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## Documentos de trabajo

# Tastes and singular economies

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# Tastes and Singular Economies

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## Abstract

The object of this paper is to show some examples of economies in which singular equilibria occur as a consequence of utility functions and where this equilibria play a crucial rolle to understand the behavior of the economy as a system. An economy will be called singular if little changes in the tastes of the consumer imply big changes in the equilibria set.

## Resumen

El objetivo de este trabajo es el de mostrar algunos ejemplos de economías en las que las singularidades aparecen como consecuencia de las funciones de utilidad, algunas de las cuales al modificarse aunque sea de manera insignificante producen grandes cambios en el comportamiento de la actividad económica.

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# 1  Introduction

An exchange economy is characterized by the set $\mathcal{E} = \{\succeq_i, w_i, I\}$, where $I$ is a finite index set, one for each agent, $\succeq_i$ represents a preference relation for the agent indexed by $i$, endowments of the agent $i$ are denoted by the symbol $w_i$. As usual preferences are binary relations in the product space $X \times X$ where $X$ is the consumption space. In our work $X = R^l_+$ that is we will deal only with economies with a finite number of agents and goods and we will assume that preferences can be represented by utility functions.

The object of this paper is to show that in some kind of economies, little changes in tastes may imply big changes in the economic behavior of the economy considered as a system.

It is well known that in a neighborhood of a singular economy in the traditional sense, there exist economies with different number of walrasian equilibria, see [Accinelli, E. (96)]. Singularity in this frame means that for fixed preferences and given endowments there exist at least one set of social weights were the excess utility function vanished: $\lambda \in \Delta$ where $e_{\succeq,w}(\lambda) = 0$ and $rankJe_{\succeq,w}(\lambda) < n - 1$, where $\lambda$ is the vector of social weights, $e_{\succeq,w}$ is the excess utility function for endowments and preferences fixed, and $Je_{\succeq,w}(\lambda)$ denotes the jacobian of this function, see [Accinelli, E.(99)].

In this paper we will show cases, where singularities appear as a property of utilities, in the sense that changes in the number of equilibria appear in a neighborhood of certain kind of utility functions, the singular utilities. We will consider endowment as given and we will show cases in which in a neighborhood of a given utility function there exist economies with a different number of equilibria. This means that if the economy is singular in this sense, a little modification in the utilities of the consumers or little mistakes in the measure or appreciation of his tastes may give raise to an unforeseen behavior. The mathematical statement of this phenomenon is that in a neighborhood of a singular utility, the system is structurally unstable.

Existence of singularities may be an answer to questions like: *Why do crisis exist ?* certainly this is an ambitious question. If this would be a real possibility to explain this topics, then a crisis would be the result of the structural conditions and not a result of exogenous movements in fundamentals. We think that this is the main argument to analyze the structural characteristics of different kind of singularities.

As our object is to show that changes in utility functions may imply big changes in the behavior of the economy, we will follow the Negishi approach. In this approach the characterization of the walrasian equilibrium set is given by the excess utility function. This function play a fundamental role in our work because:

- the utilities appear explicitly in the excess utility function, and

- zeroes of this function are in one to one correspondence with the set of walrasian equilibria.

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Then, changes in the utility functions appear directly related with changes in the walrasian equilibrium set.

In the following section we will characterize the Negishi approach and in the third section the space of utility functions, will be a metric space, next we will show some examples of economies with this kind of *singular utilities,* and we conclude with some comments about the economic meaning of this kind of singularities.

## 2    The Negishi approach and the excess utility function

Consider the social welfare function: $W_\lambda : R_+^{ln} \to R$ defined as:

$$W_\lambda(x) = \sum_{i=1}^{n} \lambda_i u_i(x_i). \tag{1}$$

where $u_i$ is the utility function of the agent indexed with $i$, and

$$\lambda = (\lambda_1, \lambda_2, ..., \lambda_n) \in \Delta_+ = \left\{ \lambda \in R_{++}^n : \sum_{i=1}^{n} \lambda_i = 1 \right\}.$$

Each $\lambda_i$ represents the social weight of the agent $i$ in the market.

