 years and 3 months CMT, representing the (opposite of the) “slope factor”, whereas the low mean reverting factor,  $x_2$ , is strongly and positively correlated with long term yields and can be associated with the “level factor”.

**Table 2**  
**Correlation between  $x_1$ ,  $x_2$  and US rates**

<b>Maturity – CMT rates</b>	<b><math>x_1</math></b>	<b><math>x_2</math></b>
3 month	0.978	-0.119
6 month	0.981	-0.080
1 year	0.975	-0.019
2 years	0.944	0.111
3 years	0.916	0.185
5 years	0.851	0.317
7 years	0.810	0.387
10 years	0.742	0.487
20 years	0.648	0.576
30 years	0.333	0.724
30 years - 3 month spread	-0.940	0.471

The model fits the 3 - year and 5 - year CMT rates reasonably well, with average mean errors of 4.7 and 6.4 b.p. respectively, similar to the ones reported by Keswani[31], Berardi et al.[6] and Pando[43]. As shown in Figure 1, the estimated US average yield curve is upward sloping.



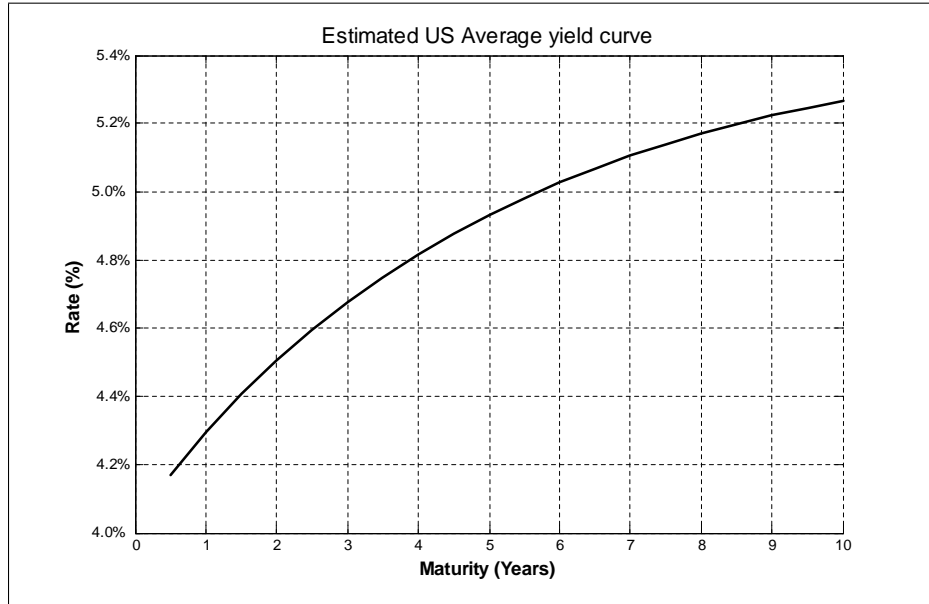


Figure 1

As to the Uruguayan term structure, both the speed of the mean reversion ( $\kappa_3$ ) and the volatility ( $\sigma_3$ ) of the idiosyncratic factor are considerably higher than those corresponding to the US factors. The estimate of  $\gamma$  is negative and significant, indicating a negative correlation between the risk – free instantaneous rate and the Uruguayan spread that could be attributed to a “flight to quality” behavior by the investors. The negative sign of  $\gamma$  is consistent with previous empirical evidence on both corporate [Longstaff and Schwartz[38]; Duffee[19]; Collin – Dufresne et al.[13]] and sovereign [Pagès[42], Duffie et al.[20]; Pando[43]] debt. The estimate of  $\lambda_3$  is positive, which implies that the term premiums associated with the idiosyncratic factor decline as maturity increases. Bond yield errors are higher than those corresponding to US yields, which could be explained by the limited liquidity and development of Uruguayan securities market, the high commissions charged by brokers and the role played by Uruguayan Pension Fund Administrators (AFAPs) in the demand of Global Bond 2027.<sup>24</sup>

