Consensus and equilibria in the presence of self-interest and conformity in social groups*

Brida, Juan Gabriel [†] Alvarez, Emiliano [‡]

Abstract

This paper analyses the decision-making processes of heterogeneous agents, when both individual preferences and group actions are taken into account. Under the assumptions of certain mechanisms of interactions and assuming cognitive and informational restrictions, an agent-based model is introduced to analyze the evolution of decisions over time.

The results of the simulations show how social pressures can determine the relationship between individual and social preferences. Societies whose agents have strong individual preferences have outcomes with fragmentation processes that generates a higher number of groups. A minor importance of the individual preferences, results in a smaller proportion of individuals maximizing their individual utility. As well, the quantity of options available and the initial proportion of each alternative are significant variables to determine the proportion of individuals selecting a particular option.

In addition, the study analyses the existence of equilibria of the dynamical system. To this aim, the notion of metastable equilibria is introduced and linked to the dynamic analysis, showing the existence of one or more stable/metastable states depending on the parameters of the model.

Keywords Agent-based models · Opinion dynamics · Social preference

JEL Classification B4 · C6 · C7 · D7 · D8

*Trabajo presentado en las XVI Jornadas de Investigación de la Facultad de Ciencias Sociales-UdelaR. Montevideo, 13, 14 y 15 de setiembre de 2017

†Grupo de investigación en Dinámica Económica, Departamento de Métodos Cuantitativos, Facultad de Ciencias Económicas y de Administración, Universidad de la República, Uruguay. Email: gbrida@ccee.edu.uy

‡Grupo de investigación en Dinámica Económica, Departamento de Métodos Cuantitativos, Facultad de Ciencias Económicas y de Administración, Universidad de la República, Uruguay. Email: ealvarez@ccee.edu.uy

1 Introduction

A main aspect in opinion formation is to study the dynamics of this process. In particular, this study aims to know how this process develops when individuals obtain information from choices made by their closest neighbors. For this purpose, it is important to formalize the relationship between individual and social preferences.

Conformity is linked to those decisions taken by agents that depend only on the observable characteristics -or decisions- of the rest of individuals. This concept, studied in depth from the pioneer experiments of Asch (1951), has been extensively analyzed experimentally. The degree of conformity is related to the degree of influence that other individuals have on agents' decisions, as shown by Bond and Smith (1996) and Hamamura (2012), between others.

The literature has focused on studying the consequences that social preferences have for social welfare. In Banerjee (1992) and Bikhchandani et al. (1992) find that the solutions may not be the socially desired. Other authors study how social pressure change agents motivations, which generates changes in their preferences (see Maness and Cirillo (2016) and Vanhée et al. (2015)).

The inclusion of social preferences leads us to study the trade-off between social and individual preferences. It may happen that local interactions allows to self-organization (see Kirman (1992), Arthur et al. (1997) and Page (2001)). The opinion formation phenomenon and the appearance of consensus and fragmentation processes are the main results in this field. See Krause (2000), Hegselmann and Krause (2002), San Miguel et al. (2005), Watts and Dodds (2007), Acemoglu and Ozdaglar (2011), Xie et al. (2016) and Sîrbu et al. (2017) for review studies in this topic.

Schelling (1971) shows that the level of tolerance of the individuals has consequences in the result. Tolerance is posed as the permissible level of individuals in the environment of different type, before modifying behavior. In addition, Schelling (1971) shows that lower tolerance leads to greater segregation.

Granovetter (1978) introduces the notion of threshold in this framework as the level from which the system changes its behavior. The model in this paper introduces agents with different thresholds, according to their decision rules. The individual threshold depends on the preferred choice and on the relationship between individual and social preferences. When individuals choose their preferred option, they have a greater tolerance to different crowding types (or options), compared with the case where individuals do not choose their preferred type. The relationship between social and individual preferences also influences the threshold, since a greater weight of individual preferences increases the level of tolerance to external pressures.

This paper introduces a dynamic version of Conley and Wooders (2001) where the solutions to the model are based on different assumptions about the behavior of the agents and their interactions. By assuming that the agents have intrinsic preferences over the actions to be taken, the study shows the mechanism by which a choice of an agent determine the actions of other individuals.

The paper contributes to the literature by introducing a different approach to the topic of intertemporal change in decision-making processes, including differentiated individuals, cognitive and processing constraints, and decisions that change over time according to the opinion spreading mechanism selected.

The simulations of the model show that coalitions between individuals in clusters arise from the iterative decision-making process of agents, based on their individual preferences (unobservable characteristics) and actions taken by other individuals (observable characteristics). Both observable and unobservable characteristics are the fundamental ingredients of the static models presented in Conley and Wooders (2001) and Brida et al. (2011). The present study introduces a dynamic framework for these models.

The performed simuations seem to show that generically the model converges towards a stable situation. To analyze this point, the concept of metastable equilibrium is introduced showing that -for some of the values of the parameters- the model converges to a stable equilibrium and for others, to a metastable equilibrium.

The research also generalizes -by introducing intrinsic preferences- the dynamic models in Castellano et al. (2009) and Axelrod (1997). This modification allows observing processes of local convergence, fragmentation and polarization when the agents decide on the basis of a single factor. Then, fragmentation processes are achieved from simpler and more realistic heuristics.

In addition, the proposed model allows to replicate the basic ideas of Thaler and Sunstein (2008). These authors expressed that it would be desirable to use as public policies, small mutations or "nudges" that cause coordinated actions. The simulation results presented in the present study shows that the conjunction of small mutations and the interaction process allow a higher frequency of coordinated actions.

This paper is organized as follows. Section 2 introduces the model, following the ODD Protocol (Grimm et al., 2006). This methodology allows to introduce the experimental design and working hypotheses. Section 3 describes the empirical results, exposing and discussing the simulations. Finally, Section 4 presents the conclusions, limitations of the study and future research.

2 Theoretical model and methodology

Economic theory in particular and the social sciences in general have benefited from the broader range of aspects that allow for agent-based models, which can not be formulated from other approaches. However, the way in which agent's behavior is designed is fundamental. Small variations can cause, given the nonlinearities of the model, very different effects. Then, the different processes involved in these models must be explained. The advances arising from the ODD Protocol (see Grimm et al. (2006), Grimm et al. (2010), Grimm et al. (2013)) go in this direction, providing an appropriate framework for describing agent-based models for purposes of scientific dissemination. This protocol was originally introduced in Ecological modelling (Grimm et al., 2006), but its use in the social sciences has been increasing in recent times (Grimm and Railsback, 2012).

2.1 Model description and documentation

In this subsection the characteristics and design of the model is introduced, following the next points:

1. Purpose: the main purpose of this model is to analyze the dynamics of decision processes of individuals and to understand how the groups of individuals are distributed and evolve over time. The model is based on certain assumptions about the behavior of the agents and topological characteristics of the configuration. It is sought to know if different types of behavior, understood as differences in the valuation of individual and social preferences, give rise to different aggregate behaviors, in particular to the emergence of consensuses or the formation of clusters.

2. Entities, state variables, and scales: The model consists of agents, which are located in cells of a two-dimensional grid. The individuals interact with other agents by means of the public information they provide through their crowding type c ($c = 1, ..., C$). In this context, there are C different crowding types.