As it is well known if $x^* \in R^{nl}$ solves the maximization problem of $W_{\lambda*}(x)$ subject to being a feasible allocation i.e.,

$$x^* \in \mathcal{F} = \left\{ x \in R_+^{nl} : \sum_{i=1}^{n} x_i \leq \sum_{i=1}^{n} w_i \right\}$$

then $x^*$ is a Pareto optimal allocation see [Mas-Colell, A. Whinston, M.]. Reciprocally it can be proved that if a feasible allocation is Pareto optimal then there exists a $\lambda^* \in \Delta$

$$\Delta = \left\{ \lambda \in R_+^n : \sum_{i=1}^{n} \lambda_i = 1 \right\},$$

such that $x^*$, maximizes $W_{\lambda*}$.

If we will consider every Pareto optimal allocation we need to consider cases where $\lambda_j = 0$ for some $j \in \{1, 2, ..., n\}$. In these cases the agents indexed in this subset will be out of the market. As utilities are strictly increasing the maximization process implies that this agent will receive $x_j = 0$. Since we consider that each agent has a non-null endowment this allocation can not be an equilibrium allocation. Then we can restrict ourselves, without loss of generality, to consider only cases where $\lambda \in \Delta_+$.

Characterized the set of Pareto optimal allocations, our next step is to choose the elements $x^*$ in the Pareto optimal set such that can be supported by a price $p$, satisfying $px^* = pw_i$ for all $i = 1, 2, ..., n$ i.e., an equilibrium allocation.

Let $\mathcal{E} = \{u_i, w_i\}_{i=1}^{n}$ be an exchange economy, to find the walrasian equilibria we will define the excess utility function.

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**Definition 1** *Let* $e_{i,u_i,w_i} : \Delta_+ \to R, i = 1, 2, ..., n$ *be the function*

$$e_{i,u_i,w_i}(\lambda) = \sum_{j=1}^{l} \frac{\partial u_i(x*(\lambda))}{\partial x_{ij}} \left( x_{ij}^*(\lambda) - w_{ij} \right), \tag{2}$$

*is the excess utility function for the agent* $i$, $u_i$ *is his utility function and* $w_{ij}$ *is the endowment of this agent in the commodity* $j$. *The bundle set* $x_i^*(\lambda)$ *is risen from* $x^*(\lambda)$ *that maximize* $W_\lambda(x)$ *s.t.:* $\mathcal{F}$, *with* $i = 1, 2, ..., n$ *and* $j = 1, 2, ..., l$.

**Definition 2 The excess utility function**, $e_{u_1,u_2,...u_n,w} : \Delta_+ \to R^n$ *is the vector*

$$e_{u,w}(\lambda) = (e_{1,u_1,w_1}(\lambda), e_{2,u_2,w_2}(\lambda), ..., e_{n,u_n,w_n}(\lambda)).$$

Assuming conditions such that the solution of the maximization program involving the welfare social function will be attained in the interior of $\Re_+^l$, it follows that if $x^*(\lambda, w)$ is the allocation that solves this problem, we have that

$$\frac{\partial u_i(x^*(\lambda, w))}{\partial x_{ij}} = \frac{1}{\lambda_i} \gamma_j(\lambda),$$

and then:

$$e_{i,u_i,w_i}(\lambda) = \frac{1}{\lambda_i} \sum_{j=1}^{l} \gamma_j(\lambda) \left( x_{ij}^*(\lambda) - w_{ij} \right). \tag{3}$$

These conditions are satisfied if for instance :

  i)  preferences are strictly monotone and quasiconvex, or else

  ii)  preferences are strictly monotone and quasiconvex in the interior of $R_+^l$, and everything in the interior is preferred to anything on the boundary.

Let $\Omega = \Pi_{i=1}^n \Re_+^l$ be the consumption space.

**Definition 3** *For fixed utility functions* $u$, *and for each* $w \in \Omega$ *one can define the set*

$$\mathcal{E}q(u, w) = \left\{ \lambda \in \Delta_+ : e_{u,w}(\lambda) = 0 \right\},$$

*it is called the set of the* **Equilibrium Social Weights**.

In [Accinelli, E.(99)] it is proved that it is a non-empty set.

**Theorem 1** *Let* $\lambda \in \mathcal{E}q(u, w)$, *and let* $x^*(\lambda)$ *be a feasible allocation, solution of the maximization problem of* $W_\lambda$ *and let* $\gamma(\lambda)$ *be the corresponding vector of Lagrange multipliers. Then, the pair* $(x^*(\lambda), \gamma(\lambda))$ *is a walrasian equilibrium and reciprocally, if* $(p, x)$ *is a walrasian equilibrium then, there exists* $\bar{\lambda} \in \mathcal{E}q$ *such that* $x$ *maximize* $W_{\bar{\lambda}}$ *restricted to the feasible allocations set, and* $p$ *will be the corresponding vector of Lagrange multipliers, i.e.,* $p = \gamma(\bar{\lambda})$).