<sup>24</sup>Uruguayan Pension Fund Administrators have become the leading institutional investors in Uruguayan securities market. Since AFAPs are bound by law to invest a considerable part of the

Figure 2 shows the average Uruguayan term structure, which can be obtained by plugging the estimated parameters and the sample means of the latent factors in equation (5). The downward sloping average term structure is consistent with previous empirical evidence on both corporate [Sarig and Warga[48], Kao[30]] and sovereign [Pando[43]] bonds , suggesting that low – quality debtors face negatively sloped yield curves.

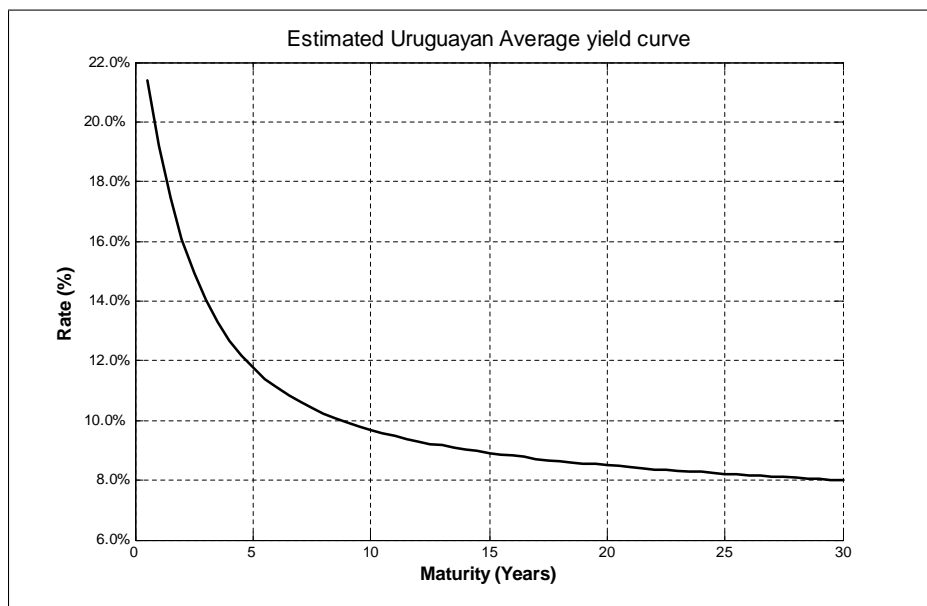


Figure 2

Figure 3 shows the evolution of the average Uruguayan term structure during the sample period. As we can see, the Uruguayan yield curve was never positively sloped between July 1997 and May 2003. In fact, the average yield curve was descending even between 1997 and 2001, which implies that although Uruguayan sovereign debt was regarded as investment grade and yield spreads paid on these bonds were quite

---

funds they manage in Government bonds, they have clear incentives to boost the prices of sovereign bonds [Bergara and Masoller [7]]. Besides, until the beginning of 1999 Global Bonds 2027 were valued in their portfolio not at market but at purchase prices. Therefore, AFAPs repeatedly sold and repurchased these bonds, causing their prices to rise in order to benefit from the capital gains resulting from higher prices. See Instituto de Economía, UDELAR[27].

low, Uruguayan term structure was typical of a low - quality debtor in that period too. The term structure of credit spreads was also clearly downward between 1997 and 2001. This is a rather surprising outcome as far as according to the “crisis at maturity theory” [Johnson[29]; Fons[25]] the yield spreads of investment grade bonds should show an upward term structure.<sup>25</sup>