3. Process overview and scheduling: In each period of time, each agent interacts with its reference group -a Von Neumann or Moore neighborhood with radius 1-. Each agent choose a type based on: i) its preferences on crowding types, ii) the type selected in the previous period and iii) the type selected by the agents of its surroundings. The decision rules determine the proportions needed of each type in the neighborhood to change or maintain the crowding type.

4. Design concepts:

   - Basic principles: This model seeks to study the dynamics resulting from the process of interaction between individuals. Following Ajzen (1985), Conley and Wooders (2001) and Brida et al. (2011), for each agent, the utility function is constructed from individual and social preferences, and represents the different rules of behavior.
   - Emergence: The distribution of crowding types in the population is an emergent feature of the system. It originates from the process of interactions and can lead to consensus (global convergence), defined clusters (local convergence and global polarization; see Axelrod (1997)) or without defined patterns (global and local fragmentation).
   - Adaptation: In order to decide the crowding type, the agents take into account which of the alternatives generates a greater utility, considering three ingredients: i) the prefered crowding type, tii) he crowding type selected in the previous period and iii) the crowding type of its surroundings.
   - Objectives: The utility function of these agents can be written as:

     $$(1) \qquad u_{i,t}(c,\tau,m) = \frac{a}{C} * f_i(c) + b * m_{c,t} - d * m_{c',t}$$

     where:

     - $f(c)$ are individual preferences, exogenous to the model, with $f(c) > 0$ if $c = \tau$ (preferred crowding type) and $f(c) = 0$ in other cases.
     - $m_{c,t}$ is the proportion of agents with crowding type c in the period t and $m_{c',t}$ is equal to the proportion of agents with a crowding type other than c in t, *in the neighborhood of i*.
     - C is the number of crowding types available.
     - a, b and d are the parameters that will allow us to study other behaviors from the initial model, with $a = b = d = 1$ (Table 1). In particular, we find:
       * maximizer behavior: $a = 1$, $b = 1$, $d = 1$,

         $$(2) \qquad u_{i,t}(c,\tau,m) = \frac{1}{C} * f_i(c) + m_{c,t} - m_{c',t}$$

       * conformist behavior: $a = 0$, $b = 1$, $d = 1$,

         $$(3) \qquad u_{i,t}(c,\tau,m) = m_{c,t} - m_{c',t}$$

       * loss aversion behavior: $a = 1$, $b = 1$, $d = 2$,

         $$(4) \qquad \frac{a}{C} * f_i(c) + m_{c,t} - 2 * m_{c',t}$$

       * exclusive -or snob- behavior: $a = 1$, $b = -1$, $d = -1$,

         $$(5) \qquad u_{i,t}(c,\tau,m) = \frac{1}{C} * f_i(c) - m_{c,t} + m_{c',t}$$

    * intrinsic behavior: $a = 1$, $b = 0$, $d = 0$,

    $$(6) \quad u_{i,t}(c, \tau, m) = \frac{1}{C} * f_i(c),$$

    where τ is its preferred type.

  From this characterization, the agents decide which of the alternatives allows them to obtain a greater utility.

- Learning: Each agent draw on information from the previous period and can change their decision, but do not change their decision rules based on this new information.
- Prediction: When agents make their decisions, they use all available past information. They do not consider the future actions of the rest of agents.
- Sensing: Agents know only their preferences about crowding types, their decision regarding crowding type in the previous period and the decisions of the agents in their neighborhood. Based solely on that information, the agents construct their decisions.
- Interaction: Interactions occur from available public information about the crowding type selected in each time period.
- Stochasticity: In this model, the crowding type preferred by each agent and the crowding type selected in the initial period are randomly distributed. As a consequence, in $\frac{C-1}{C}$ cases, the crowding type is different from the preferred crowding type in $t = 0$. Also when more than one crowding type generates the maximum level of utility among the available options, agents choose randomly between them. Note that there is also randomness when studying the model under the presence of mutations.
- Collectives: Are an emergent feature of the system. These are derived from the process of interactions, which lead to the formation of clusters and consensuses.
- Observation: From the computational model, the following information is obtained for each time period:
  - Number of groups formed, discriminated between those one individual and those with several agents.
  - Proportion of agents that opted for each crowding type.
  - Proportion of agents who selected their preferred crowding type.
  - Proportion of agents who change their crowding type, compared to the previous period.

5. Inicialization: The simulations are performed from a grid of 35 x 35 agents, in Netlogo software (Wilensky, 1999). As discussed above, the initial crowding type and their preferred crowding type are generated randomly in each simulation. As a result, the initial conditions vary between simulations.

6. Input data: The model does not use external data sources to represent the interaction processes between agents.

7. Sub models: The parameters involved the model are described in Table 1. Following Oremland and Laubenbacher (2014), to determine the total population of agents and the number of time periods, data average and dynamical changes in the configuration are examined to decide an increase in the number of runs. The model pseudo-code can be found in Appendix I.

Table 1: Model parameters

Parameter	Description	Value
N	total of agents	35 x 35 = 1225
t	time periods	800
a	individual utility parameter	0 when conformist, 1 in o/c
b	social utility parameter	-1 when exclusive, 1 in o/c
d	antisocial utility parameter	-1 when exclusive, 2 when loss aversion, 1 in o/c
c	crowding type	$c \in \{1, ..., C\} = \mathcal{C}$.
p	mutation rate	$p = 0$ o $p = 0.015$

We perform 100 replicates of each simulation, by using:

- three decision rules (maximizer, conformist and loss aversion),
- two values of p (0 & 0.015), as in Breukelaar and Bäck (2005),
- two, three or four crowding types,
- two sets of neighborhoods (Moore and Von Neumann).

3 Numerical results

In this section we describe the main results of the study, using the model parameters for the discussion.

3.1 Convergence

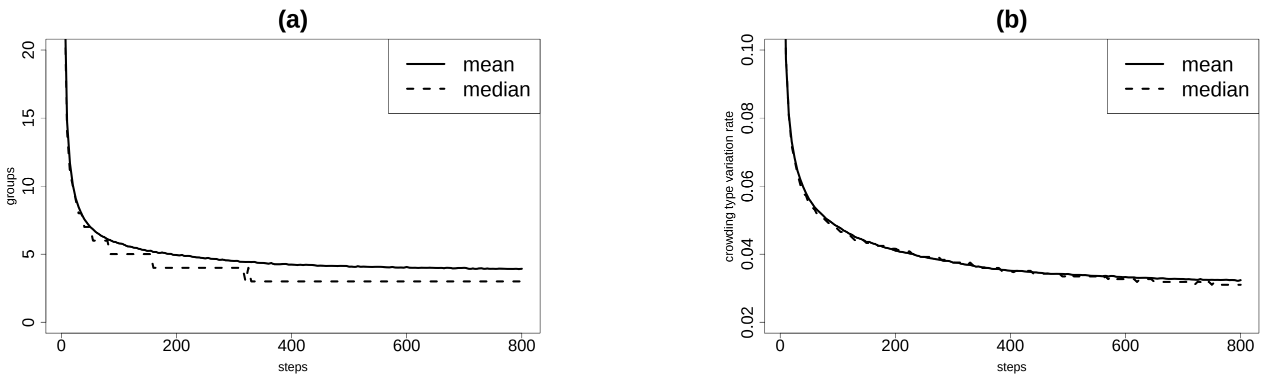

Figure 1: Convergence of the model from central tendency measures (a): Decrease in groups quantity. (b): decrease of the crowding type variation rate.