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The proof can be see in [Accinelli, E.(99)].

The allocation $x(\lambda)$ is an equilibrium allocation if the excess utility function of each agent vanishes, that is $\lambda$ is in the equilibrium set, $\mathcal{E}q(w)$ if and only if it gives rise to an allocation such that the gradient $\nabla u_i(x(\lambda)) = \left( \frac{\partial u_i(x(\lambda))}{\partial x_1}, \frac{\partial u_i(x(\lambda))}{\partial x_2}, \ldots \frac{\partial u_i(x(\lambda))}{\partial x_l} \right)$ of each utility $u_i$ evaluated in $x_i(\lambda)$, is orthogonal to the bundle set $(x_i(\lambda) - w_i)$, for each $i \in I$.

# 3  The Metric Space of the Utility Functions

In this work we will consider as consumption space $X$ only $R_+^l$, and no more general preferences than those that can be represented by utility functions. Then an exchange economy will be a set $\mathcal{E} = \{u_i, w_i, I\}$ where preferences are represented by the utility functions $u_i : R_+^l \to R, \quad \forall i \in I$.

In our work it is indispensable to have available a notion of closeness for utility functions, to make this, following [Mas-Colell, A.], we will consider the topological space of functions $C^r(X)$, where $r \geq 1$ specify that for every $0 \leq s \leq r$ the sth-derivative of $f \in C^r(X)$, is continuous on $X$. The topology considered will be the **topology of the $C^r$ uniform convergence.** By letting $f_n \to f$ if and only if every derivative of $f_n - f$ up to the $rth$ order converges uniformly to zero.

The following properties hold:

a) Every $C^r(X)$, $0 \leq r \leq \infty$, is metrizable, separable and complete.

b) In the following sequence the topologies are increasingly finer and every space is dense in the preceding one:
$$C^\infty(X) \subset \ldots \subset C^2(X) \subset C^1(X) \subset C^0(X).$$

c) If $X$ is not compact, we give to $C^r(X)$ the topology of the $C^r$ uniform convergence on compact, that is $f_n \to f$ if and only if $f_n/Y \to f/Y$ for every compact $Y \subset X$, in the previous sense.

This are well know properties of this space, see for instance [Royden, H.L.].

# 4  Regular and Singular Utilities

As we will focus in utilities and the implications that changes in utilities or tastes have for the economy, (changes in the number of equilibria when utilities change, abruptness of this changes, etc...) then we will follow the Negishi approach, see [Accinelli, E. (96)].

We will say that an utility function is singular for given endowments $w$, if in a neighborhood (considering the compact uniform convergence) of this function there exist utilities such that the respective excess utility functions have different number of zeroes, i.e., different number of walrasian equilibria.

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As the excess utility function satisfies the Walras law, $\lambda e_{u,w}(\lambda) = 0$ and as we can consider $\sum_{i=1}^{n} \lambda_i = 1$ see [Accinelli, E. (96)], then in the case of an economy with two agents, to obtain the set of points $\lambda = (\lambda_1, \lambda_2)$, such that $e_{w,u}(\lambda) = 0$ it is enough to solve only one of the two equation $e_{i,w,u}(\lambda_i) = 0$ as a function of only one variable $\lambda_i$, $i = 1, 2$. If we assume that each agent has a positive endowment, and utilities are increasing, then it is possible to find an value $\epsilon > 0$ such that all $\lambda_i$ in the equilibrium set, verify $\lambda_i \geq \epsilon$. So, in this case the, the problem to solve the equilibrium is equivalent to obtaining the zeroes of the function $e_{i,u,w} : X \to R$, where $X = [\epsilon, 1]$

We need the following definition:

**Definition 4** *Let $f$ be in $C^{\infty}(X, R)$, a function $f$ is a Morse function if all of the critical points of $f$ are non degenerates.*

Recall that $C^{\infty}(X, R)$ denotes the set of real value-functions $f : X \to R$, whose derivatives of any order are continuous.

It is a well known fact that the set of Morse functions are a residual set in $C^{\infty}(X, R)$.
[Golubitsky, M. Guillemin, V.].