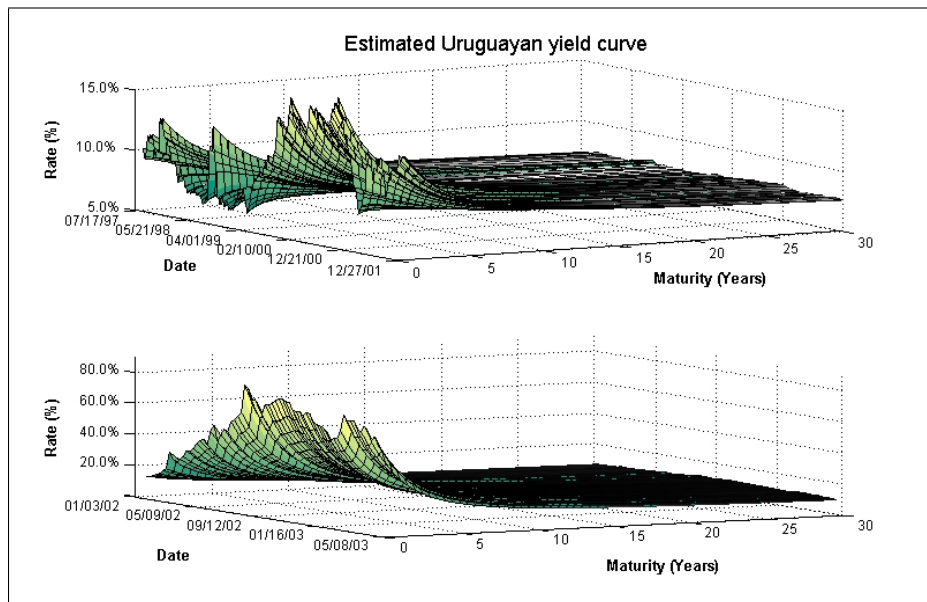


Figure 3

Figure 3 also shows that short - term yields rose sharply from 2002 onwards, probably reflecting an increase in the short - term default probability perceived by investors as a result of the downgrading of Uruguayan sovereign debt, skyrocketing country risk premiums and the highly concentrated debt repayment schedule.<sup>26</sup>

Finally, Figure 4 shows the estimated weekly Uruguayan spreads during the sample period. As we can see, this series captures the sharp hikes on spreads due to

<sup>25</sup>The observed negative slope of the term structure of yield spreads could be caused, at least in part, by the liquidity impact on yield spreads [Díaz y Navarro[17]]. Although we do not explicitly consider liquidity effects, our modeling approach is flexible enough to include a liquidity premium driving the spreads [see Duffie et al.[20]].

<sup>26</sup>The spread between 30 - year and 6 - month yields, taken as a proxy of the slope of the yield curve, went from - 6,0 b.p. in December 2001 to - 71,4 b.p. in July 2002.

the financial in crisis in 2002. The estimated spread is also compared with the most widespread benchmark of bond spreads in Uruguay, the “Uruguayan Bond Index” (UBI) developed by República AFAP.<sup>27</sup> This indicator measures the spread of a portfolio containing several US denominated Uruguayan international bonds of different maturities with respect to comparable US Treasury bond yields.<sup>28</sup>