Note that one indicator of convergence of the model is a decrease in crowding types variation. Figure 1 presents the graphical representation of the simulation results, comparing number and rates of variation of groups from $t = 0$ to $t = 800$. Figure 1(a) shows a decrease in the quantity of groups, both in mean and median. After a small number of runs, the quantity of groups stabilize in five. Figure 1(b) shows a decrease in the crowding type variation rate, converging at the end of the periods to values below 4 %. Then the system tends to a state with fewer groups and crowding type variations.

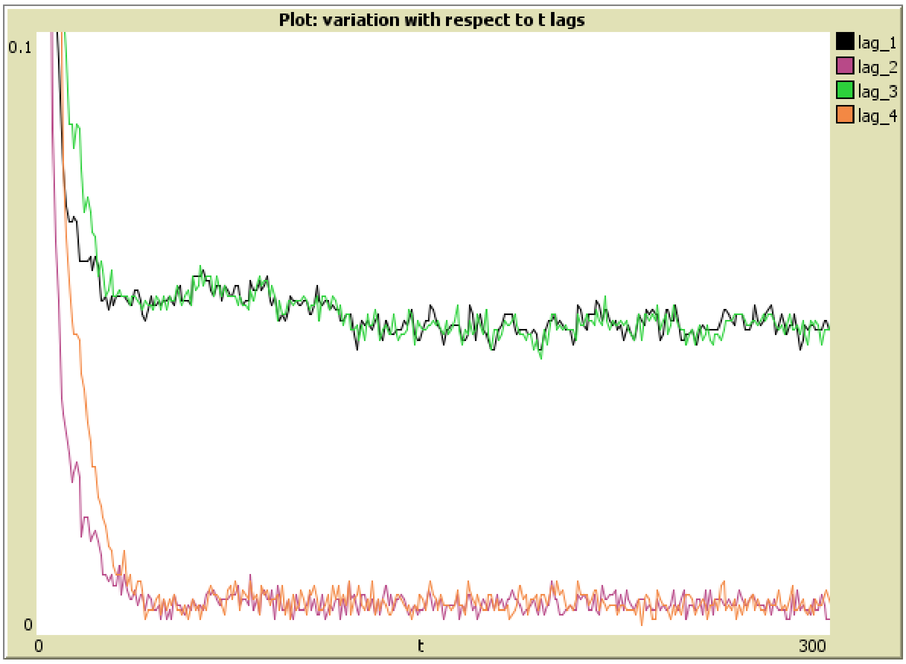

Figure 2: Crowding type variation rate, with respect to different lags. The exercise is constructed using $\mathcal{C}$=3, maximizer behavior, and a time period from $t = 0$ to $t = 300$.

Figure 2 shows a decrease in the crowding type variation rate, confirming the facts illustrated in Figure 1(b). In this case, the deviation with respect to the previous period is around 5%. This deviation occur frequently in the edge of the groups, because several different options influence agent's behavior. Note that when the number of different lags is higher than one and if randomization prevails, the crowding type variation rate with respect to *t lags* would be the same.

Figure 2 shows that there is a number of agents who remains in a stable state and other agents who cyclically changes his state -spin- between the different options. This indicates the emergence of a metastable equilibrium. Comparing the crowding type variation rate at different lags, a 5% of agents vary between two crowding types. The system fluctuates between two different states, with variations in the edges of the groups.

Kelso (1995) describes metastability as a state when the fixed points have disappeared, but visits to the remnants of those fixed points are inherent to the system. (Kelso, 2009, p. 1538) defines a significant aspect of metastability, as follows: "... is the simultaneous realization of two competing tendencies: the tendency of the components to couple together and the tendency of the components to express their intrinsic independent behavior." A metastable result is suggested by Balenzuela et al. (2015) in a voting scheme, under certain circumstances. In a two-dimensional social network grid, this is one of the outcomes of this study.

Cohesion emerges from the formation of consensus, i.e., an agreement between the people of a community. Thus the dynamic process leads to crowding types shared by the whole community and crowding types that are no longer chosen. In terms of market share, the crowding type share by the entire community has a 100% market share and the options not chosen have a 0% market share.

Figure 3 shows the average market share of each crowding type, from initial conditions -when all the options have similar market share- to final conditions, when much of the simulations arrive to a consensus -with market share equal to 0% or 100%- and several simulations have a fragmentation and polarization process -with market share different to 0% or 100%-.

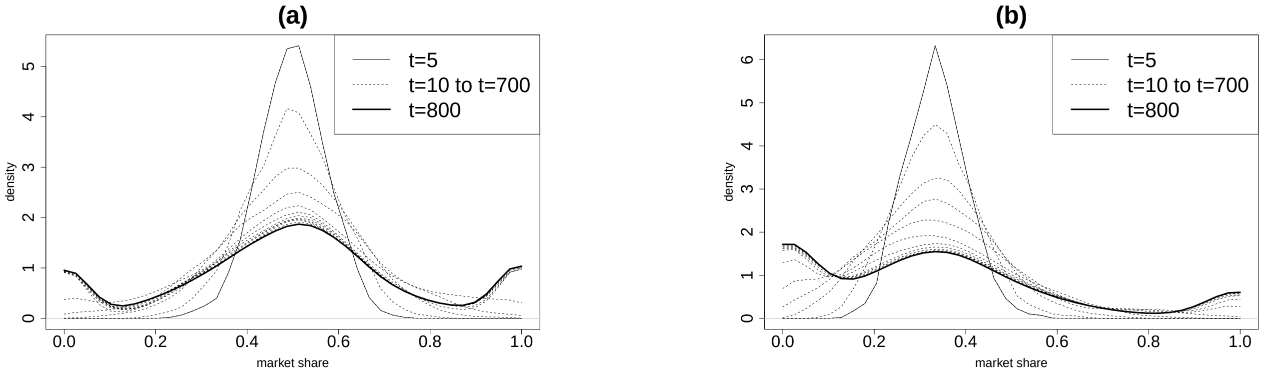

Figure 3: Opinion dynamics (market share of each crowding type) from $t = 5$ to $t = 800$, estimating the model with (a): two crowding types; (b): three crowding types

The figure shows that at the beginning of the time period, there is a balance between the different options (represented as a continuous line) and a predominance of some options over others (at $t = 800$, in bold). This is a consequence of the interaction process between individuals, that also depends on the initial conditions of the system. The behavior coincides when using two options (Figure 3(a)) or three options (Figure 3(b)).

3.2 Random decisions and consensus

Figure 4 shows the different performance of the model with and without mutations. In the case of the model in the presence of mutations, the dynamical process of interactions significantly modifies the quantity of agents who choose their preferred option (Figure 4(b)), the number of groups (Figure 4(c)) and the consensus is more frequent (Figure 4(d)). Note that the crowding type variation rate converges to the same level at $t = 800$ (Figure 4(a)) for the cases of $p = 0$ or $p = 0.015$.

Note that the random choice of crowding types, implies that options who are selected that may not be individually preferred, could be socially desirable. Due to mutation the results with different behavioral rules are similar -but different to results without mutation-, as can be seen in Figure 4(c).