We will say that an economy is stable, if the excess utility function is an stable map in the sense of [Golubitsky, M. Guillemin, V.], that is if there is a neighborhood $W_e$ (in the sense of the uniform topology above considered) of $e$ in $C^{\infty}(X, R^{n-1})$ such that all $f \in W_e$ is equivalent to $e$.

We say that $f$ and $e$ are equivalent if there exist diffeomorphisms $g : X \to X$ and $h : R^{n-1} \to R^{n-1}$ such that $f = heg^{-1}$. In this sense if an economy is stable, the graphic representation of her excess utility function will be similar to the excess utilities of the economies in which tastes are similar.

The following theorem help us to characterize the behavior of the economies in a neighborhood of a singular economy, assuming that an economy $\mathcal{E}'$ is in the neighborhood of an economy $\mathcal{E}$, if an only if the excess utility $e'$ of $\mathcal{E}'$, is in a neighborhood of the excess utility $e$, of $\mathcal{E}$.

**Theorem 2** *Let $f$ be in $C^{\infty}(X, R)$ where $X$ is a compact manifold. Then $f$ is stable if and only $f$ is a Morse function the critical values of which are distinct (i.e., if $x_1$ and $x_2$ are distinct critical points of $f$ in $X$, then $f(x_1) \neq f(x_2)$).*

This theorem is proved in [Golubitsky, M. Guillemin, V.].

The example below show a two consumer economy, whose excess utility function has one singular equilibrium and one regular equilibrium. This is a singular economy, i.e., in all neighborhood of its excess utility function there are economies with different number of equilibria.

Recall that in the two consumers case it is enough to consider only one coordinate of the excess utility function. If the excess utility function $e_i : [0, 1] \to R$ of the economy $\mathcal{E}$ is a Morse function, all of whose

critical values are distinct then this excess utility function belong to a residual set in $C^\infty(X, R)$. Moreover, from the theorem above it follows that, this excess utility is a stable function in the sense that, there is a neighborhood $W_e$ of $e_i$ such that all function $e' \in C^\infty(X, R)$ in this neighborhood, will be equivalent to $e$. In spit of the singular equilibrium will disappear if some change in the tastes of the consumer occur, all economy $\mathcal{E}'$ in a neighborhood of $\mathcal{E}$ will have equivalent excess utility function.

If the economy is singular and the excess utility function has more than one singular equilibrium, then there will be more than one critical point $\lambda_i^1 \neq \lambda_i^2$ where $e(\lambda_i^1) = e(\lambda_i^2) = 0$, and then there won't be a stable economy, that means that we can't say anything about the behavior of the economies in a neighborhood of the singular economy.

The stable functions in $C^\infty(X, R)$ have a nice form, since there are just the classical Morse functions. Such functions take on only certain type of singularity, (i.e., have only non-degenerate, critical points). See [Golubitsky, M. Guillemin, V.]. Stable maps are also dense in $C^\infty(X, R)$, this property is also verified for functions in $C^\infty(X, Y)$ if $X$ and $Y$ are manifolds such that $\dim X = \dim Y = 2$, but unfortunately this is not the case if we consider $C^\infty(X, Y)$ for an arbitrary manifold $Y$ see [Accinelli, E.; Puchet, M]. The general answer depend on relative dimensions of $X$ and $Y$. This means that, in some cases there exist one generic economy $\mathcal{E}'$ such that in all neighborhoods $W_{\mathcal{E}'}$ of $\mathcal{E}'$ there exist an economy $\mathcal{E}$, such that has a no equivalent behavior i.e., the respective utility functions are not equivalent. Such possibility depend on the relative dimensions of the consumption space $X$ and the number of agents in the economy.

However if the economy is regular, (i.e.,the excess utility function $e : R^{n-1} \to R^{n-1}$ is an immersion one to one) then $e$ is a stable map.

## 4.1   Examples of Singular and Regular Economies

Consider the following two agents two good economy:

$$u_{\alpha,1}(x_{11}, x_{12}) = x_{11} - \frac{1}{\alpha} x_{12}^{-\alpha}$$

$$u_{\alpha,2}(x_{21}, x_{22}) = -\frac{1}{\alpha} x_{21}^{-\alpha} + x_{22}$$

and the initial endowments: $w_1 = (w_{11}, w_{12})$, $w_2 = (w_{21}, w_{22})$, for consumer 1 and 2 respectively. (This example, with $\alpha = 8$ is given in [Mas-Colell, A. Whinston, M.])