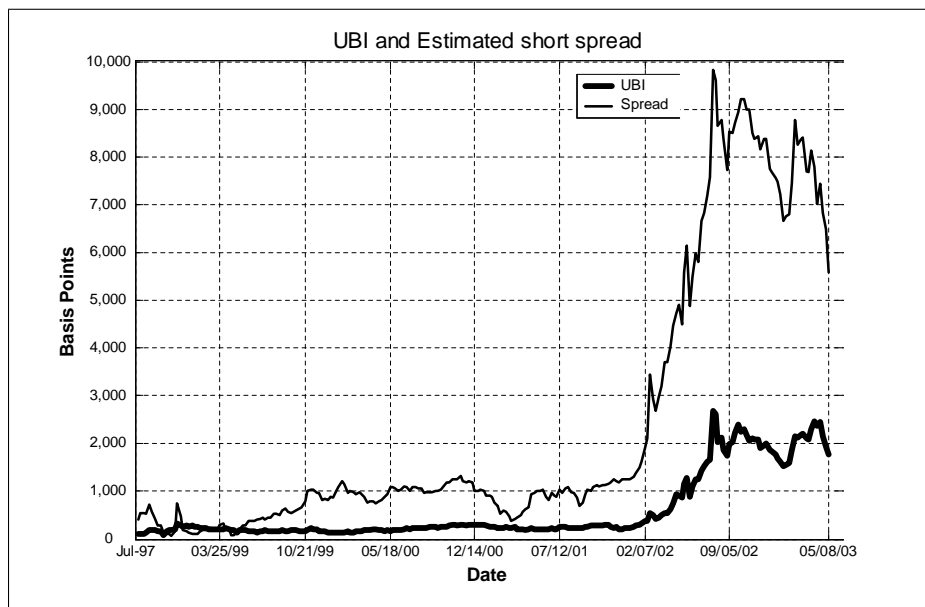


Figure 4

There is a strong direct comovement of the two series and, as we can see, the estimated spreads match the peaks and troughs featured by the UBI. The correlation coefficient between both series is 0.9794, suggesting that the fitted spreads are able to capture the behavior of the UBI despite the fact that our modeling approach requires only one Uruguayan bond. On the other hand, not only is the UBI more demanding in terms of data requirements, but the methodology used to construct it requires several assumptions and is also much more arduous. However, our fitted spreads perform rather poorly at fitting the actual levels of the UBI. This can be

<sup>27</sup>República AFAP is the leading and only state-owned Pension Fund Administrator in Uruguay.

<sup>28</sup>For an explanation of the methodology used to construct the UBI, see Laporta et al.[34].

explained by the fact that the latter is comprised of bonds with several liquidities, maturities and coupon rates that differ from those of the bond used in our estimation.

A principal components analysis shows that 60% of the variability of credit spreads can be explained by the first principal component, which is highly negatively correlated with short - term US yields (3 month -, 6 month - and 1 year - CMT rates). This result underscores the impact of changes in US short term rates on Uruguayan sovereign bond spreads and is consistent with evidence found by Berardi et al.[6] for other Latin American countries (Mexico, Brazil, Argentina).

## 6.2 The reaction of the Uruguayan term structure to recent financial shocks

In this section we analyze how the Uruguayan term structure of interest rates fluctuated in response to the effect of certain recent international crises: the Asian crisis on November 1997, the Russian default on August 1998 and the Brazilian devaluation in 1999. We also analyzed how the banking and debt crises that the country underwent during 2002 affected the Uruguayan yield curve. Based on the fitted spreads, we chose three dates for each crisis: a date prior to the beginning of the corresponding crisis, a date marking the peak of the crisis and a third date in which the crisis had already finished. Table 3 shows the Uruguayan yields for maturities of one, five and thirty years during the three chosen dates for each crisis.

**Table 3**  
**Uruguayan yields for maturities of 1, 5 and 30 years**  
**In percentage points**

	Crisis											
	Asian			Russian			Brazil			Uruguayan		
	1	5	30	1	5	30	1	5	30	1	5	30
<b>Pre</b>	9.4	8.3	7.3	7.1	7.6	7.4	7.7	7.8	7.4	12.8	9.6	7.5
<b>Peak</b>	12.1	9.3	7.5	12.3	9.5	7.8	8.5	8.1	7.4	69.5	29.0	10.9
<b>Post</b>	9.4	8.4	7.5	7.5	7.8	7.5	7.6	7.8	7.4	47.9	21.7	9.7

The table shows that short term rates are more sensitive to the crises than long - term rates. The three international financial crises cause parallel shifts in yield

curves, but once the effect of a crisis vanishes, they return nearly to their pre – crisis levels.<sup>29</sup> As we expected, in the case of the Uruguayan financial crisis of 2002, post – crisis yields remain much higher than the pre – crisis yields. Based on the yield values, it is evident that the local financial crisis had the biggest impact, with returns of nearly 70% p.a. for the shortest maturity during the crisis peak. On the other hand, Brazilian devaluation had the least effect. This result might seem rather surprising, in view of the strong trade and economic links between Uruguay and Brazil. Nonetheless, it is consistent with Rigobon[46], who argues that real linkages such as trade contribute only marginally to the international transmission of financial shocks. The author asserts that the main difference between crises that are contagious from the ones that are not lies in the degree of anticipation of the crisis: unanticipated crises are contagious, while anticipated ones are not (or less). In this sense, Rigobon finds evidence that neither the Asian crisis nor the Russian default were as predictable as the Brazilian devaluation, and this could explain the reduced effect of the latter on the Uruguayan term structure.