3.3 Opinion dynamics and decision rules

This section analyzes how the behavioral type exhibited by individuals (maximizer, conformist or loss aversion) affect the solutions of the model, both in the speed of convergence and in the degree of cohesion reached under each behavioral type.

The "conformist" behavioral type shows individuals only concerned about their social preferences (Eq. 3), as the individual utility parameter is zero. As mentioned earlier, this behavior has been analyzed extensively -see Asch (1951) and others for experimental results and Banerjee (1992) and others for theoretical results in economics, both in dynamic environment-. The agent observes the options taken by their neighbors and then, opt for the most popular; in the case that two options have the same demand, the agent sort at random.

The "maximizer" behavior seeks to emulate the individual?s characteristics in the Conley and Wooders (2001) model in a dynamic environment (Eq. 2). In this case, each agent knows the actions that individuals selected the previous period in their neighborhood and his preferred

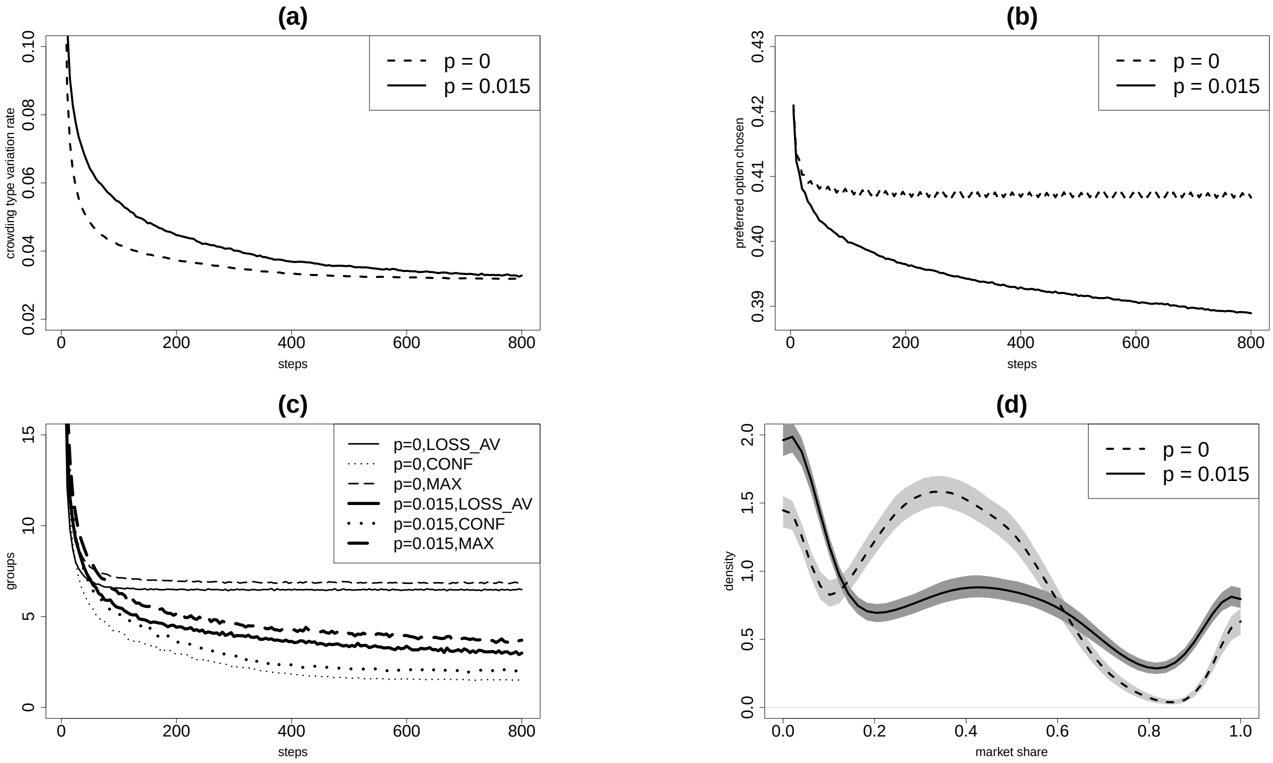

Figure 4: (a): variation rate of the crowding type. (b): Rate of individuals selecting their preferred crowding type. (c): Number of groups, according to the decision rules. (d): Opinion dynamics (market share of each crowding type), evaluated in $t = 800$.

option. Note that this behavioral type is the one that gives higher relevance to the individual preferences, as the individual parameter a is equal to the social parameters b and c.

By the contrary, the "loss aversion" behavior ponders differently the number of individuals in the surroundings that have the same crowding type than those having a different crowding type (Eq. 4). This can lead to higher variation in the actions of individuals over time and could allow to the emergence of cohesion.

For each simulation, in the initial period, the crowding types are randomly distributed (approximately 25 % of each type when there are four types, one third of each type when there are three types and 50 % of each type when there are two crowding types).

Figure 5 shows the distribution of the chosen crowding types, discriminated by behavior and quantity of crowding types. The 95 % confidence intervals is generated by bootstrap.

Figure 5(a) shows that in simulations with maximizing or loss-averse behaviors, no unanimity is generated after 800 periods -unanimity seen as a market share equal or close to one for dominant crowding types and market share equal or close to zero for dominated crowding types-. By the contrary, the conformist behavior has a bi-modal distribution: more than half of the simulations shows cohesion and the rest shows a fragmentation process, with market share close to the mean. Figures 5(b) and 5(c) shows the same results for $C = \{3, 4\}$. Note that under a conformist decision rule, individuals tend to be more cohesive, with a higher frequency of consensus over other types of behavior. This result is coherent with intuition, given that the utility function of these individuals depends solely on the actions taken by others in their surroundings, with respect to other behavioral types that also depend on individual preferences. The mode of the conformist decision rule is at the extremes, in contrast to other behavioral types. The distribution function of the other behavioral types concentrates in values close to the mean, showing also a behavior that is coherent with the initial intuition of the experiment.

Figures 7, 8 and 9 in Appendix II show the dissimilar behavior of the system under different

Table 2: Kolmogorov-Smirnov and Anderson-Darling statistic tests. Comparison of distributions between maximizer and loss-aversion behavior types.

Figure	K-S Test	A-D Test
Fig.5 (a)	0.095*	6.771***
Fig.5 (b)	0.167***	18.93***
Fig.5 (c)	0.08	2.091*
* Significant at 10%;	** significant at 5%;	*** significant at 1%.

crowding types for different periods and number of available crowding types, showing also the correlation between the initial proportion of crowding types and the final proportion of individuals choosing and option.

We can establish that different decision rules lead to different results, if the distribution function of each decision rule is significantly different from the rest. Table 2, shows the results of testing the null hypothesis that maximizers and loss-averse agents arose from the same distribution function. The Kolmogorov-Smirnov and Anderson-Darling tests indicate that we can reject the null hypothesis when there are two or three crowding types (Figure 5(a),(b)), and we can not reject it in presence of four crowding types (Figure 5(c)).