The excess utility function of this economy is:

$$e_{(u_{\alpha,1}, u_{\alpha,2}, w_1, w_2)}(\lambda) = e_{\alpha,w}(\lambda)$$

When $\alpha = 12$ we obtain for $w_1 = (2, r)$, and $w_2 = (r, 2)$ with $r = 2^{\frac{8}{9}} - 2^{\frac{1}{9}}$ an economy with three relative wight of equilibrium, $\lambda = (0.05, 0.95); (0.5, 0.5); (0.95, 0.5)$ that is three different values of $\lambda$ such that $e_{\alpha,w}(\lambda) = 0$.

If we consider $\alpha$ in a $\epsilon$ neighborhood of 12, that is $\alpha \in (12 - \epsilon, \quad 12 + \epsilon))$ the economy has the same behavior as in $\alpha = 12$, that is there is no changes in the structure of the economy, the economy has three walrasian equilibrium if utilities are chosen in this neighborhood. This is showed in figure 1 on the left.

If we assume $\alpha = 5$ we obtain only one equilibrium with $\lambda = (0.5, 0.5)$, and we won't observe significant changes for little changes of this value of $\alpha$.

But if $\alpha^* = 7.76$ and we consider an $\epsilon$ neighborhood of $\alpha^*$ for fixed endowments $w$, then there exist different values of $\alpha$ where the excess utility function $e_{\alpha,w}(\lambda)$ has different number of zeroes. For this value $\alpha^*$ we said that the utility functions are singular, because in a neighborhood of this function there exist utility functions such that the respective excess utility function has different number of zeros, and then the economy is structurally unstable in the sense that a little change in the tastes of one agent, implies a qualitative and quantitative change in the structure of the walrasian equilibrium set. The cardinality of the equilibrium set change from one to three, or reciprocally.
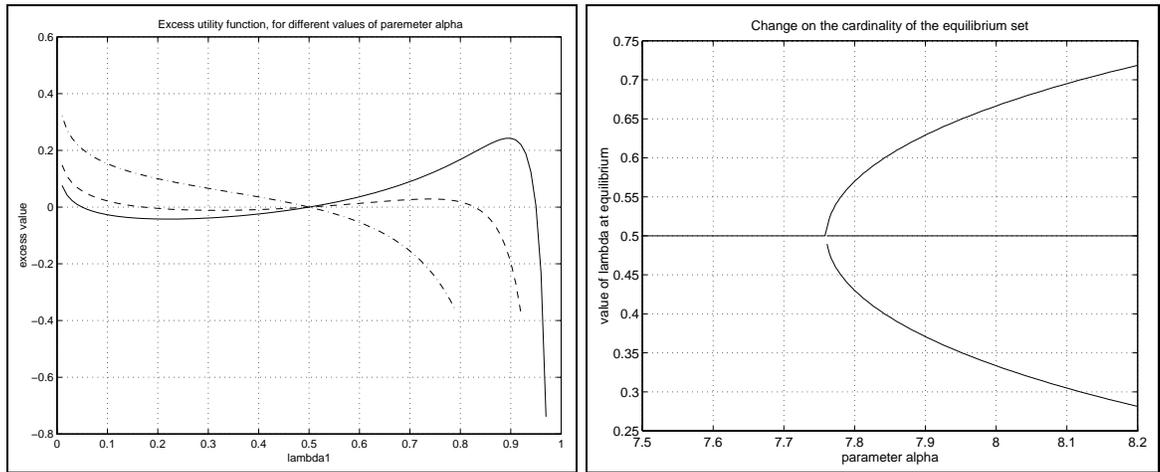


Figure 1: On the left, changes in utilities, given by little ganges in $\alpha$, imply changes in the number of the equilibria. On the right, the bifurcation point, economies with one equilibrium, after $\alpha^*$ become economies with three equilibria.

The first case is an economy with regular utility functions, and in the second case utilities are singular. In the first case, some changes in tastes or mistakes in the consideration of utilities don't imply big differences in the expected behavior of the economy. But in the second case little mistakes in the measurement of the utilities of the agents or little changes in tastes, have implicit unpredictable consequences.