## 7 Concluding remarks

We applied a reduced - form model to estimate the Uruguayan term structure and default spreads from July 1997 to May 2003. Consistent with empirical literature on defaultable term structure, we find that the average Uruguayan yield curve is downward sloping. In fact, the average yield curve was negatively sloped, typical of a low - quality debtor, even between 1997 and 2001, when Uruguayan debt was regarded as investment grade by the leading rating agencies. We also find that the estimated default spread is highly correlated with a more traditional country risk benchmark such as the UBI, although using data on a single Uruguayan bond. Finally, we studied the effect of several financial crises on the shape of the yield curve. Our results show that these crises cause parallel shifts in the Uruguayan term structure, with short rates reacting more than long – term rates, and that the local financial crisis of 2002 had the most significant and long - lasting impact.

---

<sup>29</sup>Similar results were found in Bugallo and Dabós[10] and Pando[43] for Argentina.

Possible extensions of this work include: estimating the Uruguayan term structure after the swap of Uruguay's public debt, adding more risk factors, including other Uruguayan bonds in the estimation and extending the model to account for other credit and liquidity related events.

## References

- [1] Alonso, J., M. Buscaglia and J. Cruces (2002): "The Term Structure of Country Risk and Valuation in Emerging Markets", *Anales de la Asociación Argentina de Economía Política*, N° 1682, 2002.
- [2] Babbs, S. (2002): "Conditional Gaussian models of the term structure of interest rates", *Finance and Stochastics*, Vol. 6, issue 3, 333 – 353.
- [3] Barbieri, E., J. de Brun and N. Gandelman (2001): "Investment equations and financial restrictions at the firm level: The case of Uruguay", *Interamerican Development Bank Research Network*, Washington, 2001.
- [4] Central Bank of Uruguay (2003): *Prospectus reflecting the additional filings made with the Securities and Exchange Commission on April 15, 2003 and May 9, 2003.*
- [5] Bernard, H. and S. Gerlach (1996): "Does the term structure predict recessions? The international evidence.", *Working Paper N° 37*, Bank for International Settlements.
- [6] Berardi, A., S. Ciruolo and M. Trova (2002): "The term structure of credit spreads on sovereign bonds", *Conference Proceedings, XXVI Amases Meeting.*
- [7] Bergara, M. and A. Masoller (2002): "Un Análisis del Mercado de Bonos de Uruguay", *Revista de Economía, Banco Central del Uruguay, Segunda Época*, Vol. IX, N° 1 (Mayo), 49 - 91.
- [8] Black, F. and M. Scholes (1973): "The pricing of options and corporate liabilities", *Journal of Political Economy*, 81, 637 - 654.