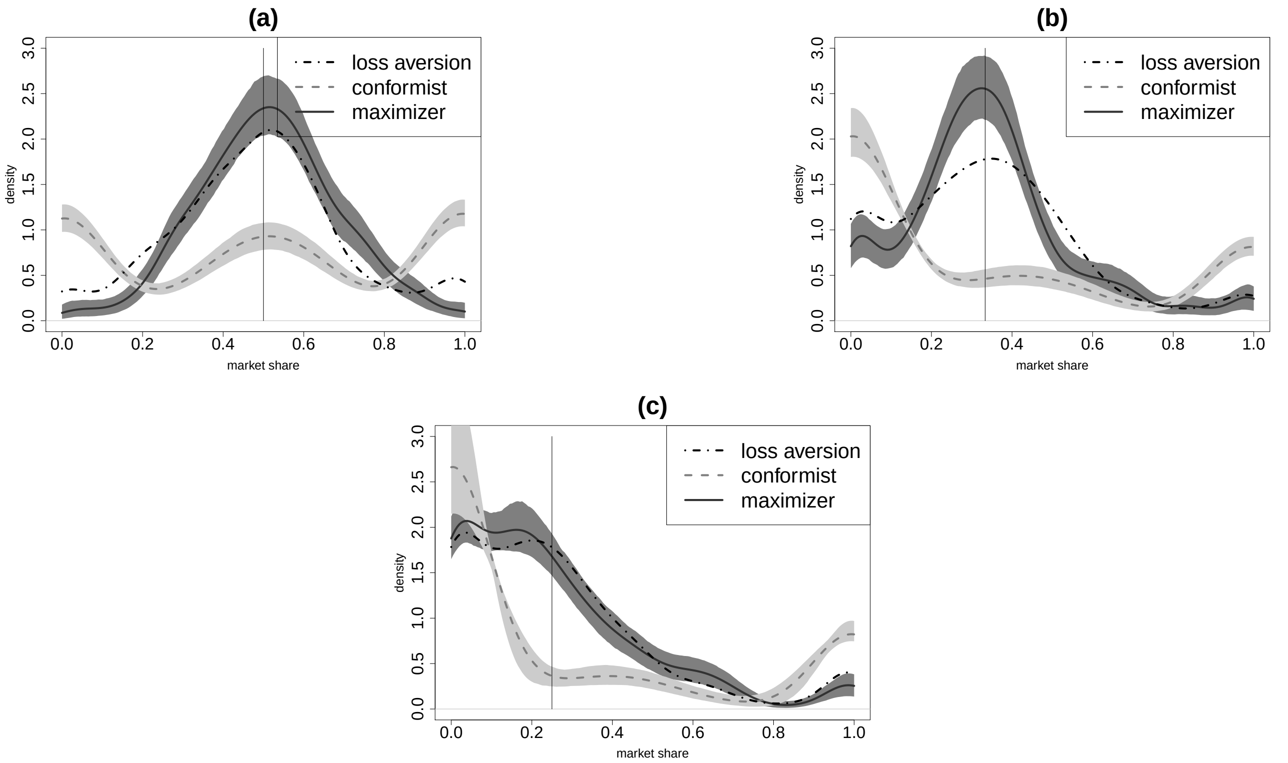

Figure 5: Frequency of each crowding type at $t = 800$, according to behavior type and number of crowding types (a): two options, (b): three options, (c): four options.

Figure 6 represents the differences between decision rules, when a dynamic environment is simulated. Figure 6(a) shows the crowding type variation rate by period, according to the different decision rules. As we mentioned before, the values ??reached (less than $\varepsilon = 5\%$, for different behaviors from t = 200 onwards) and their trajectory seems to ahow the existence of a dynamic equilibrium. However, the way in which actions evolve is different according to the decision rule: the conformist heuristic stabilizes at a variation rate of 2%, while the maximizer

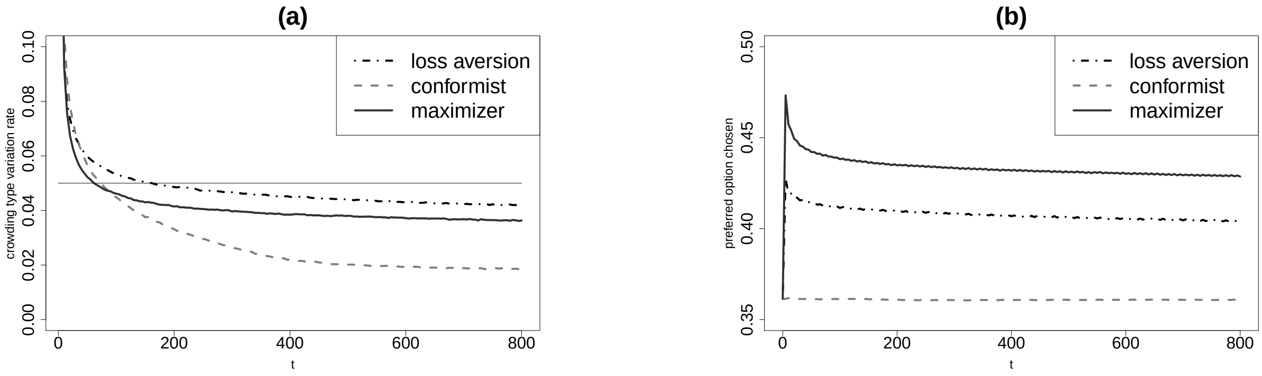

Figure 6: (a): crowding type variation by period, according to the different decision rules. (b): Percentage of population with individual preferences equal to their choice, according to the different decision rules.

rule and the loss-averse rule stabilize at 4%.

Figure 6(b), shows the proportion of individuals choosing their preferred option, for each moment t. As expected, the results show that the maximizer behavior is the one that gives greater importance to the individual preferences, whereas the conformist behavior does not weigh it. Between both extremes we have the loss-averse behavior, which gives a greater weight to the losses caused by belonging to a different crowding type compared to the neighborhood.

Note also that the conformist behavior quickly converges to its reference value. In the other cases, two phases happen: the first, dominated by individual preferences, while the second stage is determined by the pressure exerted by the environment to change the chosen option. In the first phase, the percentage of population who choose their preferred option grows; after that, social pressure produces a decay on this phenomena.

As a final comment, beggining from a simple set of rules, the model reveals a complex set of interactions and an emergent group behavior. The set of rules concerns only the relation between an agent and his neighbors and between individual, social and antisocial preferences.

4 Conclusions

4.1 Main results and discussion

The formulated model and the decision rules evaluated allow to reach different forms of equilibrium. The results show that these equilibrium are characterized by the grouping of individuals into clusters, the amount of which depends on the decision rules adopted, the number of options available, the mutation probability and the initial proportion of each option. Also, simulations show that these equilibrium can be stable or metastable.

Under a strong influence of social pressures, individuals organize themselves in a smaller number of groups and manage to converge to a lower crowding type variation rate. As a counterpart, individuals less frequently select their preferred option, as we can find in Wooders et al. (2006), Xu et al. (2015) and Beran et al. (2015).

Cohesion is common when individual preferences are not taken into account; we may even find that different weights to individual and social preferences yield different results. We can find differences between societies, according to the degree of individualism or intrinsic collectivism. This work then allows us to study the behavior of societies, in the sense of Bond and Smith (1996) and Hamamura (2012).

Different weights to individual and social preferences lead to different outcomes. With a predominance of individualism, agents execute their preferred actions more frequently compared with a predominance of conformism.