Figure 1 on the right shows changes in the equilibrium set when the parameter $\alpha$ change, the bifurcation point is $\alpha = 7.76$ for smaller values we can observe only one equilibrium $\lambda = (0.5, 0.5)$ beyond this point with observe that two new equilibrium branches appear. If we consider an economy with the same kind of utility functions that in the example, and endowments $w_{11} = 2; w_{12} = 0.75; w_{21} = r; w_{22} = 2$ we obtain the

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possibility of more sudden changes. Figure 2 on the left, show the equilibrium values of this economy, and figure 2 on the right, show the bifurcation point $\alpha^* = 8.81$. If $\alpha \geq \alpha^*$ we observe that the economy has three than $\alpha^*$ two of these branches disappear and then we obtain only one equilibrium value for $\lambda$, and only one equilibrium branch see figure 4. The possibility of a sudden and big changes in the equilibrium values is clear for economies in a neighborhood of this value. As all measure imply mistakes the behavior of this kind of economies is absolutely unpredictable.
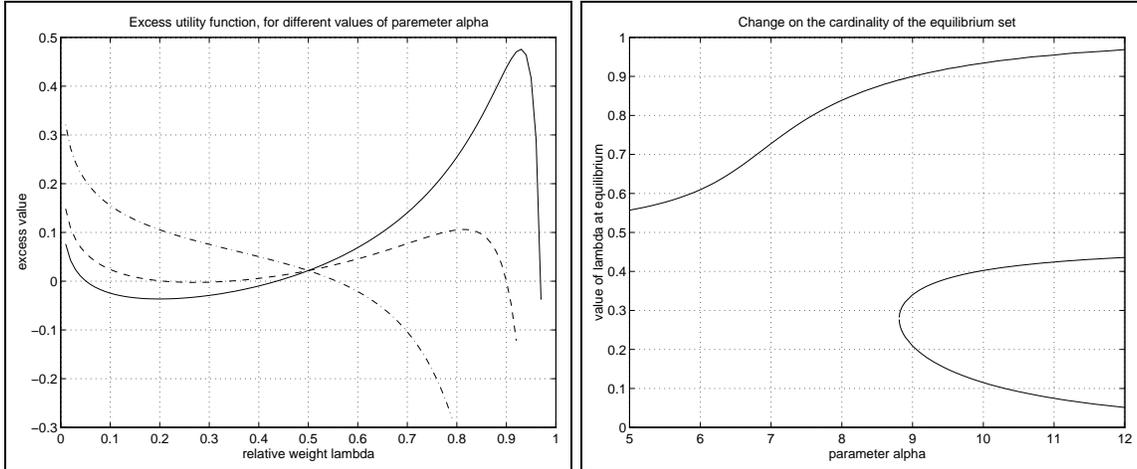


Figure 2: Another possible kind of bifurcation, the figure on the left show a dramatic situation.

The figures 1 and 2 on the left, show a generic situation in the case of a two agent economy. The excess utility function of each agent is a Morse function, these functions are regular and if they have singularities, the critical points have different values and are not degenerates. This means that if the singularities exist, generically there will be only one singular and no degenerate equilibrium, i.e., if utilities and endowment are given, there will be generically only one set of social wright, $\bar{\lambda} = (\bar{\lambda}_1, \bar{\lambda}_2)$ such that $e_{i,\alpha,w}(\bar{\lambda}_i) = 0$ and $\frac{\partial e_{i,\alpha,w}(\bar{\lambda}_i)}{\partial \lambda_i} = 0$. In all neighborhood of this function $e_{i,\alpha,w}$ there exist, excess of utility functions with three regular equilibria and excess of utility functions with only one regular equilibrium corresponding to different economies, and without singular equilibrium. To see this it is enough consider little modifications in the value of the parameter $\alpha$.

As we said above, are better know the properties of the economies in a neighborhood of a singular endowment and fixed utilities. This case is shown in our example when we consider $\alpha = 8$, $w_{21} = r$ but we allow changes in $w_{12}$. Figure 3 show this case, the singularities are economies such that $w_{21} = 0.769$ or $w_{21} = 0.7730$. In all neighborhood of this points we observe changes in the number of equilibria. Now allow us to modify the values of $\alpha$. As figure 3 on the left, show the behavior of the economy change. For values of $\alpha$ lowers than 7.767 the possibility of changes in the number of equilibria disappear and we