- [9] Bolsa Electrónica de Valores del Uruguay (2003): “Índice de Riesgo Uruguay – IRUBEVSA”, Documento de presentación, Setiembre de 2003.
- [10] Bugallo, F. y M. Dabós (2000): “Term Structure of Interest Rate Changes during International Financial Crises: The Case of Argentina vs USA”, Working Paper, Universidad de San Andrés.
- [11] Campbell, J. (1995): “Some Lessons from the Yield Curve”, Working Paper 5031, National Bureau of Economic Research.
- [12] Chen, R. and L. Scott (1993): “Maximum Likelihood Estimation for a Multi-factor Equilibrium Model of the Term Structure of Interest Rates”, The Journal of Fixed Income, 3, 14 - 31.
- [13] Collin – Dufresne, P., B. Goldstein and S. Martin (2001): “The Determinants of Credit Spreads Changes”, The Journal of Finance, Vol. LVI, N° 6.
- [14] Basel Committee on Banking Supervision (1999): “Principles for the Management of Credit Risk”, Consultative paper issued by the Basel Committee on Banking Supervision, July 1999.
- [15] Cortázar, G. et al. (2003): “Term Structure Estimation in Low – Frequency Transaction Markets: A Kalman Filter Approach with Incomplete Panel – Data”, Working Paper, Pontificia Universidad Católica de Chile.
- [16] Dai, Q. and K. Singleton (2000): “Specification Analysis of Affine Term Structure Models”, The Journal of Finance, Vol. LV, N° 5, October 2000.
- [17] Díaz, A. and E. Navarro (2002): “Yield spread and term to maturity: default vs. liquidity”, European Financial Management, Vol. 8, N° 4, 449-477.
- [18] Dickey, D. and W. Fuller (1979): “Distribution of the Estimator for Autoregressive Time Series With a Unit Root”, Journal of the American Statistical Association, 74, 427 – 431.



- [19] Duffee, G. (1999): “Estimating the Price of Default Risk”, *Review of Financial Studies* 12, 197 – 226.
- [20] Duffie, D., L. Pedersen and K. Singleton (2001): “Modeling Sovereign Yield Spreads: A Case Study of Russian Debt”, *The Journal of Finance*, (Feb., 2003), Vol. LVIII, N° 1, 119 -159.
- [21] Duffie, D. and K. Singleton (1999): “Modeling Term Structures of Defaultable Bonds”, *Review of Financial Studies* 12, 687 – 720.
- [22] Edwards, S. (1998): “Capital inflow into Latin America: A stop – go story”, Working Paper 6441, National Bureau of Economic Research.
- [23] Eichengreen, B. and A. Mody (1998): “What explains changing Spreads on Emerging – Market Debt: Fundamentals or Market Sentiment?”, Working Paper 6408, National Bureau of Economic Research.
- [24] Federal Reserve: H – 15 Report, [www.federalreserve.gov/releases/h15/data.htm](http://www.federalreserve.gov/releases/h15/data.htm).
- [25] Fons, J. (1994): “Using Default Rates to Model the Term Structure of Credit Risk”, *Financial Analysts Journal*, 50 (Sept./Oct.), 25 - 32.
- [26] Frankel, J. and C. Lown (1994): “An indicator of future inflation extracted from the steepness of the interest rate yield curve along its entire length”, *Quarterly Journal of Economics*, Vol. 109, N° 2, 519 - 530.
- [27] Instituto de Economía de la Facultad de Ciencias Económicas y de Administración de la UDELAR (1999): “Informe de Coyuntura – Uruguay 1999 – 2000”.
- [28] Jarrow, R., D. Lando and F. Yu (2001): “Default Risk and Diversification: Theory and Applications”, Working Paper, Cornell University.
- [29] Johnson, R. (1967): “The term structure of corporate bond yields as a function of the risk of default”, *The Journal of Finance*, Vol. 22, N° 2, May 1967, 313 - 345.