The loss aversion behavior has a different treatment to the gains derived from the individuals of its surroundings who choose the same crowding type, with respect to the utility losses generated by choosing a different option to its surroundings. As a consequence, these individuals will obtain a greater utility from the cohesion.

This study analyses how the results of the original computational model are modified, when we allow crowding type's mutations. In this case, a higher amount of consensus is reached and fewer groups are made, with fewer individuals choosing their preferred option. The effect is similar to conformity, although in this case the process begins with a random mutation and then is reinforced with the interaction process. However, it was observed that this effect is more important when the decision rules have an individual preference component. This can be explained by the fact that individual preferences act as a restriction, the default option in the sense of Thaler and Sunstein (2008).

Due to mutation, individuals acquire a new state, which they could not previously reach. This model allows us to understand how socially undesirable options interact over time, the possibilities of introducing a mutation in the default option, and the consequences it has on aggregate behavior. Together with decision rules and the possibility of mutations, result show that the quantity of options available and the initial proportion of the crowding type are significant variables to determine the proportion of individuals choosing a particular option. Then, the conclusions of Clark and Polborn (2006), Brida et al. (2010) and Brida et al. (2011) are fulfilled in a dynamic context[1]. In economics, it is important to understand that the initial distribution is significant, even in the case where individuals do not know the initial distribution. The model predicts that the initial market share of a given product influences its final market share, because individuals obtain a greater utility of consuming the same product as other agents, even though agents do not know the market share of the product.

4.2 Final considerations and recomendations

The simulations performed in this paper show that after an initial number of periods, larger groups emerge from the dynamical process. These groups allow to reach the necessary threshold to move from the individually-preferred option (outside the group) to the non-individually-preferred option (within the group). The decrease in the proportion of individuals choosing their preferred crowding type is an emergent behavior of the group.

The model proposed in this paper serves as a nexus between models with ordinal individual preferences and models where the actions of individuals depend only on the actions of the neighborhood. Hence, it is a generalization of several models in opinion formation dynamics and allows the comparison between the results of those.

This model supports multiple extensions. One possible improvement is to study other decision rules, allowing individuals to interact with different heuristics. In particular, one can introduce influential members between the population of agents, (Mäs and Flache (2013); Xu et al. (2015)), "fundamentalist" or "intrinsic" members who are not influenced by the rest of the society; or members who are contrary to the opinion of the majority (Galam (2004); Borghesi and Galam (2006); Makarewicz (2016)).

An additional extension of the model is to introduce other types of interactions as those in the case of "small worlds" (Watts (1999); Wang and Shang (2015)).

Finally, another direction of future research can include the introduction of these simple decision rules to study macroeconomic phenomena such as inflation (Salle et al. (2013)), the

[1]As in (Nowak et al., 1990, p. 370), the result of this type of model may not be a consensus, although the result of the interaction process is the conformation of a smaller number of groups and a dependence of the initial proportion on the final proportion.

occurrence of crisis (Heymann et al. (2004)) or investment, saving, education and consumption decisions.

Appendix I: pseudo-code

;; in the first period, the selected and preferred crowding type is randomly chosen.
set c-type (random)
set t-type (random)

;; now, we count the agents in the neighborhood who have the same crowding type or different crowding type than the agent.
for c in (1:C)
mc = (count neighbors with $c - type = c$**) ; same type**
mc' = (count neighbors with $c - type$ **!=** c**) ; different type**
for c in (1:C)
$u_c = \frac{a}{C}$ *** f(c-type) + b * mc - d * mc'**
;; then, each agent choose an option according to the comparison between the utility gains that give the different alternatives: **c-type = max (**u_c**), con** $c \in \{1, ..., C\} = \mathcal{C}$
;; finally, the system counts again the number of neighbors with each crowding type and calculates the alternative utilities. This process is iterative.

Appendix II: Convergence. Comparison of different decision rules.