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obtain economies with uniqueness of equilibrium for all value os $w_{21}$ and then there in not singularities, nevertheless in a neighborhood of $\alpha = 7.767$ and endowments given, we can observe big changes in the equilibrium set for little changes in utilities, for instance for little changes in the values of $\alpha$. This situation is well know in economics.
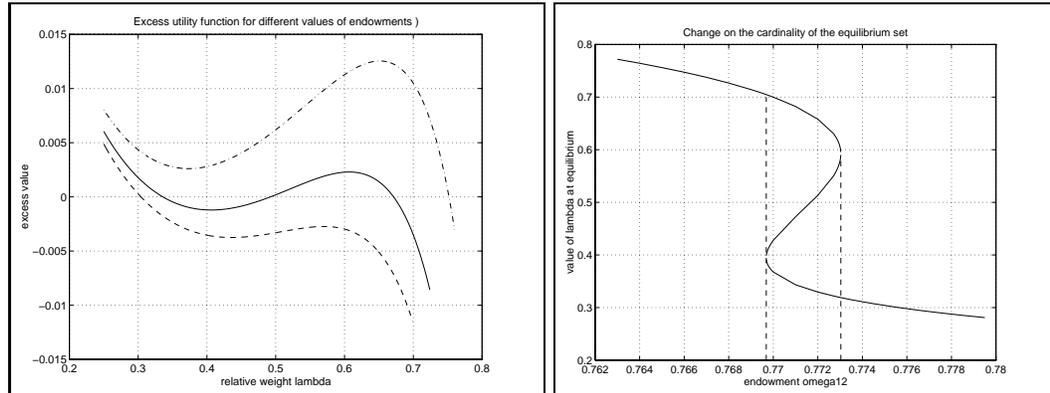


Figure 3: A well know situation, when a singular economy appear, changes in the endowments in a neighborhood of this singularity, give raise to changes in the number of equilibria.

From the point of view of the singularity theory we can consider the function $e$ as a function from the space formed by the cartesian product $\mathcal{U} \times \Omega \times \triangle$ in $R^{n-1}$ and to consider changes in tastes or in endowment, and economy will be singular if she has critical points (singular walrasian equilibria) i.e., if for some $\bar{\lambda} \in \triangle, e_{u,w}(\bar{\lambda}) = 0$ and $rankJe_{u,w}(\bar{\lambda}) < n - 1$. In a neighborhood of these economies, little changes in tastes or endowments may imply big changes in the behavior of the economy, as for instance, changes in the number of walrasian equilibria, and in his geometrical representation. Figure 4, show different kinds of equilibrium set, for economies with different set of utilities, this differences was obtained changing the parameter $\alpha$. The doted curve shown in figure 4, represent the equilibrium set of an economy with a singular equilibrium, it separe regular economies with uniqueness of equilibrium for all value of the endowments, from economies such that the number of equilibria change with the values of the endowments.

# 5   Economic Meaning

The existence of singularities is the support of the irreversibility. Little changes can imply big changes, and after theses changes to come back may be possible only if the society does big efforts.

Nevertheless the knowledge of the kind of the possible singularities in an economy, allow us to characterize the kind of the future unforeseen, in some sense this means to have additional information. Economies with the same possible singularities, will show a similar behavior. There is not big changes in the neighborhood of the regular economies, significative changes occur in a neighborhood of a singularity, and this
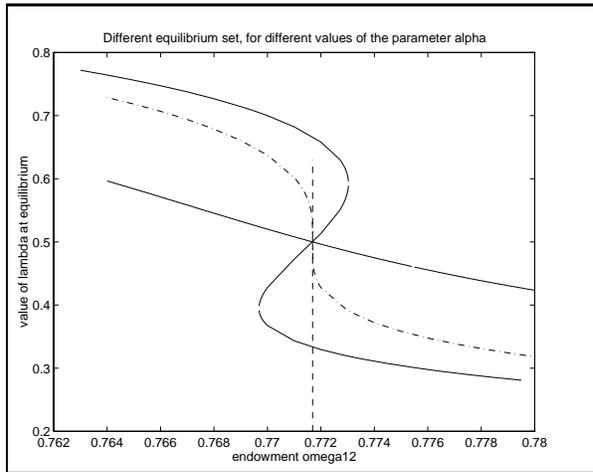
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Figure 4: Variations in the values of the parameter $\alpha$ give raise to economies with qualitatively different equilibrium set

kind of changes characterize the economy.

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