- [30] Kao, D. (2000): “Estimating and pricing credit risk: an overview”, *Financial Analyst Journal*, Vol. 56, N° 4, July/August 2000, 50 – 66.
- [31] Keswani, A. (2000): “Estimating a Risky Term Structure of Brady Bonds”, Working Paper 2000/028, Management School, Lancaster University.
- [32] Kozicki, S. (1997): “Predicting real growth and inflation with the yield spread”, *Economic Review*, Federal Reserve Bank of Kansas City, Fourth Quarter, 39 –57.
- [33] Langetieg, T. (1980): “A multivariate model of the term structure”, *The Journal of Finance*, 35, 71 – 97.
- [34] Laporta, S., M. Larzabal and M. Valdés (2001): “Spread Soberano: Evidencia empírica del caso uruguayo”, Departamento de Estudios de República AFAP.
- [35] Li, X. (2003): “Estimation of Term Structure Models with Latent State Variables”, Cap. 1 of her Doctoral Dissertation, Simon Fraser University.
- [36] Litterman, R. and J. Scheinkman (1991): “Common factors affecting bond returns”, *The Journal of Fixed Income*, 1, 54 – 61.
- [37] Longstaff, F. and E. Schwartz (1992): “A Two Factor Interest Rate Model and contingent Claims Valuation”, *The Journal of Fixed Income*, 2:3, 16-23, December 1992.
- [38] Longstaff, F. and E. Schwartz (1995): “A Simple Approach to Valuating Risky Fixed and Floating Debt”, *The Journal of Finance*, 50, 789 - 819.
- [39] Merton, R. (1974): “On the Pricing of Corporate Debt: the Risk Structure of Interest Rates”, *The Journal of Finance*, 29, 449 - 470.
- [40] Moreno, M. (1997): “Risk Management under a Two-Factor Model of the Term Structure of Interest Rates”, Working Paper 254, Centre de Recerca en Economia Financiera i Comptabilitat, Universitat Pompeu Fabra.

- [41] Oldfield, G. and R. Rogalski (1987): “The stochastic properties of term structure movements”, *Journal of Monetary Economics*, 19, 229 – 245.
- [42] Pagès, H. (2001): “Can liquidity risk be subsumed in credit risk? A case study from Brady bond prices”, Working Paper N° 101, Bank for International Settlements.
- [43] Pando, M. (2002): “Estimating the Argentinean Term Structure: A Reduced - Form Approach”, Working Paper, UCLA, November 2002.
- [44] Papageorgiou, N. (2003): “The term structure of defaultable bonds”, Working Paper, HEC Montreal.
- [45] Piazzesi, M. (2003): “Affine term structure models”, forthcoming *Handbook of Financial Econometrics*.
- [46] Rigobon, R. (2003): “News, Contagion and Anticipation”, paper presented at the 8th Annual Meeting of LACEA, Mexico, 2003. Preliminary version provided by the author.
- [47] Rogers, L. (1995): “Which Model for Term – Structure of Interest Rates should one use?”, *Mathematical Finance*, IMA, Vol. 65, 93 – 116, Springer, New York, 1995.
- [48] Sarig, O. and A. Warga (1989): “Some empirical estimates of the risk structure of interest rates”, *The Journal of Finance*, 44, 1351 – 1360.
- [49] Schönbucher, P. (1996): “The Term Structure of Defaultable Bond Prices”, Discussion Paper N° B – 384, Department of Statistics, Faculty of Economics, University of Bonn.
- [50] Spanos, A. (1986): “Statistical Foundations of Econometric Modelling”, Cambridge University Press, Cambridge.

- [51] Steneri, C. (1994): “Endeudamiento externo de Uruguay: desde la crisis de los años 80 al plan Brady”, *Revista de Economía*, Banco Central del Uruguay, 1(1), 358, Mayo 1994.
- [52] Steneri, C. (2003): “Voluntary Debt Reprofilng: The Case of Uruguay”, document prepared for the Four UNCTAD Conference on Debt Management, Geneva, 11 – 14 November 2003.
- [53] Stigum, M. (1990): “The Money Market”, Dow Jones – Irwin, Homewood, IL.
- [54] Vasicek, O. (1977): “An Equilibrium Characterization of the Term Structure”, *Journal of Financial Economics*, 5, 177 – 188.
- [55] Wilson, T. (1998): “Portafolio credit risk”, *Economic Policy Review*, Federal Reserve Bank of New York, Vol. 4, No 3, 71 – 82.
- [56] Working, H. (1960): “A note on the correlation of first differences of average in a random chain”, *Econometrica*, 28, 916 – 918.
- [57] Yang, J. (2001): “Estimating Term Structure of Interest Rates with Treasuries”, Working Paper, Department of Economics, University of Toronto.