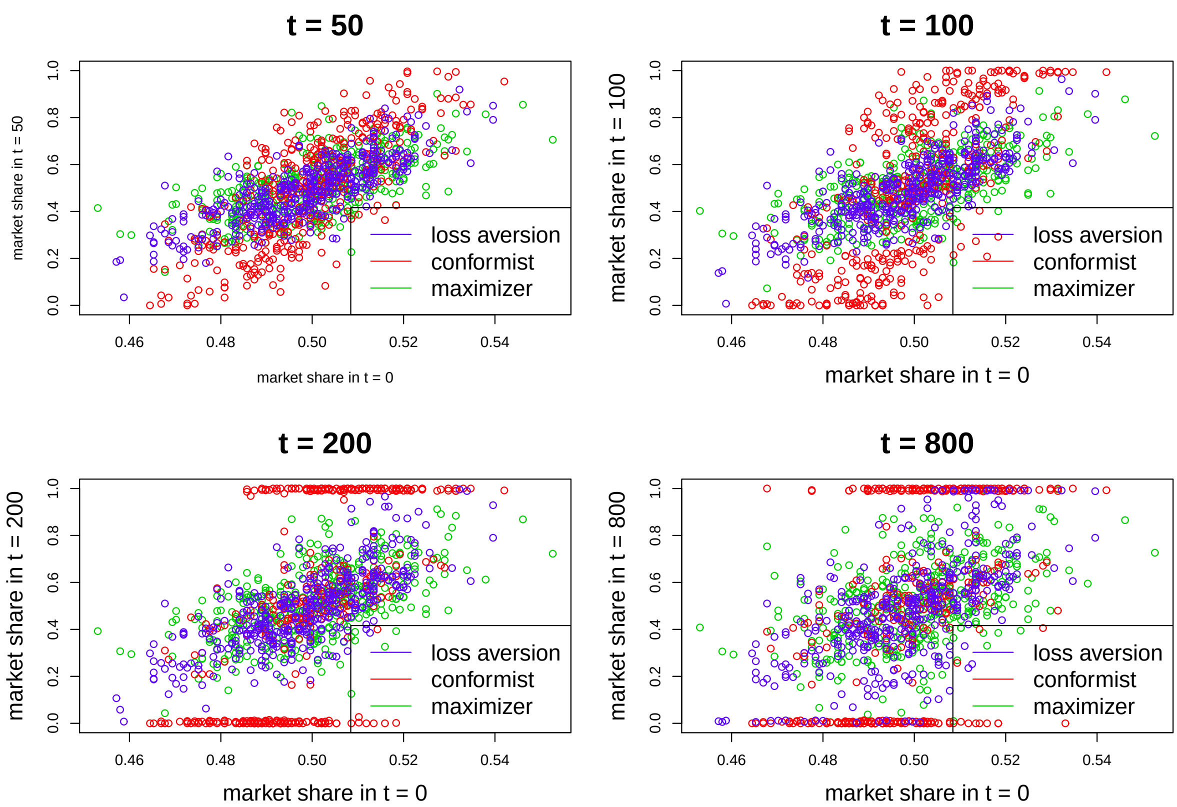

Figure 7: Convergence: market share with $\mathcal{C} = 2$, comparing to $t = 0$

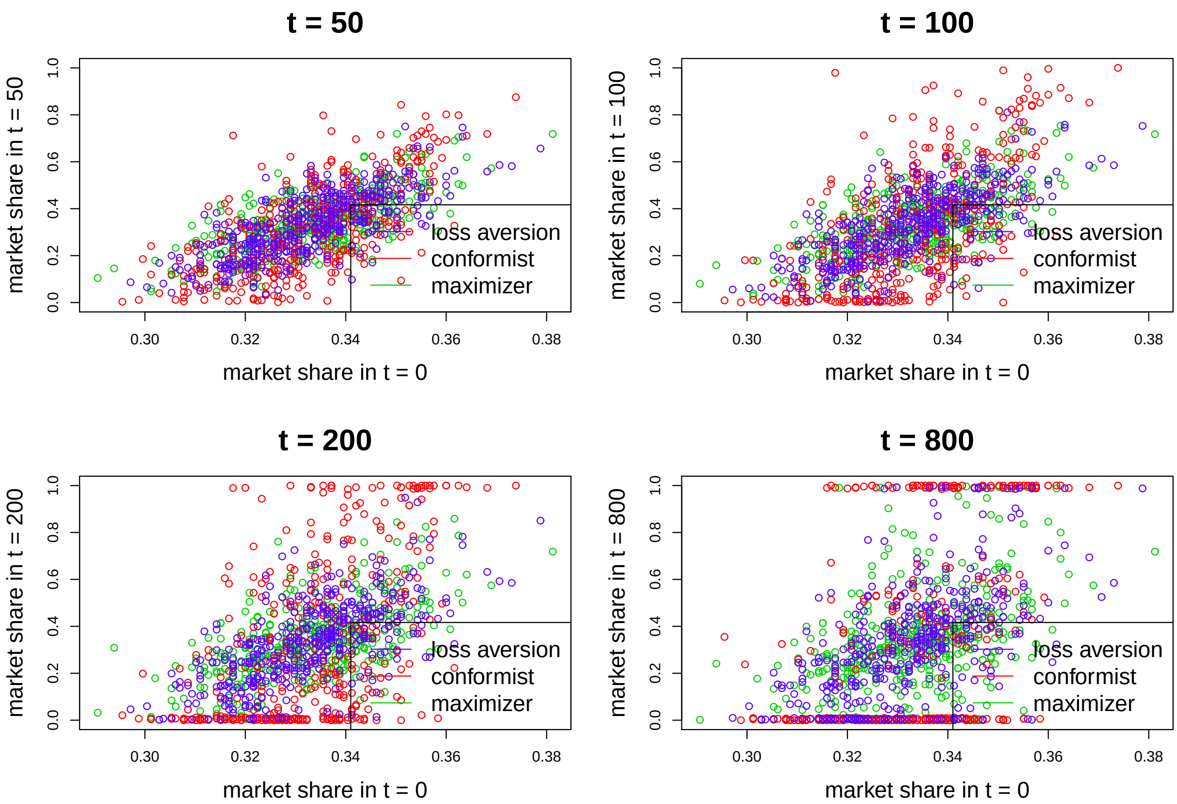

Figure 8: Convergence: market share with $\mathcal{C} = 3$, comparing to $t = 0$

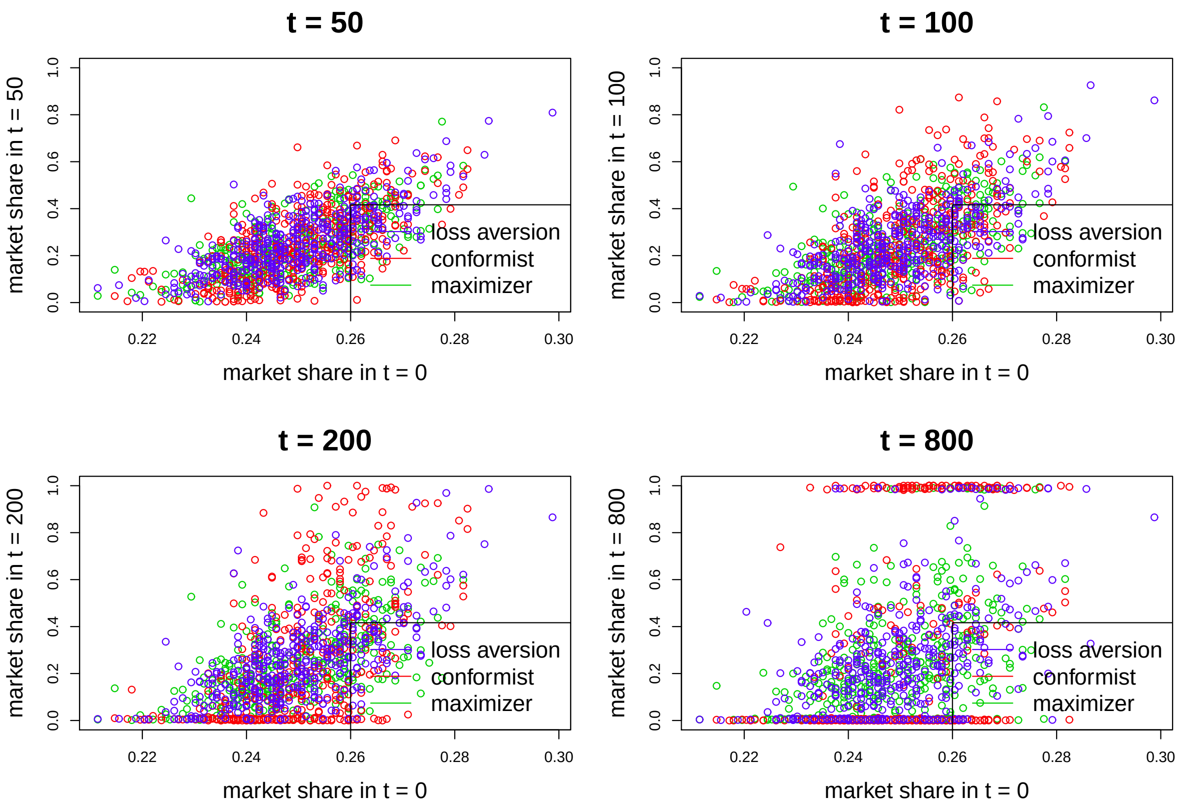

Figure 9: Convergence: market share with $\mathcal{C} = 4$, comparing to $t = 0$

References

Acemoglu, D. and Ozdaglar, A. (2011). Opinion dynamics and learning in social networks. *Dynamic Games and Applications*, 1(1):3–49.

Ajzen, I. (1985). From intentions to actions: A theory of planned behavior. In *Action Control*, pages 11–39. Springer.

Arthur, W. B., Durlauf, S. N., and Lane, D. A. (1997). *The Economy as an Evolving Complex System II*, volume 28. Addison-Wesley Reading, MA.

Asch, S. E. (1951). Effects of group pressure upon the modification and distortion of judgments. *Groups, Leadership, and Men*, pages 222–236.

Axelrod, R. (1997). The dissemination of culture: A model with local convergence and global polarization. *Journal of Conflict Resolution*, 41(2):203–226.

Balenzuela, P., Pinasco, J. P., and Semeshenko, V. (2015). The undecided have the key: Interaction-driven opinion dynamics in a three state model. *PLoS ONE*, 10(10).

Banerjee, A. V. (1992). A simple model of herd behavior. *The Quarterly Journal of Economics*, pages 797–817.

Beran, T., Drefs, M., Kaba, A., Al Baz, N., and Al Harbi, N. (2015). Conformity of responses among graduate students in an online environment. *The Internet and Higher Education*, 25:63–69.

Bikhchandani, S., Hirshleifer, D., and Welch, I. (1992). A theory of fads, fashion, custom, and cultural change as informational cascades. *Journal of Political Economy*, 100(5):992–1026.

Bond, R. and Smith, P. B. (1996). Culture and conformity: A meta-analysis of studies using Asch's (1952b, 1956) line judgment task. *Psychological Bulletin*, 119(1):111.

Borghesi, C. and Galam, S. (2006). Chaotic, staggered, and polarized dynamics in opinion forming: The contrarian effect. *Physical Review E*, 73(6):066118.

Breukelaar, R. and Bäck, T. (2005). Using a genetic algorithm to evolve behavior in multi dimensional cellular automata: emergence of behavior. In *Proceedings of the 7th Annual Conference on Genetic and Evolutionary Computation*, pages 107–114. ACM.

Brida, J. G., Defesa, M. J., Faias, M., and Pinto, A. (2010). Strategic choice in tourism with differentiated crowding types. *Economics Bulletin*, 30(2):1509–1515.

Brida, J. G., Defesa, M. J., Faias, M., and Pinto, A. (2011). A tourist's choice model. In *Dynamics, Games and Science I*, pages 159–167. Springer.

Castellano, C., Fortunato, S., and Loreto, V. (2009). Statistical physics of social dynamics. *Reviews of modern physics*, 81(2):591.

Clark, C. R. and Polborn, M. K. (2006). Information and crowding externalities. *Economic Theory*, 27(3):565–581.

Conley, J. P. and Wooders, M. H. (2001). Tiebout economies with differential genetic types and endogenously chosen crowding characteristics. *Journal of Economic Theory*, 98(2):261–294.

Galam, S. (2004). Contrarian deterministic effects on opinion dynamics: “the hung elections scenario”. *Physica A: Statistical Mechanics and its Applications*, 333:453–460.

Granovetter, M. (1978). Threshold models of collective behavior. *American Journal of Sociology*, pages 1420–1443.

Grimm, V., Berger, U., Bastiansen, F., Eliassen, S., Ginot, V., Giske, J., Goss-Custard, J., Grand, T., Heinz, S., Huse, G., et al. (2006). A standard protocol for describing individual-based and agent-based models. *Ecological Modelling*, 198(1):115–126.

Grimm, V., Berger, U., DeAngelis, D. L., Polhill, J. G., Giske, J., and Railsback, S. F. (2010). The ODD Protocol: a review and first update. *Ecological Modelling*, 221(23):2760–2768.

Grimm, V., Polhill, G., and Touza, J. (2013). Documenting social simulation models: the odd protocol as a standard. In *Simulating Social Complexity*, pages 117–133. Springer.

Grimm, V. and Railsback, S. F. (2012). Designing, formulating, and communicating agent-based models. In *Agent-based Models of Geographical Systems*, pages 361–377. Springer.

Hamamura, T. (2012). Are cultures becoming individualistic? A cross-temporal comparison of individualism–collectivism in the United States and Japan. *Personality and Social Psychology Review*, 16(1):3–24.

Hegselmann, R. and Krause, U. (2002). Opinion dynamics and bounded confidence models, analysis, and simulation. *Journal of Artificial Societies and Social Simulation*, 5(3).

Heymann, D., Perazzo, R., and Schuschny, A. (2004). Learning and imitation: Transitional dynamics in variants of the BAM. *Advances in Complex Systems*, 7(1):21–38.

Kelso, J. A. (2009). Coordination dynamics. In *Encyclopedia of complexity and systems science*, pages 1537–1564. Springer.

Kelso, J. S. (1995). *Dynamic patterns: The self-organization of brain and behavior*. MIT press.

Kirman, A. P. (1992). Whom or what does the representative individual represent? *The Journal of Economic Perspectives*, 6(2):117–136.

Krause, U. (2000). A discrete nonlinear and non-autonomous model of consensus formation. *Communications in difference equations*, pages 227–236.

Makarewicz, T. (2016). Contrarian behavior, information networks and heterogeneous expectations in an asset pricing model. *Computational Economics*, pages 1–49.

Maness, M. and Cirillo, C. (2016). An indirect latent informational conformity social influence choice model: Formulation and case study. *Transportation Research Part B: Methodological*, 93:75–101.

Mäs, M. and Flache, A. (2013). Differentiation without distancing. explaining bi-polarization of opinions without negative influence. *PloS One*, 8(11):e74516.

Nowak, A., Szamrej, J., and Latané, B. (1990). From private attitude to public opinion: A dynamic theory of social impact. *Psychological Review*, 97(3):362.

Oremland, M. and Laubenbacher, R. (2014). Optimization of agent-based models: scaling methods and heuristic algorithms. *Journal of Artificial Societies and Social Simulation*, 17(2):6.

Page, S. E. (2001). Self organization and coordination. *Computational Economics*, 18(1):25–48.

Salle, I., Yıldızoğlu, M., and Sénégas, M.-A. (2013). Inflation targeting in a learning economy: An ABM perspective. *Economic Modelling*, 34:114–128.

San Miguel, M., Eguiluz, V. M., Toral, R., and Klemm, K. (2005). Binary and multivariate stochastic models of consensus formation. *Computing in Science & Engineering*, 7(6):67–73.

Schelling, T. C. (1971). Dynamic models of segregation. *Journal of Mathematical Sociology*, 1(2):143–186.

Sîrbu, A., Loreto, V., Servedio, V. D., and Tria, F. (2017). Opinion dynamics: models, extensions and external effects. In *Participatory Sensing, Opinions and Collective Awareness*, pages 363–401. Springer.

Thaler, R. H. and Sunstein, C. R. (2008). *Nudge: Improving Decisions About Health, Wealth, and Happiness*. Yale University Press.

Vanhée, L., Dignum, F., and Ferber, J. (2015). Modeling culturally-influenced decisions. In *Multi-Agent-Based Simulation XV: International Workshop, Revised Selected Papers*, pages 55–71. Springer.

Wang, H. and Shang, L. (2015). Opinion dynamics in networks with common-neighbors-based connections. *Physica A: Statistical Mechanics and its Applications*, 421:180–186.

Watts, D. J. (1999). *Small Worlds: The Dynamics of Networks Between Order and Randomness*. Princeton University Press.

Watts, D. J. and Dodds, P. S. (2007). Influentials, networks, and public opinion formation. *Journal of consumer research*, 34(4):441–458.

Wilensky, U. (1999). NetLogo. Center for Connected Learning and Computer-Based Modeling, Northwestern University. Evanston, IL. http://ccl.northwestern.edu/netlogo/. Accessed 22 April 2017.

Wooders, M., Cartwright, E., and Selten, R. (2006). Behavioral conformity in games with many players. *Games and Economic Behavior*, 57(2):347–360.

Xie, Z., Song, X., and Li, Q. (2016). *A Review of Opinion Dynamics*, pages 349–357. Springer Singapore, Singapore.

Xu, B., Wang, J., and Zhang, X. (2015). Conformity-based cooperation in online social networks: The effect of heterogeneous social influence. *Chaos, Solitons & Fractals*, 81:78–82.